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Research Article

We need to make up for the gap: University student teachers' difficulties associated with basic algebraic manipulations

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Abstract

It is irrefutable that preparing successful mathematics teachers is a complex task, marked by a convergence of studies in content knowledge and instructional technologies. Considering the increasing number of students enrolling in South African teacher training institutions, it is essential to determine what mathematical knowledge gaps and understanding they bring from secondary school level for the purpose of configuring best strategies to prepare them to become effective teachers. This is the context for a study of first year undergraduate mathematics education students at the University of Limpopo. In this paper, we present our autoethnographical experiences of lecturing calculus courses for a teacher preparation programme. In this paper, we use autoethnography reflexivity to illustrate intersections between self and university society, the particular and the general, the personal and the politics of mathematical knowledge. The patterns that emerged from our interactions with students revealed that they experienced difficulties in understanding basic algebraic procedures and recognising structure to solve algebraic problems in the context of differentiation. This made us aware that we needed to configure effective strategies to make up for the identified elementary mathematics knowledge gaps, which we assumed students brought with from Grade 12. Our quest to make up for the algebraic knowledge gaps does not only serve the purpose of enabling our student teachers' mathematical knowledge, but to ensure that they develop good knowledge base needed to teach the subject during their training as well as once they qualify as teachers.

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Introduction

It is understandable that mathematics teacher preparation programmes characteristically includes the study of content knowledge and pedagogical knowledge that enables learners' epistemological access to mathematics content. With this in mind, it is important to note that South African universities have students from diverse schooling contexts, geographically, culturally and academically. Accordingly, it is our responsibility as teacher educators, to critically evaluate prospective teachers' background knowledge of mathematics. This is one way of gaining insight into the quality of students' mathematics understanding, to help us identify possible gaps in their content knowledge in order to configure strategies to address such gaps and ensure that we produce effective teachers. Wasserman (2018) suggests that the secondary school mathematics should inform how mathematics for teacher preparation programmes are structured and taught. In this vein, our concern is that prospective teachers enter the university system with content and procedural knowledge gaps from secondary school which we need to make up for in our programmes.

According to Wood and Solomonides (2008), it is essential to focus on preparing prospective teachers for the profession, rather than focusing on the mathematics difficulties they had in secondary school. While focusing on

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helping students develop their professional and mathematical identities is important, as teacher educators, we cannot simply ignore the knowledge gaps that students come to university with, as this plays a significant role in their learning in mathematics undergraduate courses. Speer et al. (2015) states that "evidence suggests that prospective high school mathematics teachers, who earn a mathematics major or its equivalent, do not have sufficiently deep understanding of the mathematics of the high school curriculum" (Speer et al. 2015, p. 107). One reason for this could be the oversight teacher educators make in understanding and working with students' level of content relating to secondary school curriculum. This paper focuses on illuminating some of the prospective teachers' knowledge gaps in understanding what we consider to be basic algebraic procedures, which from our experiences constrain their more abstract thinking and learning of the topics in our mathematics teacher preparation programmes.

Literature Review

Prospective teachers' limited understanding of mathematical concepts and procedures is extensively documented in various previous research studies (Ryan & Williams, 2011; Anthony et al. 2012; Tunç & Durmuş, 2012; Askew, Bowie & Venkat, 2019). The absence of preservice mathematics teachers' mathematical content knowledge absence has been described in terms of conceptual flaws that students make, mistakes in mathematics conventions as well as inappropriate computation and application of mathematical procedures (Amador et al. 2017; Putra & Winsløw, 2018). In our case, we have observed that pre-service mathematics teachers' content knowledge absence relating to algebraic rules constrain their use of mathematical contents they learn in undergraduate courses to answer questions, especially in cases where such content directly required them to draw from the secondary school mathematics we assumed they had. The prospective teachers' observed difficulties during lectures in our teacher education programmes provided us with information from which we can reinforce inferences in literature relating to prospective teachers' limited foundational mathematical content knowledge, which is required for further learning at university. We use the term foundational mathematics content to refer to mathematical content that the students are expected to come to university with from secondary school and should be able to demonstrate in teacher education programmes.

Within the concept of prospective teachers' mathematical content knowledge, both the importance of understanding students' misconceptions and errors and difficulties in engaging with mathematical thinking and processes have been documented in a number of studies (Koç & Bozkurt, 2011; Tunç & Durmuş, 2012). While this is the case, within the context of South African public universities, there are limited studies that offered insights into teacher educators' experiences of working with pre-service mathematics teachers who demonstrate conceptual flaws, mistakes in mathematics computations and/or apply mathematical procedures incorrectly (Ndlovu et al. 2017; Adendorff et al. 2019). We believe that teacher educators' reflections on students' errors or misconceptions they observe during lectures could help us understand different ways of thinking and knowing about undergraduate students' mathematical content knowledge absences and subsequently how such absences could be adequately addressed in initial teacher education programmes. This is vested on the idea that teachers' mathematical content knowledge is developed during their training and before, thus teacher educators are better positioned to critically evaluate and understand pre-service teachers' levels of conceptual and procedural knowledge for the purpose of bridging the gaps before students graduate.

Theoretical Framework

In this paper, we use social practice theory as a theoretical lens to critically analyse and understand pre-service mathematics teachers' limited understanding of foundational mathematical concepts and procedures in our initial teacher education programmes. The social practice theory foregrounds the idea that students' errors and difficulties in learning are inherently part of the teaching and learning processes, for both experienced and apprentice in a particular field or practice (Yackel & Cobb, 1996; Bell et al. 2012; Brodie, 2013). Within the social practice position, reflections and conversations about the errors and content absences students demonstrate while solving mathematical problems are essential for creating ways of thinking about the practice and the mathematics education community. Students' errors and difficulties in computing mathematical procedures during teaching and learning provides opportunities to establish the criteria to foreground what counts as "…valid or invalid production mathematically" (Brodie, 2013, p. 224). In our case, we consider the conversations we had with the students during lectures to be significant in supporting pre-service teachers to develop correct conceptual and procedural understanding, of the knowledge they are supposed to bring with to university.

In our critical analysis of our prospective mathematics teachers' errors and content absences in their procedures while solving mathematical problems, we employ three principles of social practice theory to create meanings and educational knowledge (Holland & Lave, 2009; Brodie, 2013). The first principle emphasises the idea that students' errors and lack of understanding are sensible and reveal (mathematical) thinking among students. Secondly, students' mathematical errors and absences in their foundational content are typical and integral part of mathematics learning, especially considering that our pre-service teachers come from predominantly rural schools, which are reported to have sub-standard mathematics teaching (Spaull, 2013; Masinire, 2015; Mbhiza, 2021). Thirdly, students' mathematical errors and content absences in their mathematisation processes give educators access and understanding of current students' thinking as well as their current ways of doing mathematics. In the context of understanding and problematizing prospective teachers' foundational content knowledge during lectures, this last principle gives us access to possibilities for future development in their mathematical thinking and procedural fluency when they qualify as mathematics teachers. In the current paper, we offer dense descriptions of what we learned about prospective mathematics teachers' errors and content knowledge absences relating to basic knowledge of algebra, which is typically knowledge students should know and own from secondary school. In the following section we present the methodological approach we espoused in this paper and how the approach enabled us to evaluate and illuminate various errors and absences in students' foundational content knowledge in our pre-service mathematics problems.

Methodology

An autoethnographic approach focuses on describing and systematically analysing personal experiences of authors for the purpose of sharing lived experiences (Ellis et al. 2011; Adams et al. 2017). In the current paper, we present and critically examine our experiences of teaching a mathematics education course at first year undergraduate level, to offer understanding of the patterns that emerged during lectures. We believe that the personal experiences in teaching mathematics education and preparing future mathematics teachers is an important source of knowledge in and of itself, as authors "look inward and outward, exposing a vulnerable self that is moved by and may move through, refract, and resist cultural interpretations" (Ellis, 2009, p. 10). In this paper, we use autoethnography reflexivity to demonstrate intersections between self and university society, the particular and the general, the personal and the politics of mathematics knowledge. We particularly reflect on and present our autoethnographical experiences related to the students' foundational mathematics knowledge gaps in our initial teacher educational programmes. As stated earlier, our students demonstrated conceptual and procedural knowledge gaps during lectures on different mathematics education topics. Accordingly, we use extracts from our debriefing reflections on our lectures, to illuminate critical incidences that revealed students' conceptual and procedural knowledge and understanding, positioning us as both insiders and outsiders in the system of observing our students' mathematical knowing and understanding (Boylorn, 2011; Cooper et al. 2017). In this paper, we articulate our insider experiences and knowledge of teaching prospective mathematics teachers who demonstrated limited understanding of mathematical procedures which we assumed to be foundational – knowledge we expected that they brought to university from earlier levels of schooling.

We believe that as writers, we can inform readers about aspects of mathematics education teaching that other researchers may not be able to understand and/or take for granted. As far as it can be determined, within the South African Higher Education context, there is a dearth of autoethnographical accounts of mathematics teacher educators' knowledge and experiences of lecturing prospective teachers. We argue that as mathematics teacher educators, we are best positioned to talk about prospective teachers' mathematical knowledge in ways different from others who do not have experiences with this aspect. While this is the case, "Insider knowledge does not suggest that an autoethnographer can articulate more truthful or more accurate knowledge as compared to outsiders, but rather that as authors we can tell our stories in novel ways when compared to how others may be able to tell them" (Adams et al. 2017, p. 3). For the purpose of this paper, to think with a story about prospective teachers' limited foundational knowledge is to experience such knowledge gaps affecting our teaching as teacher educators and to find in that effect particular truths of our lives as mathematics education lecturers.

Data Collection and Analysis in Autoethnography

Previous studies that used autoethnography (Adams, 2011; Duncan, 2004; Gobo, 2008; Pelias, 2011) have suggested the need for 'hard' evidence to support 'soft' impressions, and to generate interpretations and make claims about personal experiences. In this paper, we present our experiences of teaching mathematics education at undergraduate level as stories to offer insight into the patterned processes in our interactions with students, and into the knowledge gaps they demonstrated during teaching and learning. We use extracts of the errors students made during lectures and reflective lectures discussions as our data. We analysed and re-analysed the observed errors students made during learning and the notes from the lectures that we individually wrote and discussed together using thematic content analysis and using the processes of 're-memory' to identify commonalities and differences in students' difficulties associated with algebraic rules.

Ethical Consideration

In writing about subjective experiences of a particular phenomenon, Ellis (2009, 42) cautions that, "autoethnographic stories of our experiences, are not wholly our own; they implicate relational others in our lives". This statement addresses the core essence of relational ethics, which obliges researchers to make ethical considerations when speaking about others in our stories, in this paper, prospective teachers in our initial teacher prapation programme. To ensure protection of our students' identities, even though we talk about them in describing and discussing our experiences, we do not mention any identifying information throughout the paper. In this paper, we talk about our students as a group to ensure that we conceal their true identities. The rationale for this is vested in Tullis' (2013) argument that as autoethnographers, we should earnestly consider our "responsibilities to intimate others who are characters in the stories we tell about our lives" (p. 4). Thus, in the descriptions and discussions of our our experiences and observations of our students' foundational knowledge gaps for mathematics, we conceal their true identities through writing about the students as a group.

Narratives and Discussion

Prospective teachers are expected to have prompt recall of the elementary mathematics facts of addition, multiplication, division and subtraction in order to solve mathematical problems in initial teacher education courses purposefully and effectively. Our autoethnographies will demonstrate that the prospective mathematics teachers in our programme do not understand mathematical ideas and procedures that we consider to be basic to mathematics education courses. That is, we have experienced that the students do not possses the skills to engage in algebraic manipulative processes of even secondary mathematics, and in-turn this constrain their learning of new ideas. In this section, we critically present our autoethnographical experiences of working with prospective mathematics teachers' limited knowledge of the abovementioned basic facts during lectures on differentiation.

Simplifying Powers and Fractions as They Learn Rules of Differentiation

This section focuses on how prospective teachers are fraught by a lack of essential technical mathematics facility, in particular a lack of procedural fluency in algebraic simplification and manipulations during lectures on the rules of differentiation. The first critical incident that we have selected to use to highlight students' severe lack of technical facility was when we asked prospective teachers to apply the procedures to differentiate the function $f(t) = \sqrt{t}(1 - t)$ and present the derivative in its simplest form. The question required the application of the combination of two rules of differentiation: Power Rule and Product Rule. The objective was to assess whether they would be able to apply the Product Rule given the context of the problem. We gave the students time to solve the problem independently and later invited them to verbalise the steps to finding the solution and the final answer. Given the limitations we have with blackboard we decided to share what we thought would be the answer to the question to the benefit of those who might have got it wrong. Below is what we shared during the synchronous session.

$$
\begin{aligned}
f'(t) &= \sqrt{t}\frac{d}{dt}(1-t) + (1-t)\frac{d}{dt}\sqrt{t} \\
&= \sqrt{t}(-1) + (1-t) \times \frac{1}{2}t^{-\frac{1}{2}} \\
&= -\sqrt{t} + \frac{1-t}{2\sqrt{t}} = \frac{1-3t}{2\sqrt{t}}
\end{aligned}
$$

According to Aguilar and Telese (2018, p.25), "Teacher-candidates need to experience and face the struggle of solving different types of problems, which develop, not only their mathematical concepts, but also their ability to address student solutions from different perspectives". Thus, we gave propspective teachers time to go through the solution and invited them to make sense of the mathematical concentions and ask questions where they did not understand as a way of allowing them to think about mathematical processes applied in the example. To our surprise, almost all the questions asked were on how we moved from $(1 - t) \times \frac{1}{2}t^{-\frac{1}{2}}$ to $\frac{1-t}{2\sqrt{t}}$ and from $-\sqrt{t} + \frac{1-t}{2\sqrt{t}}$ to $\frac{1-3t}{2\sqrt{t}}$. We had assumed that the students would not struggle with the basic algebraic manipulations and calculations, but that they would have difficulties understanding the rule. That is, we did not expect students to lack elementary skills and knowledge for simplifying mathematical expressions. In view of this, the observed content absences during lectures gave us access and understanding of our students' mathematical thinking and difficulties, to help us configure preventative measures to help learners learn the procedures (Holland & Lave, 2009; Brodie, 2013).

The observed students' difficulties resonates with Sebsibe and Feza's (2020) iteration that, "Experiences and public evidence disclose that difficulties in calculus brought from grade 12 inhibit students' progress at university" (p. 2). In this regard, our students demonstrated two forms of elementary mathematics knowledge gaps or difficulties: rewriting powers using positive exponents and identifying a step where a sum of two fractions were simplified. At this level of studies, we often assume that our students are proficient with working with powers given the vast mathematical concepts that requires the application of laws of exponents in secondary school. Working with surds could also be a reason why they could not simplify the sum, thereby constraining their learning of abstract concept of differentiation. This was concerning for us, especially considering that the lesson had to shift from learning about the Product Rule and its application to simplifying powers, identifying the common denominator when adding fractions that involves variables, and working with surds, which are skills students should come with from Grade 12.

Our concern amplified when we noticed that the above was not an isolated case, as a similar observation was made when we introduced students to L'Hopital's Rule. In this instance, we asked prospective teachers to calculate $\lim_{x\to\infty}\frac{lnx}{\sqrt[3]{x}}$. After giving them time to try it out, we shared the solution depicted in image 1 below and invited them to have a look at it and comment on it.

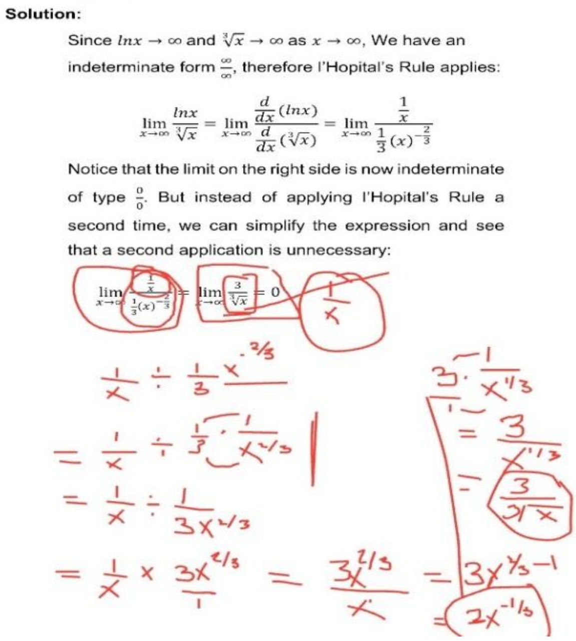

Image 1 .

Solution We Shared with the Students

Again, the questions that students asked were not on the application of L'Hopital's Rule, but on how we simplified $\frac{\frac{1}{x}}{\frac{1}{3}(x)^{-\frac{2}{3}}}$ to get $\frac{3}{\sqrt[3]{x}}$. Similar to the lesson on the Product Rule, the lesson's focus also moved from L'Hopital's Rule to focus on the calculation processes to simplify powers using positive exponents. The handwriting in image 1 was an attempt to explain the calculation processes. In relation to social practice theory, modelling this calculation processes to the students was essential in supporting the students to develop correct conceptual and procedural understanding, of the knowledge we iniallty thought they brought with to university (Brodie, 2013). This is the knowledge that we assume students come to university with and yet students fail, not because they do not understand high level mathematics, instead because they struggle with the algebraic manipulations that they should have mastered in their secondary school mathematics. Existing literature (Pillay, 2008; Maharaj, 2010; Siyepu, 2015), has also demonstrated that most students' knowledge gaps in algebraic manipulation skills from lower levels of schooling limits their performance in university mathematics.

Prospective Teachers' Mathematical Knowledge of Equivalence

Among one of the practices we conduct as lecturers of mathematics for educators modules is the study of prospective teachers' errors or difficulties that relate to their understanding of algebra, to ensure that we configure strategies to address the errors as we prepare them to become effective mathematics teachers. When differentiating composites of transcendental functions, we noticed that students saw equivalence in cases where it did not exist. This was more evident in composite functions for which the differentiation of the inner functions involved a product rule. When determining the derivative of $f(x) = e^{x\sin x}$, among the answers we found was:

$$\frac{d}{dx}\, e^{x \sin x} = e^{x \sin x} \cdot x \cos x + \sin x \cdot 1$$

While it can be argued that the mathematics prospective teachers have learned at this level was not a problem, the same could not be said for the algebra learned in high school. It can be claimed that prospective teachers were aware that they had to multiply the derivative of the outer function by the derivative of the inner function. But the student teachers faced difficulties in using an algebraic procedure that involved multiplication of a factor $e^{x \sin x}$ by a binomial $x \cos x + \sin x$ effectively. As part of the procedure, the brackets should have been used to show that in this case, as the binomial is a compound factor. In the absence of the brackets, we argue that the prospective teachers failed to realise that the product of $e^{x \sin x}$ and $x \cos x + \sin x$ is not $e^{x \sin x} \cdot x \cos x + \sin x$. In its basic form the structure of this multiplication is the same as $a(b + c) = ab + ac$. Rittle-Johnson et al. (2011, p. 2) states that "Understanding mathematical equivalence requires understanding that the values on either side of the equal sign are the same". Accordingly, our interpretation is that the students were unable to translate distributive law, to a different context within the mathematics domain, which is elementary computational manipulative. Knowledge of mathematical equivalence should be well developed when students enter university, as it is a prerequisite for learning higher level algebra effectively. Considering the importance of students' understanding of mathematical equivalence, it was concerning that students failed to notice the application of this concept.

A similar 'error' was made when students were asked to determine the derivative of $y = \ln(\cos x \tan 5x)$. Among the answers that were given is that

$$y' = \frac{1}{\cos x \tan 5x}\, \tan 5x \frac{d}{dx} \cos x + \cos x\, \frac{d}{dx} \tan 5x = \frac{-\sin x \tan 5x + 5 \cos x\ \sec^2 5x}{\cos x \tan 5x}$$

This answer shows that the students were not aware that structurally $\frac{1}{\cos x \tan 5x} \tan 5x \frac{d}{dx} \cos x + \cos x\, \frac{d}{dx} \tan 5x$ and $\frac{-\sin x \tan 5x + 5\cos x\ \sec^2 5x}{\cos x \tan 5x}$ are not the same and therefore not equivalent. This lack can be linked to students' poor sense of mathematical structure. Structure sense in this instance is taken as an ability to "recognise which manipulations is possible to perform" (Hoch & Dreyfus, 2005, p. 51). Again in this case, the brackets should have been used to group the terms that formed the binomial so that the multiplying factor $\frac{1}{\cos x \tan 5x}$ could be multiplied with each term in the binomial. Thus, the students were not able to "recognise an algebraic expression or sentence as a previously met structure" (Hoch & Dreyfus, 2005, p. 51). Put differently, the students failed to recognise similar mathematical properties in different forms (Hawthorne, & Druken, 2019). In contrast to applying set of procedures automatically as discoraged by Hawthorne and Drunken, by applying the rules of logarithms, the function $y = \ln(\cos x \tan 5x)$ could have been written as its equivalent $y = \ln(\cos x) + \ln(\tan 5x)$ before finding its derivative.

In view of the above, we argue that the students lacked comparative relational thinking, which is a prerequisite for solving equations and evaluating structures of equations through comparing the expressions on both sides of the equal sign. To move beyond the mere *noticing* of the mathematics knowledge gaps in prospective teachers, such errors presents opportunities for us as teacher educators to configure effective teaching strategies to help remediate prospective teachers' basic algebraic knowledge and computational skills throughout the trajectory of training them to become effective mathematics teachers. This position resonates with the social practive theory's first principle, which advocates that students' errors and lack of understanding are sensible and reveal (mathematical) thinking among students, which in turn teachers can use as information when transforming knowledge for future lessons.

Conclusions

In this article, we have illustrated our experiences as mathematics education lecturers and researchers mathematics knowledge gaps that prospective teachers bring with to university from secondary school. To our knowledge, there is no study within the South African educational context that discusses the issues of undergraduate mathematics education students'knowledge gaps from personal experiences of lecturers. This article illustrates how the students' foundational mathematics knowledge gaps constrain their effective learning of algebra at undergraduate level from an insider perspective. We also call for research that unearth the lived experiences of mathematics education lecturers in other universities and contexts, to understand the nature of students' understanding of mathematics content and how their knowledge gaps shapes their learning of mathematics at undergraduate level.

Autoethnographical approach to writing and research bears critical implication to our professional learning as teacher educators. We have learned through this writing that autoethnographical reflections fosters selfawareness and knowledge about a field, as it helped us to identify students' algebraic knowledge gaps, rethink our instructional practices and configure strategies to facilitate their learning of mathematics. We therefore recommend that further research and writing explore how autoethnographical approach to writing and research may be incorporated regularly in university lecturers' professional development programmes.

Recommendations

In view of our experiences detailed above, we argue that observed prospective teachers' difficulties associated with elementary mathematical calculation skills could provide valuable learning opportunities for their learning, provided lecturers employ remediation strategies to address the knowledge gaps from Grade 12. Accordingly, it is important to assess students' foundational knowledge and configure preventive strategies to address observed difficulties and improve prospective teachers' level of conceptual knowledge. If the urgency of improving the standard of Mathematics Education in South Africa is seriously considered, addressing prospective teachers' knowledge of basic computational skills and sense of structure should given much attention in initial teacher education programmes. We recommend that basic algebraic skills should be infused in mathematics education content courses. Algebraic manipulation skills and structure noticing are crititical abilities to hone in mathematics, as such, prospective teachers should be given opportunities to develop and improve such skills to ensure that they become prepared to enable their own learners' epistemological access to mathematical knowledge.

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