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Erratum to "On β-Local Functions in Ideal Topological Spaces" and New Results

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Abstract

In 2020, Powar et al. [P.L. Powar, T. Noiri and S. Bhadauria, *On β-local functions in ideal topological spaces*, European Journal of Pure and Applied Mathematics, **13**(4), (2020), 758-765] introduced β-local functions and studied related properties. The purpose of this paper is to show that Theorem 1(2), Theorem 2(4) and Theorem 3 of Powar et al. [P.L. Powar, T. Noiri and S. Bhadauria, *On β-local functions in ideal topological spaces*, European Journal of Pure and Applied Mathematics, **13**(4), (2020), 758-765] are not true. To disprove their claims, we provide a suitable example and justifications. Thus, we rectify their claims using our results and establish the correct results.

Keywords: *Ideal topological space, β-open set, β-local function, operation cl^*_β, τ^*_β-sets.*
2010 Mathematics Subject Classification: *54C10, 54A05*

1. Introduction

Topology is an area of mathematics which has several applications in many interdisciplinary fields. Thus, the applicability of topology has attracted attention of several researchers to develop various generalized structures. The process of generalizing existing topological structures is not new. This goes back to decade of 1930's when Kuratowski [1] introduced ideal $\mathscr{I}$ on a topological space (X,τ) which resulted ideal topological space $(X,\tau,\mathscr{I})$. The concept of ideal on a topological space is a generalized version of the concept filter on the same topological space in an interconnecting way. Complement of every element of an ideal becomes an element of a filter on X and vice-versa.

An ideal $\mathscr{I}$ [1] on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) $A\in\mathscr{I}$ and $B\subset A$ implies $B\in\mathscr{I}$ and (ii) $A\in\mathscr{I}$ and $B\in\mathscr{I}$ implies $A\cup B\in\mathscr{I}$. In an ideal topological space $(X,\tau,\mathscr{I})$, the local function $(\cdot)^*$[1] is defined as $A^*(\mathscr{I},\tau)$ (or simply A^*) $=\{x\in X|\,(\forall U\in\mathscr{U}(x))\,(U\cap A\notin\mathscr{I})\}$ where $\mathscr{U}(x)$ is the collection of all open subsets containing $x\in X$. A Kuratowski closure operator cl^* for a topology $\tau^*(\mathscr{I},\tau)$ called the $*$-topology, finer than τ is defined by $cl^*(A)=A\cup A^*$.

There are several generalized forms of local function available in topology. One of them is β-local function, which was introduced by Powar et al. [2] in 2020. In this paper we prove that Theorem 1(2), Theorem 2(4) and Theorem 3 of Powar et al. are not true. We provide suitable examples and justifications to disprove their results.

2. Preliminaries

In this section we consider some notions which are important for the rest part of the paper.

Definition 1. [3] Let (X,τ) be a topological space and $A\subset X$. A is said to β-open [3] if $A\subset cl(int(cl(A)))$.

The complement of a β-open set is called a β-closed set. The family of all β-open subsets of X is denoted by $\beta O(X)$ and the family of all β-open subsets containing a point x of X is denoted by $\beta O(X,x)$.

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Definition 2. [2] Let $(X,\tau,\mathscr{I})$ be an ideal topological space and $A\subset X$.

(i) $A^*_\beta(\mathscr{I},\tau)=\{x\in X|\,(\forall U\in\beta O(X,x))\,(U\cap A\notin\mathscr{I})\}$ is called the β-local function of A with respect to $\mathscr{I}$ and τ.

(ii) $cl^*_\beta(A)=A\cup A^*_\beta$ and the topology generated by cl^*_β is denoted by τ^*_β that is $\tau^*_\beta=\{A\subset X|cl^*_\beta(X\setminus A)=X\setminus A\}$.

Theorem 1. ([2], Theorem 1) Let $(X,\tau,\mathscr{I})$ be an ideal topological space and A,B be subsets of X. Then, the following properties hold:

(1) If $A\subset B$, then $A^*_\beta\subset B^*_\beta$,

(2) $(A\cup B)^*_\beta=A^*_\beta\cup B^*_\beta$,

(3) $(A\cap B)^*_\beta\subset A^*_\beta\cap B^*_\beta$,

(4) $(A^*_\beta)^*_\beta\subset A^*_\beta$,

(5) $A^*_\beta=\beta\text{-}cl(A^*_\beta)\subset\beta\text{-}cl(A)$.

Theorem 2. ([2], Theorem 2) Let $(X,\tau,\mathscr{I})$ be an ideal topological space. Then, the following properties hold:

(1) $A\subset cl^*_\beta(A)$,

(2) $cl^*_\beta(\emptyset)=\emptyset$ and $cl^*_\beta(X)=X$,

(3) $A\subset B$ implies $cl^*_\beta(A)\subset cl^*_\beta(B)$,

(4) $cl^*_\beta(A)\cup cl^*_\beta(B)=cl^*_\beta(A\cup B)$,

(5) $(cl^*_\beta(A))^*_\beta\subset cl^*_\beta(A)=cl^*_\beta(cl^*_\beta(A))$.

Theorem 3. ([2], Theorem 3) Let $(X,\tau,\mathscr{I})$ be an ideal topological space. Let $\tau^*_\beta=\{U\subset X|cl^*_\beta(X\setminus U)=X\setminus U\}$. Then, τ^*_β is a topology for X such that $\tau^*\subset\tau^*_\beta$ and $\beta O(X)\subset\tau^*_\beta$.

3. New results

In this section we provide the following example to prove that Theorem $1(2)$, Theorem $2(4)$ and Theorem 3 of Powar et al. [2] are not true. Later, we establish new results based on the following example.

Example 1. Let $(X,\tau,\mathscr{I})$ be an ideal topological space, where $X=\{a,b,c\},\ \tau=\{\emptyset,X,\{a,b\}\}$, and $\mathscr{I}=\{\emptyset,\{c\}\}$. Then, we calculate $\beta O(X)=\{\emptyset,X,\{a\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$. Let $A=\{a\}$ and $B=\{b\}$.

(a) It is not difficult to see that $A^*_\beta\cup B^*_\beta=\{a,b\}$, and $(A\cup B)^*_\beta=X$. Then, $A^*_\beta\cup B^*_\beta=\{a,b\}\neq X=(A\cup B)^*_\beta$.

(b) It is clear that $cl^*_\beta(A\cup B)=(A\cup B)\cup(A\cup B)^*_\beta=\{a,b\}\cup X=X$, and $cl^*_\beta(A)\cup cl^*_\beta(B)=(A\cup A^*_\beta)\cup(B\cup B^*_\beta)=\{a\}\cup\{b\}=\{a,b\}$. Then, $cl^*_\beta(A\cup B)\neq cl^*_\beta(A)\cup cl^*_\beta(B)$.

(c) $\tau^*_\beta=\beta O(X)$ is not a topology on X.

Thus, the correct versions of Theorem 1 and Theorem 2 are given below.

Theorem 4. Let $(X,\tau,\mathscr{I})$ be an ideal topological space and A,B be subsets of X. Then, the following properties hold:

(1) If $A\subset B$, then $A^*_\beta\subset B^*_\beta$,

(2) $A^*_\beta\cup B^*_\beta\subset(A\cup B)^*_\beta$,

(3) $(A\cap B)^*_\beta\subset A^*_\beta\cap B^*_\beta$,

(4) $(A^*_\beta)^*_\beta\subset A^*_\beta$,

(5) $A^*_\beta=\beta\text{-}cl(A^*_\beta)\subset\beta\text{-}cl(A)$.

Theorem 5. Let $(X,\tau,\mathscr{I})$ be an ideal topological space. Then, the following properties hold:

(1) $A\subset cl^*_\beta(A)$,

(2) $cl^*_\beta(\emptyset)=\emptyset$ and $cl^*_\beta(X)=X$,

(3) $A\subset B$ implies $cl^*_\beta(A)\subset cl^*_\beta(B)$,

(4) $cl^*_\beta(A)\cup cl^*_\beta(B)\subset cl^*_\beta(A\cup B)$,

(5) $(cl^*_\beta(A))^*_\beta\subset cl^*_\beta(A)=cl^*_\beta(cl^*_\beta(A))$.

Moreover, it is clear from Example 1 that Theorem 3 is not correct, and thus we would like to suggest our readers to be cautious to use Theorem 3 to develop any new result based on it or to use it anywhere in future researches of topology with β-local function.

4. Conclusion

In this paper, we showed that Theorem $1(2)$, Theorem $2(4)$ and Theorem 3 of [2] are not true. We provided suitable example to prove our claims and thus, we provided correct results of Theorem 1 and Theorem 2. Moreover, we discarded Theorem 3, which can not be corrected by the available information given in [2]. Thus, we do not suggest our readers to use Theorem 3 for the development of any new theory related to β-local function. At the end, we hope that the results of Section 3 will find their suitable roles in research topics related to β-local function in future.

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Author's contributions

The author contributed to the writing of this paper. The author read and approved the final manuscript.

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