Sigma J Eng & Nat Sci 38 (4), 2020, 2057-2067

Sigma Journal of Engineering and Natural Sciences
Sigma Mühendislik ve Fen Bilimleri Dergisi

Research Article

ANALYSIS OF LOCAL THERMAL EQUILIBRIUM ASSUMPTION IN TRANSIENT FORCED CONVECTION IN A GRAPHITE FOAM CHANNEL

Gürşah GÜRÜF[1], İsmail SOLMAZ*[2], Özgür BAYER[3]

[1]*Department of Mechanical Engineering, Karabük University, KARABÜK;* ORCID: 0000-0003-3602-8710
[2]*Department of Mechanical Engineering, Atatürk University, ERZURUM;* ORCID: 0000-0002-3020-4798
[3]*Dept. of Mechanical Eng., Middle East Technical University, ANKARA;* ORCID: 0000-0003-0508-2263

Received: 09.06.2020 Revised: 20.08.2020 Accepted: 02.12.2020

ABSTRACT

In this study, the validity of Local Thermal Equilibrium (LTE) assumption in the transient forced convection of a rectangular channel filled with a block of graphite foam is examined numerically. The governing macroscopic energy conservation equations for solid and gas phases are derived by taking the average of the microscopic one over the averaging volume. Initially, LTE is in existence between the phases and then, the fluid temperature at the channel inlet is suddenly raised. Besides, an appropriate insulation is provided for the wall of the channel. Hence, a transient one-dimensional Local Thermal Non-Equilibrium (LTNE) model is considered in the numerical investigation. Thermo-physical properties of the solid and fluid phases are presumed to be constant. The graphite foam porosity is spatially uniform and constant. The impact of two dimensionless variables such as fluid to solid Nusselt number (Nu_{fs}) and Reynolds number (Re) on the LTE assumption is extensively investigated. . It was found that the dimensionless time required to attain LTE between the phases (τ_{LTE}) increases with the increasing value of *Reynolds* number. However, the real-time (σ_{LTE}) corresponding to τ_{LTE} was found to be nearly 4 sec over the range of Re numbers studied. Additionally, an increase in the Nu_{fs} resulted in a decrease in τ_{LTE} for a constant value of Re number and σ_{LTE} varied from 1.5 to 5 sec. As a result, the obtained findings showed that it is reasonable to assume the LTE between the phases under the investigated conditions.

Keywords: Foam, forced convection, graphite, LTE, LTNE, porous media, transient.

1. INTRODUCTION

Overheating is one of the most critical criteria for the manufacturer of electronic components. Concerning the fact that in the components, the complete elimination of the dissipated heat is not possible by employing heat-sinks that utilize air-cooled metal fins, as the traditional cooling method, the graphite foam-based heat-sink has been introduced to solve this issue thanks to its high surface area to volume ratio, high thermal conductivity value, low density, and strong flow mixing potentials [1].

A two-phase porous medium is typically composed of solid and gas phases and the temperature distributions of those inside the computational domain are commonly assumed to be identical. In this situation, temperature fields of either phase are well demonstrated by a single

* Corresponding Author: e-mail: solmus@atauni.edu.tr, tel: (442) 231 48 45

energy equation and this kind of approach is identified as Local Thermal Equilibrium (LTE) assumption. However, in most cases, the application of the LTE assumption contribute to insubstantial consequence of the simulation unless its validity is shown over the parameters investigated. If the LTE approach is not applicable, different energy conservation equations for solid and gas phases are needed for the sake of the calculation of the phases' separate temperature fields in question. This approach is known as the Local Thermal Non-Equilibrium (LTNE) model. Several investigations have been carried out for the verification of the LTE supposition in forced convection flow in the porous channel. Abu-Hıjleh et al. [2] reported that the LTE supposition is valid in channels in which Nusselt number is significantly high and the thermal diffusivity ratio and Peclet number provide large values as well. On the other hand, the non-dimensional channel length along with the thermal capacity of solid to fluid brings low amounts where the LTE assumption is verified in the channels. Al-Nımr et al. [3] found that four dimensionless parameters such as porosity, volumetric Biot number, non-dimensional channel length, and solid to the fluid total thermal caspacity ratio control the local thermal equilibrium assumption. However, in this study, axial conduction in the domains of either fluid or solid is not considered. Rees et al. [4] investigated the impact of LTNE on the infiltration of hot fluid into a cold porous medium. They reported that LTE is always attained as time goes to infinite. Kim et al. [5] conducted a numerical analysis to describe the temperature distributions in the microchannel heat sink by using both one-equation and two-equation models for heat transfer. The obtained findings showed that an increase in the effective thermal conductivity ratio or a decrease in the Darcy number approaches the fluid temperature to the solid temperature, in which case the LTE assumption and the corresponding one-equation model is applicable. Kim and Jang [6] carried out a numerical study to present a general criterion for local thermal equilibrium in terms of parameters of engineering importance such as Darcy number, Prandtl number, and Reynolds number. The authors pointed out that as $Pr_{eff,f}Re_{dp}Da^{1/2}\varepsilon Nu \ll 1$ criterion is satisfied, the LTE in a porous medium including convective and conductive heat transfer can be assumed. Khashani and Al-Nımr [7] performed a numerical study to assess the validity of the local thermal equilibrium assumption in the non-Newtonian forced convection flow through channels filled with porous media. Their findings revealed that LTE between the phases vanishes for the circumstances of a higher modified Peclet number, a lower modified Biot number, a lower fluid-to-solid thermal conductivity ratio, a lower power-law fluid index, and a lower microscopic and macroscopic frictional flow resistance coefficients. Haddad et al. [8] analytically investigated the validity of the LTE assumption in natural convection over a vertical flat plate embedded in porous medium. The authors reported that volumetric Biot number, modified Rayleigh number, modified Darcy number and the ratio of effective to dynamic viscosity manage the LTE assumption. They also proposed two correlation equations to determine the region where the LTE assumption is applicable. Zhang and Liu [9] presented a general criterion for LTE in forced convection flows through porous media under a constant heat flux boundary condition as function of boundary heat flux, the area of cross section, effective thermal conductivity ratio, characteristic pore length, porosity, Nusselt number, fluid conductivity and the heat source of solid phase. The presented criterion was also validated by the existing experimental and numerical results for convection heat transfer in a porous medium.

The conditions, in which the LTE assumption is valid in porous media, have been studied extensively by several researchers up to now and some of them are summarized above. Graphite foam heat sink is a promising alternative for the thermal management of electronic components due to its unique thermal and physical properties as it is compared with the conventional heat sinks [11,12]. This study, unlike in the literature, attempts to provide a numerical investigation toward the impacts of fluid-to-solid Nusselt number (Nu_{fs}) and Reynolds (Re) number on the verification of the LTE supposition in transient forced convection in a rectangular channel filled with a block of graphite foam.

2. MATHEMATICAL FORMULATION

A schematic view of the physical problem is presented in Fig. 1. A block of graphite foam heat sink is inserted in a rectangular channel. Table 1 represents the graphite foam's thermal and physical features which are taken into account in the present study. Air enters the channel with uniform velocity and temperature and flows through the block of foam. Initially, LTE is in existence between the solid graphite and fluid air phases and then, the fluid temperature at the channel inlet is suddenly raised to the desired value T_i. In addition to that, the channel walls are well insulated and hence, a transient one-dimensional LTNE approach is employed in the numerical study.

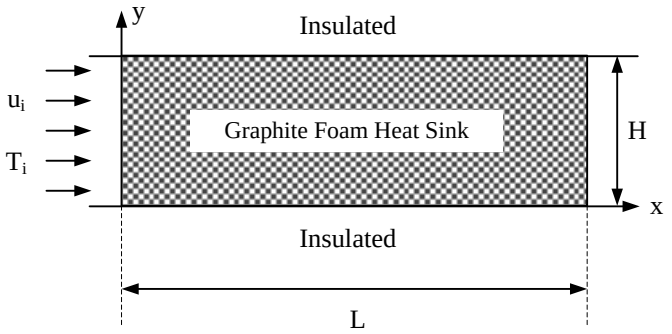

Figure 1. A schematic view of the problem.

Table 1. Thermal and physical properties of POCO graphite foam [10]

ε	k_s (W/mK)	C_p (kJ/kgK)	ρ (kg/m^3)	d_p (mm)	K (m^2)	C_F
0.728	1100	0.7	550	0.31	7.9243e-10	0.071

The porosity (ε), permeability (K) and pore dimameter (d_p) of the graphite foam material are defined as the fraction of the volume of voids over the total material volume, measure of the resistance of the material to the fluid flow and the average diameter of each of the voids in the material, respectively.

The following items are considered for providing more simplicity in the mathematical model:

- The foam porosity is spatially uniform and constant.
- The velocity and temperature of air at the channel inlet are assumed to be uniform.
- The fluid is incompressible.
- Thermo-physical properties of the solid and fluid phases are the constant.
- Radiative heat transfer and buoyancy forces are neglected [11-20].

2.1. Governing Equations

Energy conservation equation for the solid phase is as follows:

$$(1-\varepsilon)\left(\rho c_p\right)_s \frac{\partial T_s}{\partial t} = h_v\left(T_f - T_s\right) + k_{se}\frac{\partial^2 T_s}{\partial x^2} \tag{1}$$

where, ε, ρ, c_p, T_s, h_v, T_f and k_{se} are porosity, density, specific heat at constant pressure, solid temperature, heat transfer coefficient between solid and fluid phases, fluid temperature and solid effective thermal conductivity respectively.

Energy conservation equation for the gas phase is as below:

$$\varepsilon\left(\rho c_p\right)_f \frac{\partial T_f}{\partial t} + \left(\rho c_p\right)_f u_D \frac{\partial T_f}{\partial x} = h_v\left(T_s - T_f\right) + \left(k_{fe} + k_d\right)\frac{\partial^2 T_f}{\partial x^2} \tag{2}$$

where, u_D, k_{fe} and k_d *are* darcy veclocity, fluid effective thermal conductivity and thermal dispersion coefficient respectively.

Solmuş [11] has shown that the flow through the block graphite foam material placed in a channel can be modeled as a slug flow ($U(u_D/u_i)\approx 1$). This result is consistent with the results obtained from Xu et al. [21], Feng et al. [22], Lua et al. [23], and Yang et al. [24] studies. Therefore, the Darcy velocity in the Eq. (2) can be roughly identical to the flow velocity of the channel inlet [25-27].

$$u_D = u_i \tag{3}$$

The initial and boundary conditions which are necessary for solving Eq. (1) and (2) are given below:

$$T_s(0,x) = T_f(0,x) = T_a \tag{4}$$

$$T_s(t,0) = T_i \tag{5}$$

$$\frac{\partial T_s}{\partial x}(t,L) = 0 \tag{6}$$

$$T_f(t,0) = T_i \tag{7}$$

$$\frac{\partial T_f}{\partial x}(t,L) = 0 \tag{8}$$

The afore-mentioned equations make dimensionless by using the following non-dimensional parameters.

$$\theta = \frac{T - T_a}{T_i - T_a} \tag{9}$$

$$X = \frac{x}{L} \tag{10}$$

$$\tau = \frac{t}{L/u_i} \tag{11}$$

$$C_1 = \frac{\left(\rho c_p\right)_f}{(1-\varepsilon)\left(\rho c_p\right)_s} \tag{12}$$

$$C_2 = \frac{\left(\rho c_p\right)_f k_{se}}{(1-\varepsilon)\left(\rho c_p\right)_s k_f} \tag{13}$$

$$C_3 = \frac{k_{fe} + k_d}{\varepsilon k_f} \tag{14}$$

$$C_4 = \frac{k_{fe}}{k_{se}} \tag{15}$$

$$Pr = \frac{\mu\left(c_p\right)_f}{k_f} \tag{16}$$

$$Re = \frac{\rho_f u_i L}{\mu} \tag{17}$$

$$Nu_{fs} = \frac{h_v L^2}{k_f} \tag{18}$$

Non-dimensional energy conservation equation for the solid phase is as follows:

$$\frac{\partial \theta_s}{\partial \tau} = \frac{C_1 Nu_{fs}}{RePr}\left(\theta_f - \theta_s\right) + \frac{C_2}{RePr}\frac{\partial^2 \theta_s}{\partial X^2} \tag{19}$$

Non-dimensional energy conservation equation for the solid phase could be calculated as below:

$$\frac{\partial \theta_f}{\partial \tau} + \frac{1}{\varepsilon}\frac{\partial \theta_f}{\partial X} = \frac{Nu_{fs}}{\varepsilon RePr}\left(\theta_s - \theta_f\right) + \frac{C_3}{RePr}\frac{\partial^2 \theta_f}{\partial X^2} \tag{20}$$

The dimensionless initial and boundary conditions are as follows:

$$\theta_s(0, X) = \theta_f(0, X) = 0 \tag{21}$$

$$\theta_s(\tau, 0) = \theta_i \tag{22}$$

$$\frac{\partial \theta_s}{\partial X}(\tau, 1) = 0 \tag{23}$$

$$\theta_f(\tau, 0) = \theta_i \tag{24}$$

$$\frac{\partial \theta_f}{\partial X}(\tau, 1) = 0 \tag{25}$$

The thermal dispersion coefficient in Eq. (2) is neglected since the difference between the thermal conductivity coefficients of the phases is significant [28].

The effective thermal conductivity of the carbon foam material is calculated by the following equation reported by Tee et al. [29], assuming that the thermal conductivities of the structure in every direction is similar.

$$k_{se} = k_f(1-t)^2 + k_s t^2 + \frac{2k_s k_f t(1-t)}{k_f t + k_s(1-t)} \tag{26}$$

where, *t* is the normalized thickness of the graphite foam.

$$t = 0.5 + cos\left(\frac{cos^{-1}(2\varepsilon - 1)}{3} + \frac{4\pi}{3}\right) \tag{27}$$

2.2 Numerical Solution

The system of coupled non-dimensional PDEs under consideration is solved by using the numerical finite difference technique. A uniform grid arrangement is employed. The second-order accurate central differencing, first-order upwind scheme, and the implicit Crank-Nicolson method are used for the discretization of the second-order spatial derivatives, convective term, and time derivative, respectively. The resulting set of linear algebraic equations is solved by a block tridiagonal matrix solver algorithm (Thomas algorithm). The presented physical problem is simulated in MATLAB environment with the help of the described mathematical model and numerical procedure above. To have trusted results, mesh independence and time step studies are done. It is clear from the Fig.2 that a uniform grid size of 20 and a dimensionless time step 0.003 is adequate to have precise numerical results.

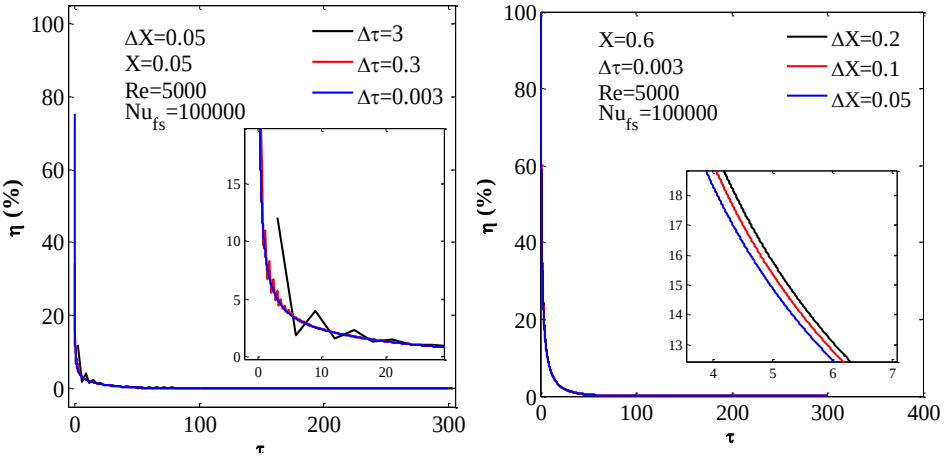

Figure 2. The impact of grid size and time step on the numerical results.

3. RESULTS AND DISCUSSION

The simulation results are given as a percentage deviation of the non-dimension solid-phase temperature from the non-dimension gas-phase temperature. The percentage deviation (*η)* is calculated as below:

$$\eta = \left| \frac{(\theta_f - \theta_s)}{\theta_f} \right| * 100 \tag{28}$$

In simulations, the length of the porous material is kept constant at 0.05 m. In the case where η is less than about 1%, it is figured out that the LTE assumption between the phases is valid. The dimensionless time required to attain LTE between the phases is labeled as τ_{LTE} and the real-time (σ_{LTE}) corresponding to τ_{LTE} is equal to $\tau_{LTE}*(L/u_i)$. The impact of Reynolds number on the variation of η with τ at different axial locations is shown in Fig. 3 for a specified value of Nu_{fs}. It is obvious from this figure that τ_{LTE} is prolonged with the increasing value of the Reynolds number. However, σ_{LTE} for each Reynolds value is found to be about 4 sec. To justify this matter it is better to mention the increase of τ_{LTE} and u_i with increasing value of *Re* . For this reason, σ_{LTE} remains almost constant in the considered *Re* interval . In other words, the effect of the *Re* number in the examined range on the LTE assumption is negligible for graphite foam. On the other hand, for small values of *Re* number, the change of the η with time especially in the region outside the inlet section of the graphite foam is almost the same.

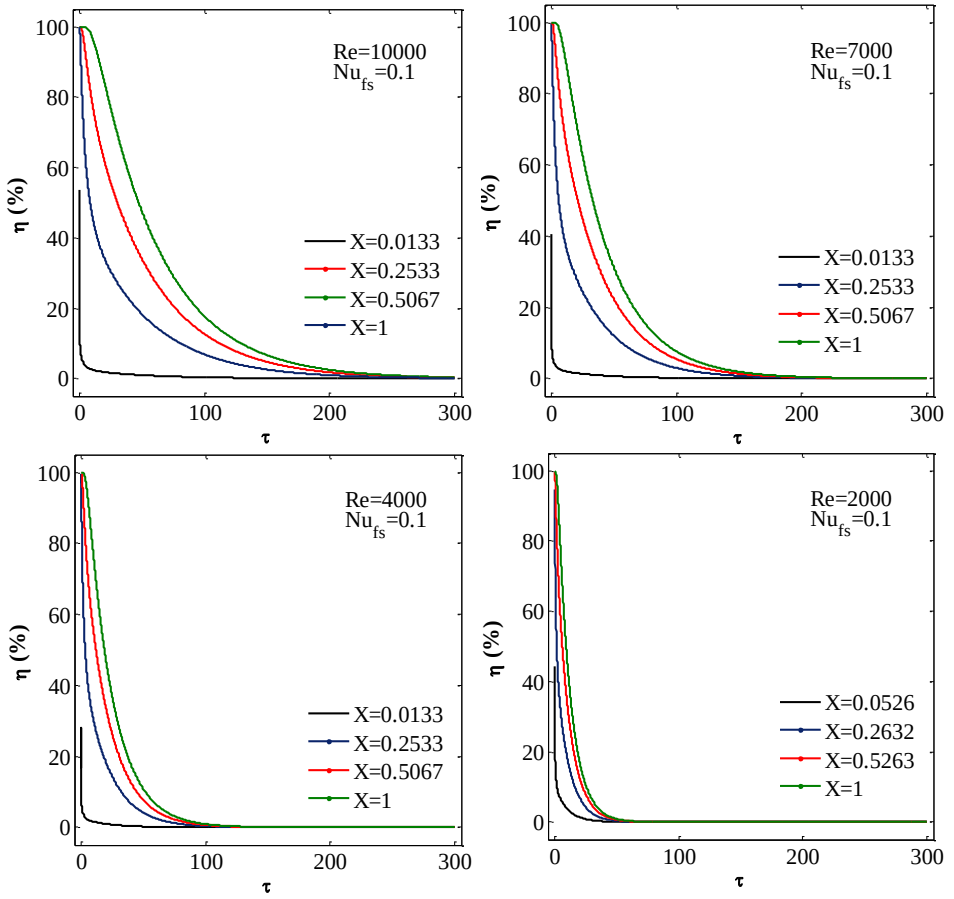

Figure 3. The variation of η with τ as a function of Re number.

Fig. 4 shows the impact of Nu_{fs} on the variation of η with τ at different axial locations for a constant value of Reynolds number. In the simulations, Nu_{fs} is assumed to be constant and uniform throughout the computational domain. It is obvious from Fig. 4 that growth in Nu_{fs} contributes to a decrease in τ_{LTE}. The justification behind this is that the heat transfer between the phases is positively affected by the increase in Nu_{fs} and as a result, τ_{LTE} value is shortened. The σ_{LTE} value ranges from 1.5 to 5 seconds under the conditions investigated. The rate of decrease in σ_{LTE} increases with the increasing value of Nu_{fs} until a certain point. After that point, the rate of decrease in σ_{LTE} starts to reduce. Generally, the LTE assumption between the phases becomes valid after a few seconds, and the effect of Nu_{fs} and Re numbers on σ_{LTE} is negligible referring to the high values of the diffusivity ratio of solid to fluid phase.

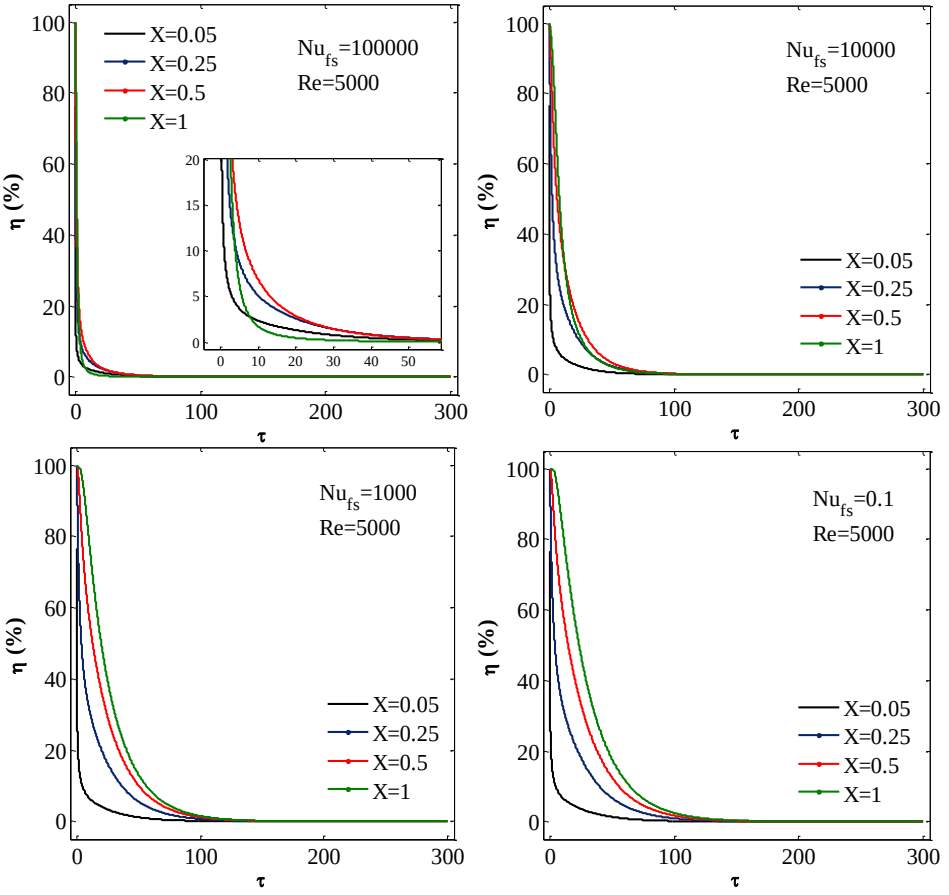

Figure 4. The variation of η with τ as a function of Nu_{fs} number.

4. CONCLUSIONS

A numerical analysis is carried out to study the impact of Nu_{fs} and Re on the conditions under which the LTE assumption is verified in the transient forced convection in a rectangular channel filled with a block of graphite foam. Generally, the presumption of the thermal equilibrium between the graphite foam and air (solid and fluid phase) is valid a few seconds after the start and

Nu_{fs} and *Re* have an insignificant effect on the σ_{LTE} referring to the high value of diffusivity ratio of solid to fluid phase. Consequently, it is found that it is reasonable to assume the LTE between the phases under the investigated conditions and thus, the mathematical model under the single energy equation could be appropriately employed to represent the temperature distribution for either phases in the computational domain.

Acknowledgements

The authors would like to thank the Scientific & Technological Research Council of Turkey (TÜBİTAK) for financially supporting this research under Project No. 315M136.

Nomenclature

C_F	inertial coefficient
c_p	specific heat at constant pressure, J/kg^{o}C
d_p	pore diameter, m
H	height of the porous insert, m
h_v	heat transfer coefficient between solid and fluid phases, W/m^{3}K
k	thermal conductivity, W/mK
K	permeability, m^2
L	length of the porous insert, m
Nu_{fs}	fluid-to-solid Nusselt number, $h_v L^2/k_f$
Pr	Prandtl number, υ/α
Re	Reynolds number, $\rho_f u_i L/\mu$
T	temperature, ^{o}C
t	time, s
u	velocity component in the x-direction, m/s
u_D	darcy velocity, m/s
X	dimensionless cartesian coordinate
x,y	cartesian coordinates, m

Greek Letters

θ	dimensionless temperature
τ	dimensionless time
ε	porosity
ρ	density, kg/m^3

Subscripts

a	ambient
f	fluid
fe	fluid effective
s	solid
se	solid effective
i	inlet

REFERENCES

[1] Lin W., Sunden B., Yuan J., A performance analysis of porous graphite foam heat exchangers in vehicles, *Applied Thermal Engineering*, 50, 2013, pp.1201-1210.

[2] Abu-Hıjleh B.A., Al-Nımr M.A., Hader M.A., Thermal equilibrium in transient forced convection porous channel flow, *Transport in Porous Media*, 57, 2004, pp.49-58.

[3] Al-Nımr M.A., Abu-Hıjleh B.A., Validation of thermal equilibrium assumption in transient forced convection flow in porous channel, *Transport in Porous Media*, 49, 2002, pp.127-138.

[4] Rees D.A.S., Bassom A.P., Sıddheshwar P.G., Local thermal non-equilibrium effects arising from the injection of a hot fluid into a porous medium, *Journal of Fluid Mechanics*, 594, 2008, pp.379-398.

[5] Kim S.J., Kim D., Lee D.Y., On the local thermal equilibrium in microchannel heat sinks, *International Journal of Heat and Mass Transfer*, 43, 2000, pp.1735-1748.

[6] Kim S. J., Jang S. P., Effects of the darcy number, the prandtl number, and the reynolds number on local thermal non-equilibrium, *International Journal of Heat And Mass Transfer*, 45, 2002, pp. 3885-3896.

[7] Khashani S.A., Al-Nimr M.A., Validation of the Local Thermal Equilibrium Assumption in Forced Convection of Non-Newtonian Fluids through Porous Channels, *Transport in Porous Media*, 61, 2005, pp. 291-305.

[8] Haddad O.M., Al-NimrA M.A., Al-Khateeb N., Validation of the local thermal equilibrium assumption in natural convection from a vertical plate embedded in porous medium: non-Darcian model, *International Journal of Heat And Mass Transfer*, 47, 2004, pp.2037-2042.

[9] Zhang X., Liu W., New criterion for local thermal equlibrium in porous media, *Journal of Thermophysics and Heat Transfer*,22,2008.

[10] Leong K.C., Li H.Y., Jin L.W., Chai J.C., Numerical and experimental study of forced convection in graphite foams of different configurations, *Applied Thermal Engineering*, 30, 2010, pp.520-532.

[11] Solmuş İ., Numerical investigation of heat transfer and fluid flow behaviors of block type graphite foam heat sink inserted in a rectangular channel, *Applied Thermal Engineering*, 78, 2015, pp.605-615.

[12] Gürüf G., Solmuş İ., Bilen K., Bayer Ö., Experimental based numerical approach for determination of volumetric heat transfer coefficients of modified graphite foams, *Applied Thermal Engineering, 174, 2020, 115310.*

[13] Mahjoob S., Vafai K., A synthesis of fluid and thermal transport models for metal foam heat exchangers, *International Journal of Heat And Mass Transfer*, 51, 2008, pp. 3701-3711.

[14] Lu W., Zhao C.Y., Tassou S.A., Thermal analysis on metal-foam filled heat exchangers. Part I: Metal-foam filled pipes, *International Journal of Heat And Mass Transfer*, 49, 2006, pp. 2751-2761.

[15] Teamah M. A., El-Maghlany W.M., Dawood M.M.K., Numerical simulation of laminar forced convection in horizontal pipe partially or completely filled with porous material, *International Journal of Thermal Sciences*, 50, 2011, pp.1512-1522.

[16] Hetsroni G., Gurevich M., Rozenblit R., Sintered porous medium heat sink for cooling of high-power mini-devices, *International Journal of Heat and Fluid Flow*, 27, 2006, pp.259-266.

[17] Dukhan N., Quin˜ones-Ramos P.D., Cruz-Ruiz E., Ve´lez-Reyes M., Scott E.P., One-dimensional heat transfer analysis in open-cell 10-ppi metal foam, *International Journal of Heat and Mass Transfer*, 48, 2005, pp. 5112-5120.

[18] Dukhan N., Chen K.C., Heat transfer measurements in metal foam subjected to constant heat flux, *Experimental Thermal and Fluid Science*, 32, 2007, pp.624-631.

[19] Hsieh W.H., Wu J.Y., Shih W.H., Chiu W.C., Experimental investigation of heat-transfer characteristics of aluminum-foam heat sinks, *International Journal of Heat and Mass Transfer*, 47, 2004, pp.5149-5157.

[20] Izadpanah M.R., Muller-Steinhagen H., Jamialahmadi M., Experimental and theoretical studies of convective heat transfer in a cylindrical porous medium, *International Journal of Heat and Fluid Flow*, 19, 1998, pp. 629-635.

[21] Xu H., Gong L., Huang S., Xu M., Non-equilibrium heat transfer in metal-foam solar collector with no-slip boundary condition, *International Journal of Heat and Mass Transfer*, 76, 2014, pp.357-365.

[22] Feng S.S., Kuang J.J., Wen T., Lu T.J., Ichimiya K., An experimental and numerical study of finned metal foam heat sinks under impinging air jet cooling, *International Journal of Heat and Mass Transfer*, 77, 2014, pp.1063-1074.

[23] Lua W., Zhao C.Y., Tassou S.A., Thermal analysis on metal-foam filled heat exchangers. Part I: metal-foam filled pipes, *International Journal of Heat and Mass Transfer*, 49, 2006, 2751-2761.

[24] Yang Y.T., Hwang M.L., Numerical simulation of turbulent fluid flow and heat transfer characteristics in heat exchangers fitted with porous media, *International Journal of Heat and Mass Transfer*, 52, 2009, 2956-2965.

[25] Ando K., Hirai H., Sano Y., An accurate experimental determination of interstitial heat transfer coefficients of ceramic foams using the single blow method, *The Open Transport Phenomena Journal*, 5, 2013, pp.7-12.

[26] Dukhan N., Chen K.C., Heat transfer measurements in metal foam subjected to constant heat flux, *Experimental Thermal and Fluid Science*, 32, 2007, pp.624-631.

[27] Hwang J.J., Hwang G.J., Yeh R.H., Chao C.H., Measurment of interstitial convective heat transfer and frictional drag for flow across metal foams, *Journal of Heat Transfer*, 124, 2002, pp.120-129.

[28] Calmidi V.V., Mahajan R.L., Forced convection in high porosity metal foams, *ASME Journal of Heat Transfer*, 122, 2000, pp.557-565.

[29] Tee C.C., Yu N., Li H., Modeling the overall heat conductive and convective properties of open-cell graphite foam, *Modeling Simulation Materials Science Engineering*, 16, 2008, 075006.