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Research Article

A Note on Bipartite Graphs with Domination Number 2 and 3

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Abstract

When each edge of a connected G graph is replaced by a unit resistor, the resistance distance is computed as the effective resistance between any two vertices in G. The Kirchhoff index of G is given by the sum of resistance distances between all pairs of vertices. The multiplicative eccentricity resistance-distance (*MERD*) of a connected graph G is defined as $\xi_R^*(G)=\sum_{\{v_i,v_j\}\subseteq V_G}\varepsilon_G(v_i)\varepsilon_G(v_j)r_G(v_i,v_j)$, where V_G is the set of vertices of G, $r_G(v_i,v_j)$ is the resistance-distance between the vertices v_i and v_j, $\varepsilon_G(v_i)$ and $\varepsilon_G(v_j)$ are the eccentricity of the vertices v_i and v_j, respectively. The *MERD* of the G can be obtained by using Kirchhoff index. In this paper, we characterize the bipartite graphs which have the smallest and largest *MERD* with domination number 2 are given. We also characterize the bipartite graphs which have the smallest *MERD* with the domination number 3.

Keywords: Electric circuits, Kirchhoff index, Bipartite graphs, Resistance-distance.

Baskınlık Sayısı 2 ve 3 Olan İki Parçalı Graflar Üzerine Bir Not

Öz

Bağlantılı bir G grafının tüm kenarları birim direnç ile değiştirildiğinde, direnç mesafesi G'nin herhangi iki köşesi arasındaki efektif direnç olarak hesaplanır. G'nin Kirchhoff indeksi tüm köşe çiftlerinin direnç mesafelerinin toplamı olarak tanımlanır. V_G, G'nin köşelerinin kümesi, $r_G(v_i,v_j)$ ise v_i ile v_j köşeleri arasındaki direnç mesafesi ve $\varepsilon_G(v_i)$, $\varepsilon_G(v_j)$ de sırasıyla v_i ve v_j köşelerinin eksantriği olmak üzere, bağlantılı bir G grafının çarpımsal eksantrik direnç mesafası *(ÇEDM)* $\xi_R^*(G)=\sum_{\{v_i,v_j\}\subseteq V_G}\varepsilon_G(v_i)\varepsilon_G(v_j)r_G(v_i,v_j)$ olarak tanımlanır. G grafının *ÇEDM*'i Kirchhoff indeksini kullanarak hesaplanabilir. Bu makalede, baskınlık sayısı 2 olan iki parçalı graflardan en küçük ve en büyük *ÇEDM*'e sahip olanlar karakterize edilmiştir. Ayrıca baskınlık sayısı 3 olan iki parçalı graflardan en küçük *ÇEDM*'e sahip olanlar karakterize edilmiştir.

Anahtar Kelimeler: Elektik devreleri, Kirchhoff indeks, İki parçalı graflar, Direnç mesafesi.

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1. Introduction

All graphs are considered connected, simple and finite in this study. Let $G=(V_G,E_G)$ be a graph where $V_G=\{v_1,v_2,...,v_n\}$ denotes the vertex set of G and E_G denotes its edge set. The distance between any two vertices v_i and v_j in G is denoted by $d_G(v_i,v_j)$ is the length of shortest path between v_i and v_j. The eccentricity of a vertex v_i is denoted by $\varepsilon_G(v_i)$ is the maximum distance between v_i and a vertex in V_G. The diameter D of G is $\max\{\varepsilon_G(v_i):u\in V_G\}$. The resistance distan-ce is denoted by $r_G(v_i,v_j)$ which is defined as the effective resistance between any two vertices v_i and v_j, in G when all edges of G is replaced by a resistor of 1 Ohm. Let a battery be connected between vertices v_i and v_j; and let $I>0$ be the net current. The effective resistance $r_G(v_i,v_j)$ between vertices v_i and v_j is defined by

$$r_G(v_i,v_j)=\frac{v_i-v_j}{I}.$$

Suppose that G is a connected weighted graph and w_{ij} is the weight of the edge between v_i and v_j vertices. The resistance distance is defined as

$$r_G(v_i,v_j)=\frac{1}{w_{ij}}$$

when G is edge weighted. Firstly, Klein and Randic [11] introduced the resistance distance, resistance distance matrix and Kirchhoff index [3, 11] on the basis of electrical network theory.

The *MERD* $\xi_r^*(G)$ of a graph G is

$$\xi_r^*(G)=\sum_{v_i,v_j\in V_G}\left(\varepsilon_G(v_i)\varepsilon_G(v_j)r_G(v_i,v_j)\right)$$

where $\varepsilon_G(v_i)$ and $\varepsilon_G(v_j)$ are the eccentricity of the related vertices [8].

The minimum cardinality set $S\subseteq V_G$ of vertices is called the domination number of graph G when every vertex in V_G-S is adjacent to a vertex in S. Domination number can be denote by $\gamma(G)=\gamma$. Domination number is well studied in graph theory [1, 2, 4, 5, 6, 7, 12, 14].

In this paper, we characterize the bipartite graphs of domination number $\gamma=2$ and $\gamma=3$ which have the smallest and largest *MERD*.

2. Material and Method

We give some theoretical background related to bipartite graphs and the notion of domination number.

The graph $B_\gamma(n_1,n_2,...,n_{2\gamma-1})$ with order $N=\sum_{i=1}^{2\gamma-1}n_i$ is defined by changing the i th vertex on the path $P_{2\gamma-1}$ by independent sets V_i of n_i vertices. If any two vertices in the path $P_{2\gamma-1}$ are adjacent, these two vertices are also adjacent when they are in distinct sets. The graph $B_\gamma(n_1,n_2,...,n_{2\gamma})$ with order $N=\sum_{i=1}^{2\gamma}n_i$ is defined by changing the i th vertex on the path $P_{2\gamma}$ by independent sets V_i of n_i vertices. If any two vertices in the path $P_{2\gamma}$ are adjacent, these two vertices are also adjacent when they are in distinct sets. We should note that $B_\gamma(n_1,n_2,...,n_{2\gamma-1})$ and $B_\gamma(n_1,n_2,...,n_{2\gamma})$ are bipartite graphs with the same domination number γ.

In the following, we give some useful lemmas for the proof of our main results.

Lemma 2.1.

Let $G^+=G+e$ where e is a new edge between any two distinct vertices of G. Then $\xi_r^*(G)>\xi_r^*(G^+)$ [9].

Lemma 2.2.

Let G be a bipartite graph of diameter $D\geq 3$. Then G is a subgraph of a member in $B_D(n_1=1,n_2,...,n_D,n_{D+1}=1)$ [10].

We obtain the following result by using Lemma 2.2.

Corollary 2.3.

If $\gamma\geq 2$, all bipartite graphs with domination number γ is a subgraph of a member in the class of graphs

$$B_\gamma(n_1=1,n_2,n_3,...,n_{2\gamma-1}=1)$$

or

$$B_\gamma(n_1=1,n_2,n_3,...,n_{2\gamma}=1).$$

Lemma 2.4.

Let $G=B_D(n_1,n_2,...,n_{D+1})$ where D is the diameter of graph G. The Kirchhoff index $Kf(G)$ of the graph G is

$$Kf(G)=\sum_{i=1}^{D+1}\left(\frac{N-\sum_{k=1}^{i-1}n_k}{n_{i-1}n_i}\sum_{k=1}^{i-1}n_k\right)+N\sum_{j=1}^{D+1}\frac{n_j-1}{n_{j-1}+n_{j+1}}$$

where $n_0 = n_{D+2} = 0$ [10].

Using the above lemma, the following corollary can be given.

Corollary 2.5.

If we use the domination number γ of graph G instead of the diameter, then we calculate the Kirchhoff index of G as follows:

$$Kf(G) = \sum_{i=1}^{q}\left(\frac{N-\sum_{k=1}^{i-1} n_k}{n_{i-1}n_i}\sum_{k=1}^{i-1} n_k\right) + N\sum_{j=1}^{q}\frac{n_j - 1}{n_{j-1}+n_{j+1}}$$

where

$$q = \begin{cases} 2\gamma \text{ and } n_0 = n_{2\gamma+1} = 0, & \text{if } N = \sum_{i=1}^{2\gamma} n_i, \\ 2\gamma - 1 \text{ and } n_0 = n_{2\gamma} = 0, & \text{if } N = \sum_{i=1}^{2\gamma-1} n_i. \end{cases}$$

3. Results

In this section we give our main results for $B_\gamma(n_1, n_2, ..., n_{2\gamma})$ with order $N = \sum_{i=1}^{2\gamma} n_i$. Similar results can be given for $B_\gamma(n_1, n_2, ..., n_{2\gamma-1})$ with order $N = \sum_{i=1}^{2\gamma-1} n_i$.

Theorem 3.1.

Let G be a bipartite graph with $N (N \geq 6)$ vertices and domination number $\gamma = 2$. The graphs

$$B_2(1, \lfloor N/2 \rfloor, \lceil N/2 \rceil, 1)$$

have the smallest *MERD*.

Proof. Let G be a bipartite graph with domination number $\gamma = 2$ and N vertices. The graph G should be in the class $B_2(1, n_2, n_3, 1)$. Then by Corollary 2.5 we can calculate $Kf(B_2(1, K, N-K-2, 1))$ where $1 \leq K \leq 3$.

The *MERD* of $B_2(1, K, N-K-2, 1)$ (see, Fig. 1) is given by

$$\xi_R^*(B_2(1,K,N-K-2,1)) = 6Kf(B_2(1,K,N-K-2,1))$$
$$+3r_G(a_1,d_1) - 2\left[\sum_{x_i \in V_2, y_j \in V_3} r_G(x_i, y_j) + \sum_{x_i, x_j \in V_2} r_G(x_i, x_j)\right]$$
$$-2\sum_{y_i, y_j \in V_3} r_G(y_i, y_j).$$

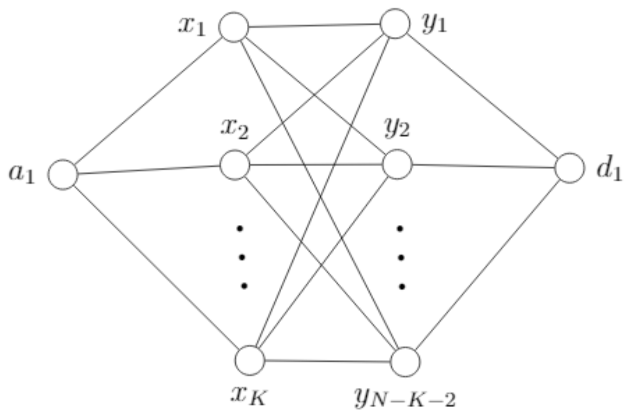

Fig. 1 $B_2(1, K, N-K-2, 1)$.

We calculate $r_G(a_1, d_1)$ at first. We put a battery between a_1 and d_1. The voltages at $x_1, x_2, ..., x_K$ and $y_1, y_2, ..., y_{N-K-2}$ are the same. So, we contract them into the vertices x' and y', respectively. The contracted graph has new resistance distance values on its each edge:

$$r_G(a_1, x') = 1/K,\ r_G(x', y') = 1/(K(N-K-2)),$$
$$r_G(y', d_1) = 1/(N-K-2).$$

Then we can easily obtain $r_G(a_1, d_1)$:

$$r_G(a_1, d_1) = \frac{1-N}{K(K-N+2)}.$$

Now, we should find $\sum_{x_i \in V_2, y_j \in V_3} r_G(x_i, y_j)$ (Fig. 2). In this step, it is enough to find $r_G(x_1, y_1)$. We delete $r_G(x_1, y_1)$ edge to obtain $B_2^-(1, K, N-K-2, 1)$. We put a battery between x_1 and y_1. The voltages at $x_2, x_3, ..., x_K$ and $y_2, y_3, ..., y_{N-K-2}$ are the same. So we contract them into the vertices x' and y', respectively.

This yields a new contracted graph and the resistance distance of the new graph is given as follows (Fig. 3):

$$r_G(a_1, x') = r_G(y_1, x') = 1/(K-1),$$
$$r_G(a_1, x_1) = r_G(d_1, y_1) = 1,$$
$$r_G(x', y') = 1/((K-1)(N-K-3)),$$
$$r_G(d_1, y') = r_G(a_1, y') = 1/(N-K-3).$$

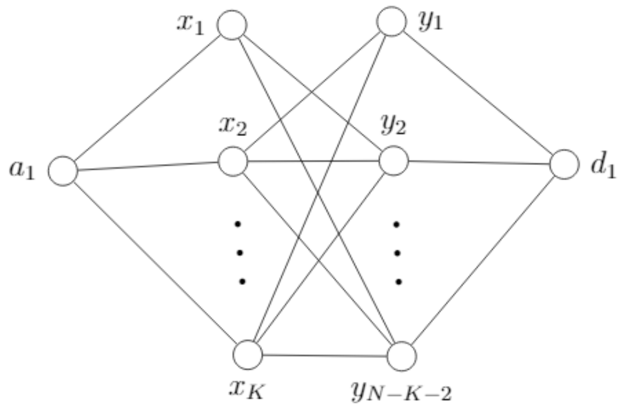

Fig. 2. Graphs used to find $\sum_{x_i \in V_2, y_j \in V_3} r_G\left(x_i, y_j\right)$.

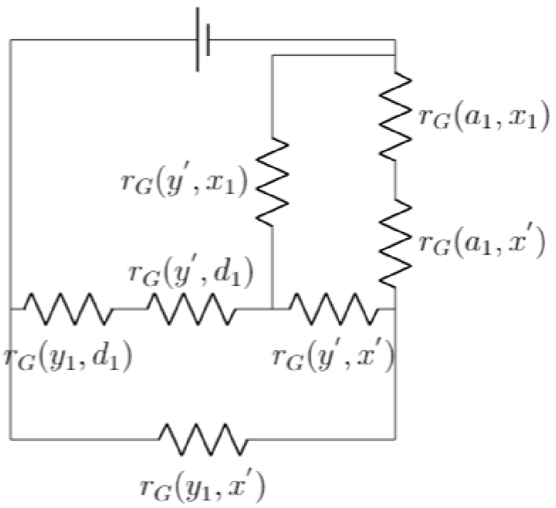

Fig. 3. The circuit diagram of Fig.2.

We compute $r_G^-\left(x_1, y_1\right)$ by using $Y-\Delta$ equivalent transformation:

$$r_G^-\left(x_1, y_1\right)=\frac{K^2\left(1-N\right)+2K+KN\left(N-3K\right)+1}{\left(K^2-K\left(2+N\right)+1\right)\left(K^2+K\left(2-N\right)-1\right)}$$

and

$$\sum_{x_i \in V_2, y_j \in V_3} r_G\left(x_i, y_j\right)=\frac{K^2N-K^2-KN^2+3KN-2K-1}{\left(K-N+1\right)\left(K+1\right)}.$$

We calculate $r_G\left(x_1, x_2\right)$ to find $\sum_{x_i, x_j \in V_2} r_G\left(x_i, x_j\right)$. Now, we put a battery between x_1 and x_2. Contracting $x_3, x_4, \ldots, x_K$ and $y_1, y_2, \ldots, y_{N-K-2}$ into the vertices x' and y', respectively. Then we obtain $r_G\left(x_1, x_2\right)$ by using $Y-\Delta$ equivalent transformation:

$$r_G\left(x_1, x_2\right)=2/\left(N-K-1\right)$$

and

$$\sum_{x_i, x_j \in V_2} r_G\left(x_i, x_j\right)=\frac{K\left(K-1\right)}{N-K-1}.$$

Lastly, we compute $\sum_{y_i, y_j \in V_3} r_G\left(y_i, y_j\right)$ by using circuit diagrams. We use a method similar to the one used to calculate $\sum_{x_i, x_j \in V_2} r_G\left(x_i, x_j\right)$. Thus we have

$$\sum_{y_i, y_j \in V_3} r_G\left(y_i, y_j\right)=\frac{\left(N-K-2\right)\left(N-K-3\right)}{K+1}.$$

Finally we calculate the *MERD* of $B_2\left(1, K, N-K-2, 1\right)$ as follows:

$$\xi_R^*\left(B_2\left(1, K, N-K-2, 1\right)\right)=6Kf\left(B_2\left(1, K, N-K-2, 1\right)\right)$$
$$+\frac{3\left(1-N\right)}{K\left(K-N+2\right)}-2\left[\frac{K^2N-K^2-KN^2+3KN-2K-1}{\left(K-N+1\right)\left(K+1\right)}\right]$$
$$-\frac{2K\left(K-1\right)}{N-K-1}-\frac{2\left[\left(N-K-2\right)\left(N-K-3\right)\right]}{K+1}.$$

It can be easily seen that $\xi_R^*\left(B_2\left(1, K, N-K-2, 1\right)\right)$ reaches its minimum value for $K=\left(N-2\right)/2$. If N is even

$$\xi_R^*\left(B_2\left(1, K, N-K-2, 1\right)\right)$$
$$=\frac{K^3\left(8K-28\right)+N\left(44N+60\right)-128}{N\left(N^2-4N+4\right)}.$$

If N is odd we have

$$\xi_R^*\left(B_2\left(1, K, N-K-2, 1\right)\right)=\frac{8N^3-28N^2+52N}{N^2-4N+3}.$$

Theorem 3.2.

Let G be a bipartite graph with N vertices and domination number $\gamma=2$. Then the graphs

$$B_2\left(\lfloor N/2\rfloor-1, 1, 1, \lceil N/2\rceil-1\right)$$

have the largest *MERD.*

Proof. Since the bipartite graphs with domination number $\gamma = 2$ which have the largest Kirchhoff index are tree, the bipartite graphs which have the largest *MERD* should be a member in $B_2(K,1,1,N-K-2)$ $(1 \le K \le N-3)$. From Corollary 2.5, we have

$$Kf\left(B_2\left(K,1,1,N-K-2\right)\right) = N^2 - \left(K+N\right)\left(K+2\right)+1.$$

We take

$T = \{t_1, t_2, ..., t_{N-K-2}\}$, $X = \{x_1, x_2, ..., x_K\}$, $Y = \{y_1\}$ and $Z = \{z_1\}$ where T, X, Y and Z are independent sets of n_i vertices.

Hence,

$$\xi_R^*\left(B_2\left(K,1,1,N-K-2\right)\right) = 6Kf\left(B_2\left(K,1,1,N-K-2\right)\right)$$
$$-2r_G\left(y_1,z_1\right)+3\left(\sum_{x_i\in X, t_j\in T} r_G\left(x_i,t_j\right)+\sum_{x_i,x_j\in X} r_G\left(x_i,x_j\right)\right)$$
$$+3\sum_{t_i,t_j\in T} r_G\left(t_i,t_j\right).$$

Clearly,

$$r_G\left(y_1,z_1\right)=1, \quad \sum_{t_i,t_j\in T} r_G\left(t_i,t_j\right) = \frac{2\left(N-K-2\right)\left(N-K-3\right)}{2},$$
$$\sum_{x_i\in X, t_j\in T} r_G\left(x_i,t_j\right) = 3K\left(N-K-2\right),$$
$$\sum_{x_i,x_j\in X} r_G\left(x_i,x_j\right) = \frac{2K\left(K-1\right)}{2}.$$

Thus, we have

$$\xi_R^*\left(B_2\left(K,1,1,N-K-2\right)\right)$$
$$= 9K\left(N-K\right)-18K+9N\left(1-3N\right)+22.$$

One can easily see that $\xi_R^*\left(B_2\left(K,1,1,N-K-2\right)\right)$ has the maximum value for $K = \lfloor N/2 \rfloor - 1$.

In the following theorem, we consider the almost complete bipartite graphs $G(m,p) = K_{m,m} - pK_2$ for convenience. The vertex set of the almost complete bipartite graph is $V = \{\alpha_1, \alpha_2, ..., \alpha_m\} \cup \{\beta_1, \beta_2, ..., \beta_m\}$ and its edge set is $E = \{\alpha_i\beta_j \mid 1 \le i, j \le n\} \setminus \{\alpha_1\beta_1, \alpha_2\beta_2, ..., \alpha_p\beta_p\}$ [13].

Now, we obtain the bipartite graphs which have the smallest *MERD* with domination number $\gamma = 3$ and order $N\left(N = 4\alpha,\ \alpha \in ¢^+ - \{1\}\right)$.

Theorem 3.3. The graphs

$$B_3\left(1, \frac{N}{4}, \frac{N}{2}-2, \frac{N}{4}, 1\right)$$

have the smallest *MERD* in almost complete bipartite graphs of order $N\left(N = 4\alpha,\ \alpha \in ¢^+ - \{1\}\right)$ with domination number $\gamma = 3$.

Proof. Since the bipartite graphs with domination number $\gamma = 3$ is a subgraph of

$$B_3\left(n_1 = 1, n_2 = M, n_3 = K, n_4 = N-M-K-2, n_5 = 1\right)$$

where $N = 4\alpha,\ \alpha \in ¢^+ - \{1\}$ and $1 \le M+K \le N-2$, by Corollary 2.5. we have

$$Kf\left(B_3\left(1,M,K,N-M-K-2,1\right)\right)$$
$$= \frac{N-1}{M} - \frac{N-1}{K+M-N+2} - N\left(\frac{K-1}{K-N+2} - \frac{M-1}{K+1}\right)$$
$$+ \frac{\left(K+M+1\right)\left(K+M-N+1\right)}{K\left(K+M-N+2\right)} - \frac{\left(M+1\right)\left(M-N+1\right)}{KM}$$
$$+ \frac{N\left(K+M-N+3\right)}{K+1}.$$

Let

$$X = V_1 \cup V_3 \cup V_5 = \{a_1, y_1, y_2, .., y_K, d_1\}$$

and

$$Y = V_2 \cup V_4 = \{x_1, x_2, ..., x_K, z_1, z_2, ..., z_{N-M-K-2}\}$$

where (X, Y) is the bipartition of the graph. Since the graph is an almost complete bipartite graph we have $X = Y$.

The eccentricities of vertices a_1 and d_1 are $\varepsilon(a_1) = \varepsilon(d_1) = 4$. Also, the eccentricities of vertices in V_2, V_4 and V_3 are $\varepsilon(V_2) = \varepsilon(V_4) = 3, \varepsilon(V_3) = 2$. Thus, the *MERD* of $B_3\left(1,M,K,N-M-K-2,1\right)$ is

$$\xi_R^*\left(B_3\left(1,M,K,N-M-K-2,1\right)\right)$$
$$= 12Kf\left(B_3\left(1,M,K,N-M-K-2,1\right)\right)$$
$$+4r_G\left(a_1,d_1\right) - 6\left[\sum_{x_i\in V_2, y_j\in V_3} r_G\left(x_i,y_j\right) + \sum_{y_i\in V_3, z_j\in V_4} r_G\left(y_i,z_j\right)\right]$$

$$-8\sum_{y_i,y_j\in V_3} r_G\left(y_i,y_j\right)-3\left(\sum_{x_i,x_j\in V_2} r_G\left(x_i,x_j\right)+\sum_{z_i,z_j\in V_4} r_G\left(z_i,z_j\right)\right)$$
$$-3\sum_{x_i\in V_2,z_j\in V_4} r_G\left(x_i,z_j\right).$$

Clearly, the graphs

$$B_3\left(1,\frac{N}{4},\frac{N}{2}-2,\frac{N}{4},1\right)$$

have the smallest *MERD* in almost complete bipartite graphs of order $N\left(N=4\alpha,\ \alpha\in ¢^{+}-\{1\}\right)$ with domination number $\gamma=3$.

4. Conclusions and Recommendations

Jiang et al. [10], identified bipartite graphs of diameter 3 with the largest and smallest Kirchhoff index. In this paper, we identify the bipartite graphs with domination number $\gamma=2$ having the smallest, largest and with domination number $\gamma=3$ having the smallest *MERD* by using the results in [10]. Our main method consists of basic electric circuit rules. Future works include finding the largest *MERD* of the bipartite graphs with domination number $\gamma=3$ and the smallest, largest *MERD* of the bipartite graphs with domination number $\gamma\geq 4$.

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References

[1] S. Artmann, and A. Pruchnewski, "Constructing a Dominating Set for bipartite graphs in several Rounds", *Techn. Univ., Inst. für Mathematik*, 2009.

[2] S. Artmann and J. Harant, "Random procedures for dominating sets in bipartite graphs", *Discussiones Mathematicae Graph Theory,* vol. 30, pp. 277−288, 2010.

[3] D. Bonchev, A.T. Balaban, X. Liu, D. J. Klein, "Molecular cyclicity and centricityof polycyclic graphs I. Cyclicity based on resistance distances or reciprocal distances", *Int. J. Quantum Chem*. vol. 50, pp. 1-20, 1994.

[4] T. Gerlach and J. Harant, "A note on domination in bipartite graphs", *Discussiones Mathematicae Graph Theory* vol. 22, pp.229-231, 2002.

[5] J. Harant and D. Rautenbach. "Domination in bipartite graphs." *Discrete mathematics* vol. 309, pp. 113-122, 2009.

[6] J. Harant and A. Pruchnewski, "A note on the domination number of a bipartite graph", *Annals of Combinatorics* vol. 5, pp.175-178, 2001.

[7] M. A. Henning, I. Schiermeyer and A. Yeo, "A new bound on the domination number of graphs with minimum degree two", *the electronic journal of combinatorics* pp.12-12, 2011.

[8] Y. Hong, Z. Zhu, A. Luo, "Some transformations on multiplicative eccentricity resistance-distance and their applications", *Appl. Math. Comput*. vol. 323, pp. 75-85, 2018.

[9] Y. Hong, Z. Zhu, A. Luo, "Extremal graphs with diameter 2 for two indices on resistance-distance", *Discrete Math.* vol. 342, pp. 487-497, 2019.

[10] X. J. Jiang, W. H. He, Q. Liu, J. P. Li, "On the Kirchhoff index of bipartite graphs with given diameters", *Discrete Appl. Math.* vol. 283, pp. 512-521, 2020.

[11] D. J. Klein, M. Randic, "Resistance distance", *J. Math. Chem.* vol. 12, pp. 81-95, 1993.

[12] N. J. Rad, "New Probabilistic Upper Bounds on the Domination Number of a Graph", *The Electronic Journal of Combinatorics* pp.3-28, 2019.

[13] L. Ye, W. Yan, "Resitance between two vertices of almost complete bipartite graphs", Discrete Applied Mathematics, vol. 257, pp. 299-305.

[14] H. M. Xing, L. Sun, X. G. Chen, "An upper bound for domination number of 5-regular graphs", *Czechoslovak Mathematical Journal* vol. 56, pp. 1049-1061, 2006.