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Research Article

Output regulation for time–delayed Takagi–Sugeno fuzzy model with networked control system

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Abstract

This article studies H_∞ control problem based on the event–triggered scheme with time delays for the synchronization of an chaotic system represented by delayed Takagi–Sugeno models. Firstly, this method depending on two scenarios: a) Each local subsystem integrated that the delayed T–S fuzzy model for the same value of input matrices for the networked system and b) This is near steady-state zero-error diversification has to all be the same local subsystems. Generally, in the case of fuzzy regulation, these in lieu of generating the fuzzy regulator as a result of linear local controllers, circumstances were adjusted by addressing the issue of fuzzy regulation for the delayed Takagi–Sugeno models fuzzy model. Then, a delayed Takagi–Sugeno uses a fuzzy system to model the non–linear regulator. On the other hand, communication delays are a vital factor that cannot be ignored. To tackle the networked induced delay initially, author attempt to implement the event–triggered scheme for output regulation which reduce the cost of network transmission. By constructing a Lyapunov functional and making use of event–triggered method, some suitable circumstances that ensure asymptotic stability of H_∞ performance index for the resulting model were derived. Additionally, as the variations of the aforementioned results, two scenarios were presented. Our developed approaches are demonstrated by a final example illustrating their superiority, usefulness and reliability.

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Keywords. Event-triggered scheme, fuzzy output regulation, delayed T–S fuzzy system, Francis equations

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1. Introduction

The latest development, delayed nonlinear systems have attained huge their widespread use in practically every industry, including but not limited to associative memories, target tracking, pattern recognition, and signal processing, fuzzy techniques [29, 30, 39] along with combinatorial optimization [17, 20, 22], when the activation functions, connection weights, and system states all have stochastic values. Thus, studies into the dynamics of nonlinear systems are of great importance, and several outcomes have been published in recent literature [15, 43]. The limited speed for signal traveling through this link [2] also contributes to time delays when gathering information storage and transmission, and ignoring these delays may lead to undesirable dynamics like oscillations and bifurcations. Dynamic analysis of nonlinear delayed networks has yielded many results. For instance, Sriraman et al.[41] discusses the asymptotic stability problem of stochastic systems, for which time-varying delays are probabilistic. For coupled nonlinear systems with bounded asynchronous delays, the anti-synchronization problem is addressed in [26].

There has been a growth in the popularity of nonlinear systems since it has a powerful modelling dynamic systems that random flaws or structural possibilities mutations faults of actuators, environmental noise. In the same consequences, tracking the desired signal is imperative in control of non-linear systems. Since last decade, numerous methods have been developed [24, 38, 47]. Based upon the previous research the regulation theory attained significant attention to design the controllers to track the desired signal along with the disturbance [34]. For the regulation problem, meanwhile, the goal is to determine a state feedback controller that possesses a stable equilibrium point asymptotically independent of external excitations. An exo-system generates the external reference signals and perturbation signals, each of which makes the tracking error zero.

The linear regulator equation was initially solved by [12]. Francis [12] has given the answer to a robust linear regulator equation keeping the system stability when there is uncertainty system parameters. To achieve robust regulation, internal model based dynamic controller is required to be designed. As a result, Isidori and Byrnes [19] generalized this findings in a nonlinear setting, as well as their has demonstrated which the nonlinear regulator shall be derived from a series recognised as the class in a partial differential equations Francis–Isidori Byrnes (FIB) equations [23, 37]. The presence of an adapted version in the nonlinear case was seen to be a sufficient prerequisite for maintaining robustness to parametric uncertainties. This internal model is generated by immersing this exosystem in an dynamical system that produces all of the potential inputs for any parameter's steady–state variance that is permitted.

In many applications, several researchers design networked controllers with distributed parameter agents, since the temporal and spatial dynamics for the system should also be taken into account. Furthermore, this authors developed an sliding mode observer for fractional super–twisting (FSTW) controllers [6]. Additionally, this paper develop an systematic strategy for the networked control of heterogeneous multi–agent systems (MASs) with distributed parameter agents in an infinite-dimensional setting [25]. The general framework for output regulation problems is established by considering both spatial and temporal parameters. According to the literature, parabolic partial differential equations (PPDEs) with spatiotemporal coefficients do not necessarily pose a challenging output regulation problem. This outcome may also be applied to systems that varying parameters. Further, Gao et al.[13] combine adaptive dynamic programming theory with output regulation theory in [28]. The second method could be used to control plant uncertainty by using both linear and nonlinear tight-feedback methods [7]. The output regulation of dynamic systems has been studied many times since then. The output networked control regulation has not been fully studied. This paper, we are analysing this kind of issue positively.

Networked Control Systems (NCSs) on the other hand, have been extensively employed in large-scale power systems, satellite systems, economic management systems, and other control domains. It displays a number of positive qualities, including ease of installation and upkeep, affordable, and dependability. It exhibits numerous positive characteristics, including ease of installation and maintenance, low cost, and great reliability. The transmission of information in the network, however, introduces a number of issues related to the introduction of the network into the control systems, including cyber attacks, packet dropouts, tracking, network-induced time delays, etc. Networked control systems have experienced rising interest in recent years. Choosing the information transmission scheme and the frequency of controller execution is crucial when designing networks of control systems [3, 16], and sliding mode control [31]. Time-triggered control and event-triggered control are two popular and useful technologies. The output of the controller is applied to the system which time-triggered control framework at sampling time, which is established by a set sampling period [21, 42]. Event-triggered control belongs to a kind of control strategies to execute the signal update when a certain event occurs. Event-driven controls consume significantly less communication resources than time–driven controls [1, 4]. This has led to the popularity of event-triggered control. Unfortunately, this scheme was not fully investigated for the output regulation problems.

Fuzzy regulation theory is related on crossed terms and derivative of membership function which are not taken into account by local linear regulators. So the solution of fuzzy regulators is not similar to local linear regulators. In specific cases, these two characteristics can be ignored. In [33] fuzzy regulators based on local linear regulators are discussed in detail. Another approach concerned with tracking signals is developed in [9]. To explain the constrained nonlinear output regulation problem, researchers explored in a properties of model predictive control (MPC) framework [23,37]. In which fuzzy integral controller is proposed for non-linear systems to identify inaccurate references. This formation of a fuzzy integral regulator for both discrete- and continuous–time non–linear systems is founded on integration of an integrator and parallel distributed compensator (PDC). These types of controllers are robust and guarantee the H_∞ performance index for external disturbances. Computation of gain for the performance index based on the feasible linear matrix inequalities (LMIs) methods. This research takes benefits for the delayed Takagi-Sugeno (T-S) fuzzy model with a networked control system, which has the ability to describe in a very useful approach, nonlinear dynamics.

Inspired by above observation, in this article, our goal is to address the H_∞ control for output regulation problem with networked control systems. The following are some examples of this article's primary novelties:

(1) To overcome the tracking error, a stabilizer is introduced, which takes the states that model yield in a steady-state error. Furthermore, we also design LMIs which have the capability to investigate the H_∞ control for nonlinear system with time delays for the synchronization of the chaotic system based on delayed Takagi–Sugeno models with appropriate LMIs.

(2) These are the first few attempts to introduce the event-triggered scheme for the regulation problem to eliminate the communication burden over the networked control system.

(3) The networked control system, the steady–state input responsible to keep tracking of the reference signals by using the steady–state zero–error manifold. Furthermore, two issues are introduced in the regulation problem which consists of determining the steady–state input as well as the steady–state zero–error manifold (See Figures 1 and 2.)

2. Formulation of the problem

Suppose the nonlinear plant is given:

$$\begin{cases} dx(t) &= g(x(t), u(t), \omega(t)) \\ y(t) &= c(x(t)) \\ \dot{\omega}(t) &= s(\omega(t)) \\ y_{ref}(t) &= p(\omega(t)) \\ e(t) &= h(x(t), \omega(t)) \end{cases} \tag{2.1}$$

From above equation, $x(t)$ presents the state of the plant with dimension $\mathbf{R}^n$, $u(t)$ denoted by the control input with length $\mathbf{R}^m$, while $\omega(t)$ is the state vector for the exosystem which belongs to $\mathbf{W} \subset \mathbf{R}^s$, which produces the perturbation and/or reference signal. Output tracking which is denoted by $e(t)$ and presented in Eq. (2.1) along with the dimension $\mathbf{R}^m$. This definition shows the difference between reference signal and output of the plant, that is

$$\begin{aligned} h(x(t), \omega(t)) &= y(t) - y_{ref}(t) \\ &= c(t) - p(\omega(t)). \end{aligned}$$

This is worth mentioning which in [5], having been proved that regulation problems it cannot determined especially whenever the size $u(t)$ can be smaller greater than the size of $e(t)$. Alternatively, when the length of the $e(t)$ is smaller than $u(t)$, the tracking problem is not more challenging. In addition, it is made-up that the nonlinear system in Eq. (2.1) is not a time variant. According to this, the formation of an internal plant is not examined [5, 32].

To make a good estimate especially to the nonlinear system, T-S fuzzy model was firstly suggested in [44]. According to this definition, we characterized the subsystem as indicated by rules related to some physical information. After defining these subsystems properly, the behavior of nonlinear framework in a predefined area of the state space. One of the main themes of this work is to incorporate the time delays in the T-S fuzzy system to get better tracking, which is still an open and challenging problem. Further proceeding of this paper, we will be considered outputs of the system which depend upon or concur the accurately with states $x(t)$. The other assumptions in this paper are as $g(0, 0, 0) = 0$, $h(0, 0) = 0$ and $s(0)$.

At this stage, in the problem of regulation, to express the fuzziness of the plant's nonlinearity, we have to use different eligibility criteria for the fuzzy model of the exosystem. This volume of fuzzy the exosystem's laws could be changed due to the number of fuzzy rules of the fuzzy plant. These assumptions were compulsory to get a better result in the analysis. In this paper, the tracking problem is expressed in Eq. (2.1) but for the other case in which nonlinear terms like $g(\cdot, \cdot, \cdot)$, $s(\cdot)$, $q(\cdot)$, $c(\cdot)$ and $h(\cdot, \cdot)$ are presented using the T-S fuzzy plant as given:

Plant Rule i: IF $\delta_1^1(t)$ is ξ_{i1}^1 and,$\cdots$,and $\delta_p^1(t)$ is ξ_{ip}^1, THEN

$$\begin{cases} \dot{x}(t) &= \mathscr{A}_i x(t) + \mathscr{A}_{di} x(t-d) + \mathscr{B}_i u(t) + \mathscr{P}_i \omega(t) \\ y(t) &= \mathscr{C}_i x(t), \quad for \quad i = 1, \cdots, r^1 \end{cases} \tag{2.2}$$

Exo-system (Fuzzy) i: IF $\delta_1^2(t)$ is ξ_{i1}^2 and$\cdots$and $\delta_p^2(t)$ is ξ_{ip}^2, THEN

$$\begin{cases} \dot{\omega}(t) &= \mathscr{S}_i \omega(t) \\ y_{ref}(t) &= \mathscr{Q}_i \omega(t), \quad for \quad i = 1, \cdots, r^2 \end{cases} \tag{2.3}$$

where r^1 and r^2 represent the number of fuzzy rules in the form (IF–THEN) for the fuzzy plant and exosystem respectively. The sets ξ_{ip}^1 and ξ_{ip}^2 present the fuzzy sets which are based upon the prior understanding of the dynamic of the two system. The concept of linearizing is applied to obtained the matrices $\mathscr{A}_i$, $\mathscr{A}_{di}$, $\mathscr{B}_i$, $\mathscr{P}_i$, $\mathscr{C}_i$, $\mathscr{Q}_i$ and $\mathscr{S}_i$based on the suitable approximation points (x, u, ω)=(x^i, u^i, ω^i), i.e.,

$$\begin{bmatrix} \mathscr{A}_i & \mathscr{A}_{di} & \mathscr{P}_i \\ \mathscr{B}_i & \mathscr{S}_i & \mathscr{C}_i \\ \mathscr{Q}_i & - & - \end{bmatrix} = \left[\begin{array}{c|c|c} \frac{\partial g(x,u,\omega)}{\partial x}\Big|_{(x^i,u^i,\omega^i)} & \frac{\partial g(x,u,\omega)}{\partial x}\Big|_{(x^i,u^i,\omega^i)} & \frac{\partial g(x,u,\omega)}{\partial \omega}\Big|_{(x^i,u^i,\omega^i)} \\ \hline \frac{\partial g(x,u,\omega)}{\partial u}\Big|_{(x^i,u^i,\omega^i)} & \frac{\partial s(\omega)}{\partial \omega}\Big|_{(\omega^i)} & \frac{\partial h(x,\omega)}{\partial x}\Big|_{(x^i,\omega^i)} \\ \hline \frac{\partial h(x,\omega)}{\partial \omega}\Big|_{(x^i,\omega^i)} & - & - \end{array}\right]$$

Taking into account phenomena about the nonlinear sector approach which is proposed in [45]. It is observed that this phenomenon produced an accurate demonstration of nonlinear model especially in the local region rather than the estimate which is obtained by the aforementioned local linearization technique before. By using this center average approach for de–fuzzifier with the dynamics of the fuzzy system Eq. (2.1) are inferred using a singleton fuzzifier and fuzzy inference as

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) \left[\mathscr{A}_i x(t) + \mathscr{A}_{di} x(t-d) + \mathscr{B}_i u(t) + \mathscr{P}_i \omega(t)\right] \\ \dot{\omega}(t) &= \sum_{i=1}^{r^2} \lambda_i^2(\delta_2(t)) \mathscr{S}_i \omega(t), \\ e(t) &= \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) \mathscr{C}_i x(t) - \sum_{i=1}^{r^2} \lambda_i^2(\delta_2(t)) \mathscr{Q}_i \omega(t) \end{cases} \tag{2.4}$$

where $x(t)$ denotes the state of the model with dimension $\mathbf{R}^n$; $u(t) \in \mathbf{R}^m$ presents the control signal; exosystem is expressed with $\omega(t) \in \mathbf{R}^s$; $e(t) \in \mathbf{R}^m$ is the error tracking; function for $x(t)$ and/or $\omega(t)$ is $\delta^\star(t) = [\delta_1^\star(t), \delta_1^\star(t), \cdot\cdot\cdot, \cdot, \delta_p^\star(t)]^T$. In the same way, membership function for exosystem and plant fulfill the $\bar{\chi}_i^\star(\delta^\star(t)) = \prod_{j=1}^{p^\star} \xi_{ij}^\star(\delta_j^\star(t))$ with $\lambda_i^\star(\delta^\star(t)) = \frac{\bar{\chi}_i^\star(\delta^\star(t))}{\sum_{i=1}^{r^\star} \bar{\chi}_i^\star(\delta^\star(t))}$, for all $t > 0$ in the term $\xi_{ij}^\star(\delta_j^\star(t))$ is the membership values for $\xi_{ij}^\star$ at $\delta_j^\star(t)$. In addition:

$$\sum_{i=1}^{r^\star} \bar{\chi}_i^\star(\delta^\star(t)) > 0 \tag{2.5}$$

$$\bar{\chi}_i^\star(\delta^\star(t)) \geq 0, \quad i, \cdot\cdot\cdot, r^\star \tag{2.6}$$

On the other side:

$$\sum_{i=1}^{r^\star} \lambda_i^\star(\delta^\star(t)) = 1 \tag{2.7}$$

$$\lambda_i^\star(\delta^\star(t)) \geq 0, \quad i, \cdot\cdot\cdot, r^\star \tag{2.8}$$

for all $t > 0$. For the simplicity, '1' & '2' for the plant and exosystem respectively. At this stage, overall system delayed T-S fuzzy system Eq. (2.4) can be written as

$$\begin{cases} \dot{x}(t) &= \mathscr{A}(\lambda)x(t) + \mathscr{A}_d(\lambda)x(t-d) + \mathscr{B}(\lambda)u(t) + \mathscr{P}(\lambda)\omega(t) \\ \dot{\omega}(t) &= \mathscr{S}(\lambda)\omega(t), \\ e(t) &= \mathscr{C}(\lambda)x(t) - \mathscr{Q}(\lambda)\omega(t) \end{cases} \tag{2.9}$$

$$\mathscr{A}(\lambda) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t))\mathscr{A}_i$$

$$\mathscr{A}_d(\lambda) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t))\mathscr{A}_{di}$$

$$\mathscr{B}(\lambda) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t))\mathscr{B}_i$$

$$\mathscr{P}(\lambda) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t))\mathscr{P}_i$$

$$\mathscr{C}(\lambda) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t))\mathscr{C}_i$$

$$\mathscr{S}(\lambda) = \sum_{i=1}^{r^2} \lambda_i^2(\delta_2(t))S_i$$

$$\mathscr{Q}(\lambda) = \sum_{i=1}^{r^2} \lambda_i^2(\delta_2(t))Q_i$$

The ground reality behind the above demonstration rests on this fact which membership functions in the exosystem and the plant sequentially rely upon t. Before proceeding further for the existence conditions of regulation, we assume same assumptions regarding networked fuzzy control system as mentioned [49].

2.1. Event-triggered mechanism

The sampler in this article is time-driven, that a sampling interval $\hbar > 0$, which is an sample of the real input $x(t)$. In the conventional approach, after each sample, the controller will receive the sampled data sampling cycle, requiring additional band resources. To overcome this limitation, we add a processor that responds to events between the sampler and the controller that determines if the current to the controller should be delivered sampled data. If the most recently released input is $\hat{i}_k$, the following instant, $\hat{i}_{k+1}$, is defined in the subsequent condition:

$$\hat{i}_{k+1}\hbar = \hat{i}_k\hbar + \underbrace{min}_{j\geq 1}\{j\hbar \mid e_k^T(t_k^n\hbar)\digamma_\ell e_k(t_k^n\hbar) \geq \rho y^T(t_k\hbar)\digamma_\ell y(\hat{i}_k\hbar)\}, \tag{2.10}$$

where $\rho \in (0,1)$ and $\digamma_\ell > 0$, $\ell = 1, 2$ are event–triggered parameters.

$$e_k(t_k^n\hbar) = y(t_k^n\hbar) - y(\hat{i}_k\hbar), \tag{2.11}$$

where $t_k^n\hbar = \hat{i}_k\hbar + j\hbar, \quad n \in \mathbb{N}$.

The event-triggered condition (5) therefore results in:
Control rule i: IF $\delta_1^1(t)$ is ξ_{i1}^1 and,$\cdots$,and $\delta_p^1(t)$ is ξ_{ip}^1, THEN

$$u(t) = K_i(\hat{i}_k h), \quad t \in [\hat{i}_k\hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}), \tag{2.12}$$

where $K_i (i = 1, 2, \cdots, r^1)$ are to determine controller gains. From the Eq. (2.4), controller design will be calculated later.

Remark 2.1. This event–triggered condition Eq. (2.9), defined this paper is similar with [10]. In this paper, we assume that the sampler samples the output measurements with a period of $\hbar > 0$. It suggests that only at the sample instants is the event triggering condition Eq. (2.9) validated, so event triggering control directly offers a minimum inter–event time that is guaranteed (at least $\hbar > 0$) [48]. So, the Zeno behaviour will not occurs in our design method.

Using the same idea as in [46], taking the network–induced time delay into account. Let $\tau(t) = t - \hat{i}_k\hbar$, this yields to $0 \leq \tau_{\hat{i}_k} \leq \tau(t) \leq \hat{i}_{k+1}\hbar - \hat{i}_k\hbar + \tau_{\hat{i}_k} \triangleq \tau_M$. One might represent the transmitted state as $x(\hat{i}_k h) = x(t-\tau(t)) + e_k(t_k^n\hbar)$. The controller's defuzzified output is then:

$$u(t) = \textstyle\sum_{j=1}^{r^1} \lambda_j^1 K_j \left[x(t-\tau(t)) + e_k(t_k^n\hbar)\right], \quad t \in [\hat{i}_k + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}). \tag{2.13}$$

Prior to presenting this paper's main theorem, we simplify the notation for the main model e.g. $\mathscr{A}(\lambda)$ presents $\mathscr{A}_i$. Furthermore, it is recalled the Isidori's regulation theory especially for nonlinear system from Eq. (2.1) is given with the event–triggered effect bellow as

$$\begin{cases} \dot{x}(t) &= \mathscr{A}_i x(t) + \mathscr{A}_{di} x(t-d) + \mathscr{B}_i K_j x(t-\tau(t)) + \mathscr{B}_i e_k(t_k^n) + \mathscr{P}_i \omega(t) \\ \dot{\omega}(t) &= \mathscr{S}\omega(t), \qquad t \in [\hat{i}_k + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}) \\ e(t) &= \mathscr{C}_i x(t) - \mathscr{S}\omega(t) \end{cases} \tag{2.14}$$

and regulation problem for nonlinear system (RNS) includes to determine the controller:

$$\dot{x}(t) = \beta(x(\hat{i}_k \hbar), \omega(t)), \qquad t \in [\hat{i}_k + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}) \tag{2.15}$$

which is closed–loop system:

$$\dot{x}(t) = \mathscr{A}x(t) + \mathscr{A}_d x(t-d) + \mathscr{B}\beta(x(\hat{i}_k\hbar), 0) + \mathscr{B}e_k(t_k^n), \qquad t \in [\hat{i}_k\hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}) \tag{2.16}$$

has an asymptotic stability with equilibrium point and the answer to close–loop system from the Eq. (2.15) and Eq. (2.16) fulfills:

$$\lim_{t\to\infty} e(t) \triangleq 0$$

Therefore, $\Gamma\omega(t)$ presents the steady–state zero-error manifold and $\zeta\omega(t)$ denotes the steady–state input, then the preceding theorem provides an existence in the solution of RNS [18].

Theorem 2.2. *Assume the below conditions hold:*
Σ_1 : *The exosystem $d\omega(t) = s(\omega(t))$ is Poisson stable.*
Σ_2 : *There is controller gain K_j exist such that delayed system Eq. (2.14) stable under the event-triggered scheme Eq. (2.9).*
Σ_3 : *There exist scalings such that:*

$$\begin{aligned} x^{ss}(t) &= \pi(\omega(t)), \\ u^{ss}(t) &= \gamma(\omega(t)), \end{aligned}$$

with initial conditions $(\pi(0), \gamma(0)) = (0, 0)$, satisfying

$$\begin{cases} \frac{\partial \pi\omega(t)}{\partial \omega(t)} s(\omega(t)) &= f(\pi\omega(t), \omega(t), \gamma\omega(t)) \\ p(\pi\omega(t), \omega(t)) &= 0. \end{cases} \tag{2.17}$$

Furthermore, the RNS is solvable, and controller is given as

$$u(t) = \sum_{j=1}^{r^1} \lambda_j^1 K_j(x(\hat{i}_k\hbar) - \pi\omega(t)) + \gamma\omega(t), \qquad t \in [\hat{i}_k\hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}). \tag{2.18}$$

Proof: *With necessity and sufficient condition follows immediately from Lemma 1 in [19]. It should be observed that by Assumption 2, there exists a matrix K_j such that in Eq. (2.14) has eigenvalues. Let us assume that the Eq. (2.17) are fulfilled with $\pi(\omega(t))$ and $\gamma(\omega(t))$, and set:*

$$\eta(x, \omega) = \gamma\omega(t) + \sum_{j=1}^{r^1} \lambda_j^1 K_j(x(\hat{i}_k\hbar) - \pi\omega(t)), \qquad t \in [\hat{i}_k\hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1} + \tau_{\hat{i}_{k+1}}).$$

This option obviously satisfies Ia) In fact, the Jacobian matrix of $f(\pi\omega(t), \omega(t), \gamma\omega(t))\eta(x, 0)$ is exactly same to Eq. (2.14). Furthermore, by construction:

$$\eta(\pi(\omega), \omega) = \gamma(\omega)$$

and therefore, first part of Eq. (2.17) reduces to (5.la) in [19]. On the other hand, second part of Eq. (2.17) is identical to (5.Ib) in [19]. Thus, by Lemma 1 in [19], requirement Eq. (2.17) is also fulfilled.

It is evident from the Theorem 3.1, regulation problem for nonlinear plant consists of partial differential equations, that rely on convolution for the exosystem and/or plant which possibly very hard, and in several situations difficult to handle it. Because of this research another a method to handling the regulation problem for nonlinear plant is set on the foundation of the delayed T–S fuzzy models under networked control system.
The mappings for the linear example are readily apparent:

$$\begin{aligned} x^{ss}(t) &= \pi(\omega(t)), \\ u^{ss}(t) &= \gamma(\omega(t)), \end{aligned}$$

convert into

$$\begin{aligned} x^{ss}(t) &= \pi\omega(t), \\ u^{ss}(t) &= \gamma\omega(t). \end{aligned}$$

Therefore, conditions Eq. (2.17) to a linear matrix equation reduction (Francis equations).

$$\begin{aligned} \Pi\mathscr{S} &= \mathscr{A}_i\Pi + \mathscr{B}_i\Gamma + \mathscr{P}_i \\ \mathscr{C}_i\Pi - \mathscr{Q} &= 0 \end{aligned}$$

Therefore, regulation problem to nonlinear system is well characterised as the issue to find:

$$u(t) = \beta(x(\hat{i}_k\hbar), \omega(t)), \tag{2.19}$$

such that:

- Equilibrium point $x(t) = 0$ for closed–loop without external signal

$$u(t) = \beta(x(\hat{i}_k\hbar), \omega(t)) \tag{2.20}$$

  is called asymptotically stable.
- Find the solution of closed–loop system Eq. (2.9) and Eq. (2.20) fulfils:

$$\lim_{t\to\infty} e(t) \triangleq 0,$$

  when the model is under behavior of exosystem with networked control system.

Remark 2.3. From the Eq. (2.4), the control signal is presented with a very simple way by presenting the weight sum with local regulation:

$$\begin{aligned} u(t) &= \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t))K_i \left[x(t) - \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) \times \sum_{j=1}^{r^2} \lambda_i^1(\delta_2(t))\Pi_{ij}\omega(t) \right] \\ &+ \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) \sum_{j=1}^{r^2} \lambda_i^1(\delta_2(t))\Gamma_{ij}\omega(t). \end{aligned} \tag{2.21}$$

From the controller Eq. (2.18) is presented in the form of K_j, which alter by the fuzzy stabilize and also doing the mapping $\pi(\omega(t))$ and $\gamma(\omega(t))$. This mapping is further solved by the exact output fuzzy regulation problem (EOFRP) for the plant Eq. (2.4).

In the next section, we will design the nonlinear regulator, such that the methodology can be implemented to get an efficient result than the traditional method which is based on the local regulator.

2.2. Fuzzy output regulation problem

Let us consider the delayed fuzzy plant Eq. (2.4), it can be seen from the previous result design, procedure for the fuzzy stabilizer is compulsory. While in the article, we design the fuzzy stabilizer with the new Lyapunov matrix theory for the networked control system, which can be implemented without losing generality. According to the Remark (2.3), the overall control input will be

$$u(t) \quad = \quad \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) K_i \left[x(t) - \pi(\omega(t))\right] + \gamma(\omega(t)). \tag{2.22}$$

On the other hand, in this case the steady–state zero-error manifold like $x_{ss}(t) = \pi(\omega(t))$, while steady–state input $u_{ss}(t) = \gamma(\omega(t))$. So overall steady-state error can be written as

$$\begin{aligned} e_{ss}(t) &= x(t) - x_{ss}(t) \\ &= x(t) - \pi(\omega(t)), \end{aligned} \tag{2.23}$$

where $e_{ss}(t) \in \mathbf{R}^n$ with proper dimension like Eq. (2.9).

It can be seen that $e_{ss}(t)$ presents the error between the steady–state zero-error and the model's states $x(t)$ manifold $\pi(w(t))$, on the other aspects, $e(t)$ shows the difference between reference signal and output of the model. So, $\pi(w(t))$, can be calculated in the frame of asymptotic stability. Now, we describe the design procedure in detail.
Using the delayed T–S fuzzy plant Eq. (2.4), the definition of steady state–error is

$$\pi(w(t)) \quad = \quad \breve{\Pi}(w(t)), \tag{2.24}$$

with incorporated the steady–state input

$$\gamma(w(t)) \quad = \quad \breve{\Gamma}(w(t)), \tag{2.25}$$

where $\breve{\Pi}(t)$ and $\breve{\Gamma}(t)$ are time–varying matrices with proper dimensions, while $\pi(w(t)) = \breve{\Pi}(w(t))$ is the part of C^{k-1} function. Now, we take differentiation for the steady–state error Eq. (2.23) with respect to time $'t'$ with consideration of Eq. (2.24), Eq. (2.25).

$$\dot{e}_{ss}(t) \quad = \quad \dot{x}(t) - \dot{\breve{\Pi}}(w(t)) - \breve{\Pi}(\dot{w}(t)) \tag{2.26}$$

$$\begin{aligned} \dot{e}_{ss}(t) \quad &= \quad \mathscr{A}_i x(t) + \mathscr{A}_{d_i} x(t-d) + \mathscr{B} i K_j x(t-\tau(t)) + \mathscr{B}_i e_k(t_k^n) + \mathscr{P}_i \omega(t) \\ &- \quad \dot{\breve{\Pi}}(w(t)) - \breve{\mathscr{S}}(t)(w(t)), \qquad t \in [\hat{i}_k \hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1}\hbar + \tau_{\hat{i}_{k+1}}) \end{aligned} \tag{2.27}$$

Therefore, from Eq. (2.14) and Eq. (2.22)–(2.25) with considering

$$u(t) = u_s(t) + u_{ss}(t),$$

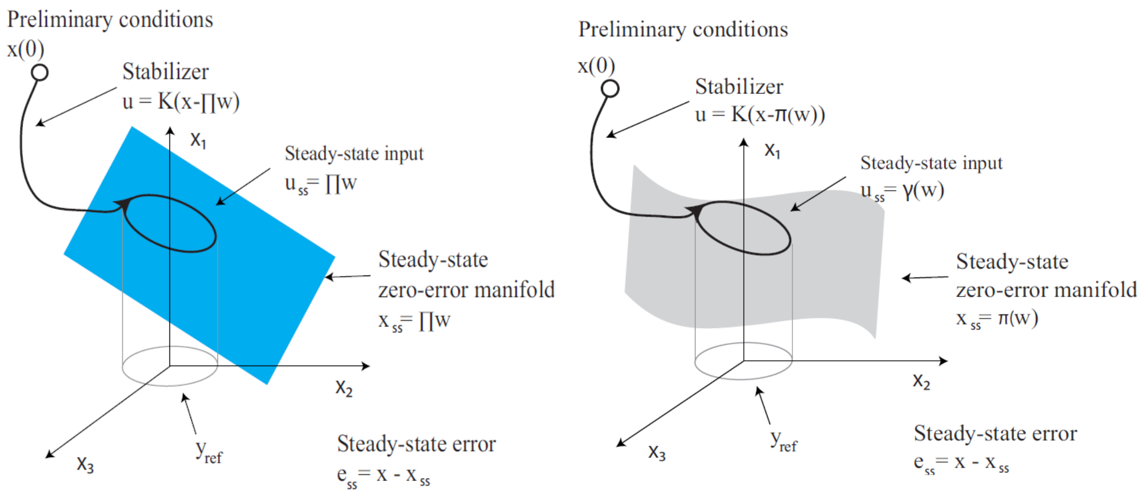
while

$$u_s(t) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) K_i \left[x(t) - \breve{\Pi}(\omega(t))\right],$$

considering at a stable part, while $u_{ss}(t) = \Gamma(t)(\omega(t))$, as a steady–state input, then following parts can be written as

$$\begin{aligned} \dot{e}_{ss}(t) \quad = \quad & \mathscr{A}_i e_{ss}(t) + \mathscr{A}_{d_i} e_{ss}(t-d) + \mathscr{B}_i K_j e_{ss}(t-\tau(t)) + \mathscr{B}_i e_k(t_k^n) + \mathscr{P}_i \omega(t) \\ & - \dot{\breve{\Pi}}(w(t)) - \breve{\mathscr{S}}(t)(w(t)), \qquad t \in [\hat{i}_k \hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1}\hbar + \tau_{\hat{i}_{k+1}}). \end{aligned} \tag{2.28}$$

Conversely, $\omega(t)$ is assumed to maintain the reference signal's ability to resist zero decay as time goes on, it must be Poisson stable. Similarly, we follow the same procedure which is given in [35]. To understand the procedure, we also introduced Figure 1 and Figure 2. Figure 1(a) and 1(b) represent graphical representations of challenges with linear and nonlinear regulation, respectively. This instance, without regard for linear subsystems, both the steady–state zero–error representation and the steady–state input are built solely for the overall T–S fuzzy plant. In Figure 1, the authors present the control law with incorporated steady-state, then evaluate the steady–state zero–error which is presented by e_{ss}. Stable–state zero-error manifolds are center manifolds that become invariant when steady–state inputs are applied. From the above assumption, we integrate the Francis equation and make the relax stabilization conditions. To understand the hierarchy of the system, we also present the block diagram (see Figure 2).



(a) $y = x$ **(b)** $y = 3sinx$

Figure 1. Typical regulation scheme [35].

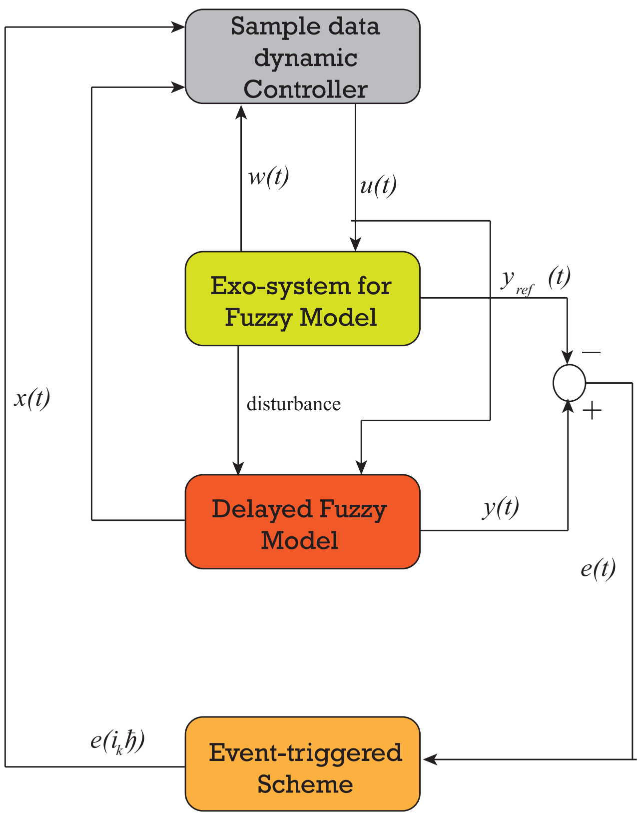


Figure 2. Control scheme for output regulation.

3. H_∞ control design

As a part of this section, NCSs are aimed at obtaining asymptotically stable non–linear systems with time–varying delays. Let's analyze the non-linear system that follows from system Eq. (2.1).

Theorem 3.1. *Considered the following the conditions:*
$\bar{\Sigma}_1$: *For the exosystem $\dot{\omega}(t) = \sum_{i=1}^{r^2} \lambda_i^1(\delta_2(t))\mathscr{S}_i\omega(t)$ is Poisson stable.*
$\bar{\Sigma}_2$: *There exit matrices $P > 0$, $Q_v > 0$, $F_\ell > 0$, $v, \ell = 1, 2$ and K_j, such that*

$$\theta_{ij} + \theta_{ji} \quad < \quad 0, \tag{3.1}$$

where

$$\theta_{ij} \quad = \quad \begin{bmatrix} \theta_{ij}^{11} & P\mathscr{A}_{d_i} & P\mathscr{B}_iK_j & 0 & P\mathscr{B}_i & P\mathscr{P}_i \\ \clubsuit & -Q_1 & 0 & 0 & 0 & 0 \\ \clubsuit & \clubsuit & -Q_2 + \rho\mathscr{C}_i^T F_\ell\mathscr{C}_i & 0 & 0 & 0 \\ \clubsuit & \clubsuit & \clubsuit & -Q_3 & \rho\mathscr{C}_i^T F_\ell & 0 \\ \clubsuit & \clubsuit & \clubsuit & \clubsuit & -F_\ell & 0 \\ \clubsuit & \clubsuit & \clubsuit & \clubsuit & \clubsuit & -\gamma^2 I \end{bmatrix}$$

$$\theta_{ij}^{11} \quad = \quad P\mathscr{A}_i + \mathscr{A}_i^T P + \sum_{v=1}^{3} Q_v + \mathscr{C}_i^T\mathscr{C}_i$$

$\bar{\Sigma}_3$: *The solution of $\breve{\Pi}(t)$ and $\breve{\Gamma}(t)$ can be obtained by the function of $\lambda_{\diamondsuit,i}^1(\delta_1(t)) \geq 0$, $i = 1, 2, \cdots, r_\diamondsuit$ for all the values of $t \geq 0$ (1 denoted for model and 2 for exosystem):*

$$\breve{\Pi}\breve{\mathscr{S}} \quad = \quad \mathscr{A}_i\breve{\Pi} + \mathscr{B}_i\breve{\Gamma} + \mathscr{P}_i, \tag{3.2}$$

$$\mathscr{C}_i\breve{\Pi} - \breve{\mathscr{Q}} \quad = \quad 0, \tag{3.3}$$

such that $\breve{\Pi}$ is directly obtained, while on the other side $\breve{\Pi}_0$ is calculated by putting the x_0 and ω_0 into $\breve{\Pi}_0$. Then, EOFRP is solvable.

Proof: Let's considered the following Lyapunov–Krasovskii functional for system Eq. (2.4):

$$\mathcal{G}(t) = x(t)^T Px(t) + \sum_{v=1}^{3} \int_{t-a_v}^{t} x(\alpha)^T Q_v x(\alpha)d\alpha, \tag{3.4}$$

where $(a_1, a_2, a_3) = (d, \tau(t), d_M)$. Calculate the time derivative of $\mathcal{G}(t)$ along with the solutions to Eq. (2.4). The result obtained is

$$\begin{aligned} \dot{\mathcal{G}}(t) \quad = \quad & 2x(t)^T P\dot{x}(t) + x^T(t)\left(\sum_{v=1}^{3} Q_v\right)x(t) - x(t-d)^T Q_1 x(t-d) \\ & -x(t-\tau(t))^T Q_2 x(t-\tau(t)) - x(t-d_M)^T Q_3 x(t-d_M) - e_k^T(t_k^n\hbar)F_\ell e_k(t_k^n\hbar) \\ & +e_k^T(t_k^n\hbar)F_\ell e_k(t_k^n\hbar). \end{aligned}$$

From the event–triggered condition Eq. (2.10), $t \in [\hat{i}_k\hbar + \tau_{\hat{i}_k}, \hat{i}_{k+1}\hat{+}\tau_{\hat{i}_{k+1}})$ we have

$$e_k^T(t_k^n\hbar)F_\ell e_k(t_k^n\hbar) \leq \rho y(\hat{i}_k\hbar)^T F_\ell y(\hat{i}_k\hbar),$$

which yields

$$[x^T(t-\tau(t)) \quad e_k^T(t_k^n\hbar)] \begin{bmatrix} \rho\mathscr{C}_i^T F_\ell\mathscr{C}_i & -\rho\mathscr{C}_i^T F_\ell \\ \clubsuit & \kappa\rho \end{bmatrix} \begin{bmatrix} x(t-\tau(t)) \\ e_k^T(t_k^n\hbar) \end{bmatrix} \leq 0. \tag{3.5}$$

Augmented matrix define as

$$\zeta(t) = \{x(t), x(t-d), x(t-\tau(t)), x(t-d_M), e_k^T(t_k^n\hbar), w(t)\}.$$

From the above analysis, we can say that

$$\begin{aligned} \dot{\mathcal{G}}(t) &- \mathcal{J}(t) \leq \zeta(t)^T\theta_{ij}\zeta(t) \\ \dot{\mathcal{G}}(t) &- y(t)^Ty(t) - \gamma^2\omega(t)^T\omega(t) \leq \zeta(t)^T\theta_{ij}\zeta(t) \end{aligned} \tag{3.6}$$

The following theorem, which offers sufficient circumstances, is based on the conditions derived in Theorem 3.2 and supplies those conditions. □

Theorem 3.2. *Considered the following the conditions:*
$\tilde{\Sigma}_1$: *For the exosystem* $\dot{\omega}(t) = \sum_{i=1}^{r^2} \lambda_i^1(\delta_2(t))\mathscr{S}_i\omega(t)$ *is Poisson stable.*
$\tilde{\Sigma}_2$: *There exit matrices* $X > 0$, $\bar{Q}_v > 0$, $\bar{F}_\ell > 0$, $v, \ell = 1, 2$ *and* $\bar{K}_j$, *such that*

$$\bar{\theta}_{ij} + \bar{\theta}_{ji} < 0, \tag{3.7}$$

where

$$\bar{\theta}_{ij} = \begin{bmatrix} \bar{\theta}_{ij}^{11} & \mathscr{A}_{d_i}X & \mathscr{B}_i\bar{K}_j & 0 & \mathscr{B}_iX & \mathscr{P}_iX & \mathscr{C}_i^T \\ \clubsuit & -\bar{Q}_1 & 0 & 0 & 0 & 0 & 0 \\ \clubsuit & \clubsuit & -\bar{Q}_2 + \rho\mathscr{C}_i^T\bar{F}_\ell\mathscr{C}_i & 0 & 0 & 0 & 0 \\ \clubsuit & \clubsuit & \clubsuit & -\bar{Q}_3 & \rho\mathscr{C}_i^T\bar{F}_\ell & 0 & 0 \\ \clubsuit & \clubsuit & \clubsuit & \clubsuit & -\bar{F}_\ell & 0 & 0 \\ \clubsuit & \clubsuit & \clubsuit & \clubsuit & \clubsuit & -\gamma^2 I & 0 \\ \clubsuit & \clubsuit & \clubsuit & \clubsuit & \clubsuit & \clubsuit & -I \end{bmatrix}$$

$$\theta_{ij}^{11} = \mathscr{A}_iX + X\mathscr{A}_i^T + \sum_{v=1}^{3}\bar{Q}_v.$$

$\tilde{\Sigma}_3$: *The solution of* $\breve{\Pi}(t)$ *and* $\breve{\Gamma}(t)$ *can be obtained by the function of* $\lambda_{\diamondsuit,i}^1(\delta_1(t)) \geq 0$, $i = 1, 2, \cdots, r_\diamondsuit$ *for all the values of* $t \geq 0$ *(1 denoted for model and 2 for exosystem):*

$$\breve{\Pi}\breve{\mathscr{S}} = \mathscr{A}_i\breve{\Pi} + \mathscr{B}_i\breve{\Gamma} + \mathscr{P}_i \tag{3.8}$$

$$\mathscr{C}_i\breve{\Pi} - \breve{\mathscr{Q}} = 0 \tag{3.9}$$

such that $\breve{\Pi}$ *is directly obtained, while on the other side* $\breve{\Pi}_0$ *is calculated by putting the* x_0 *and* ω_0 *into* $\breve{\Pi}_0$. *Then, EOFRP is solvable.*

Proof: Let suppose $X^{-1} = P$, then pre-multiplying by $\mathfrak{H}$ and post-multiplying $\mathfrak{J}$ to Eq. (3.1) which yields to Eq. (3.7) with implementing the Schur complement, where

$$(\mathfrak{H}, \mathfrak{J}) = \left[\{X^{-1}, X^{-1}, X^{-1}, X^{-1}, X^{-1}, I\}, \{X, X, X, X, X, I\}\right].$$

Furthermore, $\bar{Q}_v = X^{-1}Q_vX$, $\bar{F}_\ell = X^{-1}F_\ell X$, and $\bar{K}_j = K_jX$. The proof is completed. □

4. Simulation example

In this part, together fuzzy regulation method based upon the linear regulator and the techniques designed in the last section are implemented with 2 by 2 fuzzy rules with no external noise.

Exosystem in linear for fuzzy regulation: In this example, we select the delayed T-S fuzzy plant with Plant Rule i: IF ξ_{i1}^1 and,$\cdots$, $\delta_p^1(t)$ is ξ_{ip}^1, THEN

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r^1} \lambda_i^1(\delta_1(t)) \left[\mathscr{A}_ix(t) + \mathscr{A}_{di}x(t-d) + \mathscr{B}_iu(t)\right] \\ \dot{\omega}(t) = \mathscr{S}\omega(t), \\ e(t) = \mathscr{C}_ix(t) - \mathscr{Q}\omega(t), \end{cases} \tag{4.1}$$

where

$$\begin{bmatrix} \mathscr{A}_1 & \mathscr{A}_{d_1} & \mathscr{B}_1 \\ \mathscr{A}_2 & \mathscr{A}_{d_2} & \mathscr{B}_2 \end{bmatrix} = \left[\begin{array}{cc|cc|c} 0 & 1 & 0.5 & -1 & 0 \\ 2 & 0 & -0.1 & 0 & 2 \\ \hline 0 & 1 & -0.9 & 0 & 0 \\ 3 & 0 & -0.85 & -0.25 & 1 \end{array}\right]$$

$$\mathscr{S} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \mathscr{C}_1 = \mathscr{C}_2 = \mathscr{Q} = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Now, we define the initial conditions for $x_0 = \begin{bmatrix} 0.5 \\ -2.5 \end{bmatrix}$ $\omega_0 = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}$. Membership functions used are

$$\begin{aligned} \lambda_1^1(\delta_1(t)) &= \frac{1}{2}(1 + \frac{\delta_1(t)}{30}), \\ \lambda_2^1(\delta_1(t)) &= 1 - \lambda_1^1(\delta_1(t). \end{aligned} \tag{4.2}$$

It can be observed that reference signal is produced in a linear system.

Using matrices Eq. (4.1), which is seen the results of the delayed fuzzy model is $y(t) = x_1(t)$, on the other side about the reference signal e.g. exosystem's output $y_{ref}(t) = \omega(t)$. Its mean regulation problem can be expressed by a $u(t)$ such that $x_1(t)$ coincide to $w_1(t)$ as times grow. Using the same technique of [33], it can be found that the regulation problem will be encountered when $x_1 = w_1$, when we fuzzy mappings turn to:

$$\pi(\omega(t)) = \Pi_1(\omega(t)) = \Pi_2(\omega(t)) = diag\{1, 1\}\omega(t) = \Pi_1(\omega(t)).$$

In the same consequences

$$\gamma(\omega(t)) = \sum_{i=1}^{2} \delta_i(\omega_1(t))\Gamma_i\omega(t) = -\frac{3}{2}h_1(\omega_1(t))\omega_1(t) - 4h_2(\omega_1(t))\omega_1(t).$$

Then, set the $\gamma = 5.5$, $(\bar{F}_1, \bar{F}_2) = (0.4520)$ and solving the LMI Eq. (3.7), with the feasible solution of controller gain:

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} 0.1648 & 1.3270 \\ 1.3270 & 18.5162 \end{bmatrix}$$

Then, the following controller can be directly calculated:

$$u(t) = \sum_{i=1}^{2} \lambda_i^1(x_1(t))K_i\left[x(t) - \Pi(\omega(t))\right] + \sum_{i=1}^{2} \lambda_i^1(x_1(t))\Gamma_i(\omega(t)). \tag{4.3}$$

Figure 3 presents the simulation result after implementing the Eq. (4.3), while the inter-event intervals and release instants are displayed by Figure 4.

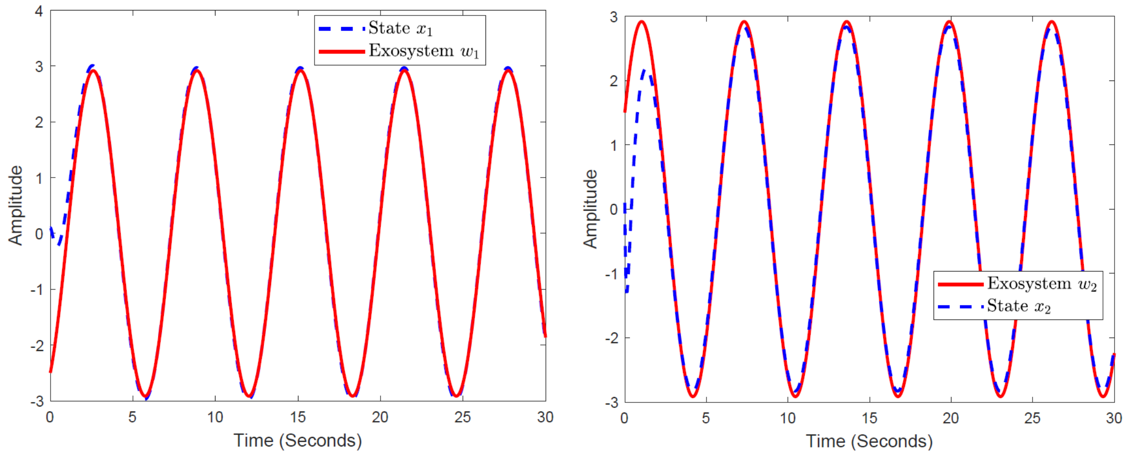
So, the new approach which is designed for the networked control system is implemented. In this scenario, mapping issue is attained by considering dynamic adaptation of Francis Equations Eq. (3.2) and Eq. (3.3).

Calculating the steady-state zero-error: This scenario, the method for design $\bar{\Pi}(t)$ can proceed as below.

$$x_1(t) = \omega_1(t), \qquad \text{with differentiating} \qquad \dot{x}_1(t) = \dot{\omega}_1(t) \tag{4.4}$$

Alternatively, it is easily inferred from the delayed T–S fuzzy model and its suitable matrices that

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{\omega}_1(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ \omega_2(t) \end{bmatrix}. \tag{4.5}$$



(a) $y = x$ **(b)** $y = 3sinx$

Figure 3. Trajectory of output versus reference.

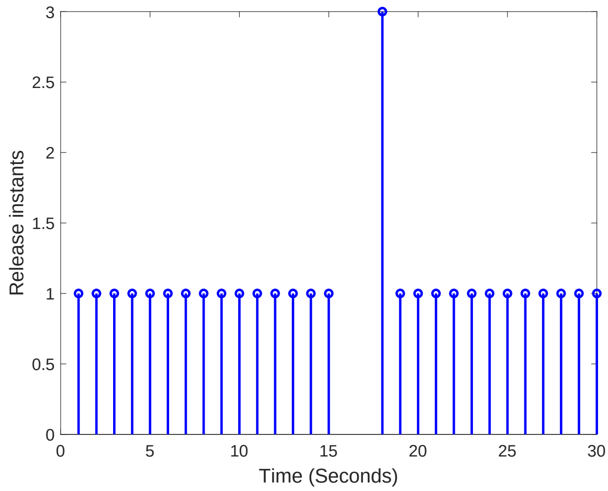


Figure 4. Release instant and interval.

Furthermore, from Eq. (4.4) and Eq. (4.5), the steady-state error shall be written as

$$\begin{aligned} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} &= \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \end{bmatrix} \\ &= \bar{\Pi}\omega(t) = \pi(\omega(t)) = x_{ss}(t). \end{aligned}$$

Calculating the steady-state input: Differentiating the Eq. (4.5) w.r.t time with considering the delayed T–S fuzzy plant Eq. (2.9), one can get:

$$\begin{aligned} \dot{x}_1(t) &= \dot{\omega}_2(t) \\ &= 2\lambda_1^1(x_1(t)) \cdot x_1(t) + 3\lambda_2^1(x_1(t)) \cdot x_1(t) + 2\lambda_1^1(x_1(t)) \cdot u(t) + \lambda_2^1(x_1(t)) \cdot u(t) \qquad (4.6) \\ &= -\omega_1(t). \end{aligned}$$

So, this investigation must be performed in steady–state. Consequently, according towards the steady-state manifold, $\pi(\omega(t))$, that is computed above, we found that $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \end{bmatrix}$.

According to the Eq. (4.6)

$$2\lambda_1^1(\omega_1(t))\cdot\omega_1(t)+3\lambda_2^1(\omega_1(t))\cdot\omega_1(t)+2\lambda_1^1(x_1(t))\cdot u_{ss}(t)+\lambda_2^1(\omega_1(t))\cdot u_{ss}(t)=-\omega_1(t). \quad (4.7)$$

Construction of the steady–state input can be followed as

$$\begin{aligned} u_{ss}(t) &= -\frac{2\lambda_1^1(\omega_1(t))+3\lambda_2^1(\omega_1(t))+1}{2\lambda_1^1(\omega_1(t))+\lambda_2^1(\omega_1(t))}\omega_1(t) \quad (4.8)\\ &= \left[\begin{array}{cc} -\frac{2\lambda_1^1(\omega_1(t))+3\lambda_2^1(\omega_1(t))+1}{2\lambda_1^1(\omega_1(t))+\lambda_2^1(\omega_1(t))} & 0 \end{array}\right]\left[\begin{array}{c}\omega_1(t)\\ \omega_2(t)\end{array}\right], \end{aligned}$$

which is equivalent to $\bar{\Gamma}(t)\omega(t)=\gamma(\omega(t))$.
Finally, we can say:

$$\begin{aligned} \pi(\omega(t)) &= \bar{\Pi}(t)\omega(t)=diag\{1,1\}\omega(t)=\Pi\omega(t) \quad (4.9)\\ \gamma(\omega(t)) &= \bar{\Gamma}(\omega(t))\left[\begin{array}{cc} -\frac{2\lambda_1^1(\omega_1(t))+3\lambda_2^1(\omega_1(t))+1}{2\lambda_1^1(\omega_1(t))+\lambda_2^1(\omega_1(t))} & 0 \end{array}\right]\left[\begin{array}{c}\omega_1(t)\\ \omega_2(t)\end{array}\right] \quad (4.10) \end{aligned}$$

It is noted that Eq. (4.8) and Eq. (4.9) are the solution of dynamic version of Francis equation. Detail comparison of the event–triggered scheme (ETS) is given in Table 1. In the same consequences, after implementing the controller on the delayed T–S fuzzy system plant is shown in Figures 5 and 6. It can be noted in designed controller through implementing the time–varying parameters to achieve the solution of a dynamic version of Francis equation outperform the controller for the linear regulators.

Table 1. An analysis of event-triggered schemes.

ρ	Time–triggered	Event–triggered	Savage use of resources
0.15	300	41	13.65%
0.25	300	47	18.42%
0.40	300	52	21.17%
0.55	300	55	24.85%

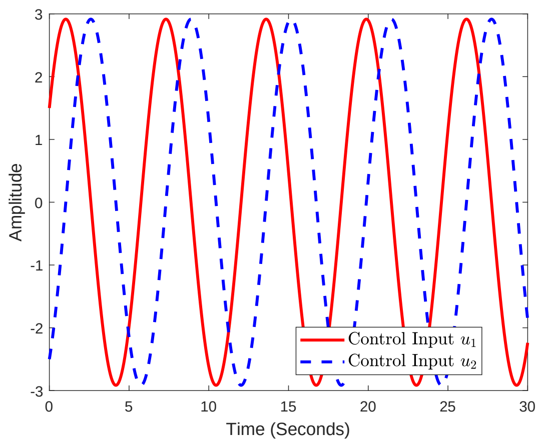


Figure 5. Control inputs.

Remark 4.1. In this research, we investigate the output regulation for H_∞ control over the networked control system. However, due to the inherent delay in the networked control system, it is difficult to achieve the exact tracking for the nonlinear system. So this problem can be investigated further for the exact output regulation by using the optimal control [40], adaptive control [27], and neural network [11].

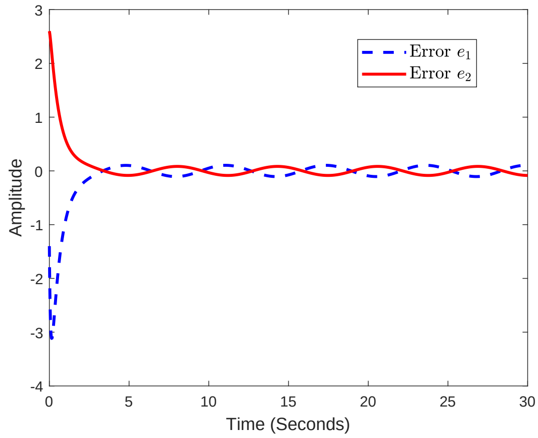


Figure 6. Trajectory error.

Remark 4.2. In this section, authors describe the procedure of the control scheme and present the theoretical results obtained using the proposed algorithm. First initialize the system parameters $\mathscr{A}_i$, $\mathscr{A}_{di}$, $\mathscr{B}_i$, $\mathscr{P}_i$, and $\mathscr{C}_i$. For the output, define the maximum allowable delay $\hbar$, and communication delay d_M. Then obtain the feasible solution by applying the LMI Eq. (3.7) of Theorem 3.2. After calculating the controller gain using LMI Eq. (3.7) taking into account output regulation theory. Using Simulink in Matlab, we can obtain simulation results.

Example: We will describe the chaotic Synchronization problem. So this scenario, the system can be explained through fuzzy Rössler attractor, the while fuzzy Lorenz attractor is regarded as the response mechanism [35] as shown.

Fuzzy Plant
Plant Rule i: IF ξ_{i1}^1 and,$\cdots$, $\delta_p^1(t)$ is ξ_{ip}^1, THEN

$$\dot{x}(t) = \mathscr{A}_i x(t) + \mathscr{A}_{di} x(t-d) + \mathscr{B}_i u(t), \quad i = 1, \cdots, 2. \tag{4.11}$$

Exo-system (Fuzzy)
Plant Rule i: IF $\delta_1^2(t)$ is ξ_{i1}^2 and$\cdots$ $\delta_p^2(t)$ is ξ_{ip}^2, THEN

$$\dot{\omega}(t) = \mathscr{S}_i \omega(t), \qquad i = 1, \cdots, 2. \tag{4.12}$$

Hence, the nonlinear dynamics were presented as

$$\left\{ \begin{array}{lcl} \dot{x}(t) & = & \sum_{i=1}^{r1} \lambda_i^1(\delta_1(t)) \left[\mathscr{A}_i x(t) + \mathscr{A}_{di} x(t-d) + \mathscr{B}_i u(t)\right], \\ \dot{\omega}(t) & = & \sum_{i=1}^{r2} \lambda_i^2(\delta_2(t)) \mathscr{S}_i \omega(t), \\ e(t) & = & \sum_{i=1}^{r1} \lambda_i^1(\delta_1(t)) \mathscr{C}_i x(t) - \sum_{i=1}^{r2} \lambda_i^2(\delta_2(t)) \mathscr{Q}_i \omega(t). \end{array} \right. \tag{4.13}$$

Parameters for the system are defined as

$$\begin{bmatrix} \mathscr{A}_1 & \mathscr{A}_{d1} \\ \mathscr{A}_2 & \mathscr{A}_{d2} \end{bmatrix} = \left[\begin{array}{ccc|ccc} -a_L & a_L & 0 & -(1-a_L) & (1-a_L) & 0 \\ c_L & -1 & -d_L & (1-c_L) & -1 & -(1-d_L) \\ 0 & d_L & -b_L & 0 & (1-d_L) & -(1-b_L) \\ \hline -a_L & a_L & 0 & -(1-a_L) & (1-a_L) & 0 \\ c_L & -1 & -d_L & (1-c_L) & -1 & -(1-d_L) \\ 0 & d_L & -b_L & 0 & (1-d_L) & -(1-b_L) \end{array} \right]$$

$$\begin{bmatrix} \mathscr{S}_1 & \mathscr{B}_1 \\ \mathscr{S}_2 & \mathscr{B}_2 \end{bmatrix} = \left[\begin{array}{ccc|c} 0 & -1 & -1 & 0 \\ 1 & a_R & 0 & 1 \\ b_R & 0 & -d_L & 0 \\ \hline -0 & -1 & -1 & 0 \\ 1 & a_R & 0 & 1 \\ 0 & d_L & -b_L & 0 \end{array}\right], \qquad \mathscr{C} = \mathscr{Q} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

with $(a_L, b_L, c_L) = (10, \frac{8}{3}, 28)$, $(a_R, b_R, d_R, d_L) = (0.34, 0.4, 10, 30)$, with $x_0 = \begin{bmatrix} 15 \\ 5 \\ -7 \end{bmatrix}$,

and $\omega_0 = \begin{bmatrix} 2.5 \\ 0 \\ -1.5 \end{bmatrix}$. The membership function for the above systems are given below:

Fuzzy Plant:

$$(\lambda_1^1(\delta_1(t)), \lambda_2^1(\delta_1(t))) = \left(\frac{1}{2}\left[\frac{x_1(t)+d_L}{d_L}\right], \frac{1}{2}\left[\frac{-x_1(t)+d_L}{d_L}\right]\right)$$

Exo-system (Fuzzy):

$$(\lambda_1^1(\delta_2(t)), \lambda_2^1(\delta_2(t))) = \left(\frac{1}{2}\left[1 + \frac{c_R-\omega_1(t)}{d_R}\right], \frac{1}{2}\left[1 - \frac{c_R-\omega_1(t)}{d_R}\right]\right),$$

with $c_R = 4.5$. Furthermore, in this example for the delayed T–S fuzzy system and their appropriate matrices that can reduce the plant output, e.g. $y(t) = x_1(t)$. Now we use the controller in the form Eq. (2.21), which comes from the foundation of the linear regulator [33], can be designed as

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix}, = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -0.1 & -0.1 & 1 & -0.1 & -0.1 \\ 10.47 & 2.48 & -1.02 & 10.26 & 2.41 & 0.57 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -0.1 & -0.1 & 1 & -0.1 & -0.1 \\ 10.47 & 2.48 & -1.02 & 10.26 & 2.41 & 0.57 \end{array}\right].$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \\ K_{11} & K_{12} \end{bmatrix} = \left[\begin{array}{ccc|ccc} 287 & 73.41 & -30.66 & 280.62 & 71.341 & 15.1 \\ 287 & 73.41 & -30.66 & 280.62 & 71.341 & 15.1 \\ -159.12 & 0.4147 & 121.89 & -159.12 & 0.4147 & 121.89 \end{array}\right]$$

Behavior of the synchronization can be calculated from the controller which is based on the local regulator (linear), this concept can be observed in the Figures 7–9, which is necessary for investigating the robust fuzzy control problems.

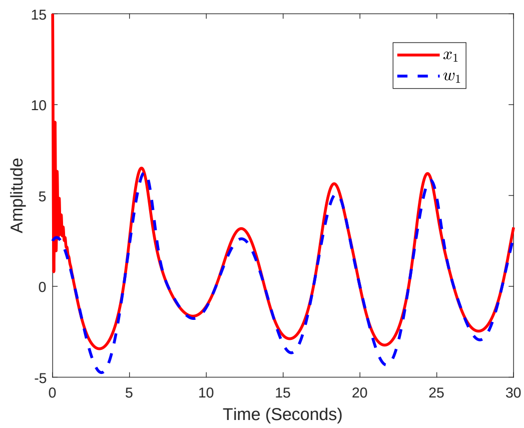


Figure 7. Synchronization based upon the fuzzy controller for Example

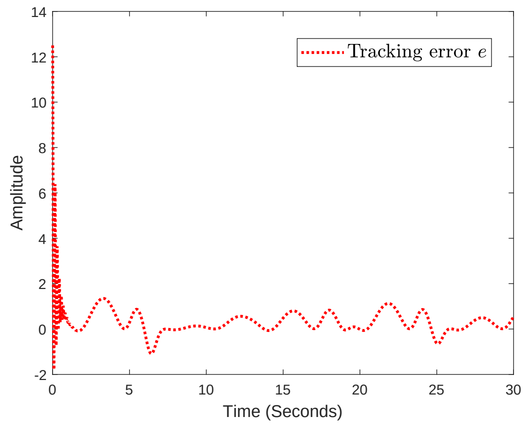


Figure 8. Trajectory error

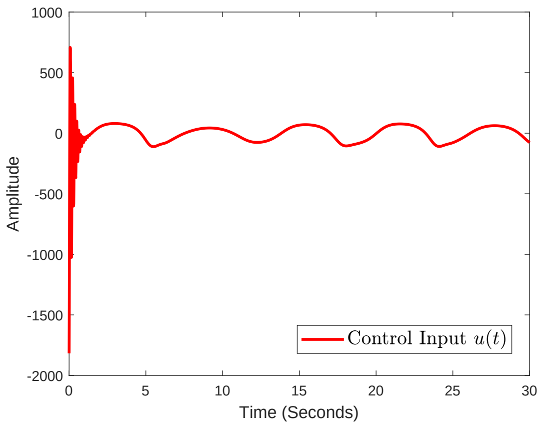


Figure 9. Control signal for Example.

Remark 4.3. In this section, authors compare their developed algorithms in order to demonstrate their superiority. The paper examines output regulation of fuzzy systems under networked control systems with a H_∞ performance index. So, we provide a numerical comparison by considering the same problem. However, we note that Chen et al. [8], Gnaneswaran and Joo [14] and Pam and Yang [36] does not address output regulation design with H_∞ performance, which is a special problem in our paper. So, we considered this special case and provided a numerical comparison between our method and that in [8,14,36]. The comparison is given in the following Table 2. It can clearly be seen that our newly proposed approach gives less conservative results than the existing ones in [8,14,36].

Table 2. Optimal performance for minimum performance with different sampling period.

$\hbar$	0.05	0.1	0.2	0.4
Pam and Yang	0.8821	0.9823	1.2964	2.4591
Gnaneswaran and Joo	0.5818	0.6317	0.7145	2.1574
Chen et al.	0.4602	0.5389	0.6893	1.6201
Theorem 3.2	0.1731	0.1937	0.2109	0.2367

5. Conclusions

A delayed T–S fuzzy modeling with H_∞ performance index has been investigated over the networked control system with utilising both linear regulation theory and (dynamical Francis equations). In an effort to minimize the burden of network transmission, an event-triggering mechanism has been proposed. New stability conditions in the form of EOFRPs with NCS have been defined. To take the advantage of global convergence can be attained, based upon the nonlinear regulator's regional characteristics technique. In addition, the mapping process gives assurance the regulation property can be calculated by solving the dynamical linear equations (dynamical Francis equations).

By constructing an Lyapunov functional and making use of event-triggered method, some are sufficient requirements that ensure asymptotically stability of H_∞ performance index for the resulting model is constructed. This result, the designed the tracking issue can be identified as a result in practice. In general, the fuzzy regulation method is more difficult as compared to the simple regulation method which is designed by the linear local regulators. To show the two numerical examples are shown, applicability, validity, benefits of the suggested methodology over traditional approaches.

Remark 5.1. In the future, this proposed algorithm can also be used to design dissipative controllers for multi-agent systems. In light of this article, fuzzy Markov jump systems can also be analyzed with a fault isolation delay and actuator saturation. In addition, automation systems and grid–connected photovoltaic plants, as well as mobile robots and cascaded H-bridge converters, have all been implemented using networked control systems in industrial systems. Communication networks, however, present considerable challenges with respect to modeling, analyzing, and synthesizing NCSs. These include network-induced delays, data packet dropouts, limited widths, and quantization.

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