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Research Article

PREDICTION OF THE INFLUENCE OF GEOMETRICAL IMPERFECTION TO LOAD CARRYING CAPACITY OF CONICAL SHELLS UNDER AXIAL LOADING

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ABSTRACT

In the present study, the effect of geometrical imperfections on the load carrying capacity of conical shells under axial loading was investigated. In accordance with this purpose, limit loads for both ideal and imperfect conical shells were utilized via a series of numerical analysis. Numerical analyses were made for radially constrained conical shells at various shell thicknesses and semi-vertex angles. Thus, it is aimed to evaluate the effect of the geometric imperfections on shells with various geometrical parameters. In all numerical simulations, nonlinear geometry influences were included (GNIA – Geometrically nonlinear analysis with imperfection included). It is found that with increasing semi-vertex angle, the sensitivity of the structures to imperfection also reduces. Shell thickness $tshell$ has almost no effect on the sensitivity to imperfection.
Keywords: Conical shell, elastic buckling, axial compression, imperfection.

1. INTRODUCTION

One of the main circumstances to take into consideration for conical shells under axial loading is the loss of stability. When constituting a design of conical shells, it must be paid sufficient attention to any probable imperfection of the structure. Aforementioned imperfections might be either geometrical or material basis. Geometrical imperfections are mostly originated from the production stage. A structure with a possible geometrical imperfection can lose its stability at a much lower load value contrary to calculations. Another point to take into consideration is the characteristics of the imperfection. The effect of the magnitude of the imperfection on the stability must be known. Thus, the production tolerances can be specified for different cases.

The effect of the geometrical imperfections on the loss of stability of truncated conical shells has been investigated by many prominent authors.

Cooper and Dexter investigated the effect of a particular type of local imperfection on the buckling of an axially compressed thin-walled conical shell. A two-dimensional shell analysis program was used in the buckling calculations. The study presented for bifurcation buckling

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analysis of a highly localized imperfection resulted in no significant drop in buckling load. However, linear static stress analyses revealed that the imperfection causes a local stress rise over 50% higher than the maximum stress in the ideal cone. Additionally, increase in the circumferential arc length of the imperfection causes a reduction up to 50% of the critical load of the ideal shell.

Wunderlich and Albertin studied the effect of four different boundary conditions and initial geometric imperfections on the load carrying capacity of conical shells, numerically. Initial geometrical imperfections were modeled with respect to the shape of the lowest bifurcation mode. It was seen in the results that both boundary conditions and magnitudes of the initial geometric imperfections have a significant effect on the load carrying capacity. Also excepting one case (cone-cylinder case), the lowest bifurcation mode has a great usability as the initial imperfection shape for limit load estimation of imperfect conical shells.

Zielnica performed an analysis and a numerical study for inelastic large deformation instability of conical shells under axisymmetric loading. The Prandtl-Reuss incremental plastic flow theory was considered for the bilinear elastic-plastic material model. It was found that the incremental flow theory was quite easy to apply in the current case because of its simplicity in the mathematical representation. The study pointed out that the inelastic deformations and buckling loads of conical shells under axisymmetric load can be determined by the currently developed iterative method.

Goldfield et al. studied the imperfection sensitivity of isotropic conical shells using initial post-buckling analysis. Three different shell theories were considered and a general code was developed to use in the parametric analysis with respect to the cone semi-vertex angle. Cone vertex half angle was found to be highly influent on the imperfection sensitivity of the structure. The most sensitive behavior was detected up to $\propto=20°$ and beyond this value (up to $\propto=80°$), the structure becomes more insensitive to imperfections. It is stated that the level of sensitivity depends on the angle, length to radius ratio and the boundary conditions.

Jabareen and Sheinman developed post-buckling analyses for imperfect conical shells. By including the geometrically nonlinear behavior of the structure, a Fortran code which uses Galerkin, Newton-Rapson and arc-length procedures and the finite-differences scheme were developed. A Fortran code was used for a parametric study to investigate the nonlinear behavior of conical shells in a wide range of cone semi-vertex angles. The study stated that the imperfection shape and amplitude has a strong effect on the limit load of the structure. Additionally, it was pointed out that a small imperfection amplitude leads to a localized buckling pattern while a large imperfection amplitude causes a global buckling behavior.

Goldfield investigated the imperfection sensitivity of filament wound laminated conical shells with respect to three different shell theories using initial post-buckling analysis. The influence of the variation of the stiffness coefficients on the buckling behavior was also studied. The study states that laminated conical shells are usually sensitive to imperfection and the sensitivity does not reduce at higher semi-vertex angles contrary to isotropic conical shells.

Jabareen and Sheinman investigated the influence of the nonlinear pre-buckling deformations on the buckling load of stiffened conical shells. Ideal and imperfect conical shells under random axial, torsional and hydrostatic-pressure loading were examined with a user-defined computer code in order to estimate the bifurcation point. It was found that external stiffening tended to increase the buckling load and sensitivity.

Ifayefunmi and Blachut analyzed three types of imperfections, initial geometric imperfections, variations in wall thickness and imperfect boundary conditions. Buckling strength of imperfect cones was investigated under three types of loading: axial compression, radial pressure and combined loading with compression and pressure. In numerical analyses, cones were modeled as mild steel with elastic-perfect plastic material behavior. Results of the current numerical study showed that cones analyzed in the study are more sensitive to imperfection under combined loading than the case under single loading case. Loss of buckling strength of cones were about 64% for axial c ompression and 34% for radial pressure in the worst cases.

Blachut investigated the effect of axisymmetric bulge-type shape imperfections on the buckling strength of conical shells. Numerical examinations to mild-steel cones subjected to axial load and the lateral pressure were made. Inward, outward and both inward and outward shape imperfections were investigated in order to obtain information about the imperfection sensitivity of buckling load of the structures. The author used Tabu search algorithm and found that both inward and outward shape imperfections caused a significant reduction in the load carrying capacity of the structures.

In the present study, the effect of geometrical imperfections on the load carrying capacity of conical shells under axial loading was investigated. In accordance with this purpose, limit loads for both ideal and imperfect conical shells were utilized via a series of numerical analyses. Numerical analyses were accomplished for radially constrained conical shells with various shell thicknesses and semi-vertex angles. Thus it is aimed to observe the effect of the geometric imperfections on the shells at various geometrical parameters.

2. MATERIAL AND METHOD

Numerical analyses for the current study were carried out with two different commercial finite element package programs; Cosmos/M and Abaqus. FE models were generated using the user interface of the both programs and a basic sketch of the models are presented in Figure 1.

In Figure 1, the geometrical parameters are called as; r_1: upper radius, r_2: bottom radius, h: height of the stiff pipe, L: conical shell length, r_e: equivalent cylinder radius, α_c: angle of lower edge, β_c: semi-vertex angle, t_{shell}: shell thickness and F: axial load. Upper radius "r_1" and bottom radius "r_2" were defined as 50 mm and 250 mm, respectively. A relatively stiff pipe is modelled the with a height "h=10mm" located at the top of the truncated conical shell.

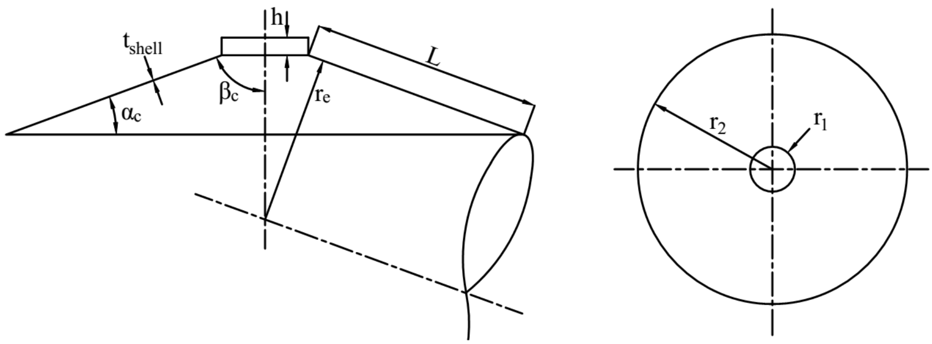


Figure 1. Front (on the left) and top view (on the right) of the conical shell.

Equivalent cylinder radius r_e is individually calculated by recommendations stated in "Buckling of steel shells European design recommendations (ECCS)". According to the ECCS recommendation, it is firstly needed to be determined either short or long conical shell is being studied based on the equations given below [1,2].

$$l_e = min\left[L; \left(\frac{r_2}{sin\beta_c}\right)(0.53 + 0.125\beta_c)\right] \quad (1)$$

where β_c is the semi vertex angle of conical shell in [Rad],

$$\beta_c = \frac{\pi}{2} - \alpha_c \quad (2)$$

if, $l_e = L$,, it means this structure is a short conical shell, equivalent radius is;

$$r_e = \frac{0.55r_1 + 0.45r_2}{cos\beta_c} \tag{3}$$

if, $l_e = \left(\frac{r_2}{sin\beta_c}\right)(0.53 + 0.125\beta_c)$, it means this structure is a long conical shell and the equivalent radius r_e is;

$$r_e = 0.71r_2 \frac{1 - 0.1\beta_c}{cos\beta_c} \tag{4}$$

According to the abovementioned equations taken from ECCS, for the current case, all models were determined as a long conical shell and each equivalent cylinder radius was calculated using equation (4). Calculated equivalent cylinder radius r_e were later used to obtain the r_e/t_{shell} dimensionless parameter. The range of the shell thickness t_{shell} and the dimensionless parameter r_e/t_{shell} are presented in Table 1 for each individual semi vertex angle β_c.

Table. 1 Variable geometrical parameters of the models.

Semi Vertex Angle β_c [°]	**Shell Thickness t_{shell}** [mm]	r_e/t_{shell} [-]
10	0.6 – 1	177 – 295
20	0.6 – 1	182 – 304
30	0.6 – 1	194 – 324
40	0.6 – 1	215 – 359
50	0.6 – 1	252 – 420
60	0.6 – 1	317 – 530
70	0.6 – 1	455 – 759
80	0.6 – 1	879 – 1465

Geometrical imperfections were formed via numerical model in the central region of the wall of the conical shell. The shape of the dent is circular with diameter $D_{imp} = l_g$. The length of the imperfection was considered as $l_g = 4\sqrt{r_2 t_{shell}}$ and characteristic depth of the imperfection was assessed to be equal to the shell thickness; $\Delta w = t_{shell}$. Geometrical parameters of the geometrical imperfection generated on the model is given in Figure 2 (on the left).

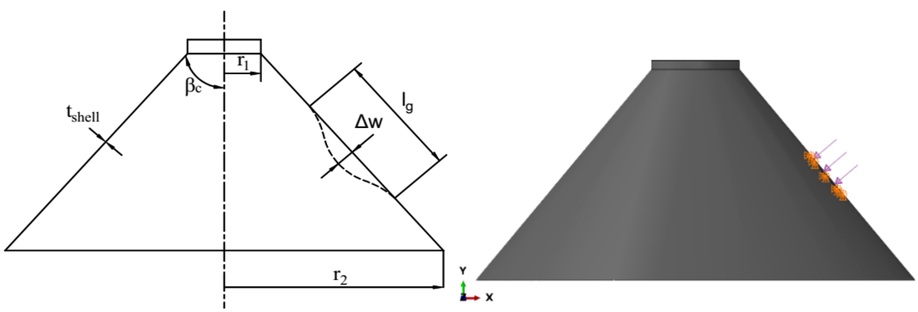


Figure 2. Geometrical parameters of the geometrical imperfection (on the left) and loads and boundary conditions to generate the imperfection (on the right).

The geometrical imperfection was created using an anticipatory static analysis. Edge of the imperfection was clamped in the simulations. Proper pressure value for generation of the imperfection was applied to only imperfection area in order to get the proper depth of the imperfection. Applied load and boundary conditions for static analysis are given in Figure 2 (on the right).

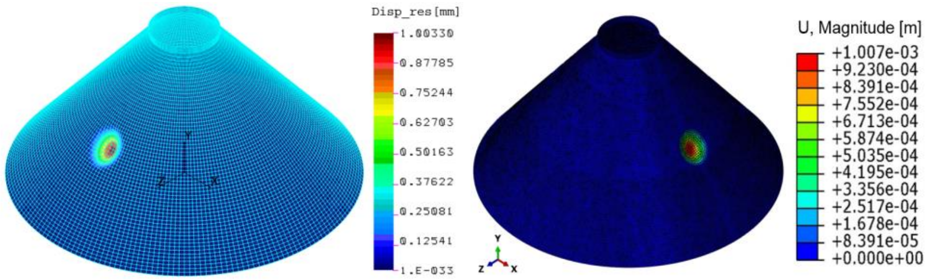


Figure 3. Mesh structure of the models in Cosmos/M (on the left) and Abaqus (on the right). (Deformation Scale factor = 15)

Generated imperfections through Cosmos/M and Abaqus using static analysis are exhibited in Figure 3. Characteristic depth of the imperfection was adjusted equal to the shell thickness of 1mm for the geometry corresponding 1mm of shell thickness and 40° of semi-vertex angle as mentioned above.

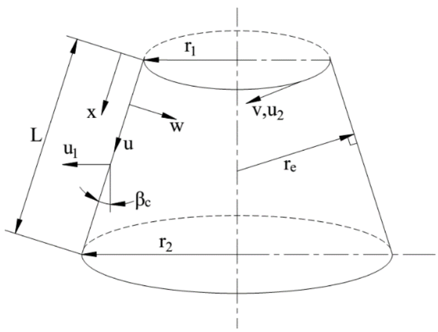


Figure 4. Basic truncated conical shell geometry and notations [12].

In all models, upper and lower circular ends of the truncated conical shells were constrained in the radial direction. In other words, these constraints are defined using the coordinates given in Figure 4 and 5. Boundary conditions are also shown in Figure 5 in detail.

$$u_1 = u \sin \beta_c - w \cos \beta_c = 0 \quad and \quad v = 0 \tag{5}$$

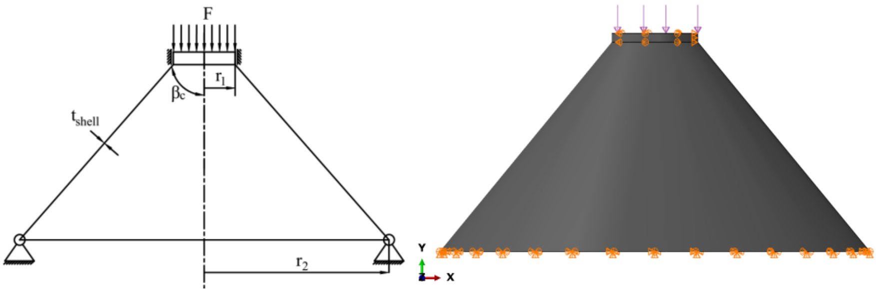


Figure 5. Schematic representation of the boundary conditions of FE models.

Material assigned for the models was chosen S235 steel and assumed to have a linear elastic and isotropic material behavior. The mechanical properties of the material for numerical analysis are; modulus of elasticity "E" of $200\ GPa$, Poisson's ratio "υ"of 0.3, and mass density "ρ"of $7850\ kg/m^3$.

All simulations were performed by considering the nonlinear geometry influence, so large displacement formulation was enabled in both FEA solutions. All analyses were performed with a mesh consisting of quadrilateral shell elements. These elements are called "SHELL4" [13] in Cosmos/M and "S4" [14] in Abaqus. In Figure 6, mesh structures of the model with a 40° of semi-vertex angle are illustrated for the both programs.

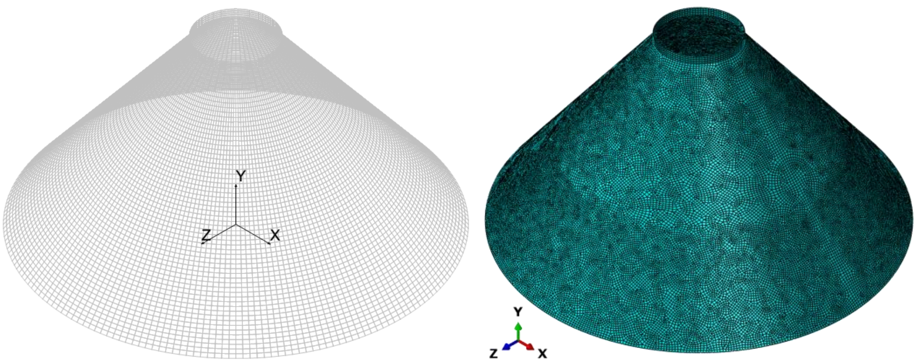


Figure 6. Mesh structure of the models in Cosmos/M (on the left) and Abaqus (on the right).

3. RESULTS AND DISCUSSION

Load vs. end-shortening curves of imperfect conical shells under axial loading were obtained via commercial finite element package programs Cosmos/M and Abaqus.

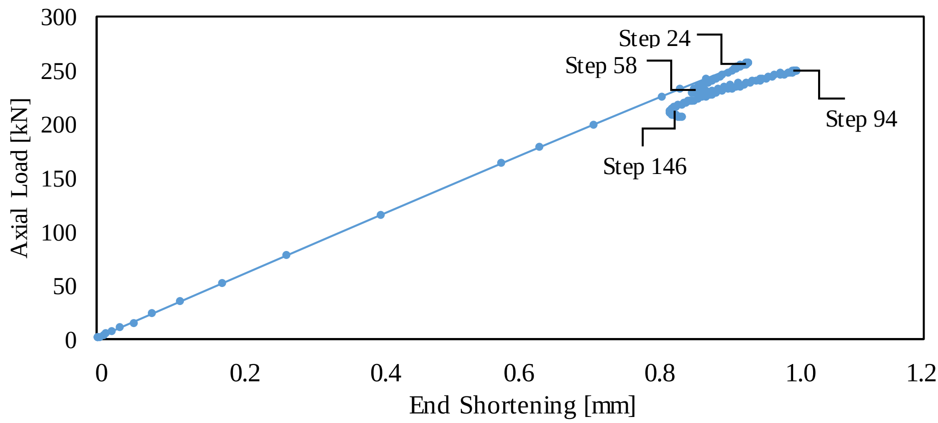
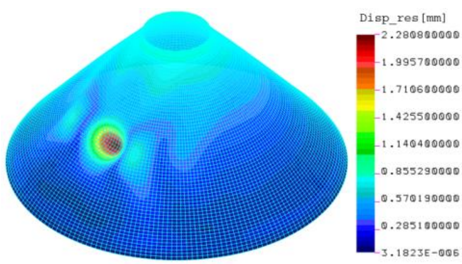
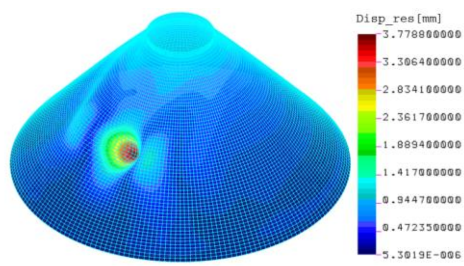


Figure 7. F – End-shortening curve obtained from Cosmos/M for $\beta_c = 60°$ and $t_{shell} = 1mm$

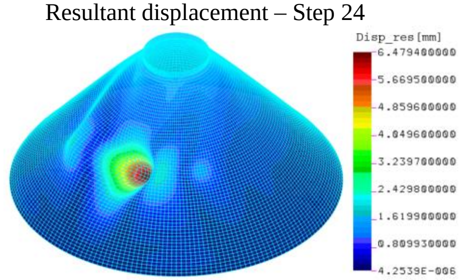
The FE analysis of the models was performed considering large displacement formulation after creating the imperfections in the previous step. Load values were recorded from a specified reference point are plotted in Figure 7 with respect to end-shortening of the cone.



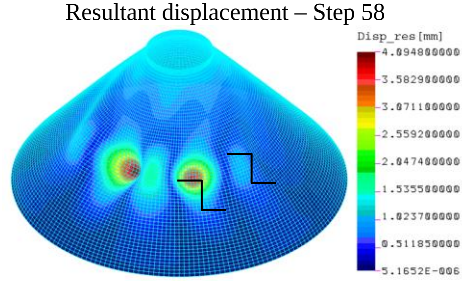
Resultant displacement – Step 24



Resultant displacement – Step 58



Resultant displacement – Step 94



Resultant displacement – Step 146

Figure 8. Resultant displacements of the models in Cosmos/M for individual steps. (Deformation scale factor = 15)

The load is increased in the simulations until the point at which the structure loses its stability at limit load. At this nonlinear buckling point, non-symmetrical deformation was observed, unlike the ideal shell structure. Geometrical imperfection on the structure caused a non-uniform deformation even though under uni-axial loading. At step 24, structure reaches its maximum load carrying capacity and a sudden drop occurs in the cones structural stiffness. At step 58, after some deformation, the strength of the structure begins to increase once again until step 94. However, after this point structure loses its stability again and a second imperfection on the structure begins to be visible. At step 146, the existence of the second imperfection can be seen clearly. All the critical points of progressively deformed structure, where the stiffness has a local maximum and minimum (inflection) value, are captured and illustrated in Figure 8.

Numerical simulations were performed for all shell thicknesses and semi-vertex angles which are determined by the current study. Limit load values obtained from the simulations are illustrated in Figure 9. Results are taken from the both finite element package programs and give a perfect match with a maximum deviation of 1%. Load carrying capacity of proposed shell structure has an inverse proportionality with the r_e/t_{shell} parameter. The curves at in Figure 9 have the characteristic of a power function with a decreasing trend. If a curve is fitted to the current data taken from the analyses, it is possible to develop an empirical expression with a deviation between -5% and +3%. This expression can be written as;

$$F_{lim,imp} = 7x10^6x(r_e/t_{shell})^{-1.9} \qquad [\text{kN}] \qquad (6)$$

where, $F_{lim,imp}$ is the limit load value for the conical shell structure with imperfect geometry.

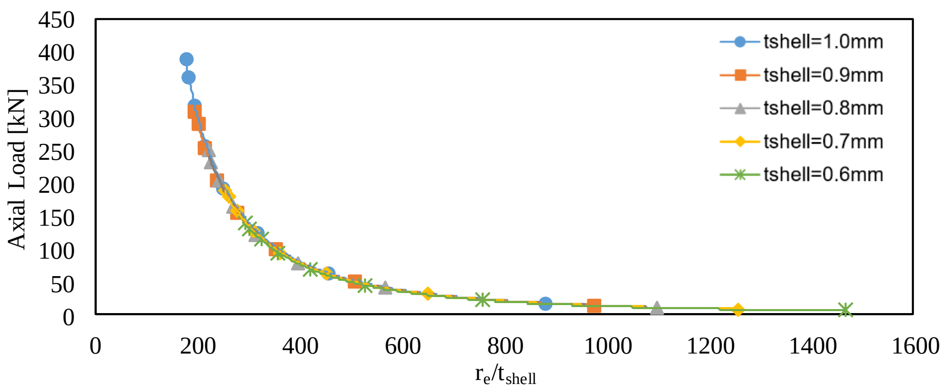


Figure 9. Limit load vs. r_e/t_{shell} curves

It is possible to calculate the load carrying capacity of a conical shell structure which has the described boundary conditions, equivalent radius and the shell thickness in the present study. In other words, the r_e/t_{shell} dimensionless parameter indicates the load carrying capacity of a structure under given boundary conditions and material properties. In Figure 9, it is obvious that the structures having the same r_e/t_{shell} value should have the same load carrying capacity but limited to parameters estimated in the study.

The effect of an imperfection which can occur either in operating conditions or in the manufacturing process is investigated in this study with respect to different shell thicknesses and semi-vertex angles. Structures without ideal geometric conditions have definitely lower load carrying capacity than the ideal shell structures. This reduction in the load carrying capacity can be represented with an "α" reduction coefficient and can be expressed as;

$$F_{lim,imp} = \alpha F_{lim,p} \qquad (7)$$

where, $F_{lim,p}$ is the limit load of an ideal conical shell structure. Hence, the lower the reduction coefficient means the lower the load carrying capacity of an imperfect structure. By considering the results obtained from two different finite element programs, calculated reduction coefficients are given in Figure 10.

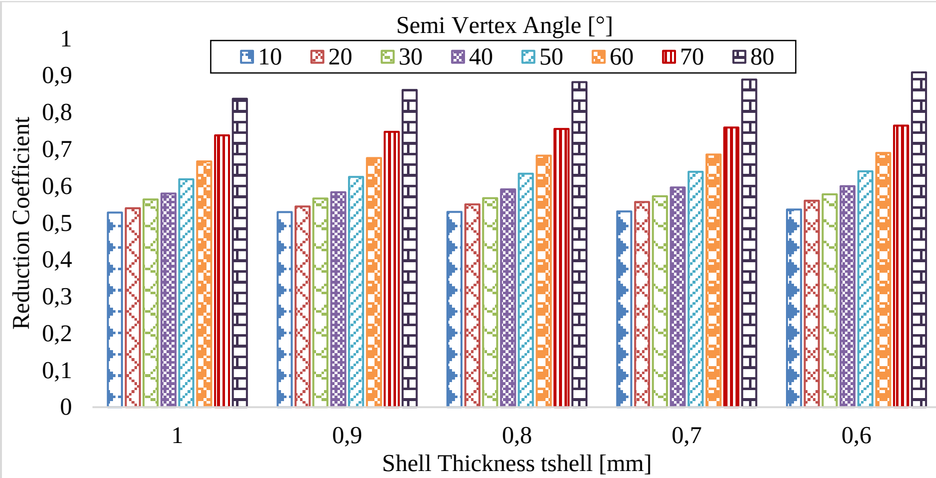


Figure 10. F vs. r_e/t_{shell} curves for the results of Abaqus

It is obviously seen in the bar graphs that the reduction coefficient of the structure increases with increasing semi-vertex angle. In the case of semi-vertex angle β_c is equal to 10°, reduction coefficient of the structures for all shell thicknesses are nearly the same and equal to 0.5. In this case, load carrying capacity of a conical shell reduces nearly to the half load carrying capacity of an ideal shell structure. With increasing semi-vertex angle, sensitivity of the structures to imperfection also reduces.

Shell thickness t_{shell} has almost no effect on the sensitivity to imperfection. Only in higher semi-vertex angles such as $\beta_c = 80°$, thickness barely affects the imperfection sensitivity of the structure. For instance, for the structure with semi-vertex angle $\beta_c = 80°$, if shell thickness increases from $t_{shell} = 0.6mm$ to $t_{shell} = 1mm$, reduction coefficient reduces from $\alpha = 0.91$ to $\alpha = 0.84$.

The magnitude of the semi-vertex angle of the conical shell strongly affects components of the applied axial load. With increasing semi-vertex angle, the load component which tends to bend the structure increases and the other component(meridional) in pure compression decreases. The reason for high sensitivity values at lower semi-vertex angles can be explained as the followings;

Imperfection on the shell surface distorts the stress flow on the side surface of the conical shell and causes stress concentrations on the imperfect area. At lower semi-vertex angles, the geometry of the structure approaches cylindrical shape which corresponds extremely high load carrying capacity. Therefore, an imperfection in the meridional direction leads severe amount of bending stresses under elevated loads. For this reason, amount of bending stresses due to imperfection (eccentricity in loading path) becomes highly dominant at lower semi-vertex angles. It is seen that the more meridional force acting on the structure makes the structure more sensitive to imperfections.

4. CONCLUDING REMARKS

In this study, the effect of a geometrical imperfection on the surface of a conical shell structure subjected to axial compression was investigated. All results obtained from numerical simulations (GNIA- geometrically nonlinear analysis with imperfection included) were compared. Main concluding remarks obtained from the present study are listed below.

- In accordance with the numerical simulations, the effect of the geometrical imperfections on the load carrying capacity of the conical shells can be represented by an empirical expression. This expression is based on the dimensionless parameter r_e/t_{shell} and characterised by a power function.
- The sensitivity of the structure against imperfections with a characteristic depth is not affected by the shell thickness.
- Influence of the imperfection quite changes with the semi-vertex angles. Especially, in the design process of a conical shell, it is necessary to take into consideration of a possible geometrical imperfection that the structure can include.

Acknowledgements

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