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Research Article

# HIERARCHICAL MATHEMATICAL MODELLING APPROACH FOR A CASE STUDY IN UNIVERSITY TIMETABLING

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**ABSTRACT**

This work presents a mathematical modelling based approach including many constraints encountered at Universities in Turkey. As in many other countries also in Turkey it appears that universities are too autonomous, and they have medley of individual requirements and constraints. This condition makes it very difficult to suggest a generalized model and a solution algorithm for university timetabling problem (UTP). In general, in this paper it is aimed to design compatible and flexible approach to generate timetable so as to meet all the requirements of Turkish universities. First, by reviewing the studies in which mathematical modelling approaches were used, comprehensive information about the subject has been presented. Then, proposed hierarchical mathematical modelling approach is described (HMMA). Since (UTP) is highly context-dependent the results of this study couldn't been compared to those of the studies which are published already. Proposed approach is tested 2015-2016 academic year winter term data of Atatürk University Engineering Faculty and obtained results are presented comparatively with results obtained from manual preparation in terms of seven objectives.
**Keywords:** University timetabling, operational research, mathematical programming.

## 1. INTRODUCTION

Since UTP is a problem requiring a long time and huge human resources and to be solved every term, it is an area receiving great interest by both researchers and practitioners. In general sense, UTP can be defined as the appointment of the courses having a number of features such as mandatory-elective, single-group under certain constraints to a limited number of time periods and classrooms. Since the courses are scheduled over more than one source such as curriculum group, classroom and lecturers and under many limitations, the problem has quite a complicated structure. In general, this problem which is proved to be NP-Hard [1] shows difference from country to country even in the universities at same country due to the diversity in academic systems. For this reason, UTP has been studied for 25 years in operations research literature and it has still been maintaining its popularity [2].

In the literature, UTP is seen to be examined under two main headings in general. The first one of the problem types subjected international time tabling competitions ITC 2002 and ITC 2007 is post enrolment based course timetabling problem (PEBTP) [3-8] and the other one is

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curriculum-based timetabling problem (CBTP). While course schedule in PEBTP is being conducted according to enrolment information, it is made according to the curriculum identified by university administration in CBTP [9]. The most important diffe rence between these two types of problem is that constraints and objective functions are tailored according to the curriculum in CBTP and according to the students enrolled in PEBTP [10].

In this study, it was focused on CBTP. Primarily, the studies solving CBTP in various versions with mathematical modelling technique were scanned and the information was given in detail by being classified according to constraints and objectives they contain. Then proposed HMMA is introduced.

The on-going portion of the study is organized as follows. In Chapter 2, the studies using mathematical modelling technique for the solution of CBTP are classified according to their constraints and objectives after making a detailed scan. In Chapter 3, the problem at hand is introduced. In Chapter 4, firstly studies reviewed in which some arrangements conducted on mathematical models to solve big sized real life academic timetabling problems. Then proposed hierarchical approach is introduced. In Chapter 5, mathematical formulations prepared for both session assigning and classroom assigning are given. In Chapter 6 timetables composed by proposed approach is compared with timetables composed by hand in terms of seven criterions. In Chapter 7 a general assessment is made.

## 2. LITERATURE REVIEW

UTP is a widely studied area in literature. Many mathematical models have been developed for various versions of this problem. However, due to the changing nature of the problem for each academic system, there is not a general model that will appeal to the solution of all problems in the literature.

In this section, the studies seeking solution to UTP with mathematical modelling technique were examined and classified according to the constraints and the objectives tried to be optimized. The classification of the examined studies according to the constraints expressing some specific features they contain is given in Table 1 and the classification of them according to the objectives they tried to optimize are given in Table 2. The general constraints that exist in all of the problems examined and that are given below are not given in Table 1:

- The courses given by the same lecturers cannot overlap.
- More than one course cannot be given in a classroom.
- Course sessions are given uninterruptedly as much as predetermined course time.
- A course can be assigned to a classroom that is suitable technically (computer laboratory, technical drawing room,... etc.) and that has enough capacity.
- All courses must be scheduled. Although this constraint was expressed as hard constraint (it cannot certainly be violated) in most of the examined studies, in some studies [11,12] it was evaluated as soft constraint.

Some distinctive constraints detected in the first column of Table 1 are expressed. In the other columns, the studies examined are submitted with the constraints they contain. The constraints are arranged as single option and multi-options. Multi-options have been distinguished with the letters like a, b, c. While "□" sign was being used in the relevant cell of single option constraint, in multi-option constraints, the letter belonging to the relevant constraint was used. Empty cells denote there is no such a constraint in the relevant work.

Table 2 in which the classification is made according to their objectives has been arranged similarly. On this table, the detailed description of the objective given in the second row and marked with the sign * has been made in Table 3.

**Table 1.** The classification of the studies based on mathematical modelling according to some prominent distinctive constraints.

| Constraints / Authors | [13] | [14] | [15] | [11] | [16] | [17] | [18] | [19] | [20] | [21] | [12] | [22] | [23] | [24] | [25] | [26] | [27] | [28] | [29] | [30] | **This paper** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Single (a), Multi (b) department timetabling. | a | a | b | b | b | b | b | b | a | b | b | a | b | b | b | b | b | | | | b |
| The courses in a curriculum group must be scheduled at different time periods. | □ | □ | □ | □ | | | □ | | □ | □ | | □ | | □ | □ | □ | | | | | □ |
| Some special courses cannot be assigned to the same period even though they are given by different lecturers and they belong to different curriculum groups. (For example, in order that the students can take courses from upper and lower curriculum group, their own curriculum group courses cannot overlap with upper/lower curriculum group lessons). | □ | | | □ | | □ | | | □ | | | □ | | □ | □ | | | | | | □ |
| Lecturers can prioritize the days and periods they will give courses (α), they can prevent them (β) or limit their numbers[1] (γ). | β, γ | α | | α, γ | α | α, β, γ | α | | α | α, β | β | | α | α | α | | α | | | | α, β |
| That the courses will be given by which lecturers is known beforehand (a), it is defined in the model (b). | a | b | a | b | a | a | a | b | a | a | a | a | a | a | a | a | a | | | | a |
| Faculty common courses, cannot conflict with courses of all curriculum groups in faculty and/or that are preassigned. | | | □ | | □ | | | | □ | | | | | | | | | | | | |
| Some courses taught by multiple instructors. | | | | □ | | □ | | | | | | | | | | | | | | | |
| Two sessions of one course cannot be assigned on one day or two consecutive days. | | | | | □ | | | □ | | □ | | □ | | | □ | □ | | | | | □ |
| Each curriculum group is divided into evening and morning groups (or more groups) which both groups follow the same program and there are some regulations about that[2]. | | | | | □ | | | | □ | | | | | | | | □ | | | | □ |
| Some lectures must be assigned to the consecutive periods of the same day (for instances theoretical and lab sessions). | | | | | | □ | | | | | □ | | □ | | | □ | | | | | □ |
| There is a priority relationship between some courses in daily timetable. | | | | | | □ | | | □ | | | | | | | | | | | | |
| There are some limitations about daily/weekly lecture loads of lecturers or students. | | | | | | | | □ | | □ | | □ | | | | | □ | | | | □ |
| There are some limitations regarding distribution of courses in the weekly lecturer timetable (e.g. A lecturer cannot teach in the first and last timeslot of a day). | | | | | | | | | | | | | | | | | □ | | | | □ |
| ITC 2007 problem | | | | | | | | | | | | | | | | | | □ | □ | □ | |

The number of the days on which the lecturer will give course or the maximum course load they can give must be limited.

In general morning (normal) group students are the higher grade students and are therefore not required to pay a fee. Students in evening (secondary) group, however, have lower grades upon entry and must pay a fee [20].

**Table 2.** The classification of the studies based on mathematical modelling according to objectives

| Objectives / Authors | [13] | [14] | [15] | [11] | [16] | [17] | [18] | [19] | [20] | [21] | [12] | [22] | [23] | [24] | [25] | [26] | [27] | [28] | [29] | [30] | **This paper** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Satisfaction of lecturers or students of expressed preferences regarding teaching periods or days of the week (a), classrooms (b) | a | a, b | a b | a, b | a | a, b | a | a | a | a,b | | a, b | a, b | a, b | a | | a | | | | a, b |
| Satisfaction of expressed preferences defined by lecturer (α), student (β) or department (γ) regarding distribution of courses in the weekly timetable.* | α, β | | | α | γ | γ | | | | | | β, γ | | | | | γ | | | | β, γ |
| Satisfaction of expressed preferences regarding courses that teachers are willing to teach. | | | | □ | | | | □ | | | | | | | | | | | | | |
| Minimal classroom changeovers between courses or minimal travelling time one classroom to another. | | □ | | | | | | | | | | | | □ | | □ | □ | | | | |
| To schedule courses approximately equal for all days of the week. | | | | □ | | | | | | | | | □ | | | | | | | | □ |
| Minimal number of courses by which instructor is underassigned (duration of course below the specified level). | | | | □ | | | | | | | | | | | | | | | | | |
| Minimal number of courses assigned to rooms outside the faculty's own building. | | | | □ | | □ | | | | | | | | | | | | | | | |
| Minimal collusion between courses for the students required to take them. | | | | | □ | □ | | | □ | | | □ | | | | □ | | | | | □ |
| Minimal violation of room capacity. | | | | | | □ | | | | | | | | | | | | | | | |
| Maximum number of course scheduled. | | | | | | | | | | | □ | | | | | □ | | | | | □ |
| Efficient utilization of available facility resources | | | | | | | | | | | | | □ | | | □ | | | | | |
| ITC 2007 problem | | | | | | | | | | | | | | | | | | □ | □ | □ | |

* The details belonging to this objective are submitted in Table 3.

**Table 3.** The details of objective: "Satisfaction of expressed preferences regarding distribution of lectures in the weekly timetable"

| Lecturer (a) | Student (b) | Department (c) |
|---|---|---|
| Lecturers may indicate a preference on the length of the interval between their courses **(13; 11)** | There should not be free day in student's timetable [22]. | Two sessions of one course cannot be assigned on one day or two consecutive days [16, 22]. |
| Lecturers may wish to have certain courses distributed to different days [17] | Student schedules should be as compact as possible [13, 27]. | Some courses must be scheduled at the same day and time periods (parallelism group courses) [17] |
| Lecturers may wish to have certain courses assigned to the same day [17] | For students whose attendance is limited to one or two days a week certain course may be desirable to be scheduled close together [13]**.** | Some courses should be assigned to the same intra-day time period of one or several days of the week (same-time-of-day-group courses) [17] |
| | Students prefer an empty slot around lunch time [13]. | |

## 3. DEFINITION OF THE PROBLEM

Engineering faculty consist of eight department giving four-year education. Each department accept students in two programs as primary or secondary. Both program follow the same curriculum but students of secondary program are educated out of office hours and they are required to pay fee. In each term approximately seven lectures are submitted to the students. The student who fails a course is compulsory to retake the course in the following year. Faculty administration asks for not putting the lessons belonging to the successive classes on the same day in the same period in order that the students who had failed can retake the lesson. The lectures consist of 2, 3 or 4 course hours. The lectures can be given in unique session or more than one session upon lecturers' request. Some lectures having more students can be given more than one groups. Besides the compulsory lectures, each department submits elective lectures to 3rd and 4th grade students. While lectures of primary program are given from 08.00 to 17.00 every day on weekdays, lectures of secondary program are given from 17.00 to 22.00 every day on weekdays and 08.00 a.m. to 10.00 p.m. on weekends. Each period lasts for 50 minutes and there are break times for 10 minutes. The break time and period are considered together in the problem and period has been identified as 1 hour (lecture hour).

The assumptions and constraints in this study are as follows:

### 3.1. Assumptions

- The student numbers taking each lecture is known beforehand.
- Which lecture will be given by which lecturer is known beforehand.
- Classroom capacities and type of classrooms (normal, laboratory) are known beforehand.
- Which lecture can be given at which type of classroom is known beforehand.
- The length (lecture time) of each lecture is known beforehand.
- Number of sessions of each lecture is known beforehand.
- Some lectures have one or more pre-conditional lectures and they are known beforehand.

Constraints are discussed under four sub-headings as regarding the lectures, students, lecturers and classrooms.

### 3.2. Constraints on the lectures

L1: Each lecture are taught uninterruptedly as much as the predetermined session time.
L2: All lectures in the curriculum must be assigned to the timetable.
L3: The sessions belonging to a lecture cannot be assigned to the same day.
L4: Some sessions definitely cannot overlap because of conditions defined below (Hard constraint):

- The sessions of the mandatory lectures belonging to the same curriculum group cannot overlap.
- The sessions of the lectures belonging to the same lecturer cannot overlap.

L5: Some sessions cannot overlap as much as possible because of conditions defined below (Soft constraint):

- The sessions of the pre-conditional lectures belonging to successive curriculum groups can overlap.
- The sessions of the elective and multi-group lectures belonging to the same curriculum group cannot overlap as much as possible.
- The sessions of the lectures belonging to successive curriculum groups cannot overlap as much as possible.

L6: Some lectures on which day, on which period and in which classroom will be taught are known beforehand (pre-assignment).
L7: No any session can be assigned to some predefined periods (because of lunch break, meetings planned previously, etc.)
L8: Some lectures should be assigned to consecutive periods (linked lectures, for instances theoretical and lab sessions).

### 3.3. Constraints on the students

S1: To ensure balanced distribution into weekly timetable, daily minimum $tm$ lecture hours (this is adjustable by user) should be assigned for each curriculum group.
S2: Sessions should be assigned into daily timetable as compact as possible. (Absence of free hours between consecutive lectures shouldn't exceed more than $tn$ lecture hour)

### 3.4. Constraints on the lecturers

T1: Because of administrative tasks of lecturers or lectures that they have to teach at other faculties/departments, sessions belong to related lecturers cannot be assigned to some periods.
T2: Maximum daily lecture load of a lecturer can be limited by faculty administration.
T3: Maximum lecture time in a day without interruption of a lecturer can be limited by faculty administration.
T4: The lecturers have a right to choose the period they want to give lectures by grading it from 1 to 3. The titles of the lecturers are used in the objective function determining the satisfaction level.
T5: A lecture that is assigned to the first period of normal program (08.00) also should be assigned to first period of secondary program (17.00) as much as possible (first in primary first in secondary).
T6: A lecture should be possibly assigned to same day for both normal and secondary program.

### 3.5. Constraints on the classrooms

R1: Maximum one lecture can be given in a classroom at the same period.

R2: A lecture can be assigned to a classroom that is technically feasible (computer laboratory, drawing room, ... etc.) and have a sufficient capacity.
R3: Pre-assignment is possible as in session assigning.
R4: Classroom assigning should be done to all sessions as much as possible.
R5: Classrooms can be prioritized due to requirements mentioned below:

- Sessions should be firstly assigned to classrooms that are allocated to related department as much as possible (optional).
- Lecturers can define the order of precedence for classrooms that they want to teach in (This constraint also optional. If any order isn't defined, sessions will be assigned to classrooms according to capacity in ascending order).

## 4. HIERARCHICAL MATHEMATICAL APPROACH

Mathematical modelling approach is frequently referring technic to solve academic timetabling problems. But it is seen that in the literature classical mathematical modelling approach based on binary variables which identifying whether lectures are assigned to periods and classrooms are either used for only representation of the problem [18, 26] or uttermost to solve only small sized problems [16,22]. Due to the complexity of academic timetabling problems, researchers made various arrangements on mathematical models that they proposed to solve the real-time problems in polynomial time.

In this context, mostly striking arrangement is reducing the size of variables. Akkoyunlu [13], used the set of timetable patterns for assigning sessions. By pattern author mean, assignment of sessions to previously defined day-period pairs. Qualizza, and Serafini [24] created their model based on columns which means a weekly timetable of a single course. [15] proposed mathematical model based on concept of grouping periods and courses. Martin [11] specified time slots which can be of any configuration as options for course times. Bakır, and Aksop [20] ignored index of classroom. To protect collusion they added extra constraints. Even so they reached narrowly feasible solution in 5 minutes for only one department's timetable. These approaches in which dimension of the variable is reduced, even made positive contribution in terms of solution time, they limit the flexibility in point of expression of various constraints in the model.

Another arrangement made on mathematical models is reducing the number of variable. Hence Daskalaki et. al. [14], Sánchez-Partida et.al. [12], Martin [11] used sets those are defined a priori so that certain indices use those as domains instead of the original ones.

In some studies mathematical model is used as a part of hybrid heuristic. Gunawan, Ng, and Poh [19] applied langarian relaxation to mathematical model that they proposed for generating initial population. Benli, and Botsalı [31] used mathematical model at third stage of their three phase approach to assign classrooms to previously timetabled sessions.

In this research it is used an integer programming model including multidimensional variables. Such modelling approaches have high flexibility in terms of rendering constraints of problem in detail. But it is almost impossible solving big sized problems. Thus in this research a hierarchical approach composed three stages is proposed for timetabling problem of universities in Turkey. To the best of our knowledge such a hierarchical approach is applied in two works. Aizam, and Liong [21], decomposed their model into three main stages which are the allocation of class meetings to days, formation of the daily course timetable and lastly the assignment of courses to a suitable rooms. The other paper by Vermuyten et al. [27]. They decomposed their model into two stages. The first stage minimizes the violation of the teacher and educational preferences by assigning lectures to timeslots and rooms. The second stage reassigns classrooms to lectures of the timetable of the first stage and minimizes the student flow.

In this study firstly to reach solution in polynomial time main problem is divided into two sub problems as session assigning and classroom assigning. Mathematical models are developed for both sub problem. Then an automatic process is designed to link each stage and get the data from database. As a result of this arrangement in which problem is divided into two sub problem, it couldn't reached any feasible solution in eight hour run time.

Thereupon M1 which is the model developed for session assignment part (stage 1) of the problem is applied to each program one by one. Not to collide sessions of lecturers those are teaching lesson in more than one department and to ensure the requirements of secondary programs (T2, T3, T5, T6) session assigning process of each program linked to each other as shown in Fig. 1.

After the completion of session assignment (stage 1) for all programs, in stage 2 M2 which is developed for classroom assigning is applied to all faculty as a whole. Due to models M1 and M2 are run independently from each other, there remained even if just a few, classroom unassigned sessions. To overcome this problem in stage 3, M3 which is composed from joining M1 and M2 is applied to overall faculty. While classroom assigned sessions in the timetable produced by M2 is defined to M3 as parameter (as pre-assigned sessions), classroom unassigned sessions are defined decision variable. Although M3 handles the problem as a hole it can reach solution in a short time due to there are few sessions remained that couldn't be found classroom. Overall solution process which is illustrated in Fig. 1 is carried out automatically with prepared software.

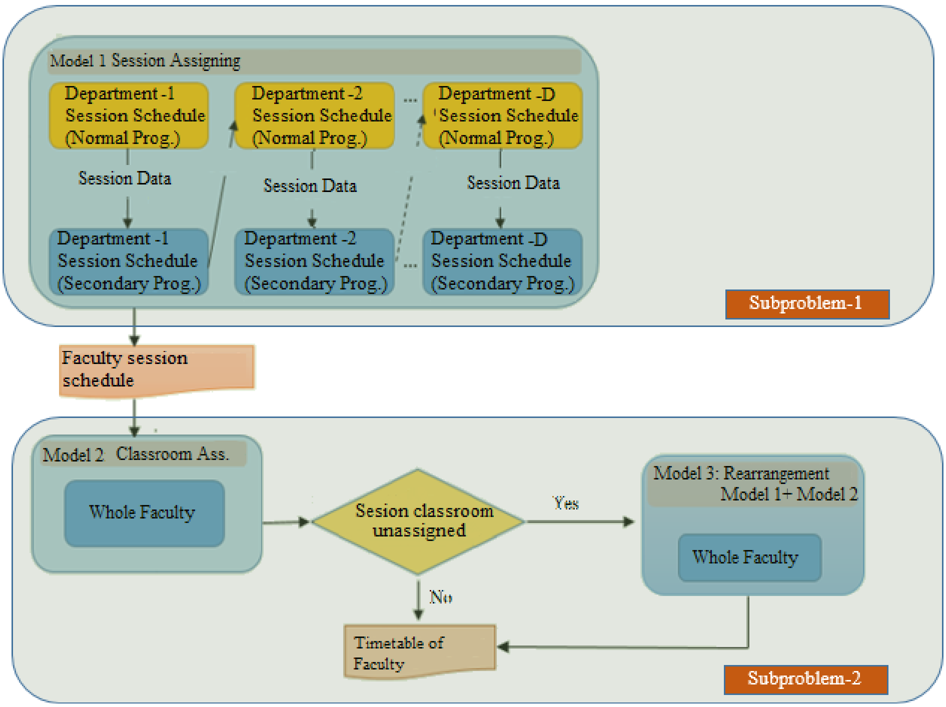


**Figure 1.** Process flow chart of proposed hierarchical approach.

It had been mentioned at the outset that the aim of this study is to find a good quality solution to timetable problem of universities in Turkey. As in many other countries also in Turkey it appears that universities are too autonomous, and they have medley of individual requirements. These requirements are identified with interviewing the stuff tasked with timetabling and necessary constraints was added to models according to these determinations. Since each university wants to run proposed timetabling procedure according to their requirements, models are designed on a modular basis. The illustrated interface in Fig. 2 is designed to allow users easily add or remove constraints. With this window it is also allowed to users defining targets for some objectives. In this way it is prevented wasting more effort once the target for a problem criterion has been met.

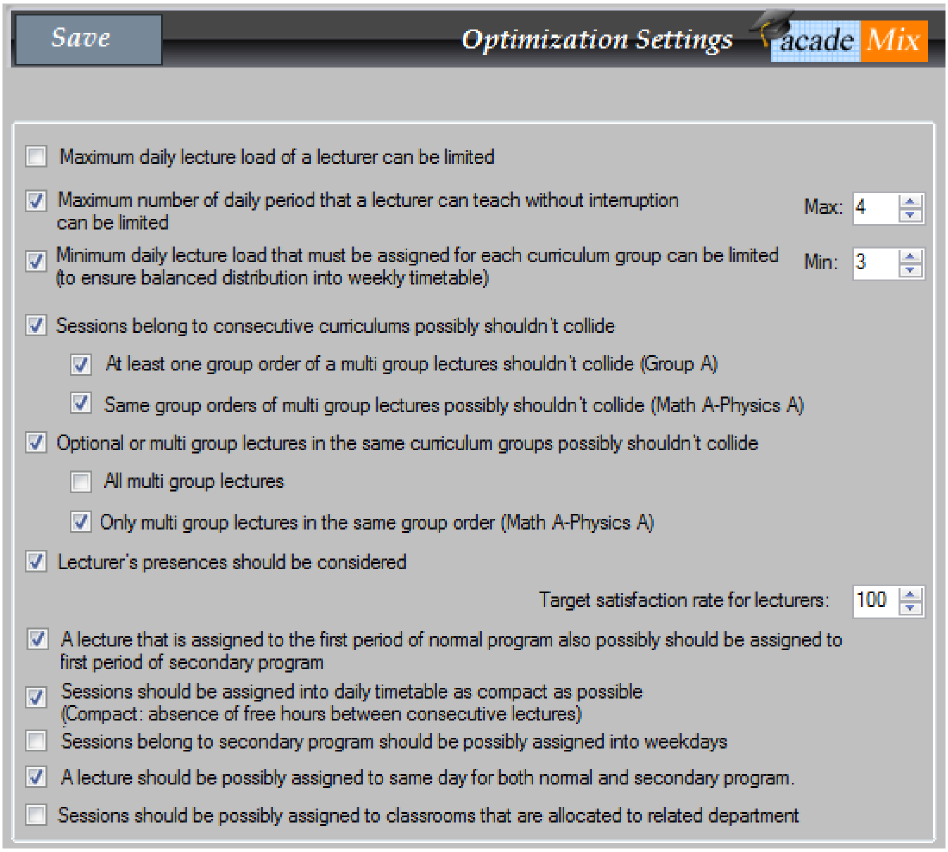


**Figure 2.** Optimization settings window.

Due to the autonomous structure of universities, significance level of objectives, in other words quality criterion of a timetable also may change from one university to another. In order to take account of this, a user interface has been designed that allows the user to weight objectives from 0 to 9 (weighting with 0 means related objective won't be considered) for each of the problem criterion as seen in Fig. 3.

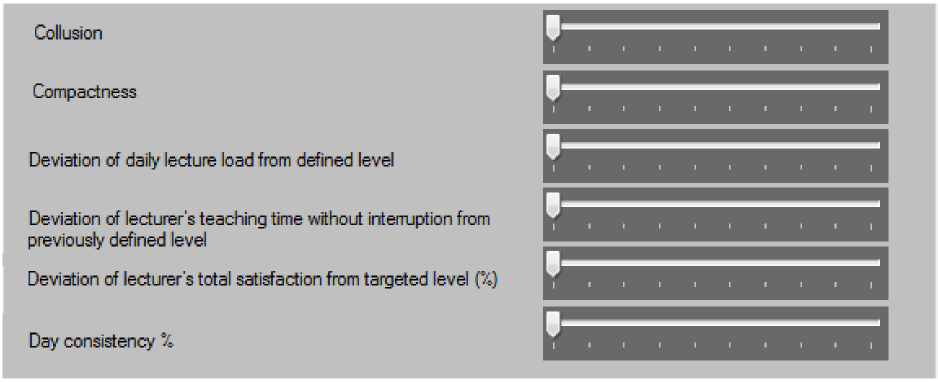


**Figure 3.** Window that is prepared to edit weights of objectives.

## 5. MATHEMATICAL MODELS DESIGNED FOR SESSION AND CLASSROOM ASSIGNING

While constructing the mathematical model of the problem, similar manner is applied Daskalaki, Birbas, and Houses [14]. To keep down the variables, sets are defined a priori so that certain indices use those as domains instead of the original ones. Definition of subsets is done with a special procedure as per information that is gotten from database and rule base. Indices subsets, parameters and decision variables that are used in both models are submitted below:

### 5.1. Indices and sets

i, f : Sessions
j, jj : Days
g : Lectures
l : Curriculum groups
d : Departments
h : Lecturers
p, pp : Periods
r, rr : Classrooms

$NoCls_{i,f}$ : They are the session pairs (i, f) consisting of the sessions belonging to the mandatory courses in the same curriculum group or given by the same lecturer.
$FlxCls_{i,f}$ : It is the set consisting of the session pairs (i, f) that are not wanted to collide and that belongs to elective or multi-group lectures in the same curriculum group or the lectures in consecutive curriculum groups within the scope of flexible collusion.
$Link_{i,f}$ : Set of linked session pairs (i, f).
$NoFeas_{j,p}$ : Prohibited day and periods for education defined by faculty administration (lunch break, meetings, seminars, ...etc).
$NoLec_{h,j,p}$ : It is the set consisting of days and periods that lecturer h cannot teach because of his/her administration functions or earlier planned sessions at the other programs.
$preS_{i,j,p}$ : Set of pre-assigned sessions to day j and period p.
$preR_{i,r}$ : Set of pre-assigned sessions to classroom r.

### 5.2. Parameters

$t_i$ : Duration of session i (lecture hour)

tm : Minimum lecture hour that must be assigned for each curriculum group
tn : Maximum amount of free hours between consecutive lectures.
$R_i$ : Classroom set that session i can be done in it
$I_r$ : Sessions can be done in the classroom r
$I_g$ : Session set belong to lecture g
$I_l$ : Session set belong to curriculum group l
$I_h$ : Session set belong to lectures that are taught by lecturer h
$N_l$ : Number of sessions belong to curriculum group l
$alw_{r,j,p}$ : This parameter shows if any classroom r is available for teaching at day j and period p (If available 1, otherwise 0).
$xx_{i,j,p}$ : This parameter is used in M2 and represents assigned day and period of session i (Value of this parameter is defined after run M1 is completed and takes the value of decision variable $x_{i,j,p}$).
$coef_h$ : Title coefficient of lecturer h.
$pref_{h,j,p}$ : Precedence coefficient that is specified by lecturer h for day j and period p (1 to 3).
$lecload_{h,j}$ : Lecture load (lecture hour) of lecturer h on day j. Due to the session assigning process of each department is done independently of one another, this parameter is a dynamic parameter that changes after compilation of each department's session assigning process.
maxLoad : Predefined daily maximum lecture load level (lecture hour) that a lecturer can teach.
$Busy_h$ : Set of periods and days that lecturer h is busy. If lecturer h has session on day j and period p at formerly timetabled programs, this parameter takes 1, otherwise 0.
csls: Predefined maximum daily lecture time that a lecturer can teach without interruption.
Tsts : Target satisfaction level defined by faculty administration.
$coeff_{i,j}$ : Objective function coefficients of M1 express the desirability of the assignment defined by variable $y_{ij}$. This parameter will be used to ensure constraint T6.
$c_{i,r}$ : Objective function coefficients of M2 express the desirability of the assignment defined by variable $w_{ir}$. This parameter will be used to ensure constraint R5.
$eight_{i,j,p}$: This parameter will be used to ensure constraint T5.

### 5.3. Decision variables

$x_{i,j,p}$ : Binary variable which equals 1, if session i is assigned to day j, period p, 0 otherwise
$y_{i,j}$ : Binary variable which equals 1, if curriculum group i is assigned to day j, 0 otherwise
$cls_{i,f,p}$ : Binary variable which equals 1, if both sessions i and f is assigned to period p, 0 otherwise.
$z_{l,j,p}$ : Binary variable which equals 1, if any session is assigned to timetable of curriculum l on day j period p, 0 otherwise.
$u_{h,j,p}$ : Binary variable which equals 1, if any session is assigned to timetable of lecturer h on day j period p, 0 otherwise.
$w_{i,r}$ : Binary variable which equals 1, if session i is assigned to classroom r.
$v_{i,j,p,r}$ : Binary variable which equals 1, if session i is assigned to day j, period p, classroom r, 0 otherwise.
$pnl_{l,j,p}$ : Penalty that will be applied when (p+2). period is empty while a session is assigned to time table of curriculum l on day j period p.
$dev_{l,j}$ : Deviation amount (lecture hour) of lecture load of curriculum l on day j from average daily lecture load of curriculum l
$devH_{h,j,p}$ : This variable is used to calculated the deviation of lecturer's teaching time without interruption from previously defined level.
devSts : Deviation amount from previously defined total satisfaction level.

### 5.4. Session assigning model (M1)

In this model assigning of sessions to timetable is done with ignoring classroom constraint. Objective function of this model is defined flexible so as to find always a feasible solution. Each term in objective function expresses these goals respectively: A lecture should be possibly assigned to same day for both normal and secondary program (day consistency), sum of the collided lecture hours of the session pairs within the scope of flexible collusion, total daily idle periods between sessions (compactness), total deviation of lecture load from predefined daily lecture load (3 lecture hours) that should be assigned at least for each program (balanced distribution), total deviation of lecturer's teaching time without interruption from previously defined level (4 lecture hours), deviation of lecturer's total satisfaction amount from targeted level.

$$\sum_{p} x_{i,j,p} = t_i * y_{i,j} \qquad \forall i, j \qquad (1)\ (L1)$$

$$t_i * (x_{i,j,p} - x_{i,j,p+1}) + \sum_{pp \geq ord(\mathrm{p})+2} x_{i,j,pp} \leq t_i * \mathrm{y}_{i,j} \quad \forall i, j; \forall p < card(\mathrm{p}) \qquad (2)\ (L1)$$

$$\sum_{j} y_{i,j} = 1 \qquad \forall i \qquad (3)\ (L2)$$

$$\sum_{i \in I_g} y_{i,j} \leq 1 \qquad \forall j, \mathrm{g} \qquad (4)\ (L3)$$

$$x_{i,j,p} + x_{f,j,p} \leq 1 \qquad \forall j, p; \forall i, f \in NoCls_{i,f} \qquad (5)\ (L4)$$

$$x_{i,j,p} + x_{f,j,p} - cls_{i,f,p} \leq 1 \qquad \forall j, p; \forall i, f \in FlxCls_{i,f} \qquad (6)\ (L5)$$

$$x_{i,j,p} = 1 \qquad \forall i, j, p \in preS_{i,j,p} \qquad (7)\ (L6)$$

$$\sum_{i} x_{i,j,p} = 0 \qquad \forall j, p \in NoFeas_{j,p} \qquad (8)\ (L7)$$

$$\sum_{pp=p}^{pp=p+t_i-1} x_{i,j,pp} + \sum_{pp=p+t_i}^{pp=p+t_i+t_f-1} x_{f,j,pp} \geq (\mathrm{t}_i + \mathrm{t}_f) * x_{i,j,\mathrm{p}}$$

$$\forall i, f \in Link_{i,f}; \forall j; \forall p \leq card(\mathrm{p}) - (\mathrm{t}_i + \mathrm{t}_f - 1) \qquad (9)\ (L8)$$

$$\sum_{i \in I_l} \sum_{p} x_{i,j,p} \geq tm - dev_{l,j} \qquad \forall j; \forall l \qquad (11)\ (S1)$$

$$\sum_{i \in I_l} x_{i,j,p} \leq z_{l,j,p} * N_l \qquad \forall j, p, l \qquad (12)\ (S2)$$

$$\sum_{i \in I_l} x_{i,j,p} \geq z_{l,j,p} \qquad \forall j, p, l \qquad (13)\ (S2)$$

$$\sum_{pp=p}^{pp=p+(tn+1)} z_{l,j,pp} \geq z_{l,j,p} * 2 - pnl_{l,j,p} \qquad \forall j, l; \forall p \leq card(\mathrm{p}) - tn - 1 \qquad (14)\ (S2)$$

$$\sum_{i\in I_h} x_{i,j,p} = 0 \qquad \forall h, j, p \in NoLec_{h,j,p} \qquad (15)\ (T1)$$

$$\sum_{i\in I_h} y_{i,j} * t_i + lecload_{h,j} \le \max Load \qquad \forall h, j \qquad (16)\ (T2)$$

$$x_{i,j,p} = 0 \qquad \forall h; \forall i \in I_h; \forall j, p \in Busy_h \qquad (17)\ (T3)$$

$$u_{h,j,p} = 1 \qquad \forall h; \forall j, p \in Busy_h \qquad (18)\ (T3)$$

$$u_{h,j,p} = \sum_{i\in I_h} x_{i,j,p} \qquad \forall h, j, \mathrm{p} \qquad (19)\ (T3)$$

$$\sum_{pp=p}^{pp=p+csls} u_{h,j,pp} - devH_{h,j,pp} \le csls \qquad \forall h, j, \mathrm{p} \qquad (20)\ (T3)$$

$$\sum_h \sum_{i\in I_h} \sum_j \sum_p x_{i,j,p} * pref_{h,j,p} * eight_{i,j,p} * coef_h + devSts \ge Tsts \qquad (21)\ (T4)\ (T5)$$

***Subjected to***

$$\mathrm{Max}(\sum_i \sum_j coeff_{i,j} * y_{i,j} - \sum_i \sum_f \sum_p cls_{i,f,p} - \sum_l \sum_j \sum_p pnl_{l,j,p} - \sum_l \sum_j dev_{l,j} - \sum_h \sum_j \sum_p devH_{h,j,p} - devSts) \qquad (22)\ (T6)$$

**5.5. Classroom assigning model (M2)**

This model tries to find proper classrooms which are technically feasible (computer laboratory, drawing room … etc.) and have a sufficient capacity for timetabled sessions. It is also being tried in this model to assign sessions to classrooms possibly which are preferred by lecturers and allocated by faculty administration to related department. All mentioned conditions above is provided by objective coefficient $c_{i,r}$.

$$\sum_{i\in I_r} v_{i,j,p,r} \le alw_{r,j,p} \qquad \forall r, j, p \qquad (23)\ (R1)$$

$$\sum_{r\in R_i} w_{i,r} \le 1 \qquad \forall i \qquad (24)\ (R2)$$

$$w_{i,r} = 1 \qquad \forall i, r \in preR_{i,r} \qquad (25)\ (R3)$$

$$w_{i,r} * xx_{i,j,p} = v_{i,j,p,r} \qquad \forall i, j, p; \forall r \in R_i \qquad (26)$$

***Subjected to***

$$Max \sum_i \sum_{r\in R_i} w_{i,r} * \mathrm{c}_{i,r} \qquad (27)\ (R4)\quad (R5)$$

## 6. NUMERICAL RESULTS

Hierarchical approach proposed in this paper has been tested with the 2016 – 2017 academic year, first term course timetabling data of Atatürk University Engineering Faculty. Problem is solved for same weight of objectives. Timetables composed by automated process is compared with composed by hand in terms of seven objective.

First of all 2016 – 2017 academic year first term timetables of Engineering Faculty made manually is evaluated in terms of seven objectives. Then problem is solved by automated process with same weights of objectives. Automated process is programmed in Microsoft C# 2010 and use the callable library of Gurobi60.net version 6. The code is executed on a PC with CoreTM 2 Quard CPU processor of 2.66 GHz and a RAM of 4 GB. Maximum run time is defined as 10 minutes for timetabling of each program at stage 1. Obtained result from both manual and automated process is submitted in Table 4**.**

**Table 4.** Comparison of automated and manual processes

| | | Primary Prog. | | Secondary Prog. | |
|---|---|---|---|---|---|
| No | **Objectives** | **Auto.** | **Manu.** | **Auto.** | **Manu.** |
| 1 | Collision time (lec.hour/program) | 2,5 | 26,25 | 21,63 | 36,25 |
| 2 | Average daily idle times between sessions (lec.hour/ program) | 0,375 | 0 | 0 | 0 |
| 3 | Deviation of daily lecture load from defined level (lec.hour/ program) | 0,375 | 2,375 | 6 | 11,75 |
| 4 | Deviation of lecturer’s teaching time without interruption from previously defined level (lec.hour/ program) | 0 | 2 | 0,5 | 1,875 |
| 5 | Deviation rate of lecturer’s total satisfaction from targeted level | 12% | 32% | 23% | 47% |
| 6 | First in primary first in secondary rate | - | - | 65,9% | 48,4% |
| 7 | Day consistency rate | - | - | 73,6% | 33,0% |

Timetables of eight departments which are composed with manual and automatic processes are evaluated in terms of 7 objectives and average comparative results are presented in Table 4 separately for primary and secondary programs. The first objective in Table 4 is average collusion time within the scope of flexible collusion. For this objective it is considerable improvement ensured. For the second objective it is seen that in primary program automated process fell behind the manual process. But if it is desired, with augmenting the weight of the related objective such condition can be eliminated. The third objective is the indicator of distribution of sessions into timetable. The less value of this objective means more balanced distribution of sessions into timetable. In this way it is protected to clustering of sessions into some days. For this objective especially for the secondary program seriously improvement is recorded. When the fourth objective is examined it is seen that while at primary programs made with automated process, defined level is never surpassed, with manual process it is surpassed average 2 lecture hours for each program. At secondary programs automated process ensured improvement more than three times according to manual process. The fifth objective is the indicator of satisfaction of lecturers from timetables. In other words it is the indicator in what rate sessions are assigned to preferred periods by lecturers. Values showed in Table 4 about this objective are the deviation rate of lecturer’s satisfaction from targeted level. Targeted level refers to calculated value on the assumption that all lecturers’ entire sessions are assigned to periods which are mostly preferred by them.

Last two objectives are related with secondary programs. With sixth objective it is aimed to generate fairer timetables in terms of lecturers. With automated procedure %65,9 of sessions which are assigned to first period (08.00) of primary program, also assigned to first period of secondary program (17.00). This rate is %48,4 while timetabling with manual process. Seventh objective which is the second performance criteria of secondary programs is day consistency rate. This value is the ratio of number of sessions that is assigned to same day for primary and secondary programs to total number of sessions. Meeting this requirement is important in terms of following the curriculum collaterally for primary and single programs. Otherwise because of unavailability of lecturers in some days or public holidays synchronization between primary and secondary programs cannot be kept. For the last objective it is seen from Table 4 more than two times improvement is ensured according to manual process.

## 7. CONCLUSION

In this research a hierarchical mathematical approach is proposed for academic timetabling problem of universities in Turkey. Proposed approach has features of the flexibility of mathematical modelling approaches consisting of multidimensional variables and high computation speed of heuristic algorithms together. The biggest difficulties encountered at this approach in which problem is solved in independent pieces are prevention of collusion and transportation of data during the solution process. In this direction an automated procedure is designed connecting each stage with the relation of input/output.

Designed hierarchical approach is tested in real life conditions. Timetables that are generated by automated and manual process is evaluated comparatively in terms of predefined criterions. The automated process outperformed manual process in terms of response time and quality. The proposed solution process was approved by the faculty administration and has been used since the first semester of 2015.

As a future work it is planned to designing a mathematical modelling based approach to solve academic course or exam timetabling problem with alternating session times.

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