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**Research Article**

# NEAR RINGS IN THE VIEW OF DOUBLE-FRAMED SOFT FUZZY SETS

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**ABSTRACT**

In this study, we aim to consider a new kind of a set called by a double framed soft fuzzy set, which is convenient to handle the real world applications and to investigate the near rings in the view of this new set. We define some of the elementary set operations of double framed soft fuzzy sets. We propose the notion of double framed soft fuzzy near rings (ideals) with several properties and characteristics. Further, we illustrate the given notions with some examples.
**Keywords:** Fuzzy set, soft set, double framed soft fuzzy set, near ring, ideal, homomorphism.

## 1. INTRODUCTION

In our daily life, many theories are presented to face uncertainty and vague concepts like crisp set theory, fuzzy set theory, intuitionistic fuzzy set theory and rough set theory (see [1-4]). These theories are used in different areas like engineering, decision making and medical diagnoses and etc. But all of these theories have their own limitations and boundaries to face uncertainty and vague concepts. To handle this problem Molodstov [5], introduced a new theory which gave us proper solution to consider uncertainty and vague concepts which is known as soft set theory. Then he applied this idea in different fields like game theory, Riemannian integral, economics, and so on.

First time in 2001 Maji et al. [6] combined the both theoretical concepts in terms of fuzzy soft set and discussed its properties. Further, they applied this idea in decision making [7]. Then many scholars applied soft sets and fuzzy soft sets in different fields (see [8-12]).

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Jun et al. [13] initiated to introduce double framed soft sets and presented its applications in BCK/BCI algebras. Also Cho et al. [14] studied the double framed soft near rings and defined its notions. Further, this concept is used by many researchers in different directions, see [15-18]. Mahmood and Bilal [19] introduced the concept of double framed T-soft fuzzy set and defined its notion. Fu rthermore, they applied this idea in BCK/BCI-algebras and discussed its properties with the support of examples.

It is known that the parameters are not crisp in real life situations because of the human thinking, and also each parameter has an effect on the given decisions in some degrees. So, the first part of the double framed soft fuzzy set in the pair, soft set, gives us the selections according to the parameters and the second part, fuzzy set, gives us the influences of the each parameter on the decision. This idea leads us to consider the double framed soft fuzzy sets in some aspects. So, we first decide to define the notions of double framed B-soft (T-soft) fuzzy near rings, and also double framed B-soft (T-soft) fuzzy ideals by giving examples. We discuss their some fundamental properties and give some examples.

## 2. PRELIMINARIES

In this section, we recall some of the basic notions with some modifications which are the backbone of the main results.

**2.1. Definition [5]** Let $U$ be a non-empty universal set and $E$ be a set of parameters. A pair $(F,E)$ is called a soft set over $U$ if and only if $F$ is a mapping from $E$ into the set of all subsets of $U$; i.e., $F: E \to \mathcal{P}(U)$. According to this definition, a soft set may be indicated as follows

$$(F,E) = \{(e, F(e)) | \ e \in E \ and \ F(e) \in \mathcal{P}(U)\}$$

The function $F$ is an approximation function of the soft set $(F,E)$. It is easy to see that soft set is a parameterized family of subsets of $U$. For simplicity, we write $F$ instead of $(F,E)$.

**2.2. Example** A soft set $F$ describes the attractiveness of the dresses which Mrs. A is going to buy for a party.

$U =$The set of all dresses under consideration in the shop. $U = [x_1, x_2, x_3, x_4\}$.
$E =$The set of all parameters. Each parameter is a word in sentence. $E = \{e_1, e_2, e_3, e_4, e_5\}$.
$e_1 \coloneqq short, e_2 \coloneqq expensive, e_3 \coloneqq colorful, e_4 \coloneqq low - cut$ and $e_5 \coloneqq basic$

According to the above criterions, the soft set $F$ is defined as follows.

$$F = \{(e_1, \{x_2, x_4\}), (e_2, \{x_1, x_4\}), (e_3, \{x_1\}), (e_4, \emptyset), (e_5, \{x_2, x_3\})\}$$

**2.3. Definition** [13] A double framed pair $((F,G); E)$ over $U$ is called a double framed soft set, where $E$ is a set of parameters and in pair form $(F,G)$ both are soft sets over $U$; i.e, $F, G: E \to \mathcal{P}(U)$. For simplicity, we write $(F,G)$ instead of $((F,G); \mathrm{E})$, unless otherwise specified.

**2.4. Example** Mrs. A will go to a party and wants to look smart in the guests. But, her husband Mr. A's decision is important for her. Despite the set of all dresses and the parameters for this couple are same, their perspectives are different. The double framed soft set $(F,G)$ shows both of their perspectives as follows.

$$F = \{(e_1, \{x_2, x_4\}), (e_2, \{x_1, x_4\}), (e_3, \{x_1\}), (e_4, \emptyset), (e_5, \{x_2, x_3\})\}$$
$$G = \{(e_1, \{x_4\}), (e_2, \{x_1, x_3, x_4\}), (e_3, \emptyset), (e_4, \{x_2\}), (e_5, \{x_4\})\}$$

As it is seen, it is easy to decide for a couple to prefer a double framed soft set.

**2.5. Definition** [20] A double framed pair $< (F,f); E >$ over $(U, [0,1])$ is called a double framed soft fuzzy set over $(U, [0,1])$, where $F$ is a soft set over $U$ and $f$ is a fuzzy set over $[0,1]$, i.e., $F: E \to \mathcal{P}(U)$ and $f: E \to [0,1]$. For simplicity, we write $(F,f)$ to denote a double framed soft fuzzy set over $(U, [0,1])$ with parameter set $E$, unless otherwise specified.

**2.6. Example** Ms. A wants to buy a dress and looks at the shops all day to find the most beautiful one which supplies her criterions. Since she is not sure for the parameters, she may give up some of them if she finds her cup of tea and also she does not spare much time for looking around.

$U =$The set of all dresses under consideration in the shop. $U = [x_1, x_2, x_3, x_4\}$.
$E =$The set of all parameters. Each parameter is a word in sentence. $E = \{e_1, e_2, e_3, e_4, e_5\}$.
$e_1 \coloneqq short, e_2 \coloneqq expensive, e_3 \coloneqq colorful, e_4 \coloneqq low - cut$ and $e_5 \coloneqq basic$

The double framed soft fuzzy set is defined as follows.

$$(F,f) = \{(F(e_1), f(e_1)), (F(e_2), f(e_2)), (F(e_3), f(e_3)), (F(e_4), f(e_4)), (F(e_5), f(e_5))\}$$
$$= \{(\{x_4\}, 0.6), (\{x_3, x_4\}, 1), (\{x_1, x_2\}, 0.3), (\{x_1, x_3\}, 0.8), (\{x_4\}, 0.7)\}$$

Here the values $f(e_i)$ indicates the influences of the parameters on the decision in what degree.

**2.7. Definition** [20] Let $U$ be a non-empty universal set and $B, C$ be the subsets of the parameter set $E$. Let $(F,f)$ and $(G,g)$ be two double framed soft fuzzy sets over $(U,[0,1])$ with the parameter sets $B$ and $C$, respectively. Then $(F,f)$ is said to be a double framed soft fuzzy subset of $(G,g)$ if for each $u \in B$, $F(u) \subseteq G(u)$ and $f(u) \geq g(u)$. It is denoted by $(F,f) \sqsubseteq (G,g)$.

**2.8. Definition** Let $U$ be a non-empty universal set and $B, C$ be the subsets of the parameter set $E$. Let $(F,f)$ and $(G,g)$ be two double framed soft fuzzy sets over $(U,[0,1])$ with the parameter sets $B$ and $C$, respectively. Then

**(1)** The uni-int of double framed soft fuzzy sets $(F,f)$ and $(G,g)$ is denoted by $(F,f) \sqcup (G,g) = (F \tilde{\cup} G, f \tilde{\wedge} g)$, where
$F \tilde{\cup} G : B \cup C \longrightarrow \mathcal{P}(U)$ defined by

$$p \mapsto \begin{cases} F(p) \text{ } if \text{ } p \in B \setminus C \\ G(p) \text{ } if \text{ } p \in C \setminus B \\ F(p) \cup G(p) \text{ } if \text{ } p \in B \cap C \end{cases}$$

and $f \tilde{\wedge} g : B \cup C \longrightarrow [0,1]$ defined by

$$p \mapsto \begin{cases} f(p) \text{ } if \text{ } p \in B \setminus C \\ g(p) \text{ } if \text{ } p \in C \setminus B \\ f(p) \wedge g(p) \text{ } if \text{ } p \in B \cap C. \end{cases}$$

**(2)** The int-uni of double framed soft fuzzy sets $(F,f)$ and $(G,g)$ is denoted by $(F,f) \sqcap (G,g) = (F \tilde{\cap} G, f \tilde{\vee} g)$, where
$F \tilde{\cap} G : B \cup C \longrightarrow \mathcal{P}(U)$ defined by

$$p \mapsto \begin{cases} F(p) \text{ } if \text{ } p \in B \setminus C \\ G(p) \text{ } if \text{ } p \in C \setminus B \\ F(p) \cap G(p) \text{ } if \text{ } p \in B \cap C \end{cases}$$

and $f \tilde{\vee} g : B \cup C \longrightarrow [0,1]$ defined by

$$p \mapsto \begin{cases} f(p) \text{ } if \text{ } p \in B \setminus C \\ g(p) \text{ } if \text{ } p \in C \setminus B \\ f(p) \vee g(p) \text{ } if \text{ } p \in B \cap C \end{cases}$$

**2.9. Definition (1)** Let $< (F,f); E >$ be a double framed soft fuzzy set over $(U,[0,1])$ and $\varphi: E \to K$ be a mapping between the parameter sets. Then the image $< (\varphi(F), \varphi(f)) ; K >$ is also a double framed soft fuzzy set over $(U,[0,1])$ which is defined as follows for each $k \in K$;

$$\varphi(F)(k) = \cup_{k=\varphi(e)} F(e) \text{ and } \varphi(f)(k) = \wedge_{k=\varphi(e)} f(e).$$

**(2)** Let $<(G,g);K>$ be a double framed soft fuzzy set over $(U,[0,1])$ and $\varphi: E \to K$ be a mapping between the parameter sets. Then the preimage $<(\varphi^{-1}(F), \varphi^{-1}(f))\,;E>$ is also a double framed soft fuzzy set over $(U,[0,1])$ which is defined as follows for each $e \in E$;

$$\varphi^{-1}(F)(e) = F(\varphi(e)) \text{ and } \varphi^{-1}(f)(e) = f(\varphi(e)).$$

**2.10. Definition** [14] Let $R_N$ be a classical near ring and $(F,G)$ be a double framed soft fuzzy set over $U$ with the parameter set $R_N$. Then $(F,G)$ is called a double framed soft near ring if the following assertions hold, for all $u, v \in R_N$

**(SN1)** $F(u+v) \subseteq F(u) \cup F(v),\ F(uv) \subseteq F(u) \cup F(v).$
**(SN2)** $G(u+v) \supseteq G(u) \cap G(v),\ G(uv) \supseteq G(u) \cap G(v).$
**(SN3)** $F(-u) = F(u),\ G(u) = G(-u).$

## 3. MAIN RESULTS

In this section, we describe the notions of double framed B-soft fuzzy near rings and double framed T-soft fuzzy near rings by considering their ideals. Further, we discuss the properties of given notions with some examples. Throughout this study, $R_N$ denotes a given classical near ring.

### 3.1. Double Framed B-Soft Fuzzy Near Rings

In this subsection, we present the basic definition of a double framed B-soft fuzzy near ring and explain this concept by using different results.

**3.1. Definition** A double framed soft fuzzy set $(F,f)$ with the parameter set $R_N$ is said to be a double framed B-soft fuzzy near ring over $(U,[0,1])$, if the following conditions hold for all $u, v \in R_N$,

**(BNR1)** $F(u-v) \subseteq F(u) \cup F(v),\ \ F(uv) \subseteq F(u) \cup F(v).$
**(BNR2)** $f(u-v) \geq f(u).f(v),\ \ \ f(uv) \geq f(u).f(v).$

**3.2. Example** Let $R_N = \{l,\ m,\ n,\ o\}$ be a given set with the sum "+" and the multiplication operations "." defined as follows

| **.** | $l$ | $m$ | $n$ | $o$ |
|---|---|---|---|---|
| $l$ | $l$ | $l$ | $l$ | $l$ |
| $m$ | $l$ | $l$ | $l$ | $l$ |
| $n$ | $l$ | $l$ | $l$ | $l$ |
| $o$ | $l$ | $l$ | $m$ | $m$ |

| $+$ | $l$ | $m$ | $n$ | $o$ |
|---|---|---|---|---|
| $l$ | $l$ | $m$ | $n$ | $o$ |
| $m$ | $m$ | $l$ | $o$ | $n$ |
| $n$ | $n$ | $o$ | $m$ | $l$ |
| $o$ | $o$ | $n$ | $l$ | $m$ |

It is routine to verify that $(R_N, +, \ .)$ is a (left) near ring. If one defines double framed soft fuzzy set $(F, f)$ over $(U, [0,1])$ in such a way that $F(n) = F(o) \supset F(m) \supset F(l)$ and $f(l) = 0.5$, $f(m) = 0.4, f(n) = 0.6 = f(o)$ then $(F, f)$ is a double framed B-soft fuzzy near ring over $(U, [0,1])$.

**3.3. Proposition** Let $(F, f)$ be a double framed B-soft fuzzy near ring over $(U, [0,1])$. Then the following properties are satisfied.

**(1)** $F(0) \subseteq F(u)$ and $f(0) \geq f(u)^2$ for all $u \in R_N$.
**(2)** $F(u) = F(-u)$.
(**3**) If $F(u - v) = F(0)$ and $f(u - v) = f(0)$, then $F(u) = F(v)$ and $f(u) \geq f(v)^3$.

Here 0 denotes the unit element of the near ring according to the sum operation.

**Proof**: **(1)** For any $u \in R_N$, we have $F(0) = F(u - u) \subseteq F(u) \cup F(u) = F(u)$ implies $F(0) \subseteq F(u)$.
And $f(0) = f(u - u) \geq f(u).f(u) = f(u)^2$ implies $f(0) \geq f(u)^2$.
(**2**) $F(u) = F(0 - (-u)) \subseteq F(0) \cup F(-u) = F(-u)$, by (1).
Now it follows that $F(-u) = F(0 - u) \subseteq F(0) \cup F(u) = F(u)$, by (1)
implies $F(u) = F(-u)$.
**(3)** Let$u, v \in R_N$ and $F(u - v) = F(0)$,$f(u - v) = f(0)$. We need to prove
$F(u) = F(v)$,$f(u) \geq f(v)^3$ *and* $f(v) \geq f(u)^3$. Now
$F(u)$= $F(u - v + v) = F((u - v) + v) \subseteq F(u - v) \cup F(v) = F(0) \cup F(v) = F(v)$,
$F(v)$= $F(v - u + u) = F((v - u) + u) \subseteq F(v - u) \cup F(u) = F(0) \cup F(u) = F(u)$ because $F(v - u) = F(-(u - v))$
Implies $F(u)$= $F(v)$.
$f(u) = f(u - v + v) = f((u - v) + v) \geq f(u - v).f(v) = f(0).f(v) \geq f(v)^3$,

**3.4. Definition** A double framed soft fuzzy set $(F, f)$ with the parameter set $A_N$ over $(U, [0,1])$ is said to be a double framed B-soft fuzzy subnear ring of the double framed soft fuzzy set $(G, g)$ with the parameter set $B_N$ over $(U, [0,1])$, if the following assertions hold.

**(1)** $A_N$ $is\ a\ subnear\ ring\ of\ B_N$
**(2)** $F(u) \subseteq G(u)$ and $f(u) \geq g(u)$, for each $u \in A_N$.

Note that a double framed soft fuzzy subset is a double framed B-soft fuzzy subnear ring if itself a double framed B-soft fuzzy near ring.

**3.5. Definition** Let $(F, f)$ and $(G, g)$ be the double framed soft fuzzy sets over the common universe $(U, [0,1])$ with the parameter sets $A_N$ and $B_N$, respectively. Then $(F, f)$ is said to be a double framed B-soft fuzzy twisted subnear ring of $(G, g)$, if the following assertions hold.

**(1)** $A_N$ $is\ subnear\ ring\ of\ B_N$.
**(2)** $F(u) \supseteq G(u)\ and\ f(u) \leq g(u)$, for each $u \in A_N$.

**3.6. Theorem** The uni-int of two double framed B-soft fuzzy near rings $(F, f)$and $(G, g)$ over $(U, [0,1])$ which has parameter set $R_N$, is a double framed B-soft fuzzy near ring over $(U, [0,1])$.

**Proof:** Let $u, v \in R_N$, then by the results above,

$$(F \tilde{\cup} G)(u - v) = F(u - v) \cup G(u - v) \subseteq \big(F(u) \cup F(v)\big) \cup \big(G(u) \cup G(v)\big)$$
$$= \big(F(u) \cup G(u)\big) \cup \big(F(v) \cup G(v)\big) = (F \tilde{\cup} G)(u) \cup (F \tilde{\cup} G)(v)$$

$$(F \tilde{\cup} G)(uv) = F(uv) \cup G(uv) \subseteq \big(F(u) \cup F(v)\big) \cup \big(G(u) \cup G(v)\big)$$
$$= \big(F(u) \cup G(u)\big) \cup \big(F(v) \cup G(v)\big) = (F \tilde{\cup} G)(u) \cup (F \tilde{\cup} G)(v).$$

$and$

$$(f \check{\barwedge} g)(u - v) = f(u - v) \wedge g(u - v) \geq \big(f(u).f(v)\big) \wedge \big(g(u).g(v)\big)$$
$$\geq \big(f(u) \wedge g(u)\big).\big(f(v) \wedge g(v)\big) = (f \tilde{\cap} g)(u).(f \tilde{\cap} g)(v)$$

$$(f \check{\barwedge} g)(uv) = f(uv) \wedge g(uv) \supseteq \big(f(u).f(v)\big) \wedge \big(f(u).f(v)\big)$$
$$\geq \big(f(u) \wedge g(u)\big).\big(f(v) \wedge g(v)\big) = (f \tilde{\cap} g)(u).(f \tilde{\cap} g)(v).$$

Hence, $< (F \tilde{\cup} G, f \check{\barwedge} g); R_N >$ is a double framed B-soft fuzzy near ring over $(U, [0,1])$.

**3.7. Theorem** Let $(F, f)$ and $(G, g)$ be the double framed soft fuzzy sets over $(U, [0,1])$ with parameter set $R_N$If $(F, f)$ is a double framed B-soft fuzzy subnear ring of $(G, g)$, then the int-uni of $(F, f)$ and $(G, g)$ is a double framed B-soft fuzzy near ring over $(U, [0,1])$.

**Proof**: Let $(F, f)$ be a double framed B-soft fuzzy subnear ring of $(G, g)$ and $u, v \in R_N$ be given. Then by the definitions the following implications are follows.

$(F \tilde{\cap} G)(u - v) = F(u - v) \cap G(u - v) = F(u - v) \subseteq \big(F(u) \cup F(v)\big)$
$=\big(F(u) \cap G(u)\big) \cup \big(F(v) \cap G(v)\big)$ because $F(u) \subseteq G(u)\ and\ F(v) \subseteq G(v)$.
$=(F \tilde{\cap} G)(u) \cup (F \tilde{\cap} G)(v)$.
$(F \tilde{\cap} G)(uv) = F(uv) \cap G(uv) = F(uv) \subseteq \big(F(u) \cup F(v)\big)$
$=\big(F(u) \cap G(u)\big) \cup \big(F(v) \cap G(v)\big)$ because $F(u) \subseteq G(u)\ and\ F(v) \subseteq G(v)$.
$=(F \tilde{\cap} G)(u) \cup (F \tilde{\cap} G)(v)$.
$(f \,\bar{\vee}\, g)(u - v) = f(u - v) \vee g(u - v) = f(u - v) \geq f(u).f(v)$
$=\big(f(u) \vee g(u)\big).\big(f(v) \vee g(v)\big)$ because $f(u) \geq g(u)$ and $f(v) \geq g(v)$
$=(f \,\tilde{\vee}\, g)(u).(f \,\tilde{\vee}\, g)(v)$.
$(f \,\bar{\vee}\, g)(uv) = f(uv) \vee g(uv) = f(uv) \geq \big(f(u).f(v)\big)$
$=\big(f(u) \vee g(u)\big).\big(f(v) \vee g(u)\big)$ because $f(u) \geq g(u)$ and $f(v) \geq g(v)$
$=(f \,\tilde{\vee}\, g)(u).(f \,\tilde{\vee}\, g)(v)$

Hence $< (F \tilde{\cap} G, f \,\bar{\vee}\, g); A_N >$ is andouble framed B-soft fuzzy near ring over $(U, [0,1])$.

**3.8. Theorem** Let $\varphi: E_N \to K_N$ be a near ring homomorphism and $\big((F, f), E_N\big)$ be a double framed B-soft fuzzy near ring of $E_N$ over $(U, [0,1])$. Then the image $\big((\varphi(F), \varphi(f)), K_N\big)$ is also a double framed B-soft fuzzy near ring of $K_N$ over $(U, [0,1])$.
Proof. It is shown by the definitions.

**3.9. Theorem** Let $\varphi: E_N \rightarrow K_N$ be a near ring homomorphism and $\big((G,g),K_N\big)$ be a double framed B-soft fuzzy near ring of $K_N$ over $(U,[0,1])$. Then the preimage $\big((\varphi^{-1}(F),\varphi^{-1}(f)),E_N\big)$ is also a double framed B-soft fuzzy near ring of $E_N$ over $(U,[0,1])$.

**Proof.** Let $u_1,u_2 \in E_N$ be given. Since $\varphi$ is a near ring homomorphism and $(G,g)$ is a double framed B-soft fuzzy near ring, we have the following inclusions.

**(BNR1)** $\varphi^{-1}(G)(u_1-u_2) = G\big(\varphi(u_1-u_2)\big) = G\big(\varphi(u_1)-\varphi(u_2)\big)$
$\subseteq G\big(\varphi(u_1)\big) \cup G\big(\varphi(u_2)\big) = \varphi^{-1}(G)(u_1) \cup \varphi^{-1}(G)(u_2).$
Similarly it is obtained that $\varphi^{-1}(G)(u_1u_2) \subseteq \varphi^{-1}(G)(u_1) \cup \varphi^{-1}(G)(u_2)$.

**(BNR2)** $\varphi^{-1}(g)(u_1u_2) = g\big(\varphi(u_1u_2)\big) = g\big(\varphi(u_1)\varphi(u_2)\big)$
$\geq g\big(\varphi(u_1)\big).g\big(\varphi(u_1)\big) = \varphi^{-1}(g)(u_1).\varphi^{-1}(g)(u_2).$
Similarly, it is obtained that $\varphi^{-1}(g)(u_1-u_2) \geq \varphi^{-1}(g)(u_1).\varphi^{-1}(g)(u_2)$.

### 3.2. Double Framed B-Soft Fuzzy Ideals

In this subsection, we present the ideal of a double framed B-soft fuzzy near ring named as a double framed B-soft fuzzy ideal.

**3.10. Definition** Let $(F,f)$ be a double framed B-soft fuzzy subnear ring of $R_N$ over $(U,[0,1])$. Then $(F,f)$ is said to be a double framed B-soft fuzzy ideal of $R_N$ if the following assertions hold for any $u,v,w \in R_N$.

**(BNI1)** $F(u) = F(v+u-v)$ and $f(u) = f(v+u-v)$.
**(BNI2)** $F(uv) \subseteq F(v)$ and $f(uv) \geq f(v)$.
**(BNI3)** $F\big((u+w)v-uv\big) \subseteq F(w)$ and $f((u+w)v-uv) \geq f(w)$.

Further, note that $(F,f)$ is said to be a left ideal of $R_N$ if it satisfies axioms of 3.1 Definition and (BNT1), (BNT2) of 3.8 Definition. $(F,f)$ is said to be a right ideal of $R_N$ if it satisfies the axioms of 3.1 Definition and (BNT1), (BNT3) of 3.8 Definition.

**3.11. Example** Let us take into consideration the near ring $(R_N,+,\dot{})$ given in 3.2 Example. If the double framed soft fuzzy set $(F,f)$ is defined in such a way that $F(n)=F(o) \supset F(m) \supset F(l)$ and $f(l) > f(m) > f(n) = f(o)$, then $(F,f)$ is a double framed soft fuzzy ideal of $R_N$ over $(U,[0,1])$.

**3.12. Proposition** Let $(F,f)$ be a double framed B-soft fuzzy ideal of $R_N$ over $(U,[0,1])$ then the followings are satisfied.

**(1)** $F(0) \subseteq F(u)$ and $f(0) \geq f(u)^2$ for all $u \in R_N$.
(**2**) If $F(u-v)=F(0)$ and $f(u-v)=f(0)$, then $F(u)=F(v)$ and $f(u) \geq f(v)^3$.

**Proof.** It is similarly proved to 3.3 Proposition.

### 3.3. Double Framed T-Soft Fuzzy Near Rings

In this subsection, we define and study the notions of double framed T-soft fuzzy near rings and ideals. Also, we give the homomorphic image and preimage of a double framed T-soft fuzzy near ring.

**3.13. Definition** A double framed soft fuzzy set $(F,f)$ over $(U,[0,1])$ with parameter set $R_N$, is said to be a double framed T-soft fuzzy near ring of $R_N$, if the following conditions hold for any $u,v \in R_N$.

(TNR1) $F(u-v) \subseteq F(u) \cup F(v)$, $F(uv) \subseteq F(u) \cup F(v)$
(TNR2) $f(u-v) \geq f(u) \wedge f(v)$, $f(uv) \geq f(u) \wedge f(v)$.

Note that 3.2 Example shows that $(F, f)$ over $(U, [0,1])$ is a double framed T-soft fuzzy near ring over $(U, [0,1])$.

**3.14. Theorem** Each double framed T-soft fuzzy near ring of $R_N$ is also a double framed B-soft fuzzy near ring of $R_N$.

**Proof** Let $(F, f)$ be a double framed T-soft fuzzy near ring of $R_N$. To prove that $(F, f)$ is a double framed B-soft fuzzy near ring, we only need to prove that $f(u - v) \geq f(u).f(v)$.
Since $(F, f)$ is a double framed T-soft fuzzy near ring, it is known that
$f(u - v) \geq f(u) \wedge f(v) \geq f(u).f(v)$ implies $f(u - v) \geq f(u).f(v)$.

Hence, the result is proved.

Note that the converse of the 3.14 Theorem is not true in general.

**3.15. Example** 3.2 Example shows that the converse of the 3.14 Theorem is not true in general since $f(on) = f(m) = 0.4 \ngeq F(o) \wedge F(n) = 0.6$.

**3.16. Definition** A double framed soft fuzzy set $(F, f)$ over $(U, [0,1])$ with parameter set $A_N$, is said to be a double framed T-soft fuzzy subnear ring of $(G, g)$ over $(U, [0,1])$ with parameter set $B_N$, if the following assertions hold.

**(1)** $A_N\ is\ subnear\ ring\ of\ B_N$
**(2)** $F(u) \subseteq G(u)\ and\ f(u) \geq g(u)$, for each $u \in A_N$.

Note that, a double framed T-soft fuzzy subset is said to be a double framed T-soft fuzzy subnear ring if itself a double framed T-soft fuzzy near ring.

**3.17. Definition** Let $(F, f)$ and $(G, g)$ be the double framed soft fuzzy sets over the common universe $(U, [0,1])$ with the parameter sets $A_N$ and $B_N$, respectively. Then $(F, f)$ is said to be a double framed T-soft fuzzy twisted subnear ring of $(G, g)$, if the following assertions hold.

**(1)** $A_N\ is\ subnear\ ring\ of\ B_N$.
**(2)** $F(u) \supseteq G(u)\ and\ f(u) \leq g(u)$, for each $u \in A_N$.

**3.18. Theorem** The uni-int of double framed T-soft fuzzy near rings $(F, f)$ and $(G, g)$ over the common universe $(U, [0,1])$ and has the common parameter sets $R_N$, is also a double framed T-soft fuzzy near ring of $R_N$.

**Proof**. Let $u, v \in R_N$ be given. Then by the assumption, the followings are true.
$$(F \tilde{\cup} G)(u - v) = F(u - v) \cup G(u - v) \subseteq \big(F(u) \cup F(v)\big) \cup \big(G(u) \cup G(v)\big)$$
$$= \big(F(u) \cup G(u)\big) \cup \big(F(v) \cup G(v)\big) = (F \tilde{\cup} G)(u) \cup (F \tilde{\cup} G)(v)$$
$$(F \tilde{\cup} G)(uv) = F(uv) \cup G(uv) \subseteq \big(F(u) \cup F(v)\big) \cup \big(G(u) \cup G(v)\big)$$
$$= \big(F(u) \cup G(u)\big) \cup \big(F(v) \cup G(v)\big) = (F \tilde{\cup} G)(u) \cup (F \tilde{\cup} G)(v).$$
$$(f \overline{\wedge} g)(u - v) = f(u - v) \wedge g(u - v) \supseteq \big(f(u) \wedge f(v)\big) \wedge \big(g(u) \wedge g(v)\big)$$
$$= \big(f(u) \wedge g(u)\big) \wedge \big(f(v) \wedge g(v)\big) = (f \tilde{\wedge} g)(u) \wedge (f \tilde{\wedge} g)(v)$$
$$(f \overline{\wedge} g)(uv) = f(uv) \wedge g(uv) \supseteq \big(f(u) \wedge f(v)\big) \wedge \big(f(u) \wedge f(v)\big)$$
$$= \big(f(u) \wedge g(u)\big) \wedge \big(f(v) \wedge g(v)\big) = (f \tilde{\wedge} g)(u) \wedge (f \tilde{\wedge} g)(v).$$
Hence, $< (F \tilde{\cup} G, f \overline{\wedge} g); R_N >$ is a double framed T-soft fuzzy near ring over $(U, [0,1])$.

**3.19. Theorem** Let $(F, f)$ and $(G, g)$ be the double framed soft fuzzy sets over the common universe $(U, [0,1])$ and has the common parameter set $R_N$. If $(F, f)$ is a subnear ring of $(G, g)$, then the int-uni of $(F, f)$ and $(G, g)$ is a double framed T-soft fuzzy near ring.

**Proof.** Let$(F, f)$ be a subnear ring of$(G, g)$and $u, v \in R_N$ be given. Then the following implications are satisfied.
$$(F \tilde{\cap} G)(u - v) = F(u - v) \cap G(u - v) = F(u - v) \subseteq \big(F(u) \cup F(v)\big)$$

$= \big(F(u) \cap G(u)\big) \cup \big(F(v) \cap G(v)\big)$ because $F(u) \subseteq G(u)$ *and* $F(v) \subseteq G(v)$.
$= (F \tilde{\cap} G)(u) \cup (F \tilde{\cap} G)(v)$
$(F \tilde{\cap} G)(uv) = F(uv) \cap G(uv) = F(uv) \subseteq \big(F(u) \cup F(v)\big)$
$= \big(F(u) \cap G(u)\big) \cup \big(F(v) \cap G(v)\big)$ because $F(u) \subseteq G(u)$ *and* $F(v) \subseteq G(v)$
$= (F \tilde{\cap} G)(u) \cup (F \tilde{\cap} G)(v)$
$(f \,\overline{\vee}\, g)(u - v) = f(u - v) \vee g(u - v) = f(u - v) \geq f(u) \wedge f(v)$
$= \big(f(u) \vee g(u)\big) \wedge \big(f(v) \vee g(v)\big)$ because $f(u) \geq g(u)$ and $f(v) \geq g(v)$
$= (f \,\tilde{\vee}\, g)(u) \wedge (f \,\tilde{\vee}\, g)(v)$
$(f \,\overline{\vee}\, g)(uv) = f(uv) \vee g(uv) = f(uv) \supseteq \big(f(u) \wedge f(v)\big)$
$= \big(f(u) \vee g(u)\big) \wedge \big(f(v) \vee g(u)\big)$ because $f(u) \geq g(u)$ and $f(v) \geq g(v)$
$= (f \,\tilde{\vee}\, g)(u) \wedge (f \,\tilde{\vee}\, g)(v)$
Hence $< (F \tilde{\cap} G, f \,\overline{\vee}\, g); R_N >$ is an int-uni double framed T-soft fuzzy near ring

**3.20. Theorem** A double framed soft fuzzy set $(F, f)$ over $(U, [0,1])$ with parameter set $R_N$ is a double framed T-soft fuzzy near ring if and only if following conditions hold for any $u, v \in R_N$,

**(1)** $F(u + v) \subseteq F(u) \cup f(v),\ F(uv) \subseteq F(u) \cup F(v)$
**(2)** $f(u + v) \geq f(u) \wedge f(v),\ f(uv) \geq f(u) \wedge f(v)$
**(3)** $F(u) = F(-u)$ *and* $f(u) = f(-u)$

**Proof.** First let us assume that the conditions (1)-(3) of the theorem are satisfied. Since
$F(u - v) = F(u + (-v)) \subseteq F(u) \cup F(-v) = F(u) \cup F(v)$, since $F(u) = F(-u)$,
$f(u - v) = f(u + (-v)) \geq f(u) \wedge f(-v) = f(u) \wedge f(v)$, since $f(u) = f(-u)$.

Hence, the result is proved.
The other direction of the theorem is clear.

**3.21. Proposition** Let $(F, f)$ be a double framed T-soft fuzzy near ring over $(U, [0,1])$. Then

**(1)** $F(0) \subseteq F(u)$ and $f(0) \geq f(u)$ for all $u \in R_N$.
(**2**) If $F(u - v) = F(0)$ and $f(u - v) = f(0)$, then $F(u) = F(v)$ and $f(u) = f(v)$.
**(3)** $\begin{cases} F(0) = F(u) \text{ if and only if } F(u+v) = F(v+u) = F(v) \\ f(0) = f(u) \text{ if and only if } f(u+v) = f(v+u) = f(v) \end{cases}$,
for a fixed $u \in R_N$ and for all $v \in R_N$

**Proof**. **(1)** For any $u \in R_N$, we have $F(0) = F(u - u) \subseteq F(u) \cup F(u) = F(u)$ implies $F(0) \subseteq F(u)$,
and $f(0) = f(u - u) \geq f(u) \wedge f(u) = f(u)$ implies $f(0) \geq f(u)$.

**(2)** Let $u, v \in R_N$ and $F(u - v) = F(0)$, $f(u - v) = f(0)$ to prove $F(u) = F(v)$, $f(u) = f(v)$. Now
$F(u) = F(u - v + v) = F((u - v) + v) \subseteq F(u - v) \cup F(v) = F(0) \cup F(v) = F(v)$,
$F(v) = F(v - u + u) = F((v - u) + u) \subseteq F(v - u) \cup F(u) = F(0) \cup F(u) = F(u)$ since $F(v - u) = F(-(u - v))$
This implies $F(u) = F(v)$.
$f(u) = f(u - v + v) = f((u - v) + v) \geq f(u - v) \wedge f(v) = f(0) \wedge f(v) = f(v)$,
$f(v) = f(v - u + u) = f((v - u) + u) \geq f(v - u) \wedge f(u) = f(0) \wedge f(u) = f(u)$ because $f(v - u) = f(-(u - v))$.

**(3)** Let $F(0) = F(u)$, $f(0) = f(u)$ to prove $F(u + v) = F(v + u) = F(v)$ and $f(u + v) = f(v + u) = f(v)$, for all $v \in R_N$, where $u \in R_N$ is fixed.

From (1) $\begin{cases} F(u) = F(0) \subseteq F(v) \\ f(u) = f(0) \geq f(v) \end{cases}$ for all $v \in R_N$

Now $F(u + v) \subseteq F(u) \cup F(v) = F(v)$ and $F(v + u) \subseteq F(v) \cup F(u) = F(v)$,
$F(v) = F(-u + u + v) \subseteq F(-u) \cup F(u + v) = F(u) \cup F(u + v) = F(0) \cup F(u + v) = F(u + v)$ and

$F(v) = F(v+u-u) \subseteq F(v+u) \cup F(-u) = F(v+u) \cup F(u) = F(v+u) \cup F(0) = F(v+u)$.
This implies $F(u+v) = F(v+u) = F(v)$. Further,
$f(u+v) \geq f(u) \wedge f(v) \geq f(v) \wedge f(v) = f(v)$ and
$f(v+u) \geq f(v) \wedge f(u) \geq F(v) \wedge f(v) = f(v)$. So,
$F(v) = F(-u+u+v) \geq F(-u) \wedge F(u+v) = F(u) \wedge F(u+v) = F(0) \wedge F(u+v) = F(u+v)$, and
$f(v) = f(v+u-u) \geq f(v+u) \wedge f(-u) = f(v+u) \wedge f(u) = f(v+u) \wedge f(0) = f(v+u)$.

**3.22. Theorem** Let $\varphi: E_N \to K_N$ be a near ring homomorphism and $\big((F,f), E_N\big)$ be a double framed T-soft fuzzy near ring of $E_N$ over $(U,[0,1])$. Then the image $\big((\varphi(F), \varphi(f)), K_N\big)$ is also a double framed T-soft fuzzy near ring of $K_N$ over $(U,[0,1])$.

**Proof.** Let $k_1, k_2 \in K_N$ be fixed. Then,

$$\varphi(F)(k_1+(-k_2)) = \bigcup_{(k_1+(-k_2))=\varphi(e_1)+\varphi(e_2)=\varphi(e_1+e_2)} F(e_1+e_2)$$
$$\subseteq \bigcup_{(k_1+(-k_2))=\varphi(e_1)+\varphi(e_2)=\varphi(e_1+e_2)} (F(e_1) \cup F(e_2))$$
$$\subseteq \left(\bigcup_{k_1=\varphi(e_1)} F(e_1)\right) \cup \left(\bigcup_{-k_2=\varphi(e_2)} F(e_2)\right) = \varphi(F)(k_1) \cup \varphi(F)(-k_2).$$
$$\varphi(F)(k_1k_2) = \bigcup_{(k_1k_2)=\varphi(e_3=e_1e_2)=\varphi(e_1e_2)} F(e_1e_2) \subseteq \bigcup_{(k_1k_2))=\varphi(e_1)\varphi(e_2)=\varphi(e_1e_2)} (F(e_1) \cup F(e_2))$$
$$= \left(\bigcup_{k_1=\varphi(e_1)} F(e_1)\right) \cup \left(\bigcup_{k_2=\varphi(e_2)} F(e_2)\right) = \varphi(F)(k_1) \cup \varphi(F)(-k_2).$$
$$\varphi(f)(k_1-k_2) = \varphi(f)(k_1+(-k_2))$$
$$= \bigvee_{(k_1+(-k_2))=\varphi(e_3=e_1+e_2)} f(e_1+e_2)$$
$$= \bigvee_{(k_1+(-k_2))=\varphi(e_1)+\varphi(e_2)} f(e_1+e_2)$$
$$\geq \bigvee_{k_1=\varphi(e_1),-k_2=\varphi(e_2)} \big(f(e_1) \wedge f(e_2)\big)$$
$$\geq \bigvee_{k_1=\varphi(e_1)} f(e_1) \wedge \bigvee_{-k_2=\varphi(e_2)} f(e_2) = \varphi(F)(k_1) \wedge \varphi(F)(-k_2).$$
$$\varphi(f)(k_1) \wedge \varphi(f)(k_2) = \bigvee_{k_1=\varphi(e_1)} f(e_1) \wedge \bigvee_{k_2=\varphi(e_2)} f(e_2)$$
$$\leq \bigvee_{k_1=\varphi(e_1),k_2=\varphi(e_2)} \big(f(e_1) \wedge f(e_2)\big) \leq \bigvee_{k_1k_2=\varphi(e_1e_2)} f(e_1e_2) = \varphi(f)(k_1k_2).$$

**3.23. Theorem** Let $\varphi: E_N \to K_N$ be a near ring homomorphism and $\big((G,g), K_N\big)$ be a double framed T-soft fuzzy near ring of $K_N$ over $(U,[0,1])$. Then the preimage $\big((\varphi^{-1}(F), \varphi^{-1}(f)), E_N\big)$ is also a double framed T-soft fuzzy near ring of $E_N$ over $(U,[0,1])$.

**Proof.** It is easily proved by the definitions.

### 3.4. Double Framed T-Soft Fuzzy Ideals

**3.24. Definition** A double framed T-soft fuzzy near ring $(F,f)$ of $R_N$ over the universe $(U,[0,1])$ is said to be a double framed T-soft fuzzy ideal of $R_N$, if it satisfies the conditions given in 3.10 Definition.

Further, note that $(F,f)$ is said to be a left ideal of $R_N$ if it satisfies axioms of 3.13 Definition and (BNT1), (BNT2) of 3.10 Definition. $(F,f)$ is said to be a right ideal of $R_N$ if it satisfies of 3.13 Definition and (BNT1), (BNT3) of 3.10 Definition.

**3.25. Example** Let $R_N = \{l,\ m,\ n,\ o\}$ be a classical set, and the sum "+" and the multiplication "." operations defined as given below.

| $+$ | $\boldsymbol{l}$ | $\boldsymbol{m}$ | $\boldsymbol{n}$ | $\boldsymbol{o}$ |
|---|---|---|---|---|
| $\boldsymbol{l}$ | $l$ | $m$ | $n$ | $o$ |
| $\boldsymbol{m}$ | $m$ | $l$ | $o$ | $n$ |
| $\boldsymbol{n}$ | $n$ | $o$ | $m$ | $l$ |
| $\boldsymbol{o}$ | $o$ | $n$ | $l$ | $m$ |
| | | | | |
| $\boldsymbol{.}$ | $\boldsymbol{l}$ | $\boldsymbol{m}$ | $\boldsymbol{n}$ | $\boldsymbol{o}$ |
| $\boldsymbol{l}$ | $l$ | $l$ | $l$ | $l$ |
| $\boldsymbol{m}$ | $l$ | $l$ | $l$ | $l$ |
| $\boldsymbol{n}$ | $l$ | $l$ | $l$ | $l$ |
| $\boldsymbol{o}$ | $l$ | $m$ | $n$ | $o$ |

It is routine to verify that $(R_N, +,\ .)$ is a (left) near ring. If the double framed soft fuzzy set $(F,f)$ is defined in such a way that $F(n) = F(o) \supset F(m) \supset F(l)$ and $f(l) > f(m) > f(n) = f(o)$, then $(F,f)$ is a left ideal of $R_N$ over $(U,[0,1])$ which is not a right ideal.

**3.26. Proposition** Let $(F,f)$ be a double framed T-soft fuzzy ideal of $R_N$ over $(U,[0,1])$ then, it satisfies the following properties.

**(1)** $F(0) \subseteq F(u)$ and $f(0) \geq f(u)$ for all $u \in R_N$.
(**2**) If $F(u-v) = F(0)$ and $f(u-v) = f(0)$, then $F(u) = F(v)$ and $f(u) = f(v)$.

**Proof.** It is similarly proved to 3.3 Proposition.

## 4. CONCLUSION

The theory of soft sets is defined by Molodtsov as a convenient tool to deal with uncertain phenomena. This theory is very useful since the parameters are considered in the calculations. But in the real life applications, it should be noted that all the parameters has their own importance degrees and these degrees change according to the decision makers and also, some parameters are omitted in some situations. So, the double framed soft fuzzy set theory is useful to solve these kinds of problems. Because, all the alternatives according to the parameters are given with their importance degrees. So, we found it reasonable to study with the double framed soft fuzzy set in algebraic aspect. We defined the double framed soft fuzzy near rings (ideals) and investigated their characteristics. For future work, we hope to apply the double framed soft fuzzy sets in decision making problems.

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**REFERENCES**

[1] Phuong, N. H., &Kreinovich, V. (2001). Fuzzy logic and its applications in medicine, International Journal of Medical Informatics, 62(2-3), 165-173.

[2] Zadeh, L. A. (1996). Fuzzy sets. In *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers by Lotfi A Zadeh* (pp. 394-432).

[3] Atanassov, K. T. (1999). Intuitionistic fuzzy sets. In *Intuitionistic Fuzzy Sets* (pp. 1-137). Physica, Heidelberg.

[4] Pawlak, Z. (1982). Rough sets. *International journal of computer & information sciences*, *11*(5), 341-356.

[5] Molodtsov, D. (1999). Soft set theory—first results. *Computers & Mathematics with Applications*, *37*(4-5), 19-31.

[6] Maji, P. K., Biswas, R., & Roy, A. (2001). Fuzzy soft sets. *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.

[7] Roy, A. R., &Maji, P. K. (2007). A fuzzy soft set theoretic approach to decision making problems. *Journal of Computational and Applied Mathematics*, *203*(2), 412-418.

[8] Mahmood, T., Ullah, M., Khan, M. B., &Ullah, K. (2017). On Twisted Soft Ideal of Soft Ordered Semigroups. *Journal of Information Communication Technologies and Robotics Applications (JICTRA). (Formally known as Journal of Computer Science of NICE). ISSN# 2226-3683*, *8*, 74-85.

[9] Khan, A., & Muhammad, N. (2016). On (∈,∈∨q)(∈,∈∨q)-intuitionistic fuzzy ideals of soft semigroups. *International Journal of Machine Learning and Cybernetics*, *7*(4), 553-562.

[10] A. Aygünoğlu, V. Çetkin, H. Aygün. (2014) An introduction to fuzzy soft topological spaces. Hacettepe Journal of Mathematics and Statistics, 43, 197-208

[11] V. Çetkin, H. Aygün. (2016) On convergence of soft nets, Journal of Multiple-Valued Logic and Soft Computing, 26(3-5), 175-187.

[12] V. Çetkin, H. Aygün. (2016) On L-soft Merotopies, Soft Computing, 20(12), 4779-4790

[13] Jun, Y. B., &Ahn, S. S. (2012). Double-framed soft sets with applications in BCK/BCI-algebras. *Journal of Applied Mathematics*.

[14] Cho, Y. U., Lee, K. J., & Jun, Y. B. (2015). A study on double-framed soft near-rings. *Applied Mathematical Sciences*, *9*(18), 867-873.

[15] Jun, Y. B., Muhiuddin, G., & Al-roqi, A. M. (2013). Ideal theory of BCK/BCI-algebras based on double-framed soft sets. *Applied Mathematics & Information Sciences*, *7*(5), 1879.

[16] Hadipour, A. R. (2014). Double-framed Soft BF-algebras. *Indian Journal of Science and Technology*, *7*(4), 491-496.

[17] Hayat, K., Cao, B. Y., Mahmood, T., & Tariq, K. U. H. (2016). Applications of double-framed soft ideals in BE-algebras. *New trends in Mathematical Sciences*, *4*(2), 285-295.

[18] Khan, A., Izhar, M., &Khalaf, M. M. (2017). Double-framed soft LA-semigroups. *Journal of Intelligent & Fuzzy Systems*, *33*(6), 3339-3353. [19] Khan, M. B., & Mahmood, T. (2018). Applications of Double Framed T-Soft Fuzzy Sets in BCK/BCI-Algebras, submitted.