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Research Article

On Prime Hyperideals of a Krasner Hyperring

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Abstract
The basis of this study, which was put forth in order to appropriate a special area in the hyperring theory, which has recently been studied as a generalization of the ring theory, which uses the module theory as an application field, is based on integrally closed Krasner hyperrings and (almost) integral dependence applications in Krasner hyperrings.

Keywords: Krasner hyperring, hyperideal, integral dependence, almost integral dependence

Krasner Yüksek Halkaların Asal Yüksek İdealleri Üzerine

Öz

Modül teorisini uygulama alanı olarak kullanan, son zamanlarda halka teorisinin bir genellemesi olarak çalışılan yüksek halka teorisinde özel bir saha oluşturmak üzere ortaya konulan bu çalışmanın temelini, tam kapalı Krasner yüksek halkalar ve Krasner yüksek halkalarda (hemen hemen) tam bağımlılık uygulamaları oluşturmaktadır.

Anahtar Kelimeler: Krasner yüksek halka, yüksek halka, tam bağımlılık, hemen hemen tam bağımlılık.

1. Introduction

Hyperrings and hypermodules categories have significant roles in hyperstructure theory. We refer to the reader having some elementary features of these theory in [1],[6] and [8]. In addition, since our study is a generalization of [9]'s work, the integral dependence in rings can be accessed from this source. Recall that some definitions and theorems from the above references are necessary to improve this article.

For an arbitrary set $A \neq \emptyset$, let $P^*(A) = P(A) \backslash \emptyset$. The couple $(A, \circ)$ is defined as *hypergroupoid* if there is $\circ: A \times A \longrightarrow P^*(A)$ is a function, namey hyperoperation. As can be understood from this definition, hypergroupoids are a proper generalization of groupoids. For two nonempty subsets $X, Y \leq A$, let $X \circ Y = \cup_{\substack{x \in X \\ y \in Y}}^{x \circ y}$. We write $\{s\} \circ X := s \circ X$ and $X \circ \{s\} := X \circ s$ whenever $s \in A$. If a hypergroupoid $(A, \circ)$ satisfies the equality $a \circ (b \circ c) = (a \circ b) \circ c$ for every $a, b, c \in A$, then $(A, \circ)$ is said to be a *semi-hypergroup*. A hypergroupoid $(A, \circ)$ is called a *quasihypergroup* in case $s \circ A = A \circ s = A$ for every element s in A. If $(A, \circ)$ is a quasihypergroup and a semi-hypergroup, then $(A, \circ)$ is defined as a *hypergroup*. Let $(A, \circ)$ be a hypergroup and B $(\neq \emptyset)$ be a subset of A. , B is said to be a *subhypergroup* of A if

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$b \circ B = B = B \circ b$ for every element b of B. In [5], a hypergroup $(A,\circ)$ is called *canonical* (i) if for every $a, b \in A$, $a \circ b = b \circ a$, i.e. $(A,\circ)$ is commutative; (ii) there is an element $e \in A$ such that $\{a\} = (a \circ e) \cap (e \circ a)$ for each $a \in A$; (iii) there is a unique $a^{-1} \in A$ such that $e \in a \circ a^{-1}$ for every $a \in A$; (iv) $s \in y \circ z$ provides that $y \in s \circ z^{-1}$ for every element s of A. In the second expression, the element e is said to be the *identity element* of the hypergroup $(A,\circ)$. In this paper, we consider some types of hyperrings and hypermodules. A triple $(S, +, .)$ is named a *Krasner hyperring* provided

1. $(S, +)$ is hypergroup which is canonical;
2. $(S, .)$ is semi-hypergroup with zero element x providing $0.x = x.0 = 0$;
3. $"."$ is distributive with respect to $" + "$.

$(S, +, .)$ is named a *hyperring* if

1. The canonical hypergroup $(S, +)$ has scalar identity 0_R;
2. $(S, .)$ is a semi-group;
3. $"."$ is distributive with respect to $" + "$.

In this place "." is a hyperoperation on S. Each hyperring $(S, +, .)$ holds following two statements: $u.0 = 0.u = \{0\}$ and $u.(-v) = \{-z | z \in u.yv\} = (-u).v = -(u.v)$ for each $u, v \in S$. We denote a hyperring $(S, +, .)$ with S for short. If S is commutative with respect to its hyperoperation ".", then S is named *commutative*. If $f \in f.1$ for every element $f \in S$, 1 is named *identity element of a hyperring* S. Assume that S is a hyperring including 1. Following [11], a *hypermodule* M *over the hyperring* S means a triple $(M, +,\circ)$ in the fact that a canonical hypergroup $(M, +)$ have a scalar identity 0_M and the operation $\circ: S \times M \longrightarrow P^*(M)$ satisfies the followings for every element $f, g \in S$ and $a, b \in M$;

1. $f \circ (a + b) = f \circ a + f \circ b$;
2. $(f + g) \circ a = f \circ a + g \circ a$;
3. $(f.g) \circ a = f \circ (g \circ a)$; $a \in 1 \circ a$

For an S -hypermodule $(M, +,\circ)$ and every $u \in S$ and $a \in M$, we can write $u \circ 0_M = \{0_M\} = 0 \circ a$ and $u \circ (-a) = (-u) \circ a = -(u \circ a) = \{-b | b \in u \circ a\}$. We denote an S-hypermodule $(M, +,\circ)$ with M for short. In [5], this notion is generalized as hypermodule over a Krasner hyperring.

For a commutative hyperring S, let $J(\neq\emptyset)$ be a subset of S. J is named *hyperideal* if $x - y \in J$ and $a.x \in J$ for any element $a \in S$ and $x, y \in J$. If J_1, J_2 are hyperideals of S, the sum $J_1 + J_2$ is also hyperideal of S. For Krasner hyperrings S and S', a function $f: S \longrightarrow S'$ is named a *strong hyperring homomorphism* if $f(x + y) = f(x) + f(y)$ and $f(x.y) = f(x).f(y)$ for every element $x, y \in M$. Assume that $(S, +, .)$ is an arbitrary ring and H is a subset of S. H is named a *multiplicative subgroup* of S if $(H, .)$ is a group. If $S = SH$ and $sH = Hs$ for every element s in S, then H is named a *normal subgroup* of S. We indicate that the rings including identity elements known as normal subgroups. A normal subgroup H of S contains an equivalence relation P in S and a part of S in equivalence classes, which inherits a hyperring structure from S. The hyperrings obtained with this structure are named *quotient hyperrings* are written by S/H. Let S be a hyperring. S is named *integral hyperdomain* if for each $f, g \in S$, $0 \in f.g$ implies that $f = 0$ or $g = 0$. S is named a *hyperfield* in which every nonzero element has a inverse in S in [8]. A hyperring S including 1 is named a *principal hyperideal hyperdomain* if S doesn't contain zero divisors and every hyperideal of S is generated by a single element ([4]). Assume that S is a hyperring. S is named *Noetherian* if S satisfies the condition (ACC) on hyperideals of S (see [3]).

Assume that M is an S-hypermodule and N $(\neq\emptyset)$ is a subset of M. N is named a *subhypermodule* of M, denoted by $N \leq M$, if N is an S-hypermodule under the same hyperoperations on M. It is shown in (Proposition 2.3, [10]) that if $x \circ a \leq N$ and $a - b \in N$ for every element $x \in S$ and $a, b \in N$, then by $N \leq M$. Let M be a canonical S-hypermodule. If $"." : S \times M \longrightarrow M$ via $(s, m) \longmapsto s.m \in M$ and $s.0 = 0$ is an externel operation, then M is named a *Krasner S-hypermodule*. Let T be a nonempty subset of a hypermodule M. $\langle T \rangle$ is defined as a *subhypermodule of M generated by T* if $\langle T \rangle$ is the smallest subhypermodule of M containing T.

2. Main Theorems

2.1. Integral Dependence in Krasner Hyperring

Let R be a subhyperring of a hyperring $R'$and $a_1, a_2, \dots, a_m \in R'$. We denote with $R[a_1, a_2, \dots, a_m]$ that is a set of polynomial expressions in $a_1, a_2, \dots, a_m$ with coefficients in R. Therefore, if $X_1, X_2, \dots X_m$ are indeterminates, then

$R[a_1, a_2, \dots, a_m] = \{f(a_1, a_2, \dots, a_m) | f(X_1, X_2, \dots X_m) \in R[X_1, X_2, \dots X_m]\}$.

The mapping $f(X_1, X_2, \dots X_m) \longmapsto f(a_1, a_2, \dots, a_m)$ is a strong hyperring homomorphism from $R[X_1, X_2, \dots X_m]$ into R', so its image $R[a_1, a_2, \dots, a_m]$ is to be a subhyperring of R'. Here $R \subseteq R[a_1, a_2, \dots, a_m]$.

2.1.1. Lemma *Given a hyperring R, let be $d = det[a_{ij}]$ where $a_{ij}, b_j \in R$ for $i, j = 1,2, \dots, m$. If $\sum_{j=1}^{m} a_{ij}.b_j = 0$ for all $i = 1,2, \dots, m$, then $d.b_j = 0$ where $j = 1,2, \dots, k$.*

Proof. Let d_{ij} be the cofactor of a_{ij} in the matrix $[a_{ij}]$. Then $\sum_{i=1}^{m} d_{ij}.\ a_{ih} = \begin{cases} d & if\ j = h \\ 0 & if\ j \neq h \end{cases}$

Hence

$$0 = \sum_{i=1}^{m} d_{ih} . \left(\sum_{h=1}^{m} a_{ih} . b_h \right)$$
$$= \sum_{h=1}^{m} \left(\sum_{i=1}^{m} d_{ih} . a_{ih} \right) . b.h = d.b_j$$

for $j = 1,2, \dots, m$. ∎

Given an R-hypermodule M. M is named *finitely generated* provided that there is a finite subset $\{m_1, m_2, \dots, m_k\}$ of M generating M, that is,

$$M = \{x | \exists r_1, r_2, \dots, r_k \in R, k \in \mathbb{N}\ such\ that\ x \in \textstyle\sum_{i=1}^{n} r_i m_i \}$$

2.1.2. Proposition *Assume that R' is a hyperring and a is an element in R'. For a subhyperring R, the followings are equivalent:*

1. *There are elements $\boldsymbol{b_0, b_1, \dots, b_{m-1} \in R}$ $(\boldsymbol{m \geq 1})$ provided that*
$\boldsymbol{b_0 + b_1.a + \cdots + b_{m-1}.a^{m-1} + a^m = 0}$ **.**

2. *Hypermodule $R[a]$ over R is finitely generated.*

3. *There exists a subhyperring R'' of R' provided that $a \in R''$ and the hypermodule R'' over R is finitely generated.*

Proof. **(1)**$\Rightarrow$ **(2)** Assume $f(X) = c_0 + c_1.X + \cdots + c_d.X^d \in R[X]$ is polynomial with $\deg f(X) = d > m$. Therefore we can write

$$f(a) = c_0 + c_1.a + \cdots + c_{d-1}.a^{d-1} + c_d.a^{d-m}b$$

$$= c'_0 + c'_1.a + \cdots + c'_{d-1}.a^{d-1} \text{ for some element } b \in R$$

It is continued similar procedures until R-hypermodule generated by $1, a, \dots, a^m$ for $f(a)$. Therefore, as a R-hypermodule $[a] = R1 + Ra + \cdots + Ra^m$; hence $R[a]$ is finitely generated.
(2)$\Rightarrow$**(3)** If R'' is taken as $R[a]$, the proof is provided.
(3)$\Rightarrow$**(1)** Suppose that $a_1, a_2, \dots, a_m$ generate R'' over R. For $i = 1,2, \dots, m$, $a.a_i = \sum_{j=1}^{m} b_{ij}.a_j$, $b_{ij} \in R$, or $\sum_{j=1}^{m}(b_{ij} - \delta_{ij}.a).a_j = 0$. If $d = det[b_{ij} - \delta_{ij}.a]$ then $d.a_j = 0$ for $j = 1,2, \dots, m$ by Lemma 2.1.1. Thus $d.c = 0$ for every $c \in R''$. With $c = 1$ we get $d = 0$. Since d is a polynomial in a with coefficients in R such that the coefficient of a^n is ± 1, this is also desired. ■

2.1.3. Definition *Assume R' is a hyperring, R is a subhyperring of $R'$and a is an element in R'.* a is named *integral on R* if the equal conditions in the above proposition are satisfied. Moreover, R' is named *integral on R* in case every element in R' is integral on R. If the elements of R are the only elements of R' that are integral on R, R is named *integral closed in R'* . If R is integral closed in its total quotient hyperring, R is named *integral closed*.

2.1.4. Proposition *Assume R is a subhyperring of a hyperring R' and $R_0 = \{a | a \in R' \text{ and } a \text{ is integral on } R\}$. Then R_0 is a subhyperring of R' and $R \subseteq R_0$.*
Proof. Clearly, $R \subseteq R_0$. Let $a, b \in R_0$. Then the R-hypermodule $R[a]$ is finitely generated and the $R[a]$-hypermodule $R[a, b] = R[a][b]$ is finitely generated. Since $a - b, a.b \in R[a, b]$, we obtain that $a - b, a.b \in R_0$. It means that R_0 is a subhyperring of R'. ■

Mentioned in Proposition 2.1.4, R_0 is named *the integral closure of R in R'*. It follows from the next proposition that R_0 is integrally closed in R'.

2.1.5. Proposition *Let $R \leq R' \leq R''$ be hyperrings. Suppose that R' is integral on R and $a \in R''$ is integral on R'. Then a is integral on R.*
Proof. Suppose $b_0 + b_1.a + \cdots. + b_{m-1}.a^{m-1} + a^m = 0$ where $b_0, b_1, \dots, b_{m-1} \in R'$. Therefore a is integral on $R[b_1, b_2, \dots, b_{m-1}]$. It follows that the krasner R-hypermodule $R[b_1, b_2, \dots, b_{m-1}, a]$ is finitely generated. This means that a is integral on R. ■

For a Krasner hyperring R, let S be multiplicatively closed subset of R with $1 \in S$. Following [7], the construction of $S^{-1}R$ is named *hyperring of fractions* if a hyperring structure is defined as follows: $\frac{a}{s} + \frac{b}{t} = \frac{a.t+b.s}{s.t}$ and $\frac{a}{s} . \frac{b}{t} = \frac{a.b}{s.t}$ for every element $\frac{a}{s}$, $\frac{b}{t}$ of $S^{-1}R$. Here a relation "$\equiv$" is on $R \times S$ defined by $(a, s) \equiv (b, t)$ if and only if $0 \in (a.t - b.s).u$ for, where $u \in S$. Then obtained equivalence class of (a, s) with $\frac{a}{s}$ and family of whole equivalence classes is denoted by $S^{-1}R$.

2.1.6. Proposition *Assume that R' is a hyperring, R is a subhyperring of $R'$and S is a multiplicative system in R. Then $S^{-1}R$ is a subhyperring of $S^{-1}R'$. Moreover, if R' is integral on R, then $S^{-1}R'$ is integral on $S^{-1}R$.*

Proof. Let 0_S and $0_S{}'$ be the S-components of 0 in R and R', respectively. We certainly have $0_S \subseteq 0_S{}' \cap R$. If $a \in 0_S{}' \cap R$, then $s.a = 0$ for some $s \in S$. Since $S \subseteq R$, we obtain that $a \in 0_{S'}$. Thus $0_S = 0_{S'} \cap R$. Hence the mapping taking $\frac{a}{s} \in S^{-1}R$ onto $\frac{a}{s} \in S^{-1}R'$ is an injective strong hyperring homomorphism; if we identify $\frac{a}{s} \in S^{-1}R$ with its image $\frac{a}{s} \in S^{-1}R'$, then $S^{-1}R$ can be considered as a subhyperring of $S^{-1}R'$. Suppose that R' is integral on R, $\frac{a}{s} \in S^{-1}R'$, $a \in R'$ and $s \in S$. There exists elements $b_0, b_1, \dots, b_{m-1} \in R$ such that we have

$b_0 + b_1.a + \cdots + b_{m-1}.a^{m-1} + a^m = 0$. Then

$$\frac{b_0}{s^m} + \frac{b_1}{s^{m-1}} \cdot \frac{a}{s} + \ldots + \frac{b_{m-1}}{s} \cdot \left(\frac{a}{s}\right)^{m-1} + \left(\frac{a}{s}\right)^m = \frac{b_0 + b_1 \cdot a + \ldots + b_{m-1} \cdot a^{m-1} + a^m}{s^m} = 0$$

So $\frac{a}{s}$ is integral on $S^{-1}R$ ■

Assume that R is a hyperring. In [2] or [11], a proper hyperideal P of R is named a *prime hyperideal* of R if whenever $AB \subseteq P$,either $A \subseteq P$ or $B \subseteq P$ where A and B are hyperideals of R. For a prime hyperideal P of R, we obtain that $S = R \setminus P$ is *multiplicatively closed* and denote by $S^{-1}R = R_P$. Let R' be a hyperring, A be a hyperideal of R and A' be a hyperideal of R' such that $A = A' \cap R$, then A' is named *lie over* A.

2.1.7. Theorem *Assume R is a subhyperring of a hyperring R', R' is integral over R and P is a prime hyperideal of R. Then there is a prime hyperideal P' of R' that lies over P. Moreover, if P' and P'' are prime hyperideals of R' that lie over P and if $P' \subseteq P''$, then $P' = P''$.*

Proof. The family of hyperideals A' of R' such that $A' \cap R \subseteq P$ is nonempty, and it follows from Zorn's lemma that this family contains a maximal element P'. Then $P' \cap R \subseteq P$. Suppose $P' \cap R \subset P$ and $a \in P$, $a \notin P'$. Then $P' \subset P' + R'a$ and consequently, by our choice of P', $(P' + R'a) \cap R \nsubseteq P$. Therefore there is an element $c \in P'$ and $r \in R'$ such that $c + ra = b \notin P$ but $b \in R$. For $d_0, d_1, \dots, d_{m-1} \in R$, $d_0 + d_1.r + \cdots + d_{m-1}.r^{m-1} + r^m = 0$. Then

$$\begin{aligned} & b^m + d_{m-1}.a.b^{m-1} + \cdots + d_1.a^{m-1}.b + d_0.a^m \\ & \quad = (c + r.a)^m + d_{m-1}.a.(c + r.a)^{m-1} + \cdots + d_1.a^{m-1}.(c + r.a) + d_0.a^m \\ & \quad = f(c) + a^m.(r^m + d_{m-1}.r^{m-1} + \cdots + d_1.r + d_0) = f(c) \in P' \cap R \subseteq P; \end{aligned}$$

where $f(c)$ is a polynomial in c with coefficients in R'. Hence, since $a \in P$, we have $b^m \in P$, so $b \in P$, a contradiction. Thus $P' \cap R = P$. Let $S = R \setminus P$. Therefore S is a multiplicative system in R'. If $P = R$, then $P' = R'$ which is prime, so we may assume $P \neq R$. Let A' be a hyperideal of R' with $P' \subset A'$. Then $A' \cap R \nsubseteq P$ so $A' \cap R$ meets S; hence $A' \cap S \neq \emptyset$. Hence P' is a maximal in the set of hyperideals of R' whose intersection with S is empty. Assume $P' \subset P''$ are prime hyperideals of R' that lie over P. Let $a \in P''$ with $a \notin P'$. Since a is integral on R there exists at least positive integer m such that there are elements $b_0, b_1, \dots, b_{m-1} \in R$ for which $a^m + b_{m-1}.a^{m-1} + \cdots + b_1.a + b_0 \in P'$. Therefore $b_0 \in P'' \cap R = P = P' \cap R$. It follows that $a(a^{m-1} + b_{m-1}.a^{m-2} + \cdots + b_1) \in P'$, but $a \notin P'$, so

$a^{m-1} + b_{m-1}.a^{m-2} + \cdots + b_1 \in P'$. This contradicts our choice of m. Thus, if $P' \subseteq P''$, we must have $P' = P''$. ■

2.1.8. Corollary *For a hyperring R, let R' be as in the Theorem 2.1.7. Let $P_0 \subset P_1 \subset \ldots \subset P_m$ be a chain of prime hyperideals of R. If the prime hyperideal P_0' of R' lies over P_0, then there is a chain $P_0' \subset P_1' \subset \cdots \subset P_m'$ of prime hyperideals of R' such that P_i' lies over P_i for $i = 0,1,\ldots,m$. If, for a given i, there is no prime hyperideal of R strictly between P_i and P_{i+1}, then there is no prime hyperideal of R' strictly between P_i' and P_{i+1}'.*

Proof. By the hypothesis, we have shown that there is $P_0' \subset \cdots \subset P_m'$ of prime hyperideals of R' such that P_i' lies over P_i for $i = 0,1,\ldots,m$. Then $\frac{R}{P_k}$ can be considered as a subhyperring of $\frac{R'}{P_k'}$ for $0 \le k \le m$. Thus, by Theorem 2.1.7. there is a prime hyperideal P_{k+1}' of R' such that $P_k' \subset P_{k+1}'$ and $\frac{P_{k+1}'}{P_k'}$ lies over $\frac{P_{k+1}}{P_k}$. So we have P_{k+1}' lying over P_{k+1}. Suppose that P' is a prime hyperideal of R' and that $P_i' \subset P' \subset P_{i+1}'$. Again by Theorem 2.1.7, P' cannot lie over either P_i or P_{i+1}. Therefore the prime hyperideal $P' \cap R$ of R is strictly between P_i and P_{i+1} . ■

2.2. Almost Integral Dependence in Krasner Hyperring

Now we shall define notions of almost integral over a hyperring and complete integral closure of a hyperring and give some properties of these subhyperrings.

2.2.1. Definition Assume that R is a subhyperring of a hyperring R'. An element $a \in R'$ is named *almost integral over R* if there is a finitely generated subhypermodule of the R-hypermodule R' which contains all powers of a.

It is clear seen that every element of R' which is integral over R is also almost integral over R. But the converse is not always true. Assume that R is an integrally closed hyperdomain with quotient hyperfield K. Let $T = R + XK[X]$. T is integrally closed and $K[X]$ is the complete integral closure of R in $K[X]$. If Krasner hyperring R is Noetherian, then the converse holds.

2.2.2. Definition Let R be a subhyperring of a hyperring R'. The set R_0 of all elements of R' which are almost integral over R is the *complete integral closure of R* in R' . If $R_0 = R$, the R is *completely integrally closed in R*.

It is immediately clear that the complete integral closure R, of R in R' is a subhyperring of R'. However, R, is not necessarily itself completely integrally closed; an example is given in the Example 2.2.3.

2.2.3.Example The complete integral closure need not be completely integrally closed. Assume that K is a hyperfield and X, Y are indeterminates. Let $R = K[\{X^{2n+1}Y^{n(2n+1)} \mid n \ge 0\}]$. Then the quotient hyperfield of the hyperfield R is $K(X,Y)$. If $R' = K[\{XY^n | n \ge 0\}]$, then $R \subset R' \subseteq R" \subseteq R^* \subseteq K[X,Y]$, where R^* is the complete integral closure of R. Since for any element of R, the exponent of Y in any of the monomials of that element is less than or equal to the square of the exponent of X in the same monomial, Y is almost integral on R', and hence is almost integral on R^*, but that $Y \notin R^*$.

Furthermore, if $R, T,$ and T' are rings with $R \subseteq T \subseteq T'$, then an element $a \in T$ may be almost integral over R as an element of T', but not almost integral over R as an element of T; it is given a counter example of this fact in Example 2.2.4.

2.2.4. Example Assume that $R \subseteq T_1 \subseteq T_2$ are hyperrings. For i = 1,2, let R_i be the complete integral closure of R in T_i. It is clear that $R_1 \subseteq R_2 \cap T_1$. If T is a subhypermodule of some T_1-hypermodule M such that T_1 is a direct summand of M, then $R_1 = R_2 \cap T_1$. In addition, the same conclusion holds if every finitely generated T_1-module M with $T_1 \subseteq M \subseteq T_2$ is a subhypermodule of a T_1-hypermodule of which T_1 is direct summand. If T is a principal hyperideal hyperdomain, then $R_1 = R_2 \cap T_1$. Assume that K is a hyperfield and X, Y are indeterminates over K. Let $R = K[\{XY^n | n \geq 1\}]$, $T_1 = R[Y]$ and $T_2 = T_1[\frac{1}{X}]$. It is continued similar procedures until R_1, and R_2 are as above, we have $R_1 \subset R_2 \cap T_1$.

Even though the complete integral closure of one ring in another may not be completely integrally closed, we have the following:

2.2.5. Proposition *Let R be a subhyperring of a hyperring R' and R_0 the complete integral closure of R in R'. Then R_0 is integrally closed in R'.*

Proof. Let $x \in R'$ be integral over R_0; $x^k + a_{k-1}.x^{k-1} + \cdots + a_0 = 0$, where $a_0, \dots, a_{k-1} \in R_0$. It follows that x is integral over the hyperring $R[a_0, \dots, a_{k-1}]$. a_i is contained in some finitely generated subhypermodule of the R-hypermodule R' for $i = 0, \dots, k-1$, say $M_i = Rx_{i1} + \cdots + Rx_{ik_i}$, where each $x_{ij} \in R'$. Then $R[a_0, \dots, a_{k-1}] \subseteq M_0 \dots M_{k-1}$, which is the subhypermodule of the R-hypermodule R' generated by all products $x_{0j_0}x_{1j_1} \dots x_{k-1,j_{k-1}}$ where each j_i runs between 1 and m_i. Hence x is contained in

$$R[x] \subseteq R[a_0, \dots, a_{k-1}, x]$$

$$= \sum_{h=0}^{k-1} R[a_0, \dots, a_{k-1}]\, x^h$$

$$\subseteq \sum_{h=0}^{k-1} \sum_{i=0}^{k-1} \sum_{j_i=1}^{m_i} R x_{0j_0} x_{1j_1} \dots x_{k-1,j_{k-1}}\, x^h \quad ,$$

which is finitely generated subhypermodule of the R-hypermodule R'. Thus x is almost integral over R, and so $x \in R_0$. Therefore R_0 is integrally closed. ■

2.2.5. Corollary *Let R, R_1, and R_2 be subhyperrings such that $R \subseteq R_1 \subseteq R_2$. If each element of R_1 is almost integral over R, and if R_2 is integral over R_1, then each element of R_2 is almost integral over R.*

3. Conclusion

In this paper, it is obtained integral dependence on krasner hyperring by using prime hyperideals of the hyperring. In this way various properties is brought in theory of Krasner hyperring. In the second section, prime hyperideals of the hyperrings are classified. In the third section, it is treated as a subject of almost integral dependence in the notion of hyperfields.

Ethics in Publishing

There are no ethical issues regarding the publication of this study.

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References

[1] Ameri, R., (2003) On categories of hypergroups and hypermodules, Journal of Discrete Mathematical Sciences and Crytography, 6:2-3, 121-132.

[2] Bordbar, H., Cristea, I., (2017) Height of prime hyperideals in Krasner hyperrings, Filomat, 31:19,6153-6163.

[3] Bordbar, H., Cristea, I., Novak, M., (2017) Height of hyperideals in Noetherian Krasner hyperrings, U.P.B. Sci. Bull.Series A, 79,2, 31-42.

[4] Bordbar, H., Cristea, I., (2021) Regular parameter elements and regular local hyperrings, Mathematics, 9, 243, 1-13.

[5] Corsini, P., (1993). Prolegomena of Hypergroup Theory, 2nd ed. Tricesimo Italy, Aviani editore Italy.

[6] Davvaz, B., Fotea, V.L., (2007). Hyperring Theory and Applications, Palm Harbor, FL, USA, International Academic Press.

[7] Davvaz, B., Salasi, A., (2006) A realization of hyperrings, Comm. Algebra, 34, 4389-4400.

[8] Krasner, M., (1983) A class of hyperring and hyperfields, IJMMS, 6:2,307-311.

[9] Larsen, M.D., McCarthy, P.J. (1971). Multiplicative Theory of Ideals, Pure and Applied athematics, A Series of Monographs and Textbooks,Volume 43, Academic Press, New York and London.

[10] Mahjoob, R., Ghaffari, V., (2018), Zariski topology for second subhypermodules, *Italian Journal of Pure and Applied Mathematics*, 39, 554-568.

[11] Siraworakun, A., Pianskool, S. (2012), Characterizations of prime and weakly prime subhypermodules, International Mathematical Forum, Vol. 7, no. 58, 2853 – 2870.