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Research Article / Araştırma Makalesi

ANALYSIS OF A LONG STRIP CONTAINING AN INTERNAL OR EDGE CRACK USING FEM

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ABSTRACT

This paper presents the elastostatic plane problem of a long strip containing an internal or edge crack perpendicular to its boundaries. The plane problem consists of an infinitely long strip of thickness h. The infinitely long strip is loaded by uniformly distributed load and rests on two simple supports. It is assumed that the effect of gravity is neglected. The finite element model of the problem is constituted using ANSYS software and the two dimensional analysis of the problem is carried out. Normalized stress-intensity factors for the cases with internal and edge crack, and normal stress (σ_x) for uncracked layer case are obtained for various dimensionless quantities. Finally, the results obtained from the finite element analysis are verified by comparing with analytical ones and it is seen that the results from finite element analysis indicate a good agreement with the analytical solution.
Keywords: Elastic layer, simple support, finite element model, stress-intensity factor.

1. INTRODUCTION

Fracture mechanics studies on the strength of a structural member containing a crack. In studying the fracture mechanics, it is assumed that all real structures have initial flaws or cracks and failure is caused by propagation of these. In application of most of the current fracture criteria, the stress-intensity factor and the crack opening displacement are mostly used quantities. The stress-intensity factor defines the stress field close to the tip of a crack and provides fundamental information of how the crack is going to behave, if it expands causing the failure of a structural element or it remains stable. This is reason why a great deal of research has been devoted to this topic in recent years. Using different methods, some of which are integral transform technique, finite element method, and other computational methods, many problems related to cracks, i.e. in a strip or layer, a half plane, bonded materials, and layered composites, have been treated [1]. Some of these studies are summarized below:

Lowengrub [2] studied the problem of determining the distribution of stress in the neighborhood of a crack in an infinitely long elastic strip. The elastostatic plane problem of an

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infinite strip containing two symmetrically located internal cracks perpendicular to the boundary was investigated by Gupta and Erdogan [3]. The plane problem for an infinite strip with two edge cracks under a given state of residual stress was considered by Bakioglu and Erdogan [4]. The elasticity problem for a long hollow circular cylinder containing an axisymmetric circumferential crack subjected to general nonaxisymmetric external loads was examined by Nied and Erdogan [5]. Hong and Chen [6] derived the integral equations of elasticity, which might be considered to be a very general formulation for solutions of (cracked and uncracked) elasticity problems. The formulation of the integral equations in plane elasticity crack problem was presented by Chen [7]. Beghini and Bertini [8] proposed a method based on the weight function for solving the problem of partially closed cracks. Kadioglu et al. [9] examined internal and edge crack problems for an FGM layer attached to an elastic foundation. Analysis of an interface crack for a functionally graded strip sandwiched between two homogeneous layers of finite thickness was considered by Shbeeb and Binienda [10]. Chen [11] studied the problem of an infinite plate with crack loaded by the remote tensile stress and a pair of concentrated forces. A perspective on the current status of the formulations of dual boundary element methods with emphasis on the regularizations of hypersingular integrals and divergent series was provided by Chen and Hong [12]. Choi [13] considered the plane problem for bonded elastic half-planes containing a crack at an arbitrary angle to the graded interfacial zone. Dag and Erdogan [14] investigated a surface crack in a graded medium loaded by a sliding rigid stamp. The transient internal crack problem for a functionally graded orthotropic strip was examined by Chen et al. [15]. Boundary element analysis of crack problems in functionally graded materials was studied by Yue et al. [16]. The static crack problem of a functionally graded coating-substrate structure with an internal or edge crack perpendicular to the interface was studied by Guo et al. [17]. Xiao et al. [18] analyzed an elliptical crack parallel to the functionally graded interfacial zone between two fully bonded solids. Kahya et al. [19] studied partial closure of a crack located in an anisotropic infinite elastic layer. The problem of a centrally cracked, linear elastic orthotropic strip loaded in bending by three point forces was investigated by Caimmi and Pavan [20]. A surface crack in a graded coating subjected to sliding frictional contact was presented by Dag et al. [21]. Zozulya [22] studied two different boundary element methods (BEM) for crack analysis in two dimensional antiplane, homogeneous, isotropic and linear elastic solids by considering frictional contact of the crack edges. Yong et al. [23] examined crack problem for superconducting strip with finite thickness. A direct traction boundary integral equation method (TBIEM) for three-dimensional crack problems was investigated by Xie et al. [24].

In the existing literature, although crack problems have been well studied by analytically and numerically, comparison of two solutions in fracture mechanics has not been explored completely. So, the main purpose of this paper is to present a comparative study of the analytical method and the finite element method (FEM) for the analysis of a crack problem. For this purpose, the elastostatic plane problem of a long strip containing an internal or edge crack perpendicular to its boundaries is examined. The finite element model of the problem is created by ANSYS software, and finite element analysis is performed. Normalized stress-intensity factors for internal and edge cracks, and normal stress (σ_x) for uncracked layer case are obtained for various dimensionless quantities, and the results are compared with analytical results [25].

2. FINITE ELEMENT ANALYSIS OF THE PROBLEM

Consider an isotropic, linear elastic long strip of thickness h containing a transverse crack. The strip is loaded by uniformly distributed load and rests on two simple supports. The effect of gravity is neglected. Thickness in z direction is taken to be unit. The geometry, coordinate system and the loading case are shown in Fig. 1.

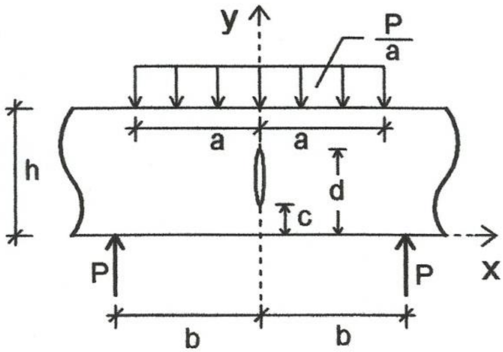
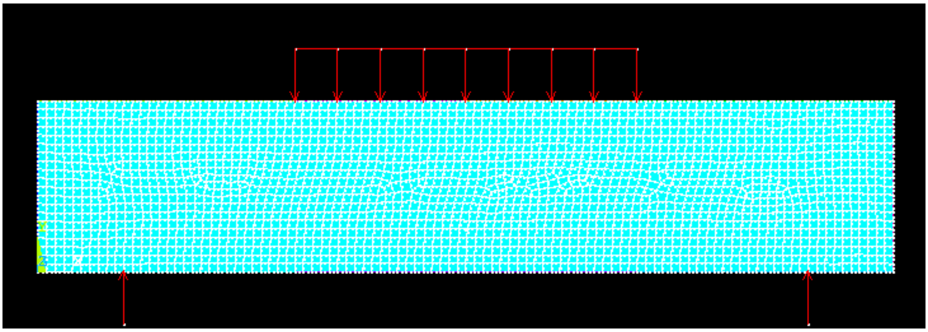


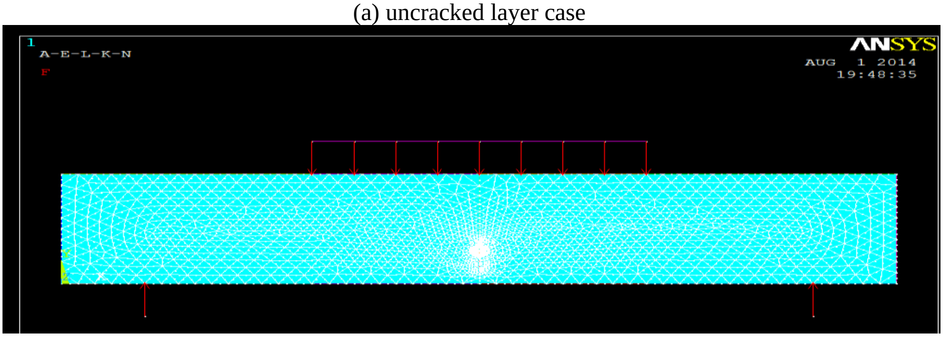
Figure 1. Geometry and loading case of the crack problem

In this paper, the finite element analysis is conducted by ANSYS software program [26]. The geometrical model is created with the standard tools in ANSYS software. Elastic layer is subjected to uniform longitudinal tensile stress from the top and clamped from the bottom. The problem is considered for three different cases such as uncracked layer, internal and edge crack. In the analysis, the length, the height, the elastic modulus, and the Poisson's ratio of the layer are taken as L = 2000 mm, h = 200 mm, E = 30 GPa and $\nu = 0.2$ for all cases, respectively. The finite element model of the problem is shown in Fig. 2.

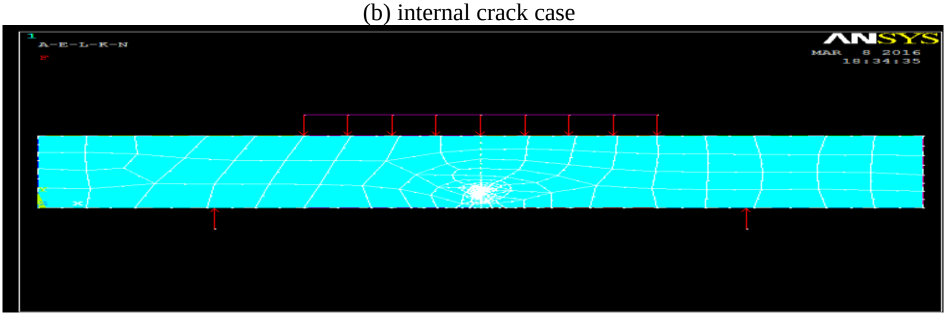
By the finite element analysis the stress intensity factor (SIFs) for the layer is calculated. The stress-intensity factor defines the stress field close to the crack tip and provides fundamental information of how the crack behaves, whether it expands causing the failure or it remains stable. To improve the efficiency of computation, a 2-D model is used. In the study of the mesh structure, the number of elements is increased until converging to theoretical results. As a result, the model consisted of 41526 eight-node triangular elements with type PLANE183. For meshing of crack tip, the crack-tip region PLANE183 eight-node triangular element is used. Deformed shapes after the analysis by using these elements are shown in Fig. 3.



(a) uncracked layer case



(b) internal crack case

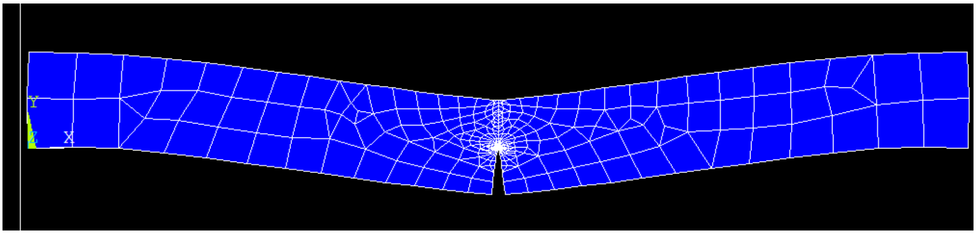


(c) edge crack case

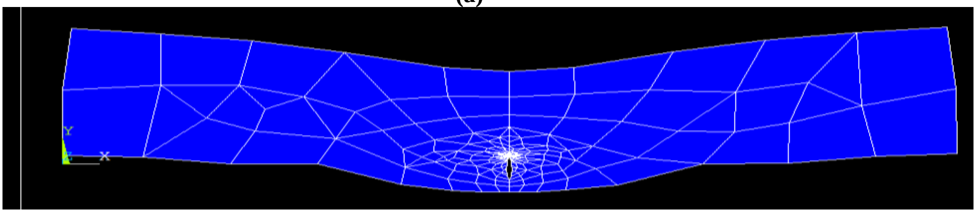
Figure 2. The finite element model of problem

3. RESULTS AND DISCUSSION

In this section, the normalized stress-intensity factors ($k(c)/\sigma_0\sqrt{(d-c)/2}$ and $k(d)/\sigma_0\sqrt{(d-c)/2}$) for the cases with internal crack and edge crack, and the normal stress (σ_x) for uncracked layer case are given for various dimensionless quantities using the finite element method. The results obtained from the finite element analysis are compared with analytical results for validation of the model. The above-mentioned k(c) and k(d) are the stress intensity factor at the crack tip c and the stress intensity factor at the crack tip d, respectively. Also,



(a)



(b)

Figure 3. Deformed shapes after finite element analysis for the case with edge crack (a) and for the case with internal crack (b)

$$\sigma_0=6P[b-(a/2)]/h^2 \quad (1)$$

Tables 1 and 2 show the variation of the dimensionless normal stress $\sigma_x(0,y)/\sigma_0$ with (y/h) for uncracked layer case. From these tables, as can be seen that tension and compression zones occur for the strip. The upper region of strip is under compression (with negative sign) and the lower region is under tension (with positive sign).

Table 1. Variation of the dimensionless normal stress ($\sigma_x(0,y)/\sigma_0$) with (y/h) for uncracked layer case (a/h=1, b/h=1, $\sigma_0=6P[b-(a/2)]/h^2$).

$\frac{y}{h}\downarrow$	($\sigma_x(0,y)/\sigma_0$)		
	Analytical [25]	FEM	Error (%)
0	1.0589	1.055	0.37
0.1	0.7949	0.795	0.01
0.2	0.5595	0.555	0.80
0.3	0.3486	0.350	0.40
0.4	0.1566	0.155	1.02
0.5	-0.0233	-0.023	1.29
0.6	-0.1933	-0.195	0.88
0.7	-0.3801	-0.380	0.03
0.8	-0.5750	-0.570	0.87
0.9	-0.7938	-0.795	0.15
0.95	-0.9135	-0.915	0.16

Table 2. Variation of the dimensionless normal stress ($\sigma_x(0,y)/\sigma_0$) with (y/h) for uncracked layer case (a/h=0.05, b/h=4, σ_0=6P [b-(a/2)]/h^2).

$\frac{y}{h}$ ↓	($\sigma_x(0,y)/\sigma_0$)		
	Analytical [25]	FEM	Error (%)
0	0.9399	0.935	0.52
0.1	0.7384	0.735	0.46
0.2	0.5491	0.545	0.75
0.3	0.3680	0.365	0.82
0.4	0.1921	0.192	0.05
0.5	0.0192	0.019	1.04
0.6	-0.1528	-0.152	0.52
0.7	-0.3259	-0.325	0.28
0.8	-0.5039	-0.505	0.77
0.9	-0.7109	-0.715	0.58
0.95	-0.9204	-0.925	0.50

In Tables 3-4, the variation of the normalized stress-intensity factors ($k(c)/\sigma_0\sqrt{(d-c)/2}$ and $k(d)/\sigma_0\sqrt{(d-c)/2}$) with (d/h) for the case with internal crack is given. As can be seen in the tables, the normalized stress-intensity factors ($k(c)/\sigma_0\sqrt{(d-c)/2}$ and $k(d)/\sigma_0\sqrt{(d-c)/2}$) decrease with increasing of (d/h).

Also, the normalized stress-intensity factor ($k(d)/\sigma_0\sqrt{(d-c)/2}$) approaches to zero about (d/h=0.6-0.7). In the case of very small crack (i.e., (c-d)/h=0.0001), ($k(c)/\sigma_0\sqrt{(d-c)/2}$ and $k(d)/\sigma_0\sqrt{(d-c)/2}$) have very close results to each other. Table 5 shows the variation of the normalized stress-intensity factor with (d/h) for the case with edge crack. It may be observed in the table that the normalized stress-intensity factor ($k(d)/\sigma_0\sqrt{(d-c)/2}$) takes the minimum value about (d/h=0.1-0.2). It is seen from Tables 1-5, there is a good agreement between the analytical [25] and FEM results.

Table 3. Variation of the normalized stress-intensity factors with (d/h) for the case with internal crack (a/h=1, b/h=1, c/h=0.1)

$\frac{d}{h}$ ↓	$k(c)/\sigma_0\sqrt{(d-c)/2}$			$k(d)/\sigma_0\sqrt{(d-c)/2}$		
	Analytical [25]	Present Study	Error (%)	Analytical [25]	Present Study	Error (%)
0.1001	0.7979	0.8000	0.26	0.7948	0.7975	0.34
0.2	0.7684	0.7625	0.77	0.6425	0.6485	0.93
0.3	0.7627	0.7620	0.09	0.5091	0.5100	0.18
0.4	0.7520	0.7475	0.60	0.3759	0.3775	0.43
0.5	0.7278	0.7215	0.87	0.2386	0.2400	0.59
0.6	0.6873	0.6825	0.70	0.0953	0.0950	0.32
0.7	0.6314	0.6275	0.62	-	-	-

Table 4. Variation of the normalized stress-intensity factor with (d/h) for the case with internal crack (a/h=0.05, b/h=4, c/h=0.1)

$\frac{d}{h}$ ↓	$k(c)/\sigma_0\sqrt{(d-c)/2}$			$k(d)/\sigma_0\sqrt{(d-c)/2}$		
	Analytical [25]	Present Study	Error (%)	Analytical [25]	Present Study	Error (%)
0.1001	0.7384	0.7375	0.12	0.7383	0.7370	0.18
0.2	0.7228	0.7225	0.04	0.6204	0.6255	0.82
0.3	0.7271	0.7275	0.06	0.5122	0.5075	0.92
0.4	0.7265	0.7325	0.83	0.3966	0.4000	0.86
0.5	0.7123	0.7101	0.31	0.2706	0.2705	0.04
0.6	0.6814	0.6825	0.16	0.1354	0.1345	0.67
0.7	0.6342	0.6375	0.52	-	-	-

Table 5. Variation of the normalized stress-intensity factor with (d/h) for the case with edge crack (a/h=0.05,b/h=4, c/h=0)

$\frac{d}{h}\downarrow$	$k(d)/\sigma_0\sqrt{(d-c)/2}$		
	Analytical [25]	Present Study	Error (%)
0.01	1.0537	1.0550	0.12
0.1	0.9731	0.9750	0.20
0.2	0.9750	0.9775	0.26
0.3	1.0375	1.0450	0.72
0.4	1.1661	1.1575	0.74
0.5	1.3907	1.3975	0.49
0.6	1.7839	1.7755	0.47
0.7	2.5127	2.5215	0.35

4. CONCLUSION

The main objective of this study is to present comparison between analytical and FEM calculations of a crack problem. In the paper, the normalized stress-intensity factors for the cases with internal crack and edge crack, and the normal stress (σ_x) for uncracked layer case are calculated by using finite element method. The conclusions drawn from the study can be presented as below:

- For the case without crack, tension and compression zones occur for the strip. The upper region of strip is under compression and the lower region is under tension.
- The normalized stress-intensity factors ($k(c)/\sigma_0\sqrt{(d-c)/2}$ and $k(d)/\sigma_0\sqrt{(d-c)/2}$) decrease with increasing of (d/h) for the case with internal crack .
- The normalized stress-intensity factor ($k(d)/\sigma_0\sqrt{(d-c)/2}$) approaches to zero about (d/h=0.6-0.7) for the case with internal crack.
- In the case of very small crack (i.e., (c-d)/h=0.0001), $k(c)/\sigma_0\sqrt{(d-c)/2}$ and $k(d)/\sigma_0\sqrt{(d-c)/2}$ have very close results to each other for the case with internal crack.
- The normalized stress-intensity factor ($k(d)/\sigma_0\sqrt{(d-c)/2}$) takes the minimum value about (d/h=0.1-0.2) for the case with edge crack.
- Finite element analysis (FEA) carried out in ANSYS software gives results which are in very good agreement with the analytical solution [25].

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