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Research Article

COMPARISON OF INTENSITY ESTIMATION METHODS FOR AN EARTHQUAKE SPATIAL POINT PATTERN

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ABSTRACT

Intensity is described as the number of points per unit area for a spatial point pattern. Intensity estimation of a spatial point pattern is necessary to determine hot spots, cold spots and clusters in a study region. Moreover, intensity may be a determinant for spatial point pattern type. It is also an indicator of risks while the points called events include the locations of the earthquakes through a fault zone, crime incidences in a district and etc. Therefore, determining the intensity provides taking precautions against possible undesirable and unexpected future incidences. In this study, several methods of intensity estimation for a spatial point pattern are given. Advantages and disadvantages of the mentioned methods are discussed. Finally, intensity images that are obtained by using different methods are compared. Adaptive kernel density estimation gave a better result in comparison to other intensity estimation methods.
Keywords: Spatial point patterns, intensity, kernel density estimation, adaptive smoothing, quadrat counts.

1. INTRODUCTION

A spatial point pattern consists of irregularly distributed set of locations within a certain region of a space and it is assumed as an outcome of a generated stochastic mechanism. The locations of the spatial point pattern are called events to separate from arbitrary events in the study region [1].

Cressie [2] made a pioneering classification for spatial data which is accepted as a valid classification up to date in the literature and categorized spatial data into three groups according to their domain features. These groups are lattice data, geostatistical data and spatial point patterns. A spatial point pattern is the realization of a process in two-dimensional space $Z(\mathbf{s}): \mathbf{s} \in D \subset \mathbb{R}^2$ while D is a random domain, $Z(\mathbf{s})$ is the attribute value dependent to location **s** and D is the domain of the study [3].

Spread of an infectious disease, crime incidences in a district, plant scattering in a forest and earthquakes occurred in a fault zone are some examples that can be given for a spatial point pattern.

There are three types of spatial point patterns. These are clustered, regular and complete spatial random patterns. Diggle [1] described complete spatial random pattern as an unreachable

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standard. It plays an important role for spatial point pattern analysis and serves as a null hypothesis for spatial point patterns. Clustered and regular patterns are the deviations from complete spatial random pattern. If a pattern is completely random, further analysis is not needed in most cases [1,3]. Spatial point patterns types are illustrated in Figure 1.

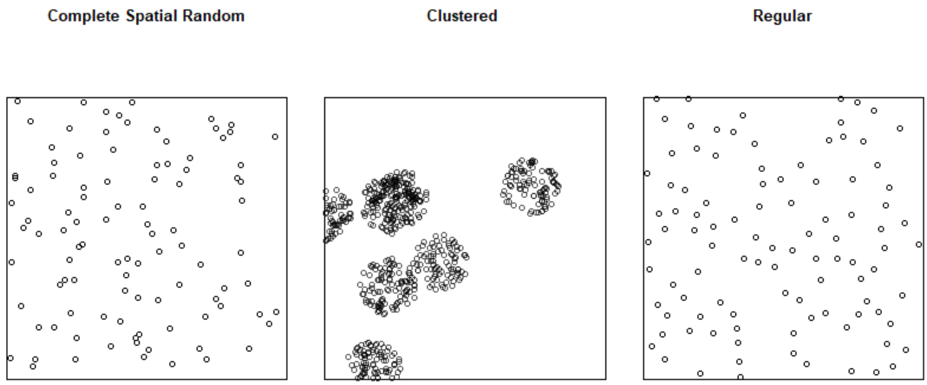

Figure 1. Spatial point pattern types.

Intensity can be spatially varying or constant over a study region. Complete spatial random pattern is the outcome of constant intensity (homogenous) over a study region. Moreover, intensity could be considered as abundance, density, prospectivity and risk [4].

Gatrell implies [5] that estimating intensity of a point pattern resembles to estimation of bivariate probability density. Intensity estimation is the primary objective in most analysis of a spatial point pattern. Schabenberger & Gotway [3] state that even for a complete spatial point pattern resulted from a homogenous poisson process, it is still useful to estimate intensity for dependency (second-order properties of a point process) structure of the pattern.

There are many methods such as kernel density estimation, adaptive smoothing, tessellations and quadrat counting for estimation of intensity for a spatial point pattern. Gatrell et al. [6] reviews methods like kernel density estimation and adaptive smoothing for exploration and modelling of spatial point pattern in epidemiology stating that quadrat counts and nearest neighbor methods fail to consider spatial variation in population density.

Stock and Smith [7] compare earthquake occurrence distributions by using adaptive kernel estimation and kernel estimation with a global bandwidth for New Zealand and Australian earthquake catalogs.

Stock and Smith [8] compare the kernel functions and global and adaptive kernel estimations for the New Zealand and Australia earthquake catalogs.

In this study, kernel estimations of intensities for a region in the North Anatolian Fault Zone are given. In intensity calculations, both global and adaptive bandwidths for Gaussian kernels are employed. In addition, the fit of these estimations are compared. Study is based on the choice of the optimum bandwidth and the best intensity estimation method. The study is organized as follows. Initially, a brief methodology is given for kernel density estimation and intensity of a point pattern. Then the properties of the data is explained. Finally, results are discussed for the given earthquake catalog.

2. METHODOLOGY

Intensity of a spatial point process is defined below as a limit of a counting process and it is the average number of points per unit area (volume) whereas ds is the infinitesimal area centered at point **s** in the study domain [3].

$$\lambda(\boldsymbol{s}) = \lim_{v(d\boldsymbol{s})\to 0} \frac{E(N(d\boldsymbol{s}))}{v(d\boldsymbol{s})} \tag{1}$$

If a point process **X** has a homogenous intensity, for a region B in the study domain expected number of points falling into region B is proportional to the area of B such as $\lambda|\mathrm{B}|$. The value of λ depends on the measurement unit. In such a case the estimation of unbiased intensity is given below like the calculation of sample mean [4].

$$\bar{\lambda} = \frac{N(B)}{|B|} \tag{2}$$

For spatially varying intensity expected number of points depends on intensity function and expected number of events for a small area in the contiguity of location is equal to $a\lambda(u)$. Expected number of events in a domain B is formulated as integral of intensity function in this domain. It is given in below equation [4].

$$E(N(B)) = \int_B \lambda(u)du \tag{3}$$

In some cases, the events are located only in the edges of the domain. Such kind of point processes do not have a closed form of intensity function. Examples of this situation can be seen in seismicity, events can occur on the edges of a study region. Therefore, a more general approach to intensity is taken into account assuming the expected number of events for a point process is known. It is given in equation (4).

$$\Lambda(B) = E(N(B)) \tag{4}$$

Quadrat counting is a simple method for estimating intensity of spatial point patterns. It can be applied to rectangular shaped or regular shaped domains easily by dividing the whole domain into quadrats of equal area. Afterwards, number of events in each quadrat are obtained and are divided to its area for the calculation of intensity for each quadrat. Quadrats are formed by dividing the sides of the domain into specific numbers. There is also a test for complete spatial randomness designed by inspiration from the idea of including approximately even number of events for each quadrat. It is not a trustworthy method because it is highly affected from the subjective choice of the quadrat number.

Kernel density estimation is a non-parametric method to estimate univariate and multivariate probability densities from the data itself. It has some advantageous of being independent from assumptions of parametric estimation however there are also problems of determining optimum bandwidth usually.

Kernel density estimation is an idea of counting the number of events per area with a moving quadrat or window instead of fixed number of regular grids. This is done by a kernel function with a bandwidth scanning the area proportionally to its size. A kernel function must satisfy the property given in (5) along with the condition of taking non-negative values. To sum up it must be a probability density function.

$$\int_{R^2} K(x)\, dx = 1 \tag{5}$$

Two dimensional kernel density estimator is given with a fixed bandwidth below in equation (6) where h is the bandwidth of the kernel function *K*, n is the number of earthquakes, **x** is the location of any location in the study area and $\mathbf{x}_\mathrm{i}$ is the location of each earthquake [9].

$$\hat{f}(\mathbf{x}) = \frac{1}{nh^2}\sum_{i=1}^{n} K\left\{\frac{1}{h}(\mathbf{x}-\mathbf{x}_\mathrm{i})\right\} \tag{6}$$

The critical point is the choice of the bandwidth in kernel density estimation rather than the choice of kernel density function. They mostly reflect the spatial earthquake epicenter distribution [7].

Gaussian kernel is given in equation (7) and is chosen as a kernel function since the objective of the study is to compare intensity estimation methods and also the kernel function choice is not a critical choice.

$$K(\mathbf{x}) = \frac{1}{2\pi}\exp\left\{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2}\right\} \tag{7}$$

Earthquakes higher than a certain magnitude (especially big earthquakes) generally have a clustering display because of foreshock, aftershock and swarm type of activity. Therefore, there are intensity higher zones compare to overall study region. Estimation of intensity with a fixed bandwidth kernel estimator might oversmooth these regions if a larger bandwidth is chosen. Moreover, estimated density can be too spiky for a too small bandwidth.

Adaptive kernel estimation or adaptive smoothing is given below as a three-step procedure [7,9].

$$\hat{f}_1(\mathbf{x}) = \frac{1}{2\pi n {h_1}^2}\sum_{i=1}^{n}\exp\left\{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{2{h_1}^2}\right\}$$
$$h_2(\mathbf{x}) = \sqrt{\frac{\mu}{\hat{f}_1(\mathbf{x})}} \tag{8}$$
$$\hat{f}_2(\mathbf{x}) = \frac{1}{2\pi n {h_1}^2}\sum_{i=1}^{n}\frac{1}{h_2(\mathbf{x}_i)^2}\exp\left\{-\frac{(\mathbf{x}-\mathbf{x}_i)^2}{h_2(\mathbf{x}_i)^2 2{h_1}^2}\right\}$$

In equation (8), $h_1$is a global parameter and μ is the global mean of earthquake activity per area. In the first step a pilot estimate for the $\hat{f}_1(\mathbf{x})$ is calculated using a kernel density estimation. It can be calculated also by using another method like nearest neighbor method. In the second step a local bandwidth is determined. In the third step adaptive density estimation is defined to estimate distribution of earthquake occurrences.

The main difference of adaptive kernel density estimation with a global bandwidth kernel density estimation is the bandwidths' property of being a function of the coordinates [10]. Baddeley et. al. [4] defines the relationship between intensity and probability density as normalized intensity is the probability density. The estimation of intensity function is given in equation (9) where $e(u)$ is the edge correction factor given as:

$$(u) = \int_W K(u-v)dv\ .$$
$$\hat{\lambda}(u) = \frac{1}{e(u)}\sum_{i=1}^{n}K(u-x_i) \tag{9}$$

Earthquakes are generally occurred in specific points of a fault line and thus the underlying mechanism would be an inhomogeneous poisson process with a varying intensity function. For this purpose, a likelihood cross validation is suggested to get an optimum bandwidth for a kernel density estimation [11].

Likelihood cross validation (LCV) is given for a gaussian kernel in equation (10) and bandwidth sigma is selected according to this method.

$$\mathrm{LCV}(\sigma) = \sum_i log\hat{\lambda}_{-i}\,(x_i) - \int_W \hat{\lambda}(u)\,du \tag{10}$$

In addition, a criterion for a better fit of an intensities is necessary to find out whether a fixed bandwidth or an adaptive bandwidth is superior to each other. However, it is still a problematic issue without one proper criterion. It is mostly left to the researchers to select the method that produce a good picture of the phenomenon. Moreover, a point pattern log-likelihood based criterion similar to equation (10) may be considered as the difference of sum of the log intensity at each point and the integral of the pixel image produced after fitting.

3. DATA

Data is taken from Republic of Turkey Prime Ministry Disaster and Emergency Management Presidency 1900-20xx earthquake catalog [12].

Cutoff magnitude is selected 4 in this study within a rectangular region between 30º-40º longitude and 39º- 42º latitude in 1900-2016 time period. This region includes the part of a North Anatolian fault line. There are 802 earthquakes in the given space and time domain.

The spatial pattern of earthquakes is given in Figure 2. As seen from the figure too, there is a high intensity of earthquake occurrences in the west side of the study region. In addition, it may seem that the earthquake pattern has a varying intensity over the region.

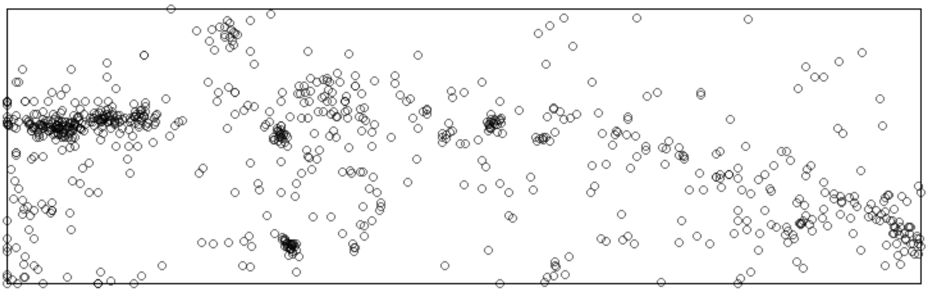

Figure 2. Earthquake pattern higher than magnitude 4 between years 1900-2016.

Magnitude distribution of the earthquakes more than magnitude 4 are illustrated in Figure 3. It can be inferred that big earthquakes only occur in some regions over the region. Also, the locations of the big earthquakes may be related to the closeness of the fault line.

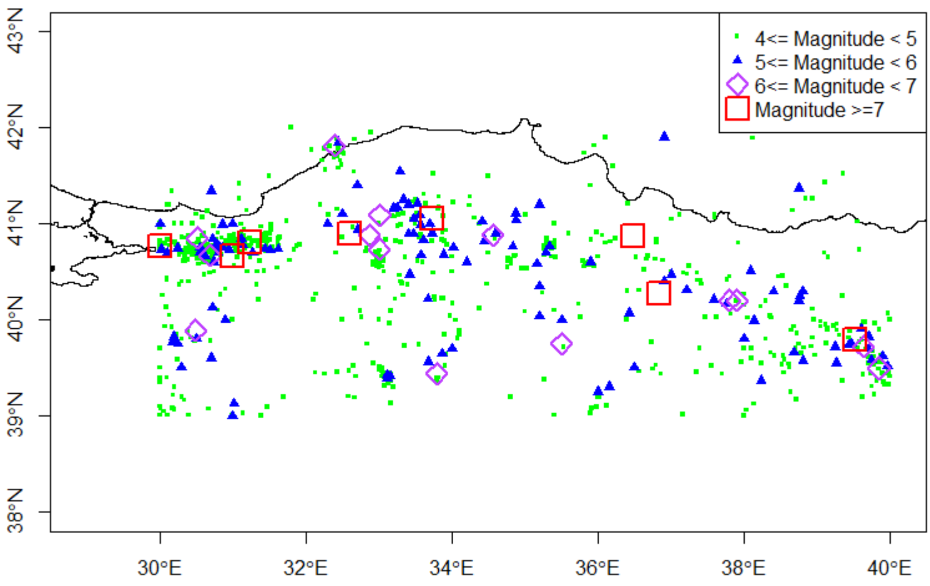

Figure 3. Magnitude distribution over the region.

In Figure 4, the active faults of Northern Anatolian Fault zone are given to support our claims about big earthquakes' occurrence locations.

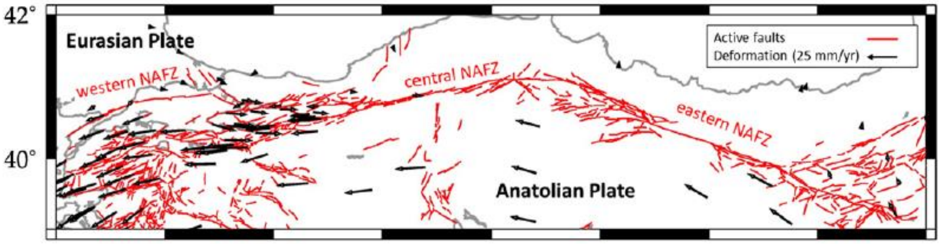

Figure 4. Northern Anatolian Fault Zone [13]

4. RESULTS

Intensities over the region are calculated by using "spatstat" and "sparr" R packages [4,10]. The edge corrected intensity estimation is applied to the earthquake pattern. In calculation process of the intensities by using kernel functions, the domain is divided into 128×128 pixels.

Quadrat counts of the pattern are given for 1º×1º in Figure 5. It is observed that the bottom right quadrat and middle left quadrats has more events compare to other quadrats.

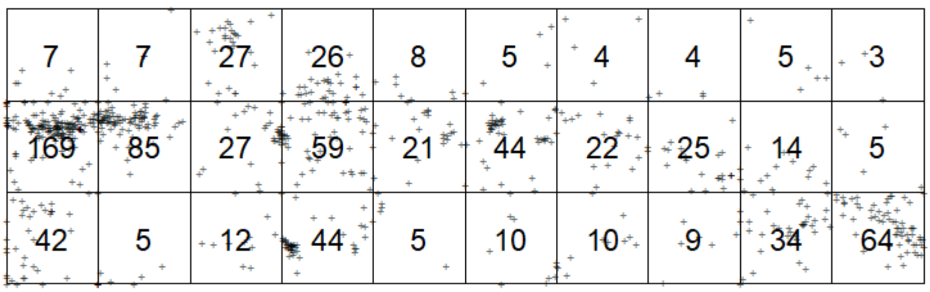

Figure 5. Quadrat counts of earthquake occurrences.

Pixel images of quadrat counts are given in Figure 6 for comparing subjective choice of number of quadrats.

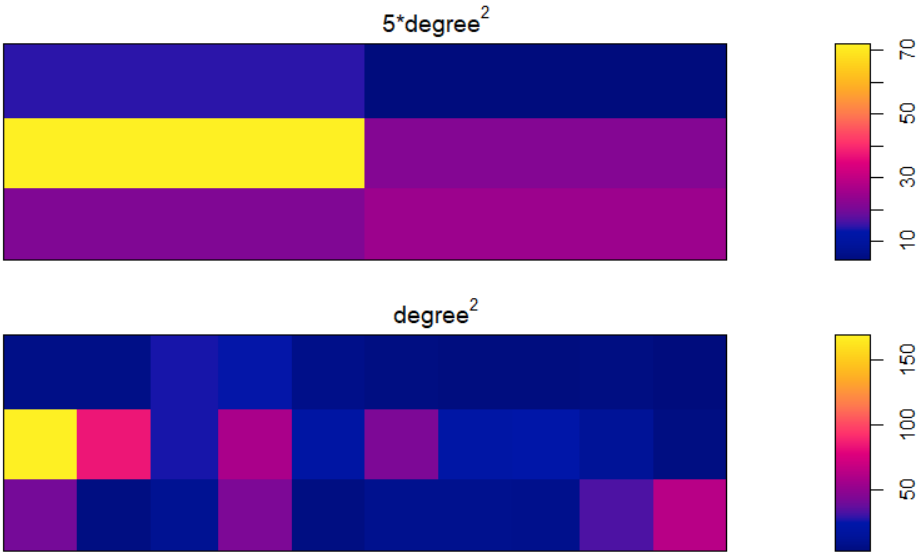

Figure 6. Intensities obtained through different number of quadrats.

It can be easily inferred from Figure 7, when the number of quadrats increases, intensity differences between quadrats increase too. Intensity is oversmoothed for lesser number of quadrats while it is undersmoothed for a higher number of quadrats.

In Figure 7, intensity estimation for two arbitrary different bandwidths are obtained. Indeed, for a large bandwidth the intensities are oversmoothed for the study region. A lower bandwidth of σ=0.5 enables high intensity zones being narrowed.

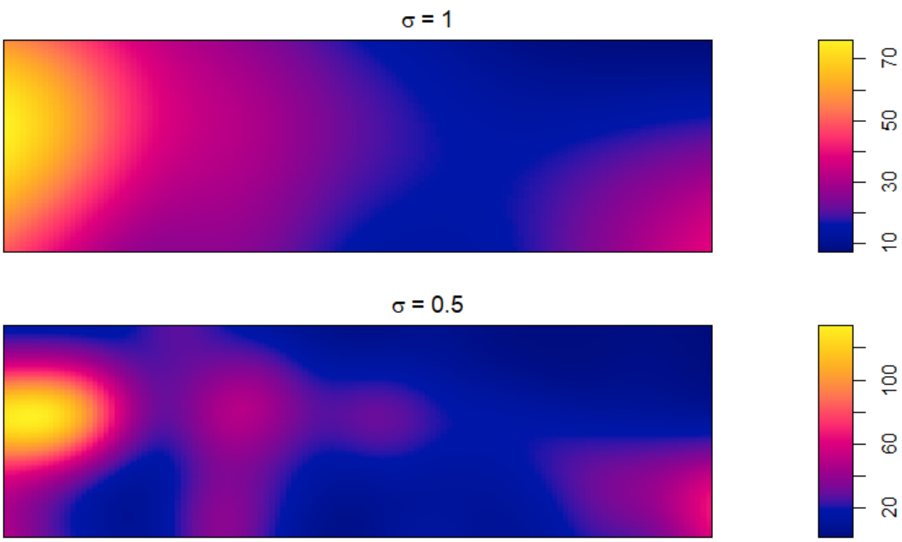

Figure 7. Intensity estimations via arbitrary bandwidths.

In Figure 8, earthquake pattern is superimposed to the pixel images of intensities in Figure 7. It seems that the pattern and intensity images has a better match for bandwidth σ=0.5.

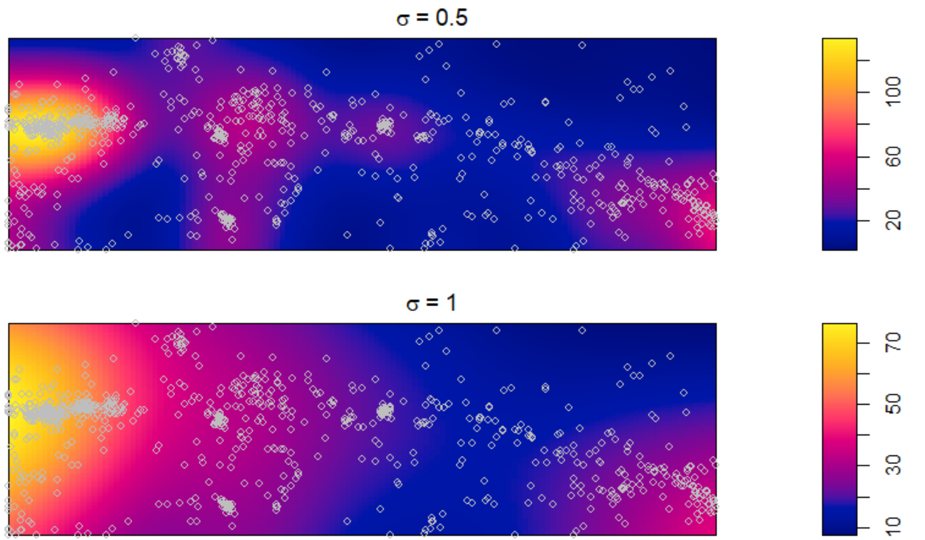

Figure 8. Earthquake patterns with pixel images of intensities.

Pixel image of intensities and intensities with the superimposed pattern is given in Figure 9 for a bandwidth σ which is obtained through likelihood cross validation. According to likelihood cross validation method bandwidth σ is found 0.17 for an assumed inhomogeneous point pattern. Intensity increase in intensity high zones and intensity high zones are getting narrowed indeed.

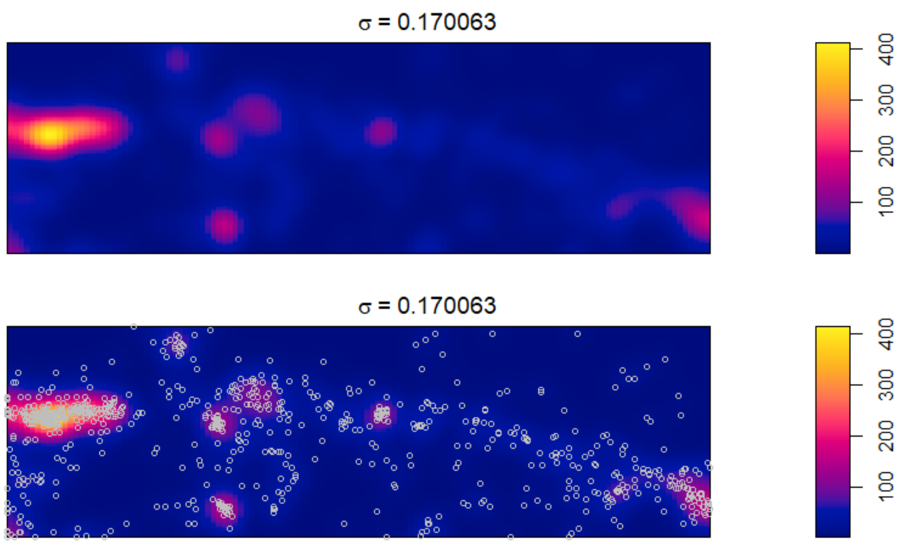

Figure 9. Earthquake pattern with pixel images of intensities obtained by using optimum bandwidth through LCV.

In Figure 10, intensity estimation of earthquake point pattern is given for the specified longitudes and latitudes. A pilot estimate which is obtained through likelihood cross validation is chosen to estimate intensities over the region via adaptive smoothing. Intensities rise up dramatically for intensity higher zones with adaptive smoothing in contrast to intensity estimation with a fixed bandwidth. It had better to interpret these dramatic intensity changes both by examining with fixed bandwidth estimation and adaptive bandwidth estimation together. In addition, it is necessary to give the varying bandwidths along the study region to understand how the adaptive smoothing works.

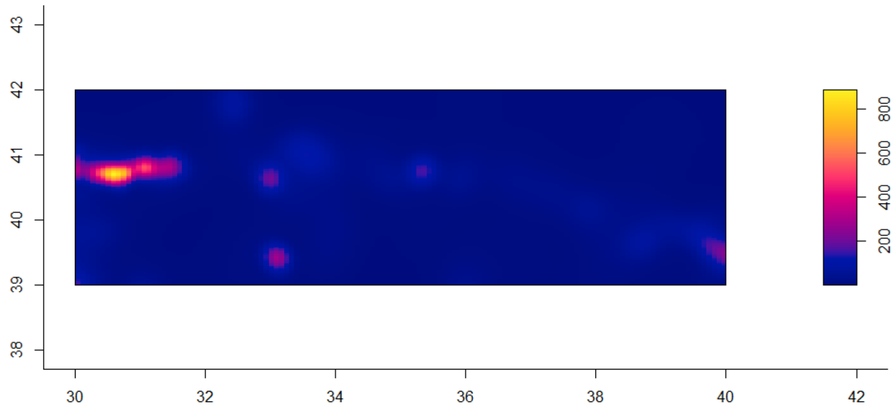

Figure 10. Intensity estimation via adaptive bandwidth in given coordinates.

The varying bandwidths and the adaptive estimation of intensity of the earthquake pattern are given in Figure 11 together. It is seen that highly-densed areas have lower bandwidths when in low-densed areas have higher bandwidths for adjusting the degree of smoothing according to them.

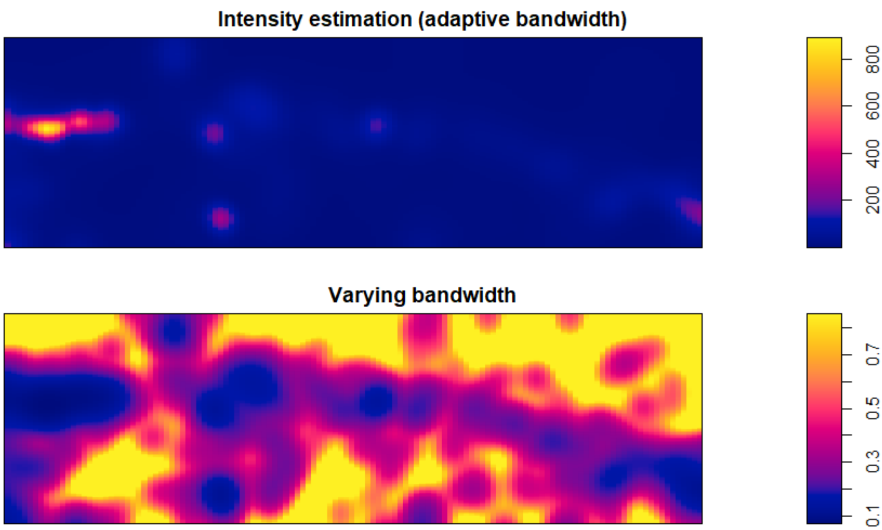

Figure 11. Varying bandwidths together with the adaptive smoothing.

In Figure 12, pixel images of two intensities with superimposed pattern is given for comparison purposes of two different estimations which are obtained through a fixed bandwidth and varying bandwidth. There is a small difference in high intensity zones if it is examined in terms of area. However, the maximum intensity of the study domain is doubled in contrast to intensities which are obtained through fixed bandwidth.

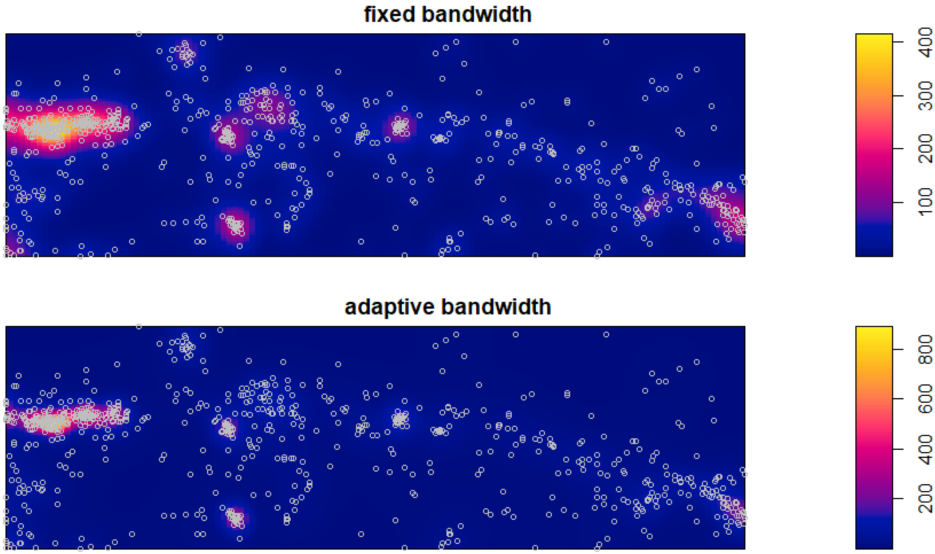

Figure 12. Comparison of intensities with superimposed point pattern via two different bandwidths.

Table 1. Model selection criterion

	Difference
Kernel function with a varying bandwidth	36618.01
Kernel function with a fixed bandwidth	41642.35

In Table 1, a value depending on a criterion for model goodness of a fit based on point process likelihood is given. The differences are not very high. Varying bandwidth performs a better fit for the intensity of the concerning earthquake pattern and one can choose adaptive smoothing to estimate the intensities over the study region for the given pattern.

5. DISCUSSION

Bandwidth selection for a point pattern and the selection of the best model to estimate intensity of point pattern is still an ongoing issue. There are methods to estimate the intensity, which have both advantages and disadvantages. Kernel estimation with different kind of bandwidths and quadrat counting is some of them. Quadrat counting is a simple method which is dramatically affected by the choice of the quadrat number. Optimum bandwidth obtained by likelihood cross validation criterion and varying bandwidths obtained by adaptive kernel estimators give better pixel images of intensities compare to pixel images obtained through arbitrary chosen bandwidths. A criterion based on a point pattern likelihood suggests to use adaptive kernel estimators to estimate the intensity. However, the higher intensity zones are approximately similar while the intensities in these zones are doubled in contrast to kernel estimation with a fixed bandwidth. Therefore, they can be both chosen to model intensities. Earthquakes that are considered as big, devastating or upper than middle value in magnitudes are tend to cluster in small zones because of mainshock-aftershock relationships. As a result, there are highly-densed locations and low-densed locations. It is better to consider adaptive bandwidth to discriminate these zones. In addition, an optimum fixed bandwidth can be chosen for these type of patterns with varying intensities along the study region.

Acknowledgements

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