The Anatomy of Mistakes: Categorizing Students' Mistakes in Mathematics within Learning Theories

Ali Türkdoğan[1]

Adnan Baki[2]

Salih Çepni[3]

Abstract

Especially, at the process part of the 4-E learning model it is natural that learners' mistakes could occur. Teacher must be guide to students who made mistake. But when the considering literature there is only a limited study about the place and role of mistakes in learning environment. But there is not seen any study related directly about learners' mistake. Most of them give only theoretical suggestions about learners' mistakes. But considering one of the most important things to implement student centered is to know how to deal with mistakes. Therefore, there is a need to describe and identify the place and role of mistakes within learning theories. It is seen that mistakes should be categorized with respect to feedback given it by teachers. Studies related mistake which give place philosophical, sociological and psychological respect of mistake should be.

Key Words: Student's mistakes, learning theories, constructivist approach.

3. Introduction

Recent global trends based on constructivism student-centered curriculums have been developed and put into implementation in Turkey since 2006. Constructivism is one of the most acceptable theories in recent years (Lerman, 1989). As most of the educational theories include constructivist approach has also a variety of models such as 4-E, 5-E and 7-E regarding how to apply this approach in the classrooms (Ayas, Akdeniz, Özmen, Yiğit, Ayvacı, and Çepni, 2008). In essence, each of these models require; students' attention on the task, exploration a fact, concept of understanding, explanation and, evaluation as a final step in learning process.

Namely constructivist models consist of four stages at the origin; introduction, exploration, explanation and evaluation (Baki, 2008). Introduction part aims at drawing the students' attention and completing the inadequacy of the students by establishing their previous

[1] Res. Asst.. KTÜ Fatih Eğitim Fakültesi, İlköğretim Bölümü,

[2] Prof. Dr. KTÜ, Fatih Eğitim Fakültesi, OFMA Eğitimi Bölümü, *abaki@ktu.edu.tr*

[3] Prof. Dr. KTÜ, Fatih Eğitim Fakültesi, İlköğretim Bölümü, *salihcepni@fenegitimi.com*

learning. The exploration as a process allows students to make inquiry. In the explanation stage, students are supposed to form hypothesis, make conjectures and share them with their friends. In this stage, the teacher is responsible for establishing and eliminating the students' inadequacy, mistakes and errors. In this process, the teacher's guidance role is of great importance. In the evaluation stage, the students generally stated the facts they had reached. On the other hand, the teacher talks about an ideal construction and try to prevent students' wrong inference from being a misconception by emphasizing the fault of making wrong inferences.

In this sense, there is a close relationship between introduction and evaluation parts because of the importance given to the pre-knowledge of the students. In other words, introduction and evaluation parts focus on examining the existence of the pre-knowledge and if there is, it is required to be eliminating them namely misconceptions while the process part deals with the mistake itself.

It is useful to mention the fact that behaviorist approach knows existence of the mistake though it accused of the students and the way of communication; it gave importance to the mistake but it was not aware of the misconception (Özmantar, Bingölbali & Akkoç, 2008). On the other hand, cognitive approaches analyzed the misconceptions, its fixation, its elimination and its effect on the process of learning but they disregarded the mistake itself.

It is not different for the constructive approach from the point of the importance of the misconception. One of the most important obstacles in creating an effective learning environment in a student-centered instruction is the misconceptions because learning requires establishing the students' pre-knowledge, reminding the main knowledge on which the newly learned things be constructed and eliminating the misconception. Moreover, a wrong construction of the knowledge will lead to new wrong constructions, so on. Even though it is supposed that a learning situation without a misconception will contribute to the following learning situations positively, that is not always true. Even after a perfect construction, it is possible for students to construct wrong knowledge and to make mistakes. Hence, as a requirement of constructivist approach, it is believed that the mistakes should also be analyzed as misconceptions.

One of the most important problems the constructivist models have to deal with is the form of mistakes. Enough strategy regarding the mistakes and dealing with them could not be established because the literature concerning the mistake mostly focuses on finding and eliminating the misconceptions (Özmantar, et al., 2008). The teacher's being lack of experience and theoretical knowledge regarding the subject cause the teacher to be in a difficult situation when it is required to identify and eliminating the mistakes. By leading to a difficult forecast, increasing anxiety and decreasing the time management ability, this situation make the teacher, who has already had some doubts related to the students' centered learning environment, stultifies his/her following studies concerning in creating a student-centered learning environment.

As a matter of fact, the teacher should accept a natural process to face with mistakes in a learning environment. However, there is not enough information in the literature for the teacher about how to react when they face with a mistake related to this subject. It is said that when the teacher face with a mistake, they should not give answer directly; they should ask questions which will enable the students to think, to make them seek, examine and create opposite examples. Unfortunately, there are no any examples of how this should be put into action through in the real classroom environment.

Therefore, it is necessary for teachers and teacher candidates to form the infrastructure for developing their point of view about the mistake through instruction they are exposed to rather than through their experience. For this reason, how the learning theories deal with the mistake should be examined by taking into consideration the nature of learning models. After all these are clarified, what should be done about that subject will appear. Hence, the basic question of this study is determined as "What is the theoretical structure of the mistake?"

There are some traditional viewpoints, which are the products of behaviorist approach, related to the mistake in the literature. Among these, compressions, making it forget and assimilation are the well known ones.

Cognitive approaches accept the misconception and suggest that some regulations are necessary to eliminate. It is possible to come across a lot of studies regarding the determination and elimination of the misconceptions. Constructivist approach thinks that it is natural for the mistake to appear in a learning environment. So, it argues that the learning environment should be analyzed well and scientific information should be obtained by seeing the mistakes as a learning opportunity. In this sense, dealing with the mistake is the main part of the constructivist approach.

It is pointed in the constructivist approach that the teacher should make alternative evaluations that is; they should make some regulations according to the student, the material used and the learning environment by assessing the development process of learning rather than giving marks to the students assessing whether they gain the aimed behavior (Baki, 2008). In the learning process, the analysis of the mistake will contribute to gain more clear information and understanding about the thinking process.

Although the existence of learners' mistakes are known very well, misconception feeding these mistakes are not taken seriously by teachers who generally focused on errors rather than the roots of mistakes. This situation also prevented the careful investigation of the concept "mistake" or "error" leading the formal education to be weak for the teachers and the students. Therefore, it is necessary for teachers and teacher candidate to form the infrastructure for developing their point of view about the mistake through the instruction they are exposed to rather than through their experience.

For this aim, it is believed that analyzing the role of the mistakes in the learning theories and making inferences depending upon this analysis will contribute to the knowledge of concept of mistake.

1.1. The knowledge of mistake in learning theories

Due to the lack of knowledge regarding how the learning process takes places, every day it is possible to see a new theory related to learning. However, given the most acceptable theories, it is possible to examine the three main ones as behaviorist, cognitive and constructive approaches (Lerman, 1989; Baki, 2008). In this sense, while we are analyzing the anatomy of the mistake, utilizing the terminology of these theories can help us to see the whole. Therefore, it can be useful to understand how the learning theories dealt with the mistake in the past in order to comprehend the concept of mistake and identify it under the light of the new developments.

Knowledge and mistake in behaviorist approaches

Behaviorist approaches argue that a behavior is acquired through the process of effect and react. The most accepted behaviorist theories are "Law of Effect" by Thorndike; "Classical Conditioning" by Pavlov and Watson; "Operant Conditioning" by Skinner and "Observation and Modeling" by Bandura. It is possible to explain the situation, on which the behavior could not be acquired, through the individual's lack of attention towards the subject, the inadequacy of the communication, the lack of effectiveness of the action and the mistakes in the source of information. In other words, an individual can react the same action in different ways. It is not possible for the students to make mistake when he listens effectively, when the action is introduced rightly and when the environment is in its ideal form (Tekin-İftar & Kırcaali-İftar, 2004).

Other students are not required to see the mistakes. If they learn wrongly, explaining them and telling them to be careful will be enough for them. However, it can be possible to weaken the relationship by applying punishment when the wrong reaction occurs as a result of action. The mistake can be prevented by establishing and changing the effect used for the reaction. Additionally punishing a student who makes mistake can enable the others not to make the same by observing that.

Hence, if a student is careless, it is not necessary for the others to see the products resulting from this carelessness. It will be enough to explain them again and to want them to be careful. Of course, the ability of the individual is important. If the individual is not adequate and intelligent enough, he or she cannot learn anyway. In other words, behaviorists neglect individual differences and give no importance to mistake.

The mistake does not occur after an ideal action, reaction, process and reinforce-behavior process. Nevertheless, there are some practices to deal with these mistakes. It aims at making this notion be forgotten. Making it be forgotten can be explained with removing the stimulus or basically deleting the action-reaction- punishment and behavior process. That is, making it be forgotten can be explained with removing a few component of the behavior development process. Making it be forgotten is like deleting a page. It will be possible to fill the page again after the deleting process is completed. Even thought if mistake occurs techniques related to the mistake in the behaviorist literature such as

suppression, assimilation, slow adjustment and giving interfering stimulus could use. Examining these terms in details will enable us to understand the concept of "making it be forgotten" well.

Fading away: The behavior of a student is not reinforced. Suppose that the dog is not fed with the meat following the bell ring, after a while, the bell itself will be also something which does not requires any reaction like the other actions.

Suppression: It is the process of making responses negative. It can be suppressed if punishment is given instead of award. Suppose that the mouse which learns to open the door by pushing the door handle is punished with the electrical current each time it touch the door handle.

Assimilation: It depends on the fact that a student who makes mistake is punished and the other students assimilate this. The other three terms related to making it be forgotten is taken from the theory by Guthrie.

Exhaustion: An individual is exhausted completely before he/she is exposed to the environment and he or she does not exhibit the same behavior. Riding a wild horse after making it run and feel exhausted.

Introducing interfering stimulus: It is introduced when the behavior could not exhibit. For example, riding horse by fastening it.

Adjustment slowly: It is a systematic desensitization. For example, you put load on the horse and make it adjust and then you ride it.

As it is seen there are some examples regarding how this theory can be applied for educational purposes. The meanings attributed by the researchers to these terms can be illustrated as follows.

Let's think that the students make mistake as $2^0=2$. A student who makes this kind of mistake can be exhausted as follows: teacher's being angry with the student

The teacher says: "$2^3=2x2x2=8$; $2^2=2x2=4$; $2^1=2$; on that condition 2^0 is 1. How did you find 2! Every number's   zero power is 1." The teacher wants the student to write many times "$2^0=1$" in his/her notebook. Moreover the teacher wants him/her to write the numbers' zero power beginning from 2 to 150 and wants him to bring and show this homework tomorrow.

Introducing interfering stimulus: Let's think that the students made mistake as $2^0=2$ and suppose that the teacher tries to correct the mistake saying it is wrong and the students goes on making the same mistakes. The teacher lists the student in a special way and while she is dealing with the zero power of the number s/he always repeats "The zero power of five is 1 ($5^0=1$), isn't it? or determining a student who comprehends this subject, s/he asks the student, Ahmet "What is 5^0?" Ahmet answers the question. By this way, the student is put into a place where she or he cannot make a mistake.

Another example can be given for introducing interfering stimulus as follows. Suppose that the students make the mistake of not using cluster bracket while deal with the cluster problems. Under these circumstances, the teacher starts solving the problem each time she or he asks the question. That is,
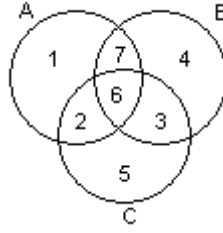
The teacher writes the question as "(AUB)/C= "instead writing it as "(AUB)C=?" or s/he writes it "(AUB)\C={ }" and s/he does not let the mistake appear.

Slowly adjustment: If the mistake of writing $2^0=0$ is repeated, the teacher writes everywhere 2^0 when "1" is necessary to be written there.
For example: What should be written instead of $4(2^0+8)=4.1+4.?=36$ "?"
When we are dealing with fractions, 2^0 is written for denominator and everywhere possible. After a while, the student who is fed up dealing with that is accustomed to the right answer.

Even though it is possible to come across with its examples, it is possible to state the fact that this notion is not true and it plays an important role in students' developing a negative attitude towards mathematics. Especially, it can be inevitable that the notion called "exhaustion" can have a deep, negative and a result impossible to compensate on the life of the students' education.

It is seen that the general point of view of the behaviorist approaches to the mistake is closely related with the action-reaction and award-punishment concepts and this is not seen adequate. At that point, what are our teachers' ways of dealing with the mistake? Asking such a question will be early to ask without analyzing viewpoints of the cognitive approaches but it will be useful too. Analyzing the cognitive approaches will enable us to understand better the deficiencies of the behaviorist approaches' viewpoints regarding mistake.

Knowledge and mistake in cognitive approaches

Cognitive approaches consider the importance of the process itself and individual differences in addition to the acquisition of the behavior in the behaviorist approach. Among these approaches, Learning through Discovery Theory by Bruner; Learning through Presentation by Ausabel; Hierarchical Learning Theory by Gagne; Absolute Learning Theory by Bloom and The Learning Theory by Jean Piaget can be listed. Cognitive theories basically concentrate on the process of learning and individual differences.

Hierarchical learning theory of Gagne: According to this theory, learning process consists of 8 stages from simple to difficult. According to this, learning takes places in the ways such as symbolic learning, action-reaction learning, successive learning, verbal learning, learning through discrimination, concept learning, rule learning and learning through problem solving. In this approach, there is an analytical taxonomic relationship. Learning through problem solving will not take place unless the first seven types of learning take place, in other words, we can say that there will not be the second type of learning without the first learning's existence, and the third one will not take place without the first and second ones.

This hierarchical learning approach, one mistake in one of the stages of this approach can cause mistakes in the following ones, which shows that learning environment has the potential for mistakes and misconceptions. Even the organization of such a learning environment can be seen as a sign of the fact that they accept the existence of the mistake in the learning environments.

The discovery learning approach by Jerome Bruner: In the approach of learning through discovery, it is argued that the students should reach the knowledge through their active participations. According to Bruner, the individual conceptualizes the stimulus he or she comes across and categorize them. In other words, the individual classifies the concepts depending on their characteristics and tries to find answers for each one with common properties (Baki, 2008).

Well, these generalizations are always scientific? Is it possible to avoid making wrong generalizations? Given the structure of the knowledge, especially when we consider the necessity of teaching a concept by making association with another concept, it is natural for us to see these generalizations face with wrong answers in the first stages of learning.

One of the most important contributions of Bruner to the education is the approach of teaching concept. According to him, the process of learning concept should be in the way of following the steps;

- firstly learning the name of the concept
- secondly the description of the concept
- thirdly the characteristics of the concept
- fourthly the importance of the concept
- and lastly the examples related to the concept.

When we have a look at the steps of concept learning approach, it is seen that concept learning approach not only includes knowing the characteristics of the concept but also requires the discrimination of the concepts from each other. Given the spiral structure of Mathematics, each newly learned concept may require the revision, expansion or restriction of most concepts (Ersoy & Ardahan, 2003). Natural numbers can be given as an example

for this process. For instance, the cluster of the numbers (excluding the complex numbers) can be completed with a cumulative structure every year until the grade eight.

A student can know the whole numbers and can know the natural numbers. But when he could not state the differences between them, we can not suppose that he really knows the natural numbers. In this sense, it is natural that the learning through discovery activities can cause the students make wrong generalizations, overgeneralizations or inadequate generalizations. Especially, when mathematics is considered with its nature, the learning process described by Bruner has the potential for the mistakes.

The meaningful verbal learning theory by David Ausabel: Ausabel who adopts teaching through presentation argues meaningful verbal learning. According to the approach of meaningful learning, an individual has got available cognitive structures. Depending upon his/her available cognitive structure, the individual chooses one which is the most suitable to him/her.

In the approach of learning through presentation, how the students conceptualize present concepts and the degree of academic comprehension of the concept which will be use for making association are of great importance. One of the previous regulators should be completely unknown for the students and the other should be known by the students beforehand for the topic to be learned. Consolidating what have been learned so far, reminding if necessary and eliminating misconceptions are essential. It is impossible for the teachers not to come across with misconceptions. However, teaching a subject again and reminding it may not be enough to remove the misconception. Misconceptions are often resistant to change (Ayvacı and Devecioğlu, 2002).

In this sense, it is necessary to ask the question again "What do the teachers do when they face with misconceptions?" Nevertheless, there are not any studies concerning what is done in the classrooms. Traditional learning approach and meaningful learning approach is similar to each other in that it is the teacher who is seen as the source of information and in that the knowledge is learned through presentation. In fact, in most of the traditional structures, reminding the previous lesson and explaining the information with its dimensions can be assessed as the teacher's effort to give a meaning to the information itself. So, there can be a relationship between the viewpoint of Ausabel and the ones of the teachers. For this reason, it is necessary to inquire what the general viewpoint regarding the mistake is.

The theory of absolute learning by Bloom: Bloom concentrates upon the individualized learning. He advocates that every student can learn if enough time is allocated for him or her and he thinks that the learning should take place according the aims and goals considering the previously learned things. He asserts that the learning environment should be organized according to the students' need and wishes given the inadequacy and lack of information the students have. At the end of the learning process, whether the students learn

the subject completely should be determined. It is also stated by this theory that there may not be an absolute and adequate learning namely mistakes or misconceptions may appear.

Given the teachers in our country depends on the theory of absolute learning in their instructions, it is a necessity that teachers' view points, their capability of dealing with misconceptions and their level of information regarding Bloom's taxonomy should be analyzed and the deficiencies should be cleared away if there are.

The learning theory of Jean Piaget: The basis of this theory is whether the individual has enough level of development. He or she construct his/her knowledge according to his/her level of development, maturation, and depending on his/her experiences as a result of his/her active participation. The learning occurs at the end of the process of equilibrium. In this sense, the culture or the society the child is living in is of great importance.

Piaget categorizes the children's cognitive development into four processes; sensor motor, preoperational, concrete operational, and formal operation period. Especially in the sensor motor period, there is a disconnection in the behaviors of the children. In the preoperational period, the child could think with the help of real objects. In the concrete operational period, the child needs objects in order to separate the whole in to parts. However, in the abstract operational period, the students can understand abstract connections. So, it is natural for primary school students to make mistakes in mathematics lesson which includes lots of abstract concepts.

For this reason, it can be said that cognitive approaches can see the mistakes as two different concepts; one is misconception (mistake with a cognitive infrastructure) and the other is mistake (a mistake without a clear cognitive infrastructure). If the mistake takes place in the learning environment as a misconception, it is not obligatory to deal with it great care whatever reason it appears out.

Inadequacy in the viewpoints of the cognitive approaches regarding mistakes

In order to make the viewpoints of the cognitive approaches regarding misconception much clearer, let's look at the relationship among the misconception, knowing, not knowing and false-true concepts again. By this way, we can see clearly the fact that the viewpoint "it is worth taking into consideration if the mistakes result from a misconception" is not enough to meet the needs of the current curriculum system because the mistakes are not only the result of the misconceptions but also they are one the most important component of the occurrence of misconception. Because;

1)   The student who comprehends the subject becomes more and more confident as long as s/he does the exercises correctly.

2)   The fact that the student who comprehends the subject make mistake will lead him/her lessen self-confidence in him/her about his knowledge, his method and algorithm. Even though this situation causes the student to state "I can not do! I do not know

anything!" this process is a sign of the shake of the student confidence rather than "not-knowing" concept. At that point, it is difficult to talk about the concept of "not-knowing".

3)   A student who has no idea about the subject and a student who is in the process of learning the truths will gradually gain self-confidence. Moreover, the mistake an individual, who has no idea about the subject, make can enable him/her to be more careful, to make him research and to make him/her reach the truth again. However, it can be one of the effective steps in forming a misconception if there is a possibility of doing rightly with the wrong algorithm.

4)   Another situation is an individual's (who has got misconception) making mistake. The individual may not accept that it is wrong. S/he can go on his/her mistake behaving as if s/he believed in the classroom. Sometimes, he can get rid of the misconception by reviewing it or it can go on even if the misconception is becoming more unclear.

There are a lot of methods regarding the determination and elimination of the misconceptions and a lot of research concerning the effectiveness of these methods is being conducted especially in science. Nevertheless, as it is seen there are also mistakes resulting from the errors which are not the products of the misconceptions in the learning environments. Especially in the student-centered learning environments, students are expected to understand the issue and to form a hypothesis explaining the issue depending upon the situation. It is necessary for the hypothesis to be shared and discussed among the students and at the end the main knowledge should be defined by the teacher depending on the knowledge of the students. So, there is a need for the studies regarding how to deal with the mistakes in learning environments.

The knowledge and mistake in constructivism

In addition to the basic excuse, constructivist approach is an approach which argues the fact that the importance of the individual's conscious regarding learning, his interests, the society he lives in and the language he speaks should not be regarded. The knowledge is constructed by the student himself through his active participation. How the learning process is carried out is as important as which behavior should be gained by the student and how it should be taught (Erdem, and Demirel, 2002; Durmuş, 2001). When the constructivist approach is assessed form the point of Turkish education system, it signals for an important change in education philosophy. Student-centered system requires changes in the components of the learning such as students, materials used in the lessons, and aimed behaviors (Demirel, 2002; Türkdoğan, 2006).

According to the student-centered instruction approach, the role of the teacher cannot provide students with readymade information because information is not something that can be transferred from one place to another place. So, the teacher is required to be a guide (Sönmez, 1993, Ersoy, 2002; Ministry of Education The Process of Curriculum

Development, 2007). While the teacher leads as a guide for the student, s/he should also analyze the learning environment and assess the learning process. The teacher could be able to organize much more effective learning environments with the help of his/her experiences s/he obtained from his applications.

With the application of the new curriculum in Turkey, it is natural for a teacher to need sample methods and sources about how and in which processes the students construct their knowledge. The process of developing suitable activities and their applications gained speed with the introduction of the new curriculum depending on the constructivist approach. Unfortunately, in literature, there is not a sample application of what is the meaning of guiding a student's understanding. Guiding the learning process is a difficult and complex process. In their studies, Turan and Sayek (2006) stated that it is difficult to write a prescription for the teacher about when and in what degree they should interfere the learning process. Furthermore, it is said "Educators should decide their own decisions depending on their knowledge and experiences." (Eggen & Kauchak, 1985). However, it is creation that teaching teacher candidates with so much open-ended approach will have a lot of disadvantages in terms of teacher quality.

It will be useful to have a look at the constructivist approach. It includes the continuous chance process of the construction, adaptation and accommodation of the schemas (Wood, 1988; Baki, 2006). Accommodation means a student's coming across with information which s/he has not seen beforehand and his allocating some place for this knowledge in his/her mind; adaptation means making association the newly learned information with the present knowledge. Schemas are the associations of the concepts with each other (Cobb, 1992).

When the individual come across a new situation including new information, s/he tries to give meaning by making associations that information with his available knowledge (Matthews, 1993; Şandır, 2002). In this process, the information the individual has just come across may not get along with his/her present knowledge (Bybee & Sund, 1990). In fact, it can signal the fact that this students can have a misconception. At the end of this process called by Piaget as disequilibrium, the individual construct his/her present knowledge according to the new knowledge he has acquired and construct his/her knowledge (Bybee & Sound, 1990). It is natural for the students to make mistakes in the construction process depending their way of perception the world. When the spiral and abstract structure of the knowledge come together with the individuals' way of thinking special to them, it is seen that it is becoming more and more impossible to have a learning environment lack of mistakes in a student-centered instruction.

Types of mistakes

The knowledge is classified in many ways in the process of construction. One of the most acceptable classifications of the knowledge is Bloom's taxonomy. In Bloom taxonomy, cognitive domain is separated into six main groups; knowledge level, comprehension, application, analysis, synthesis and evaluation and some other sub-groups. When we take

into consideration the mistakes, let's think that a student makes a mistake concerning the symbolic use of A $\cup$ B when the teacher wants him to find the intersection of A and B clusters. It is not possible for the teacher to have a lot of techniques to deal with this mistake. S/he can say it is wrong; s/he can write A $\cup$ B; s/he can want another student to write or s/he can say that it is not that symbol, the other one is necessary. Perhaps, s/he can make use of the analogy.

Now, suppose that the student ignores 1 in front of $\sqrt{3}$ while solving the question $2\sqrt{3} + 3\sqrt{3} + \sqrt{3}$ =? and find the answer as $5\sqrt{3}$. It is established by the researchers that the teachers give the following feedbacks in such situations;

1) Telling the right answer
2) Taking the right answer from another authority in the class
3) Telling that it is wrong
4) Making it concrete
5) Making association with the subject the student learned in the previous year (2x+3x+x=6x)
6) Making association with another unit learned ($2.10^2 + 5.10^2 + 10^2 = 6.10^2$)
7) Creating an analogy between the square root and prison
8) Forming a cognitive paradox.

So, it is seen that there are a lot of differences between the mistakes in symbols and the ones in conceptual questions. For this reason, the mistake is also required to be classified and analyzed again.

4. Conclusions and Suggestions

However the learning theories talk about the existence of the mistakes and their positive or negative effects on learning, their points of view regarding the nature, existence and the role of the mistake in education are seen as both inadequate and too much theoretical. The analysis of the mistakes and the determination of the ways to deal with the mistakes are necessary for the constructivist approach to be effectively put into effect. Neither the notion of the behaviorist approach about neglecting the existence of the mistake nor the notion of the cognitive approaches about the mistakes resulting from the misconceptions is important is adequate. The necessary knowledge is obtained for the students-centered curriculum to be put into effect effectively which utilizes the notion that it is not necessary to take into consideration the mistakes. It is a necessity to get rid of this notion which accepts that there is not any mistake. The role of the mistake in education should be analyzed carefully. So, the researches go on their studies about in what ways the teachers deal with the students who make mistake.

Some techniques used by the teachers when they come across mistakes are known by the researchers. Among these, there are a lot of teaching techniques such as establishing pattern, forming cognitive paradox, modeling, making association and using analogy and some other un-named techniques. The appearance of the mistake in the learning environment and its being used effectively in this environment, the students, the teacher, the time the mistake is made, whether the mistake is related with the newly learned subject or previous learned ones, where the mistake appeared on notebook or on the blackboard, whether it appears verbally or in written form should be taken into consideration and the researches should be conducted depending on these.

In essence, this study is a result of the obligation of looking at the mistake with another lens. Re-evaluation of the mistakes with the constructivist approach in a higher level can remind you a lot of concepts' gaining different meanings such as measurement-evaluation and their being used in education system. In the re-definition process of the mistake, studies regarding the source, existence, the nature of mistake and its importance in instruction should be conducted.

There is a lot of taxonomy in the classification of the knowledge. For this reason, it is clearly seen that there should also be a classification of the mistakes. It is stated by the researchers that making researches about this situation will be a turning point for the mistakes to be understood.

If the studies mentioned above were conducted, under the light of the data obtained form them, the teachers would be able to realize the mistake and analyze it; the curriculums which would help the teachers to gain adequacy in order to deal with the organization of the learning environment determining the technique and as a result of all these, the level of interference would be able to be defined.

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