An Investigation of Seventh Grade Students' Performances on Conceptual, Procedural and Graphical Problems Regarding Circles

Lütfi İncikabı[1], Abdullah Çağrı Biber[2], Müjdat Takıcak[3] and Semiha Betül Bayam[4]

Abstract: The purpose of this study is to determine seventh grade students' preferences among the procedural, conceptual and graphical questions in the subject of circles, to define their success levels in their preferences, and to compare students' success levels in one question type with their performances in other question types. The methodology adopted during this research was case study. Based on criterion-based purposive sampling strategy, 98 middle school students were selected as the participants. Data were collected through an achievement test consisting of nine questions (three per question type). The results obtained from the study indicated that students mostly preferred graphical question types. Moreover, majority of students could not succeeded high levels in their preferred question types. In addition, the students performed better in graphical question types; however, the failure in procedural question types was remarkable.

Keywords: Multiple representations, middle school students, mathematics education, circles

DOI: 10.16949/turcomat.30388

1. Introduction

Today, with the contribution of communication devices, social life is in a compound structure with different forms of the presentation of the same information. Today, prevailing public use of TV, newspapers, the Internet and other similar social information networks is undeniable (Kekule, 2008; Tairab & Khalaf Al-Naqbi, 2004). This dominancy of technological devices in daily life allows a person can obtain a piece of information from different sources in different form: From one source in verbal form, from another source in numerical form, and from another one in graphical form. Here, it is obvious that the individuals have to own the skills that will help them make sense of different presentations of the same information. The expectation from the school curricula is to grow individuals that can make sense of basic situations/problems that concern the society and can produce solutions and make decisions (NES, 1993). In this context, it can be said that the content of a lesson must be in such a form that will enable the individual to gain skills that will make him/her to compare, associate, integrate and make sense of the information.

Mathematics, like in other numerical lessons such as physics and chemistry, is a discipline that includes/requires different presentations related with a problem situation. For example, the function of the relationship between two variables can be presented to the students as mathematical equations, in graphics or in text form. Therefore, when a

[1]Assoc. Prof. Dr., Kastamonu University, Faculty of Education, Department of Elementary Mathematics Education, lincikabi@kastamonu.edu.tr
[2] Assist. Prof. Dr., Kastamonu University, Faculty of Education, Department of Elementary Mathematics Education, acbiber@kastamonu.edu.tr
[3]Graduate student, Ankara University, Faculty of Languages, History and Geography, Department of Philosopy, mujdattakicak78@gmail.com
[4] Graduate student, Ankara University, Faculty of Languages, History and Geography, Department of Philosopy, sbetulb@gmail.com

mathematical situation is handled in classes, a single view provides a single point of view for the students while different views provide opportunities for them to handle and examine the problem in different forms (Driscoll, 1999; McGowan & Tall, 2001).

Examining a problem situation by using different views gives students the opportunity to gain deeper and flexible understanding of the concepts used (Even, 1998; Hiebert & Carpenter, 1992; Keller & Hirsch 1998; Piez & Voxman, 1997) thus providing an important basis for meaningful learning (Ainsworth, 1999). Hence the multi-representational approach in NCTM standards is examined alone and the necessity for its use is explained. Multi-representation approach standard suggests that students use different representations in organizing, saving and conveying mathematical ideas. Moreover, it enables the selection, application and converting among mathematical representations while solving the problems (NCTM, 2000).

Van de Walle (1990) states that mathematics teaching should aim for students to understand mathematical concepts, mathematical procedures, and connections between conceptual and procedural knowledge. Within this regard, two terms regarding mathematics learning comes forward: Conceptual learning and procedural learning. Although it was hard to determine the distinction between procedural and conceptual learning, it is possible to find learning outcomes being specific for each learning type (Baki, 2008). According to Baki (2008), conceptual learning consists of mathematical concepts and connections among them. Rather than adopting mathematical concepts and procedures from outside (mostly from their teachers), students being accustomed of conceptual learning tries to “reinvent” mathematical facts and concepts by using their existed knowledge in a “constructivist” way. On the other hand, procedural learning origins from two knowledge sources: knowledge of mathematical symbols and procedural knowledge that were being used while making mathematics (Baykul, 2000). Procedural learners perceive mathematics as a set of rules and procedures that are being independent from each other, and believe that success in mathematics usually comes from memorization of these rules and procedures. Learning differences in mathematics also affects question types being offered to the students for measuring and evaluating their mathematical knowledge. Most literature in mathematics education focuses on procedural and conceptual learning and so they utilize procedural and conceptual questions to achieve their stated aims. Results obtained from studies usually pay attention to students’ difficulty in conceptual questions in mathematical concepts including linear algebra (Harel, 1989; Wang, 1989), absolute value (Şandır, Ubuz & Argün, 2002), sets (Incikabi, Tuna, & Biber, 2012), general mathematics (Baki & Kartal, 2004; Soylu & Aydın, 2006).

In addition to discussion about conceptual and procedural understanding, many researchers emphasized that graphical understanding is an important way for students to understand mathematics (Baki, 2008). Graphing skills are one of the critical topics that are important and fundamental to mathematics and to other disciplines (Ates & Stevens, 2003; Clement, 1985; Hadjidemetriou & Williams, 2002; Roth & McGinn, 1998; Widjaja &

Heck, 2003). Among these skills, reading, construction and interpretation have been identified as important. Recent textbooks employ a wide variety of graphs to illustrate scientific knowledge and data. Similarly, national and international standardized exams such as NAEP and TIMSS put an emphasis on reading, construction and interpretation of graphs. Because of its importance, students' deficiencies in graphing have been researched extensively in the literature mostly in science and mathematics (e.g., Berg and Smith, 1994; Bowen, Roth, & McGinn, 1999; Leinhardt, Zaslavsky, & Stein, 1990; Lenton, Stevens, & Illes, 2000). According to literature, graphics or visual representations impact on mathematical performance in two ways. Firstly, some students find various graphics difficult to interpret or decode. Secondly, students' performance is impacted upon by the type of graphic that is used. Baker, Corbett, and Koedinger (2001) reported substantial variation in eighth and ninth graders' ability to interpret informationally equivalent graphics with students' comparative success rates of 95% on a histogram, 56% on a scatterplot, and 17% on a stem-and-leaf plot.

When the test averages of the high school entrance exams are considered, the failure in mathematics is remarkable (MoNE, 2012). Angles in a circle and basic concepts in geometry are difficult subjects for students to understand in mathematics (Özsoy & Kemankaşlı, 2004). These subjects are in the curricula of the secondary schools in geometry lessons and have many different views that enable them to be shown in many fields (e.g. conceptual, procedural and graphical). Within this regard, it will be proper to analyze the academic performances of students in procedural, graphical and conceptual areas in the subject 'circles' to determine the reasons for low success levels.

Under consideration of the findings from the literature, the purpose of this study was threefold. First aim was to determine the seventh grade students' preferences among the procedural, conceptual and graphical questions in the subject of circles. Second one was to define their performances in their preferences, and the third aim was to compare students' success levels in one question type to their success levels in other question types. For this purpose, the questions were determined as:

a) What were the preferences of the seventh grade students among the procedural, conceptual and graphical question types in the subject of circles?

b) How was the distribution of the students' performances in their preferred question types?

c) How were the students' performances in one question type when compared to their performances in other question types?

2. Methodology

The methodology adopted during this research was a case study. The case that is examined within the scope of the study involves the performances of students at conceptual, procedural and graphical mathematics questions regarding the concept of circle.

2.1. Study group

Study was conducted in a middle school that has been regarded as performing at national average on national assessments. A criterion-based purposive sampling strategy was applied in forming of the study group. The criteria for selection were for all students to be in the seventh-grade and to have a score four or five (out of five) in mathematics lesson. The reason of selecting the study group from the students with high grades was to control the deviations that would be caused by unsuccessful students.

The background information on the students was collected via a demographic questionnaire, which included questions about the students' ages, genders, and their mathematics course grades. As part of their assessment for conceptual, graphical and procedural questions in circles, 98 seventh grade students participated in this study. According to the results from the demographics questionnaire, more than half of the students (n=52) were boy while the number of the girls was 46. The students' age had an average of 13.4. Their average mathematics course grade was 4.3 (based on 0-5 scale).

2.2. Data collection tool

In line with the aims, this study used an achievement test in the concept of circles consisted of question sets, which involve conceptual, graphical and procedural questions on the same subject. The achievement test involves 9 questions prepared by using different mathematics school books and question banks. In the achievement test, students were also asked about what kind of questions they preferred (at what kind of questions they felt more successful). The prepared questions were examined and approved by two field experts who studies in the field of mathematics education, in terms of its understandability and practicability. A pilot application of the achievement test conducted with 24 students, and its understandability level was found sufficient. Moreover, item analysis of the achievement test approved the test in terms of their validity. The features of the questions are given in Table 1. The data were collected with an exam-quality application in a single-session process of 90 minutes. The whole set of questions, which were directed towards students on the subject of circles, are given in Appendix A.

Table 1. Distribution of question peers

Question peers	***Topic of the question peers***
1A, 1C, 1G	Concept of Circles
2A, 2C, 2G	Chords of a Circle
3A, 3C, 3G	Angles in Circles

A: Procedural question, C: Conceptual question, G: Graphical question

2.3. Data analysis

The data that were obtained were initially examined in terms of the success and (correct answers) and failure (wrong and no answers) states of the students. During this process, the whole analysis was conducted by the researcher in company with an external expert, who holds a doctoral degree in the field of mathematics education. Then the researcher examined the distribution of preferred question types and students' performance on the question types that they preferred. In the final phase, the achievement level distributions of students, who were successful at a question type, were compared with those at other question types. The achievement levels for each question set (conceptual, procedural and graphical) were divided into three categories: High level, medium level, and low level. A student who provides correct responses for all three questions in a question set were graded as "high" while a student who provided two correct responses for the questions were categorized as "medium." Students who provided one or no correct response for each question set were graded as "low."

3. Findings

Findings obtained from the study will be divided into two sections. The former reveals students' preferences of the question types and their s performance according to their stated preferences, while the latter includes students' state of achievement among question types.

3.1. Students' preferences and their performance by their preferences

The students were asked the question "Which of the conceptual, procedural and graphical question types do you think you solve easily?" Student's responses and their performances in their preferred question types were provided in Table 2. According to the results obtained from the table, graphical questions were the most preferred question type with the percentage of 43 and were followed by the conceptual questions with 35%. Procedural questions, the least preferred question type, were preferred by 22 percentages of the students. Moreover, an analysis of students' performance in their preferred question types yielded that more than half of the students (53%) performed low; while less than one-tenth of the students performed high in their preferred question types.

Table 2. Student success in question types preferred by students

	High Level	***Medium Level***	***Low Level***	***Total***
Conceptual	2	17	15	34 (35)
Procedural	1	6	14	21 (22)
Graphical	5	14	22	41 (43)
Total	8 (8)	37 (39)	51(53)	96* (100)

*2 students did not state views. Note: Percentages were given in parenthesis.

In other words, more than half of the students have lower success levels in the field where they thought they were good at. This shows that the self-efficacy perceptions of the students in mathematics do not reflect the truth. Thinking of being successful in one field and receiving lower marks may cause for developing negative attitudes towards the field, which is mathematics in our case.

When taking Table 2 from another point of view, it becomes clear that most of the students (*f=22*) who preferred graphical question types (*f=41*) have lower success levels at graphical questions and few of them (*f=5*) have higher success levels. It has been determined that the students who have preferred conceptual question types (*f=34*) have medium success levels (*f=17*) or lower success levels (*f=15*). Similarly, majority of the students who preferred procedural questions (*f=14*) had lower success levels at procedural questions.

3.2. Students' performance comparison among question types

Table 3 presents students' overall performance (rather than their preferences) in procedural, conceptual and graphical questions. When all the question types are examined, more than half of the students showed moderate (medium level) performance on conceptual questions, while majority (54%) provided low level of achievement on the procedural questions. In addition, graphical questions were the question types that students' achievement was the highest with 37% of high level success rate. Furthermore, a closer performance distribution across the levels (high, medium, and low) was evident for the graphical questions.

Table 3. Performance distribution of students in different question types

	High Level		***Medium Level***		***Low Level***	
	f	*%*	*f*	*%*	*f*	*%*
Conceptual	14	14	59	60	25	26
Procedural	8	8	37	38	53	54
Graphical	37	38	35	36	26	26

Following tables compares the students performances for each of procedural, conceptual and graphical question types with the other question types. Table 4 associates students' success levels in procedural questions with those in the other question types. Among 53 students who had low level of success in procedural questions, 23 students also had low level success in conceptual questions, while more than half performed better in conceptual questions (23 students at medium level and 7 at high). Similarly, almost all of the students, who have shown medium level success in procedural question types, performed at medium (*f=30*) or high level (*f=6)* in conceptual question types. On the other hand, a majority of the students, who had achieved high success in procedural questions type, demonstrated lower level success (6 students at medium level and 1at low) in conceptual questions. As ageneral thought, one may deduce from above results that students performance in procedural questions were, most of the time, similar or better than their performance in conceptual questions.

Table 4. Comparison of performance levels in procedural question types with other question types

Procedural			***Conceptual***			***Graphical***	
	f	*Low*	*Medium*	*High*	*Low*	*Medium*	*High*
Low	53	23	23	7	21	20	12
Medium	37	1	30	6	4	12	21
High	8	1	6	1	1	3	4

According to table 4, among the low performing students in procedural question type, 21 students had also low level of success while majority of them (*f=32*) demonstrated higher level of success (20 medium and 12 high) in graphical question type. Moreover, more than half of the students, who succeeded in medium level in procedural questions, achieved higher level of success in graphical question type while only a few of them (*f=4*) performed worse in graphical questions. In addition, half of the high achievers (*f=4*) in procedural questions also demonstrated high achievement in graphical representations. One may conclude from the results obtained from the table that students usually had higher performance in the questions in the graphical form than those in procedural form.

Table 5 presents a comparison of students' performance in conceptual question with their performance in other question types. According to the results, students' performances (level) were, most of the time, better in conceptual questions than those in procedural questions while their performances were worse in conceptual questions than graphical questions. One may conclude from the results obtained from the table that students mostly tended to improve their achievement rate in conceptual questions through graphical questions while their achievement in conceptual questions tended to decrease in procedural questions.

Table 5. Comparison of performance levels in conceptual question types with other question types

Conceptual			***Graphical***			***Procedural***	
	f	*Low*	*Medium*	*High*	*Low*	*Medium*	*High*
Low	25	11	9	5	23	1	1
Medium	59	14	21	24	23	30	6
High	14	1	5	8	7	6	1

Table 6 associates the success levels of the students in graphical question types with the success levels in other question types. Among the students with low success level in graphical question type, 11 students demonstrated also low success level while 15 students had a better performance in conceptual questions. Most of the students (*f=21*) who were graded in the medium success level in graphical question types also performed at medium level in conceptual question type while majority high achievers leg behind of their performances in conceptual questions. Although there is no explanation for the achievement state for the intersection between graphical and conceptual question type performance, students' performances were mostly graded in the medium level for the conceptual questions. In addition, low achievers in graphical question type generally presented low performance level in procedural questions while medium and high achievers tended to present lower achievement level in procedural questions. The data obtained show that the students mostly demonstrated lower success level in procedural questions than their success level in graphical questions.

Table 6. Comparison of performance levels in graphical question types with other question types

Graphical		***Conceptual***			***Procedural***		
	f	*Low*	*Medium*	*High*	*Low*	*Medium*	*High*
Low	26	11	14	1	21	4	1
Medium	35	9	21	5	20	12	3
High	37	5	24	8	12	21	4

Table 7 shows the overall results that deduce students' performance comparison in procedural, conceptual and graphical questions. One may conclude from the results obtained from the table that students performance improved from procedural questions to conceptual and graphical questions. Similarly, students' performance in conceptual questions also improved in graphical questions. On the other hand, achievement in conceptual questions could not support achievement in procedural questions since students performed same or worse in procedural questions than conceptual questions. Similarly, achievement in the graphical questions were not positively correlated with the achievement in the procedural questions.

Table 7. Conclusions regarding students' performance comparison among question types

		To		
		Procedural	***Conceptual***	***Graphical***
From	***Procedural***	NA	Stable or improve	Improve
	Conceptual	Decline or stable	NA	Improve
	Graphical	Decline or stable	No pattern	NA

4. Conclusion and Recommendations

The current study aimed to analyze the representation preferences (procedural, conceptual and graphical) of the seventh grade students in the mathematical concept of "circles," their performance in their preferences, and to compare their achievement rates among the question types. The results of this study are limited with the participants and the achievement test that was being conducted.

According to the results of the study, graphical representations were the most preferred question types while procedural representations were the least. Although, the grade level were different, similar found Keller and Hirsch (1998), Dreyfus and Eisenberg (1982) who also studied with pupils with high ability. Among the several factors influencing students' preferences for representations are the nature of students' experiences with each representation, students' perceptions of acceptability of using a representation, and the level of the task. According to Eisenberg (1991) language of the task also affects students' preferences for representations in mathematics. Students are tended to neglect the tasks that are in formal language (symbols, equations etc. in mathematics).

This study also indicated that most of the students could not succeed in their preferred question types. The mathematical content, which is circles in our case, can be the reason for the students' bad performance in their preferred questions, since literature (e.g. Özsoy & Kemankaşlı, 2004) provides evidences for students' difficulties (mistakes and misconceptions) in the concept of circles. In addition, students' failure in their own preferences may impede developing positive attitude towards mathematics which is among the main goals of mathematics education that was stated in the official mathematics teaching program in Turkey (MoNE, 2012). A detailed and qualitative analysis of students' preferences and performances in their preferences would be beneficiary to overcome this impediment and to support/reinforce the results obtained in the current study.

The results of the current study also revealed that students' performance varied in different types of questions. Graphical questions received the highest level of success and followed by conceptual and procedural questions, respectively. This situation contradicts with the findings of Mack (1995), Baki and Kartal (2002), Moseley (2005), Yang, Li and Lin (2008) suggesting that the students in their studies are mostly successful in the questions requiring procedural knowledge. Although this study tried to distinct students'conceptual and procedural knowledge regarding circles through an achievement test, literature discusses the existence of the border line between them. Baki (1998) states that there is no strict and thick line that divides the procedural and conceptual knowledge, while Hiebert (2013) provides a distinction between procedural and conceptual knowledge "the primary relationship in procedural knowledge is "after," which is used to sequence subprocedures and superprocedures linearly. In contrast, conceptual knowledge is saturated with relationship of many kinds." (p. 8).

In addition, the current study also indicated that students' achievement in one question type was also related to their achievement in other question types. This situation also effect students' transfer knowledge form one representation type to the others. Lesh, Behr, and Post (1987a) emphasize that transiting among different types of representations are as much curial as using them in their own right. There is extensive research in mathematics education showing a positive relationship between transition ability and mathematical learning (and problem solving) (Cifarelli, 1998; Gagatsis, Michaelidou, & Shiakalli, 2000; Janvier, 1987; Kaput, 1985, Lesh, Behr, & Post, 1987b; NCTM, 2000; Owens & Clements, 1998). In order to detect students' learning deficiencies, teachers can generate a variety of useful questions by presenting an idea in one representational mode and asking the student to illustrate, describe, or represent the same idea in another mode. Moreover, the teacher education programs in colleges or universities should provide for teacher candidates a specific training on different representations and their role, features, difficulties and value in the process of teaching and learning and include courses that provide teacher candidates with the opportunities to make them use multiple representations. In addition, curriculum and curriculum sources (including textbooks) should also require uses of different type of questions and force students to compose, translate, transit among different type of representations.

References

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction, 16*, 183-198.

Ates, S., & Stevens, J. T. (2003). Teaching line graphs to tenth grade students having different cognitive developmental levels by using two different instructional modules. *Research in Science & Technological Education*, *21*(1), 55-66

Baker, R. S., Corbett, A. T., Koedinger, K. R. (2001). Toward a model of learning data representations. In J. D. Moore and K. Stenning (Eds), *Proceedings of the 23rd Annual Conference of the Cognitive Science Society* (pp. 45–50). Mahwah, NJ: Erlbaum.

Baki, A. (1998). *Balancing the conceptual and Procedural knowledge in teaching mathematics*. Declaration presented to the Mathematics Symposium at Atatürk University 40. Anniversary of Establishment.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi.* Ankara: Harf Eğitim Yayıncılığı.

Baki, A., & Kartal, T. (2004). *Assessing the algebra knowledge of high-school students in terms of conceptual and Procedural knowledge*. V. National Science and Mathematics Education Congress Declaration Book. Ankara: State Books Management Publishing House, 2002.

Baykul, Y. (2000). *İlköğretimde matematik öğretimi: 1-5. sınıflar için*. Pegem A. Yayıncılık.

Berg, C. A., & Smith, P. (1994). Assessing students' abilities to construct and interpret line graphs: Disparities between multiple-choice and free-response instruments. *Science Education, 78*(6), 527-554.

Bowen, G. M., Roth, W. M., & McGinn, M. K. (1999). Interpretations of graphs by university biology students and practicing scientists: Toward a social practice view of scientific representation practices. *Journal of Research in Science Teaching*, *36*(9), 1020-1043.

Cifarelli, V. (1998). The development of mental representations as a problem solving activity. *The Journal of Mathematical Behavior, 17*(2), 238-264.

Clement, J. (1985, July). Misconceptions in graphing. *Proceedings of the Ninth International Conference for the Psychology of Mathematics Education* (Vol. 1, pp. 369-375). Utrecht,, The Netherlands: Utrecht University.

Coştu, B. (2007). Comparison of students' performance on procedural, conceptual and graphical chemistry gas problems. *Journal of Science Education and Technology, 16*(5), 379-386.

Coştu, B. (2010). Procedural, conceptual and graphical chemistry problems. *Asian Journal of Chemistry*, *22*, 8 (11), 6013-6025.

Dreyfus, T., & Eisenberg, T. (1982). Intuitive functional concepts: A baseline study on intuitions. *Journal for Research in Mathematics Education*, 13(5), 360-380.

Driscoll, M. (1999). *Fostering algebraic thinking: A guide for teachers, grades 6-10*. Portsmouth, NH: Heinemann.

Douady, R. (1986). Jeux de cadres et dialectique outil/objet. *Recherches en Didactique des Mathématiques, 7*(2), 5-32.

Eisenberg, T. (1991). Functions and associated learning difficulties. *Advanced mathematical thinking* (pp. 140-152). Springer Netherlands.

Erkan Erkoç, N., & Coştu, B. (2011, July). Comparison of chemistry teacher candidates in terms of conceptual, Procedural and graphical success at classroom level. *II. National Chemistry Education Congress.* Ataturk University, Erzurum.

Even, R. (1998). Factors involved in linking representations of functions. *Journal of Mathematical Behavior*, *17*(1), 105-121.

Gagatsis, A., Michaelidou, E., & Shiakalli, M. (2000). *Functions: A game of changing fields of representation*. Nicosia: Erasmus IP.

Hadjidemetriou, C., & Williams, J. (2002). Children's graphical conceptions. *Research in Mathematics Education, 4*(1), 69-87.

Harel, G. (1989). Learning and teaching linear algebra: Difficulties and an alternative approach to visualizing concepts and processes. *Focus on Learning Problems in Mathematics*, *11*, 139-48.

Hiebert, J. (Ed.). (2013). *Conceptual and procedural knowledge: The case of mathematics*. Routledge.

Hiebert J., & Carpenter T. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Janvier, C. (1987). Translation processes in mathematics education. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 27-32). Hillsdale, NJ: Lawrence Erlbaum.

Incikabı, L., Tuna, A., & Biber, A. Ç. (2012). İlköğretim matematik öğretmen adaylarının kümelerle ilgili kavramsal bilgilerinin analizi. *Erzincan Üniversitesi Eğitim Fakültesi Dergisi*, *14*(2), 523-538.

Kaput, J. J. (1985). Representation and problem solving: Methodological issues related to modeling. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 381-398). Hillsdale, NJ: Lawrence Erlbaum.

Kaput, J. J. (1987). Toward a theory of symbol use in mathematics. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 159-195). Hillsdale, NJ: Lawrence Erlbaum.

Keller, B. A., & Hirsch, C. R., (1998). Student preferences for representations of functions. *International Journal of Mathematical Education in Science and Technology, 29*(1), 1-17.

Kekule, M. (2008). Graphs in physics education. *GIREP 2008 Conference physics curriculum design, development and validation*, Nicosia, Cyprus.

Leinhardt, G., Zaslavsky, O., & Stein, M. K. (1990). Functions, graphs, and graphing: Tasks, learning, and teaching. *Review of educational research, 60*(1), 1-64.

Lenton, G., Stevens, B., & Illes, R. (2000). Numeracy in science: Pupils' understanding of graphs. *School science review, 82*(299), 15-23.

Lesh, R., Post, T., & Behr, M. (1987a). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Lesh, R., Behr, M., & Post, T. (1987b). Rational number relations and proportions. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 41-58). Hillsdale, NJ: Lawrence Erlbaum.

Mack, N. (1995). Confounding whole-number and fraction concept when building on informal knowledge. *Journal for Research Mathematics Education, 26*(5), 422-441.

McGowan, M., & Tall, D. (2001). *Flexible thinking, consistency, and stability of responses:A study of divergence*. [Online]: Retrieved on 7-February 2005, at URL: http://www.warwick.ac.uk/staff/David.Tall/drafts/dot2001-mcgowen-tall-draft.pdf.

MoNE, (2012). *İlkögretim matematik dersi (6-8 sınıflar) öğretim programı* [Elementary school mathematics teaching program (grades 6-8)]. Ankara, Turkey.

Moseley, B. (2005). Students' early mathematical representation knowledge: The effects of emphasizing single or multiple perspectives of the rational number domain in problem solving. *Educational Studies in Mathematics, 60*, 37–69.

NES (1993), National education standards: *Observe, interact, change, learn*. National Academic Press, Washington, DC.

NCTM, (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Owens, K. D., & Clements M. A. (1998). Representations in spatial problem solving in the classroom. *The Journal of Mathematical Behavior, 17*, 197-218.

Özsoy,N., & Kemankaşlı, N. (2004). Basic mistakes and conceptual errors of secondary school students about circles. *Turkish Online Journal of Educational Technology, 3*(4), 140-147.

Piez, C. M., & Voxman, M.H. (1997). Multiple representations-using different perspectives to form a clearer picture. *Mathematics Teachers, 90*(2), 164-166.

Roth, W. M., & McGinn, M. K. (1998). Inscriptions: A social practice approach to "representations." *Review of Educational Research, 68*(1), 35-59.

Soylu, Y, & Aydın, S., (2006). Matematik derslerinde kavramsal ve işlemsel öğrenmenin dengelenmesinin önemi üzerine bir çalişma. *Erzincan Eğitim Fakültesi Dergisi, 8*(2), 83-95.

Şandır, H., Ubuz, B. ve Argün, Z., (2004). Ortaöğretim 9. sınıf öğrencilerinin mutlak değer kavramındaki öğrenme hatalari ve kavram yanilgılari. *V. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Bildiriler Kitabı,* (Cilt: II, ss: 1107-1112). Ankara: Devlet Kitapları Müdürlüğü Basımevi.

Tairab, H. H., & Khalaf Al-Naqbi, A. K. (2004). How do secondary school science students interpret and construct scientific graphs? *Journal of Biological Education, 38*(3), 127-132.

Van de Walle, J. A. (1990). *Elementary school mathematics, teaching developmentally*. Addison-Wesley/Longman, Route 128, Reading, MA 01867.

Wang, T. W. (1989). A Course on applied linear algebra. *Chemical Engineering Education, 23*(4), 236-41.

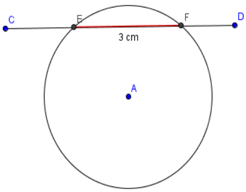
Widjaja, Y. B., & Heck, A. (2003). How a realistic mathematics education approach and microcomputer-based laboratory worked in lessons on graphing at an Indonesian junior high school. *Journal of science and mathematics Education in Southeast Asia, 26*(2), 1-51.

Yang, D. C., Li, M. N., & Lin, C.I. A study of the performance of 5th graders in number sense and its relationship to achievement in mathematics. *International Journal of Science and Mathematics Education, 6*(4), 789-807.

Appendix A Achievement Test

Procedural Questions

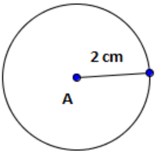
1)

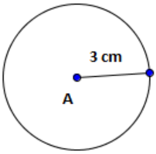


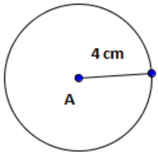
In the figure, the circle with the center A and Radius 4 cm intersects with a line segment CD which has a sufficiently big movement. When the CD line segment is moved down on to the point A, what is the length of the EF chord?

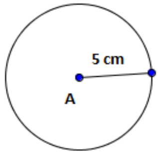
A) 3 cm B) 4 cm C) 6 cm D) 8 cm

2) Which of the following is true for a circle model with 4 cm diameter and the center A?

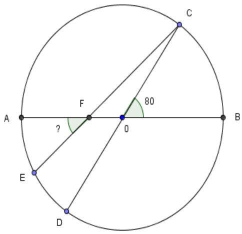
A)


B)


C)


D)


3)

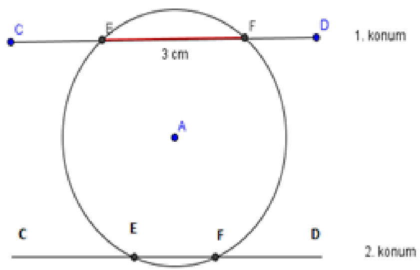


In the circle with the center O, if the [AB] and [CD] are the diameter, and measure of AE, ED arcs are equal and $s(\overset{\frown}{COB})=80$, what is the degree of $s(\overset{\frown}{AFE})$?

A) 40 B) 60 C) 80 D) 120

Conceptual Questions

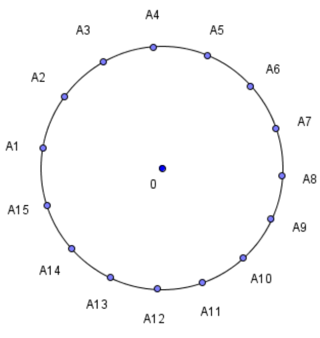
1) In the figure, the circle with the center A with the radius 4 cm intersects with the CD line segment with sufficiently big movement. As the CD line segment is moved from the 1st Location to the 2nd Location, how does the length of the EF chord change?



A. Increases continually
B. Firstly increases, then decreases
C. Firstly decreases, then increases
D. Decreases continually

2) A sheep is lashed with a piece of string, and then this string is fastened to the smooth ground in the garden. Since the string can move along all the directions in the point where it is fastened, draw the geometrical shape that shows the widest area in which the sheep can move in the garden.

3) In the figure the 15 points are marked on the circle O as A_1, A_2, A_3, … A_{13}, A_{14}, A_{15} which are equally far from each other. Which of the following acute (minor) angles have the biggest size?

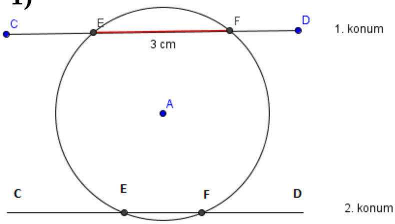


A) $s(A_1 \widehat{O} A_3)$ B) $s(A_2 \widehat{O} A_6)$

C) $s(A_3 \widehat{A_9} A_8)$ D) $s(A_8 \widehat{A_4} A_{14})$

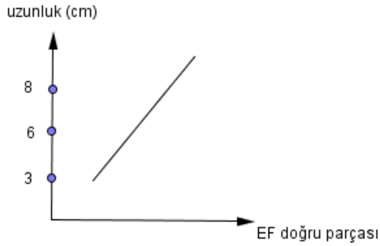
Graphical Questions

1)

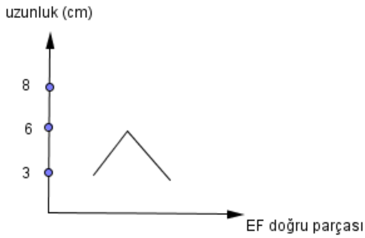


In the figure above, the circle with the center A with the radius 4 cm intersects with the CD line segment with sufficiently big movement. As the CD line segment is moved from the 1st Location to the 2nd Location, which one of the following graphics show the change in the length of the EF chord?

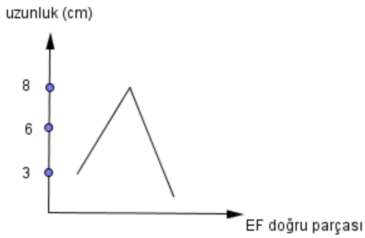
A)



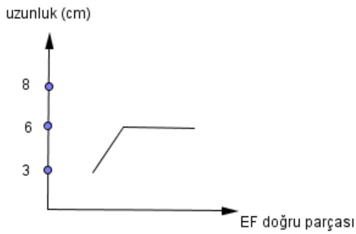
B)



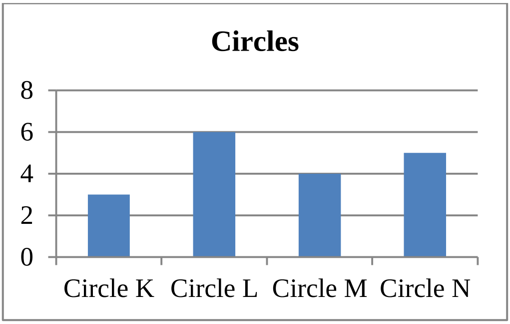
C)



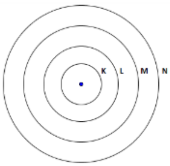
D)

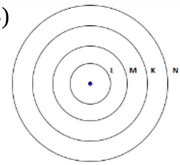


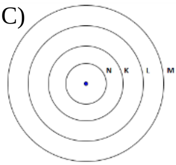
2)

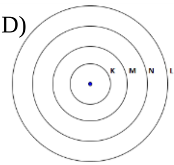


In the graphic, the lengths of the radii of 4 different circles are given. If these circles are drawn on one single center, which one of the following models may be true?

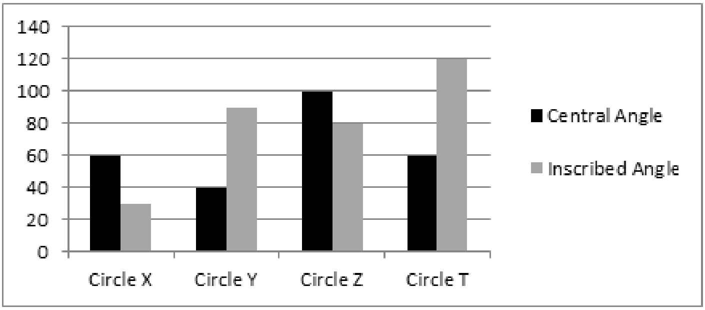
A) 

B) 

C) 

D) 

3)



In the graphic above, the values are given for the center and inscribed angles which see the same arc belonging to 4 different circles. Which of the X, Y, Z and T circles can be drawn according to the given information?

A) X B) Y C) Z D) T

Citation Information

İncikabı, L., Biber, A. Ç., Takıcak, M., & Bayam, S. B. (2015). An investigation of seventh grade students' performances on conceptual, procedural and graphical problems regarding circles. *Turkish Journal of Computer and Mathematics Education, 6*(1), 1-17.