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Research Article

VEHICLE ROUTING PROBLEM IN POST-DISASTER HUMANITARIAN RELIEF LOGISTICS: A CASE STUDY IN ANKARA

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ABSTRACT

Natural disasters have been affecting the human life and causing the death of millions since the very first day the human being came into existence. Besides, it causes physical, financial, social and environmental losses and affects societies negatively by suspending daily life and human activities. In order to minimize losses, it is necessary to plan post disaster activities effectively. One of these activities is humanitarian relief logistics activities that aim to provide sufficient amount of humanitarian relief to disaster victims as soon as possible. In this paper, a multi depot vehicle routing problem with stochastic demand (SDMD_VRP) is taken into account. A mathematical model with chance constraint approach is developed for this rarely discussed problem. The proposed non-linear mathematical model is linearized with separable programming methods and examined on test problems. Lastly, a case study was carried out for Ankara- the capital city of Turkey.
Keywords: Humanitarian relief logistics, multi-depot vehicle routing, stochastic demand, chance constraint.

1. INTRODUCTION

Disasters are extraordinary situations that cause numerous types of (human life, moral, facilities etc.) losses. An effective disaster management must be implemented in order to minimize these damages. Disaster management has 4 main functions. These functions are; information and planning, operations, logistics, financial and administrative affairs [1]. Among these functions; disaster logistics aim to mobilize people, resources, abilities and knowledge in order to assist disaster victims. The logistics of humanitarian aid in disasters is called "humanitarian relief logistics." In humanitarian relief logistics, the most important thing is to send the proper material, to the correct person, in the correct amount, with the appropriate quality, at the right time and to the place. The biggest difference between commercial logistics is that the concept of cost has secondary importance in humanitarian relief logistics. In addition, while suppliers, manufacturers and demands are determined or at least predictable in commercial logistics, the same are uncertain in humanitarian logistics [2].Vehicle routing problem is used to determination of the optimal routes for a fleet of vehicles to service a set of customers, given a set of operational constraints [3]. Vehicle routing problems has been widely used in logistics

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managem ent [4] [5]. Thus in this study, the vehicle routing problem in humanitarian logistics is taken into account. The aim is to determine vehicle routes that ensure humanitarian aid materials is transported as quickly as possible to the disaster area. After a natural disaster, aid material is transferred from aid warehouses (i.e. AFAD and Kızılay warehouses, in Turkey) to the temporary housing areas where disaster victims are located. The exact number of disaster victims (thus, the required amount of aid material) in each temporary housing area is not known beforehand. Therefore, this problem can be classified as a multi depot vehicle routing problem with stochastic demand (SDMD_VRP). The demands are assumed to belong to the normal distribution and meeting these demands is guaranteed to a certain level by developing a stochastic model with the chance constraint approach. The proposed non-linear mathematical model is linearized with separable programming methods and tested on problems. Lastly, a case study is carried out for Ankara -the capital city of Turkey.

The remainder of the paper is organized as follows. Next section, we provide an overview of the existing literature of SDMD_VRP and aid distribution in humanitarian relief logistics. In the 3rd section, SDMD_VRP is defined and the chance constraint model for the problem is explained. Then the nonlinear mathematical model is linearized with separable programming methods. In the 4th section, the mathematical model is applied to various test problems and performance of the model is evaluated. The 5th section contains a case study for Ankara; finally the 6th section summarizes the results and advices for potential future studies.

2. LITERATURE REVIEW

This section presents a brief review on the aid distribution in humanitarian relief logistics problems. In addition, existing literature for SDMD_VRP is surveyed.

2.1. Aid Distribution For Humanitarian Relief Logistics

Aid distribution problem has been a very important problem for the world for too many years. Within the scope of disaster logistics in the literature, many problems have been addressed such as; warehouse selection for aid materials, meeting points for disaster victims, inventory policies for aid warehouses, establishment of emergency intervention centers for injured people, ambulance distribution and post-disaster debris removal. Operations research techniques are used very commonly to solve these types of problems.

Altay and Green [6] revealed that the studies of operations research and management systems performed on this subject increased significantly since 1990. According to this review paper, the most frequently used methods in operations research were mathematical programming, probabilistic/statistical methods and modeling papers more than practical studies until 2005. Kovacs and Spens [2] explained the properties, challenges and general framework of the humanitarian logistics and differentiated it from the commercial logistics. Yi and Kumar [7] addressed the logistic activities involving the transportation of materials from distribution centers to disaster zones and the evacuation of injured people to medical services by using ant colony optimization. Özdamar and Demir [8] defined a hierarchic clustering and routing procedure for the post-disaster coordination of distribution and evacuation activities. Berkoune et al. [9] established a mixed integer programming model for transporting humanitarian aids to people and proposed the “branch and bound” method that provides the optimal result for small-scale problems and the genetic algorithm approach for big-scale problems. Huang et al. [10] explained the indicators of aid distribution performance. Caunhye et al. [11] explained optimization models utilized in emergency logistics. Galindo and Batta [12], as a continuation of Altay and Green's review paper, examined the humanitarian logistics papers between 2005 and 2010 and emphasized that the number of studies performed on stochastic programming has increased significantly when compared to previous years. Allahviranloo et al. [13] proposed a model for the selective vehicle routing problem, later they de veloped a parallel and a classical genetic algorithm for that model. Salman and Yücel [14] worked on a series of alternative paths from potential supply points for each demand point to find the shortest route in the post-disaster process and decided where to establish the facilities. Özdamar and Ertem [15] focused on mathematical models in humanitarian logistics and models are classified in terms of modeling features and formulation structures. Sharif and Salari [16] developed a greedy randomized adaptive search procedure for post-disaster transportation problems to meet the demands of all customers from a central warehouse. Hoyos et al. [17] explained operation research models with stochastic components (demand, demand location, transportation network, supply etc.). Rahafrooz and Alinaghian [18] are assumed that demand is in the form of fuzzy trapezoidal coefficients and they proposed a multi-objective stochastic model for relief distribution planning. Theeb and Murray [19] a vehicle routing problem has been dealt with considering delivery of aid in post-disaster humanitarian aid logistics. Here, demands have been taken into consideration as time dependent for different products. Akbari and Salman [20] created a simultaneous working program for road cleaning teams to open the road by debris removal after disaster. In this paper, it has been aimed to determine the routes of the vehicles by considering situations that the demand of disaster victims is stochastic and aids have been received from multiple depots.

2.2. Multi Depot Vehicle Routing Problem with Stochastic Demand

After examining the literature, it is seen that there are only a few studies which contains both the "multi depots" and "stochastic demands". Existing literature of the related to SDMD_VRP are given in Table 1.

Table 1. Existing literature about SDMD_VRP

Source	Content
Tillman [21]	A heuristic approach is proposed for the case, it is assumed that the demands belong to the poisson distribution.
Chan et al. [22]	The expected demands are determined by considering the demands of people who have previously received services.
Miranda and Garrido [23]	Problem is modeled as hub-and-spoke network and a heuristic procedure is proposed.
Christiansen et al.[24]	Branch-price-and-cut algorithms are implemented for the case in which depot capacity is not limited, and the demands are identified as random variables.
Calvet et al. [25]	Iterative local search and monte-carlo techniques are used for the problem, their algorithms included having safety stocks for minimizing the risk of route break-downs.

To fill this gap in the literature, our problem is defined as a Multi Depot Vehicle Routing Problem with Stochastic Demand. Next section, the problem is defined and explained comprehensively.

3. PROBLEM DESCRIPTION

Stochastic vehicle routing problem (SVRP) appears when some elements of the problem are not deterministic such as customer groups, demands or travel time. These elements are modeled as random variables received from a known probability distribution in SVRP. The objective function in these types of problems is generally the minimization of the routes. The most studied SVRP problem type is the vehicle routing problem with stochastic demand in the literature [26]. Multi-depot vehicle routing problem with stochastic demand (SDMD_VRP) is the problem of finding the minimum cost routes when serving to the customers from multiple depots, whose demands belong to a known probability distribution.

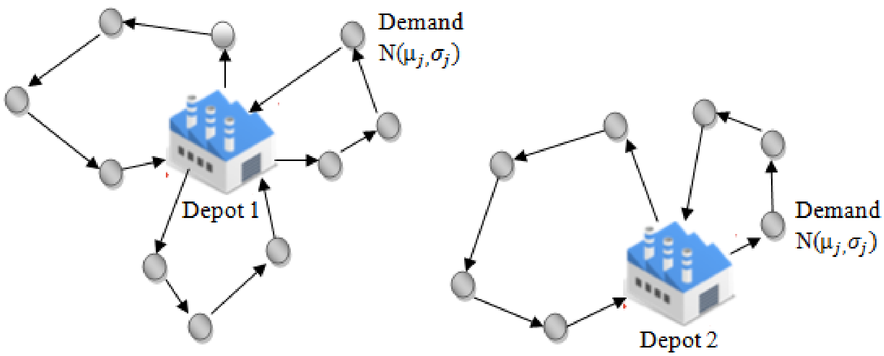

Figure 1. SDMD_VRP

There are multiple depots with limited capacities in the SDMD_VRP problems. The vehicles have limited capacities that distribute the products in depots to the customers. Each route ends at the depot where it starts. Each of the customers is visited only once by a single vehicle. Customer demands are stochastic variables received from a known probability distribution. The aim is to minimize the total expected travel time (or cost).

SDMD_VRP problem is presented on G = (V, A, D) graph. Here V is a set of the nodes and composed of two subsets. While $V_i = \{v_1, v_2, \dots v_m\}$ indicates the set of the depots, $V_j = \{v_{m+1}, v_{m+2}, \dots, v_n\}$ represents the set of the customers receiving service from the depots. A defines the set of the connections between a pair of nodes while D defines the set of the cost, travel time or distances between the nodes. Cost matrix D is symmetric and provides triangular inequality.

Here, different from the multi-depot vehicle routing problem, the customer demands are stochastic variables received from known probability distributions ($\xi_j, \mathrm{j} = \mathrm{m}+1, \dots, \mathrm{n}$). It is assumed that the customer demands (ξ_j) do not exceed the vehicle and depot capacity. Moreover, it is assumed that the real demand of each customer is obtained after reaching to the customer. Within the scope of this study; it is assumed that the customer demands belong to the normal distribution.

3.1. Mathematical Model for Deterministic Multi Depot Vehicle Routing Problem (MDVRP)

In this study, the below model in literature [27] was taken as a basis for the deterministic state of MDVRP.

Sets and Parameters:

I : Set of all depots

J : Set of all customers
K: Set of all vehicles
N : Number of customers
c_{ij}: Distance between point i and j $\quad i,j \in I \cup J$
d_j: Demand of customer j $\quad j \in J$
V_i Capacity of depot i $\quad i \in I$
Q_k: Capacity of vehicle (route) k $\quad k \in K$

Decision variables:

$$x_{ijk} = \begin{cases} 1, & if\ point\ i\ immediately\ precedes\ point\ j\ on\ route\ k \\ 0, & otherwise \end{cases} \quad i,j \in I \cup J$$

$$z_{ij} = \begin{cases} 1, & if\ customer\ j\ allocated\ to\ depot\ i \\ 0, & otherwise \end{cases}$$

U_{lk} : auxiliary variable for sub-tour elimination constraints in route k
Mathematical Model:

$$Min\ \sum_{i\in I\cup J}\sum_{j\in I\cup J}\sum_{k\in K} x_{ijk}c_{ij} \tag{1}$$

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1 \qquad \forall j \in J \tag{2}$$

$$\sum_{i\in IUJ}\sum_{j} d_j x_{ijk} \leq Q_k \qquad \forall k \in K \tag{3}$$

$$U_{lk} - U_{jk} + N x_{ljk} \leq N - 1 \qquad \forall l,j \in J\ \forall k \in K \tag{4}$$

$$\sum_{j\in IUJ} x_{ijk} - \sum_{j\in IUJ} x_{jik} = 0 \qquad \forall k \in K, i \in IUJ \tag{5}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \leq 1 \qquad \forall k \in K \tag{6}$$

$$\sum_{j\in J} d_j z_{ij} \leq V_i \qquad \forall i \in I \tag{7}$$

$$-z_{ij} + \sum_{u\in I\cup J}(x_{iuk} + x_{ujk}) \leq 1 \qquad \forall\ i \in I, j \in J, k \in K \tag{8}$$

$$x_{ijk} \in \{0,1\} \qquad \forall\ i \in I, j \in J, k \in K \tag{9}$$

$$z_{ij} \in \{0,1\} \qquad \forall\ i \in I, j \in J \tag{10}$$

$$U_{lk} \geq 0 \qquad \forall l \in J, k \in K \tag{11}$$

Objective function (1) of the model aims to minimize the total distance. Constraint (2) provides giving service to each customer at a single route. Constraint (3) is the vehicle capacity constraint which provides the total demands of the customers do not exceed the vehicle capacity. Constraint (4) provides sub-tour elimination. Constraint (5) provides equal input and output to each node in each of the routes. Constraint (6) provides a vehicle to exit from one depot at the most. Constraint (7) is the depot capacity constraint which provides the total demands of the customers assigned to a depot not to exceed the depot capacity. Constraint (8) provides a customer to be at the depot route to which customer is assigned. Constraint (9), (10), (11) are the binary and non negativity constraints of the variables.

3.2. 0-1 Integer Programming Model with Chance Constraint for SDMD_VRP

In the chance constraint approach, violation of some constraints is allowed at a predetermined probability level. In stochastic programming with chance constraint, deterministic constraints comprising stochastic information are replaced with stochastic constraint sets. This approach is illustrated as $Pr\,(Ax \leq b) \geq \varphi$. Here it is stated that realization probability of $Ax \leq b$ constraint is requested to be higher than the φ probability value [28].

The aim of this model, which assumes that the demands belong to the normal probability distribution, is to obtain minimum route length such that; the probability of the total demands of the customers in the routes exceeding the vehicle capacity Q_k and the probability of total demands of the customers assigned to the depot exceeding the depot capacity V_i will remain under the pre-determined probability level.

α : Maximum failure probability given for the route failure due to the vehicle capacity
β : Maximum failure probability given for the route failure due to the depot capacity
d_j : Random variable representing the demand of its customer and $d_j \sim \mathrm{N}(\mu_j, \sigma_j)$

For vehicle capacity:

$$Pr\left(\sum_{i \in IUJ} \sum_j d_j x_{ijk} \leq Q_k\right) \geq 1 - \alpha \qquad \forall k \in K \tag{12}$$

For depot capacity:

$$Pr\left(\sum_{j \in J} d_j z_{ij} \leq V_i\right) \geq 1 - \beta \qquad \forall i \in I \tag{13}$$

Constraints should be provided in the model instead of (3) and (7).

The probability for the total demand on a route to exceed the vehicle capacity will be maximum α. In this case $P(X > Q_k) \leq \alpha$ or $P(X \leq Q_k) \geq 1 - \alpha$ as the probability for the total demand on the route not to exceed the vehicle capacity. Here $X \sim N\left(\sum \mu_j, \sqrt{\sum \sigma_j^2}\right)$ represents the total demand of the customers in a route. If Z transformation is made here [26]:

$$P\left(z \leq \frac{\mathrm{Q_k} - \sum \mu_j}{\sqrt{\sum \sigma_j^2}}\right) \geq 1 - \alpha \tag{14}$$

$$P(z \leq z_{1-\alpha}) = 1 - \alpha \tag{15}$$

$$z_{1-\alpha} \leq \frac{\mathrm{Q_k} - \sum \mu_j}{\sqrt{\sum \sigma_j^2}} \tag{16}$$

$$\sum \mu_j + z_{1-\alpha}\sqrt{\sum \sigma_j^2} \leq \mathrm{Q_k} \tag{17}$$

When x_{ijk} 0-1 variable is added to the equation, new vehicle capacity constraint is obtained.

$$\sum \sum \mu_j x_{ijk} + z_{1-\alpha}\sqrt{\sum \sum \sigma_j^2 x_{ijk}} \leq \mathrm{Q_k} \tag{18}$$

Similar operations should also be applied for depot capacity constraint.

The probability for the total demands of the customers assigned to a depot (routes of the depot) to exceed the depot capacity will be maximum β. In this case $P(X > V_i) \leq \beta$, in other words $P(X \leq V_i) \geq 1 - \beta$.

$$P\left(z \leq \frac{\mathrm{V_i} - \sum \mu_j}{\sqrt{\sum \sigma_j^2}}\right) \geq 1 - \beta \tag{19}$$

$$P(z \leq z_{1-\beta}) = 1 - \beta \tag{20}$$

$$z_{1-\beta} \leq \frac{\mathrm{V_i} - \sum \mu_j}{\sqrt{\sum \sigma_j^2}} \tag{21}$$

$$\sum \mu_j + z_{1-\beta}\sqrt{\sum \sigma_j^2} \leq \mathrm{V_i} \tag{22}$$

When z_{ij} 0-1 variable is added to the equation, new depot capacity constraint is obtained.

$$\sum \mu_j z_{ij} + z_{1-\beta}\sqrt{\sum \sigma_j^2 z_{ij}} \leq \mathrm{V_i} \tag{23}$$

As a result, the mathematical model with chance constraint for SDMD_VRP is defined as follows:

Objective function (1)

Constraints (2), (18), [4-6], (23), [8-11]

Constraints (1), (2), (4), (5), (6) and [8-11] are the same as the constraints described before. Constraint (18) is the vehicle capacity constraint with chance constraint which provides the total demands of the customers with stochastic demand to whom the vehicle goes to be bigger than the vehicle capacity namely, allows for the route failure to a certain extent. Constraint (23) is the depot capacity constraint with chance constraint which allows the total demands of the customers with stochastic demand assigned to a depot to exceed the depot capacity to a certain extent.

3.3. Linearization of SDMD_VRP Using Separable Programming

The mathematical model developed is a nonlinear model due to the $\sqrt{\sum_{\mathrm{i}\in\mathrm{I}\cup\mathrm{J}} \sum_J \sigma_j^2 x_{ijk}}$ and $\sqrt{\sum_{\mathrm{j}\in\mathrm{J}} \sigma_j^2 z_{ij}}$ terms. The model is transformed into integer linear programming model with the use of separable programming method to solve the model more easily.

Separable programming problems are special type of nonlinear problems. To use separable programming, the function should be defined as the sum of the single variable functions. Separable programming does not guarantee an optimal solution for nonlinear models, but offers an approximate solution for the problem. To find better approximate solutions, separable functions are replaced with piecewise linear functions [29]. Piecewise linear function is used in order to transform the nonlinear programming models into a suitable form with separable programming [29]. Huang [30] describe the piecewise linear function approach in detail.

$$S_k = \sqrt{\sum_{\mathrm{i}\in\mathrm{I}\cup\mathrm{J}} \sum_J \sigma_j^2 x_{ijk}} \qquad \forall k \in K \tag{24}$$

$$S_k^2 = \sum_{\mathrm{i}\in\mathrm{I}\cup\mathrm{J}} \sum_J \sigma_j^2 x_{ijk} \qquad \forall k \in K \tag{25}$$

$$\sum_{\mathrm{i}\in\mathrm{I}\cup\mathrm{J}} \sum_J \mu_j x_{ijk} + z_{1-\alpha} \times S_k \leq \mathrm{Q_k} \qquad \forall k \in K \tag{26}$$

In this case, the value range $\left[\overline{S_k^2}, \underline{S_k^2}\right]$ to be retained by S_k^2 should be identified in order to estimate S_k, and this range should be divided into M parts. M is set of selected grid points. Since S_k^2 is the sum of the variations of the customer demands for each vehicle, minimum value it will retain is 0. Maximum value to be retained by S_k^2, however, can be found easily by solving the knapsack problem defined below.

$$q_{jk} = \begin{cases} 1, & \text{if vehicle k is allocated to customer j} \\ 0, & \text{otherwise} \end{cases}$$

$$Max \sum_{j\in J} \sum_{k\in K} \sigma_j^2 q_{jk} \tag{27}$$

$$\sum_{j\in J} \mu_j \, q_{jk} \leq Q_k \qquad \forall k \in K \tag{28}$$

$$q_{jk} \in \{0,1\} \qquad \forall j \in J, \forall k \in K \tag{29}$$

Objective function (27) maximizes the sum of the variations of the customers assigned to the same vehicle. The constraint (28) prevents the sum of the average of the customer demands assigned to the vehicle from exceeding the vehicle capacity.

When the function is divided into M range, M number of new variables should be identified. Here, the higher M values is chosen, namely the more the function is divided into parts, the better the result will be, but the solution time of the model will also increase.

Similar operations apply for $\sqrt{\sum_{j \in J} \sigma_j^2 z_{ij}}$ terms.

λ_{mk} and ψ_{mi} variables are identified as Special Order Set Type-2 (SOS2) in order to provide the value to be in only one range. The objective of the particular order set is to provide only the two adjacent variables to retain a value different from zero, here all the other λ_{mk} variables will retain 0 value. In the square root function, b_{mk} and g_{mi} shows the range values in the y-axis, c_{mk} and n_{mi} shows the values in the x-axis.

The constraints required for linearization of the inequality (18) are given below.

$$0 \leq \lambda_{mk} \leq 1 \qquad \forall m \in M, \forall k \in K \qquad (30)$$

$$\sum_m \lambda_{mk} = 1 \qquad \forall k \in K \qquad (31)$$

$$S_k = \sum_m b_{mk} * \lambda_{mk} \qquad \forall k \in K \qquad (32)$$

$$V_k = \sum_m c_{mk} * \lambda_{mk} \qquad \forall k \in K \qquad (33)$$

$$b_{mk} = \sqrt{c_{mk}} \qquad \forall m \in M, \forall k \in K \qquad (34)$$

$$V_k = \sum_{i \in I \cup J} \sum_J \sigma_j^2 x_{ijk} \qquad \forall k \in K \qquad (35)$$

$$\sum_{i \in I \cup J} \sum_J \mu_j x_{ijk} + z_{1-\alpha} * S_k \leq Q_k \qquad \forall k \in K \qquad (36)$$

$$\lambda_{mk} \in SOS2 \qquad \forall m \in M, \forall k \in K \qquad (37)$$

$$S_k, V_k \geq 0 \qquad \forall k \in K \qquad (38)$$

The constraints required for linearization of the inequality (23) are given below.

$$0 \leq \psi_{mi} \leq 1 \qquad \forall m \in M, \forall i \in I \qquad (39)$$

$$\sum_m \psi_{mi} = 1 \qquad \forall i \in I \qquad (40)$$

$$P_i = \sum_m g_{mi} * \psi_{mi} \qquad \forall i \in I \qquad (41)$$

$$R_i = \sum_m n_{mi} * \psi_{mi} \qquad \forall i \in I \qquad (42)$$

$$g_{mi} = \sqrt{n_{mi}} \qquad \forall m \in M, \forall i \in I \qquad (43)$$

$$R_i = \sum_{j \in J} \sigma_j^2 z_{ij} \qquad \forall i \in I \qquad (44)$$

$$\sum_{j \in J} \mu_j z_{ij} + z_{1-\beta} * P_i \leq V_i \qquad \forall i \in I \qquad (45)$$

$$\psi_{mi} \in SOS2 \qquad \forall m \in M, \forall i \in I \qquad (46)$$

$$P_i, R_i \geq 0 \qquad \forall i \in I \qquad (47)$$

As a result, SDMD_VRP mathematical model which is linearized by separable programming is given below.

Objective function (1)
S.t
(2), (4), (5), (6), [8-11] and [30-47].

Solution of the model linearized by piecewise function is either infeasible or optimal solution for the original problem. If the solution obtained from the model is an infeasible solution for the original problem, grid point (M) is increased and the model is solved again. If the solution obtained is not infeasible, it is optimal solution.

4. COMPUTATIONAL EXPERIMENTS

In this section the proposed mathematical model formed for SDMD_VRP is studied on a problem and the performance of the model is examined according to the customer and depot amounts.

4.1. Sample Problem

The problem data are produced randomly (in the table below) for the problem that is composed of 2 depots and 8 customers. Coordinates of the depot and customer sets are formed randomly using normal distribution in the [0,99] range of field. Average of the customer demands is obtained using normal distribution in the range of [25,75]. Standard deviation is considered in three levels as low, median and high in order to better observe the behavior of the model. Low standard deviation (dev1), median standard deviation (dev2) and high standard deviation (dev3) are determined by taking 10%, 15% and 20% of the average of the demands, respectively.

Table 2. Data of the example problem

	X coordinate	Y coordinate	Capacity
Depot 1	45	47	250
Depot 2	15	28	250

Customer No	X coordinate	Y coordinate	Demand mean	dev1	dev2	dev3
1	66	13	51	5.1	7.65	10.2
2	21	74	31	3.1	4.65	6.2
3	73	58	52	5.2	7.8	10.4
4	81	98	59	5.9	8.85	11.8
5	94	33	44	4.4	6.6	8.8
6	31	94	43	4.3	6.45	8.6
7	0	75	38	3.8	5.7	7.6
8	39	37	71	7.1	10.65	14.2

Vehicle capacity $Q_k = 180 \ (k = 1,2,3,4)$ and ranges are divided into 5-unit parts. The total routes and the route lengths calculated for the different α levels are given in the table below (α and β levels were taken as equal).

Table 3. Calculation results for the sample problem

		Route length	Routes	CPU (sec.)
$\alpha = 0.05$	dev1	399.52	D1-3-5-1-D1 D1-8-D1 D2-7-2-6-4-D2	35.12
	dev2	464.34	D1-1-5-D1 D1-4-3-D1 D2-7-6-2-D2 D2-8-D2	407.87
	dev3	487.22	D1-4-6-D1 D1-8-D1 D2-1-5-3-D2 D2-2-7-D2	540.67
$\alpha = 0.025$	dev1	399.52	D1-1-5-3-D1 D1-8-D1 D2-4-6-2-7-D2	47.03
	dev2	464.34	D1-1-5-D1 D1-4-3-D1 D2-7-6-2-D2 D2-8-D2	287.91
	dev3	506.99	D1-3-5-D1 D1-6-4-D1 D2-1-8-D2 D2-2-7-D2	1705.34
$\alpha = 0.005$	dev1	435.05	D1-3-4-6-D1 D1-8-D1 D2-1-5-2-7-D2	103.40
	dev2	464.34	D1-1-5-D1 D1-4-3-D1 D2-7-6-2-D2 D2-8-D2	279.57
	dev3	506.99	D1-3-5-D1 D1-6-4-D1 D2-1-8-D2 D2-2-7-D2	851.11

Mathematical models developed for the problem are solved with GAMS 23.3 package program (CPLEX solver) and the results given in the table above.

4.2. Performance Evaluation of Mathematical Model

In this section, the performance of model based on the number of depot and customers is examined in a confidence interval of ($\alpha = \beta = 0.05$). Standard deviation is considered in 3 levels as in the sample problem. As a data set is not available for SDMD_VRP in the literature, the data set [31] formed for MDVRP in the literature is taken as a basis in the study. Customer and depot numbers are gradually increased. The demands in the data set are taken as the average of demands. Standard deviation is determined by taking 10% (dev1), 15% (dev2) and 20% (dev3) of the demands. Ranges are divided into 10-unit parts. Vehicle capacity is taken 50 while depot capacity as 200.

Model is encoded with GAMS 23.3 package program and solved using a computer with Intel Core i5 1.8 GHz, 4GB RAM features. The results are given below.

Table 4. Calculation results

Number of customer	Level of standart deviation	2 Depot		3 Depot		4 Depot	
		CPU (sec.)	GAP(%)	CPU (sec.)	GAP(%)	CPU (sec.)	GAP(%)
8	dev1	32.04	0.00*	18.77	9.93	223.74	9.97
	dev2	117.51	0.00*	59.47	8.12	137.59	9.95
	dev3	32.13	0.00*	54.16	8.74	160.45	9.94
10	dev1	384.24	0.00*	176.60	7.29	737.78	9.99
	dev2	205.33	0.00*	209.46	9.99	943.42	9.99
	dev3	268.87	0.00*	250.28	8.65	1000.51	10.00
12	dev1	229.66	9.99	167.19	9.99	670.51	9.99
	dev2	278.79	9.99	637.03	9.99	504.76	9.99
	dev3	317.43	9.99	1000.44	13.51	1000.51	18.25
15	dev1	1000.77	14.02	736.11	10.00	1000.50	13.89
	dev2	1000.40	12.98	743.62	9.99	1000.64	13.91
	dev3	1000.90	16.73	901.10	9.84	1000.57	14.40

* Optimal solution

5. CASE STUDY

This section presents the results of the proposed model for Ankara. Model was implemented as an earthquake scenario prepared by AFAD (Republic of Turkey Prime Ministry Disaster & Emergency Management Authority).

Ankara is the capital city of Turkey and 2th most crowded city. Its population was 5.346.518 with a total acreage of 25437 km^2 in 2016. City is consisting of 24 districts as shown in figure 2. Ankara's %29 land is located on the first and second seismic hazards. Larger lands in Ankara is located in the area of third and fourth-degree earthquake zone but there are faults capable of producing large-scale earthquakes in the near vicinity of Ankara [32]. Earthquake map of city is shown in figure 3.

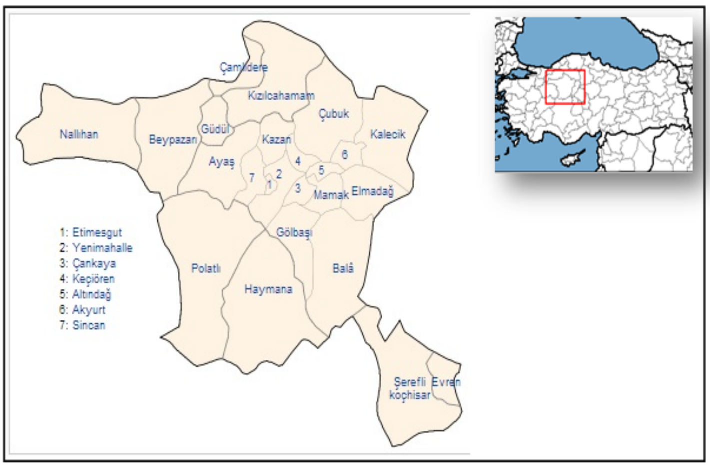

Figure 2. District map of Ankara

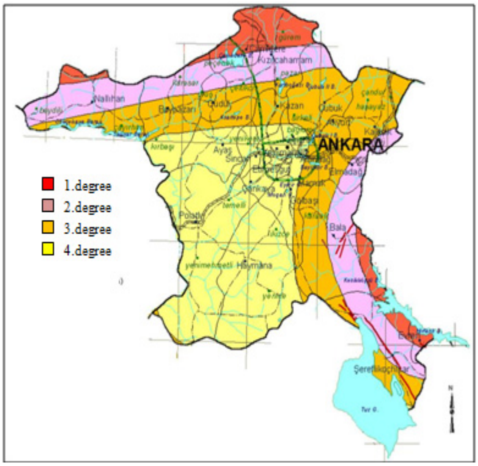

Figure 3. Earthquake map of Ankara

Estimated victim number has been obtained from the results of a scenario study carried out by AFAD for Ankara. This scenario study has been carried out by considering the fault lines in Ankara, the populations of districts, the status of structures and the details of scenario are not included in this research. Estimated victim number represents the estimated people who are in need of help and it is presented in Table 5.

Table 5. The estimated number of victims

District	Population	The number of people(in temporary shelter)	Standard deviation
Kızılcahamam	24933	1774	267
Kazan	43213	691	104
Güdül	8595	206	31
Ayaş	10785	114	18
Beypazarı	45984	951	143
Çamlıdere	5639	448	68
Çubuk	83347	1311	197
Nallıhan	22391	261	40
Kalecik	5454	75	12
Pursaklar	20345	168	26
Sincan	25667	200	30
Keçiören	12682	103	16
Akyurt	25148	224	34
Yenimahalle	14584	111	17
TOTAL	348767	6637	1003

Table 6. Temporary residential areas (TRA)

District	TRA	District	TRA
Akyurt	Kalaba area	Yenimahalle	75.Year hippodrome
Ayaş	Ayaş district stadium	Beypazarı	City stadium
Çubuk	City stadium	Çamlıdere	City stadium
Kalecik	Emeklikent park	Güdül	District stadium
Kazan	Saray Mosque Saray truck park City stadium	Pursaklar	Endüstri occupational high school Saray industrial high school Azmi Ertuğrul school
Keçiören	Ovacık sport facility	Kızılcahamam	City stadium Sport facility Anadolu high school Kazım Karabekir school Orhangazi school Çağatay school
Sincan	Water Depot Area	Nallıhan	Sport facility area

The number of people given table 5 refers to the number of people waiting for aid materials in temporary housing after disaster. However, the real numbers that occur after a disaster will show a deviation from the estimated number. Average demand is accepted as the forecasted amount of people in table 5 and standard deviation is accepted as %15 (rounded to the upper integer values). Temporary residential areas (TRA) determined by the AFAD are given in Table 6. Then, areas and depots (both AFAD and Kızılay depots) are marked using Google Earth and distances are calculated (Appendix 1).

In order to reach a solution with a mathematical model in the areas of Kazan, Pursaklar, and Kızılcahamam, they are first treated as clusters (Kazan city stadium, Pursaklar industrial occupational high school, Kızılcahamam city stadium). Later on, these districts are routed among themselves. Depot capacities are taken at varying levels. In this scenario, the values are identified in proportion to the amount of people that will need sheltering. The demands and capacities in the study are in units for uniform products that belong to normal distribution. Vehicle capacities are taken as 2500 units, and there are 4 available vehicles. The results for varying confidence levels are given in the Table 7.

Table 7. Implementation results of the scenario

Case no	Confidence Level	Depot(D) capacity	Total route lenght(km)	Routes
1	%99	D1 unlimited D2 limited(6000 unit)	854.4	1-4-13-25-15-14-1 2-6-3-5-18-10-12-2 2-9-11-2 2-19-2
2	%99	D1 limited(6000 unit) D2 unlimited	803.7	2-9-2 2-10-18-6-3-5-12-2 2-11-4-13-25-15-14-2 2-19-2
3	%99	D1-D2 equal and limited(6000 unit)	850.5	1-4-13-25-15-14-1 2-5-3-6-18-10-12-2 2-11-9-2 2-19-2
4	%95	D1 unlimited D2 limited(6000 unit)	850.5	1-4-13-25-15-14-1 2-5-3-6-18-10-12-2 2-11-9-2 2-19-2
5	%95	D1 limited(6000 unit) D2 unlimited	799.0	2-9-2 2-12-10-18-6-3-5-2 2-14-15-25-13-4-11-2 2-19-2
6	%95	D1-D2 equal and limited(6000 unit)	850.5	2-5-3-6-18-10-12-2 2-11-9-2 2-19-2 1-4-13-25-15-14-1
7	%90	D1 unlimited D2 limited(6000 unit)	849.8	1-4-13-25-15-14-11-1 2-9-2 2-12-10-18-6-3-5-2 2-19-2
8	%90	D1 limited(6000 unit) D2 unlimited	799.0	2-9-2 2-12-10-18-6-3-5-2 2-14-15-25-13-4-11-2 2-19-2
9	%90	D1-D2 equal and limited(6000 unit)	849.8	1-4-13-25-15-14-11-1 2-9-2 2-12-10-18-6-3-5-2 2-19-2

When Table 7 is examined, it is seen that longer routes are formed in situations where confidence level is high (e.g. case 1 and 4) or route length stays the same (e.g. case 3 and 6). Furthermore, when case 7 and 9 are examined, it is seen that the fact that capacity of D1 is infinite does not lead to a change in the solution and the fact that capacity of D2 is infinite (case 8) reduces the total route.

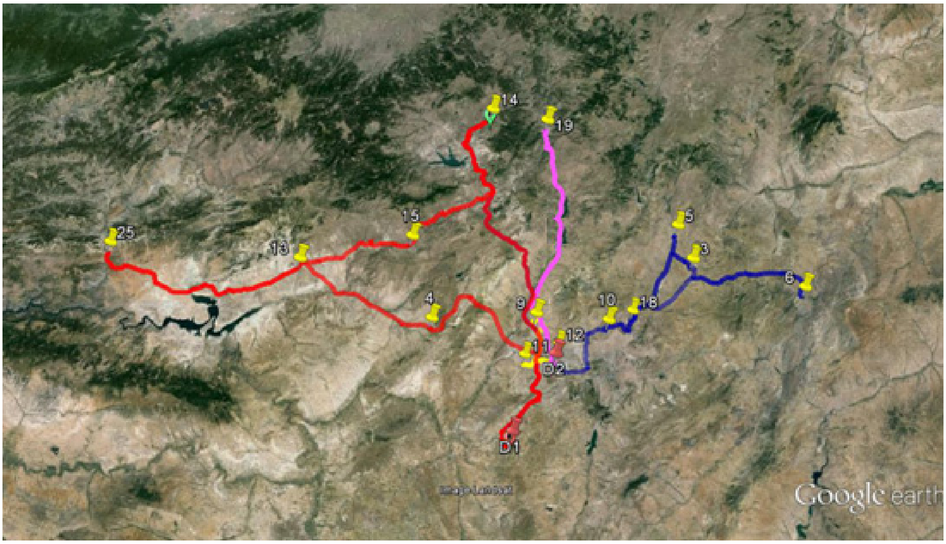

Figure 4. Routes (Case No: 6)

The results for minimum tours are given in as follows: (when Kazan, Pursaklar and Kızılcahamam districts are routed among themselves)

Kazan Saray Mosque- Saray Truck Park-City Stadium

Pursaklar Industrial Occupational High School-Saray Industrial High School-Azmi Ertuğrul School

Kızılcahamam City Stadium -Çağatay School-Kazım Karabekir School-Orhangazi School-Sport Facility-Kızılcahamam High School

6. CONCLUSION

The task of humanitarian logistics comprises acquiring and delivering requested supplies and services, at the places and times they are needed, while ensuring best costs. In the immediate aftermath of any disaster, these supplies include items that are vital for survival, such as food, water, temporary shelter and medicine, among others. Success and performance in humanitarian relief chains is very difficult to measure because of some distinct characteristics that humanitarian operations have, such as very unpredictable demand, difficulty to obtain data from operations, unpredictable working environment, lack of incentive for measurement (due to their non-profit character), very short lead time and unknown variables, like geography, political situation or weather. Stochastic systems describe the physical systems in which the values of parameters, measurements, expected input, and disturbances are uncertain. In probability theory, a purely stochastic system is one whose state is randomly determined, having a random probability distribution or pattern that may be analyzed statistically but may not be predicted precisely. Humanitarian logistics planning has stochastic elements in literature and most commonly faced parameter is the “stochastic demand”.

Thus in this paper, a rarely discussed problem namely SDMD_VRP is investigated and a mathematical model with chance constraint is suggested for the problem. Mathematical model is reformulated with separable programming and an integer linear model was obtained. The proposed models guarantee the optimal result for SDMD_VRP. In the study, model performance is examined by applying the proposed model at different variation/confidence levels and in different depot and customer numbers. Also, after conducting a case study for Ankara, routes for aid material distribution after a disaster are identified. The case study belongs to a scenario which is generated by Republic of Turkey Prime Ministry Disaster & Emergency Management Authority regarding the threats in Ankara. The obtained results are important for decision makers from the point of view of determining the vehicle routes which are going to ensure that the uncertain demands of disaster victims will be met at a certain level.

The future study may be conducted to reach more effective solutions for this NP-hard problem by using heuristic– meta-heuristic methods. The collapse of roads has not been included in this research and the model suggested in future studies can be improved to include this situation. Furthermore, the cost of unmet demand has not been reflected to the model and this situation can also be examined in further studies.

Distribution of multiple types of products can be considered or heterogenuos fleet can be applied to the same problem. Moreover, several variations of the problem such as SDMD_VRP with time window and SDMD_VRP with distance constraints can be studied by the researchers.

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Appendix 1. Distances matrix (km)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	-	92.3	63.3	93.6	124	47.8	45.2	64.8	57.9	38.4	39.8	115	111	116	64.5	72.7	68.1	97.6	97.2	97.3	87.7	97.6	97.9	168
2	-	0	59.9	48.2	61.3	95.7	19.6	17.1	37	30.1	11.2	7	88.4	84.8	78.6	36.6	44.8	40.2	69.1	68.7	68.8	69.2	69.1	69.4	147
3	92.3	59.9	0	99.5	11.7	39.4	57.5	54.9	74.8	36.8	66.9	52.7	140	124	130	26.2	20.9	25.6	82	81.6	81.7	82.1	82	82.3	198
4	63.3	48.2	99.5	0	102	131	47.2	53.1	48.7	65.8	40.1	46.8	41.4	98.3	31.6	72.3	80.5	75.9	81.8	81.4	81.5	81.9	81.8	82.1	100
5	93.6	61.3	11.7	102	0	50.6	58.3	55.8	38.8	37.7	67.8	53.5	141	125	116	27.1	21.7	26.5	68.5	68.1	68.2	68.7	68.5	68.8	199
6	124	95.7	39.4	131	50.6	0	88.9	86.3	90.1	68.2	98.3	84.1	171	156	161	57.6	52.3	57	120	119	120	120	120	139	230
7	47.8	19.6	57.5	47.2	58.3	88.9	0	5.4	22.8	24	21.1	14.3	93.9	70.2	75.7	30.5	38.7	34.1	54.9	54.5	54.6	55	54.9	55.2	153
8	45.2	17.1	54.9	53.1	55.8	86.3	5.4	0	21.8	23.5	20.6	13.8	86.6	69.2	74.7	30	38.2	33.6	53.9	53.5	53.6	54	53.9	54.2	145
9	64.8	37	74.8	48.7	38.3	90.1	22.8	21.8	0	41.6	39.1	31.9	88.8	58.4	50.8	34.5	39.4	35.3	32.8	32.5	32.6	33	32.9	33.2	147
10	57.9	30.1	36.8	65.8	37.7	68.2	24	23.5	41.6	0	32.5	17.6	108	91.4	97	9.2	16.6	9.2	73.9	74	74.3	74	73.9	34.5	165
11	38.4	11.2	66.9	40.1	67.8	98.3	21.1	20.6	39.1	32.5	0	13.3	80.6	85.2	70.8	39	47.2	42.6	72.1	71.8	71.9	72.3	72.2	72.5	139
12	39.8	7	52.7	46.8	53.5	84.1	14.3	13.8	31.9	17.6	13.3	0	86.3	81.6	76.5	24.1	32.3	27.7	63.9	63.6	63.7	64.1	64	64.3	145
13	115	88.4	140	41.4	141	171	93.9	86.6	88.8	108	80.6	86.3	0	95.4	35.7	114	123	118	92.2	91.9	92	92.4	92.3	92.6	58.8
14	111	84.8	124	98.3	125	156	70.2	69.2	58.4	91.4	85.2	81.6	95,4	0	54.6	97.9	106	102	26.4	28.4	26	27	26.4	26	154
15	116	78.6	130	31.6	116	161	75.7	74.7	50.8	97	70.8	76.5	35.7	54.6	0	104	112	107	56.7	56.3	56.4	56.9	56.7	57	92.7
16	64.5	36.6	26.2	72.3	27.1	57.6	30.5	30	34.5	9.2	39	24.1	114	97.9	104	0	5.9	0.75	66.7	66.3	66.4	66.8	66.7	79.6	171
17	72.7	44.8	20.9	80.5	21.7	52.3	38.7	38.2	39.4	16.6	47.2	32.3	123	106	112	5.9	0	5.3	69.4	69	69.1	69.5	69.4	87	178
18	68.1	40.2	25.6	75.9	26.5	57	34.1	33.6	35.3	9.2	42.6	27.7	118	102	107	0.75	5.3	0	67.5	67.1	67.2	67.6	67.5	82.2	173
19	97.6	69.1	82	81.8	68.5	120	54.9	53.9	32.8	73.9	72.1	63.9	92.2	26.4	56.7	66.7	69.4	67.5	0	1	0.35	1	0.7	0.89	149
20	97.2	68.7	81.6	81.4	68.1	119	54.5	53.5	32.5	74	71.8	63.6	91.9	28.4	56.3	66.3	69	67.1	1	0	0.60	1.2	1.1	1.1	149
21	97.3	68.8	81.7	81.5	68.2	120	54.6	53.6	32.6	74.3	71.9	63.7	92	26	56.4	66.4	69.1	67.2	0.35	0.60	0	1.2	1	0.60	149
22	87.7	69.2	82.1	81.9	68.7	120	55	54	33	74	72.3	64.1	92.4	27	56.9	66.8	69.5	67.6	1	1.2	1.2	0	0.25	0.75	149
23	97.6	69.1	82	81.8	68.5	120	54.9	53.9	32.9	73.9	72.2	64	92.3	26.4	56.7	66.7	69.4	67.5	0.7	1.1	1	0.25	0	0.50	149
24	97.9	69.4	82.3	82.1	68.8	139	55.2	54.2	33.2	34.5	72.5	64.3	92.6	26	57	79.6	87	82.2	0.89	1.1	0.60	0.75	0.50	0	150
25	168	147	198	100	199	230	153	145	147	165	139	145	58.8	154	92.7	171	178	173	149	149	149	149	149	150	0

Appendix 2. District no

No	Definition	No	Definition	No	Definition
1	Afad depot	10	Keçiören Ovacık sport facility area	19	Kızılcahamam city stadium
2	Kızılay depot	11	Sincan Water depot area	20	Kızılcahamam sport facility
3	Akyurt Kalaba area	12	Yenimahalle 75. Year hippodrome area	21	Kızılcahamam Anadolu high school
4	Ayaş stadium area	13	Beypazarı city stadium area	22	Kızılcahamam Kazım Karabekir school
5	Çubuk city area	14	Çamlıdere city stadium area	23	Kızılcahamam Orhangazi school
6	Kalecik Emeklikent park area	15	Güdül stadium area	24	Kızılcahamam Çağatay school
7	Kazan Saray mosque	16	Pursaklar industrial occupational high school	25	Nallıhan sport facility area
8	Kazan truck park	17	Pursaklar Saray industrial high school		
9	Kazan city stadium	18	Pursaklar Azmi Ertuğrul school		