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The New Wave Solutions in the Field of Superconductivity

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Abstract
In this study, the Landau-Ginzburg-Higgs (LGH) equation, which has the physically important wave solutions, is considered. This equation is discussed via the modified exponential function method (MEFM) to describe superconductivity. Some new solutions are discovered in the form of rational, hyperbolic, and trigonometric functions when compared with the ones taking part in the literature. The functions which are candidates to be the exact solutions of the nonlinear equation are tested by the Mathematica program at the end of the steps of the method and it is observed that they satisfy the LGH equation. Additionally, the 2-D and the 3-D graphs accompanying the density and contour plots are illustrated.

1. Introduction

Having an active role and the profound employment of the nonlinear partial differential equations (NLPDEs) in various fields such as fluid dynamics, electromagnetism, acoustic, optics, DNA vibration dynamics, electrical lines and etc., the exact solutions for understanding and interpretation of the nonlinear phenomenon become crucial. For instance, the Korteweg-de Vries equation used in fluid dynamics, aerodynamics, and continuum mechanics as a model for shock wave formation, solitons, turbulence, boundary layer behavior, and mass transport. The nonlinear Schrödinger equation describes the propagation of optical pulses in optic fibers. The Zakharov–Kuznetsov equation arises in number of scientific models including fluid mechanics, astrophysics, solid state physics, plasma physics, chemical kinematics, chemical chemistry, optical fiber and geochemistry [1]-[3]. Due to its significant role in applied sciences, it is very important to obtain the solution of an NLPDE which allows us to analyze the phenomenon modeled via such equations. Therefore, various methods have been proposed to find exact solutions to NLPDEs, such as the $(G'/G\,,1/G)$-expansion method [4], He's variational methods [5], $\exp(-\varphi(\xi))$-expansion method, and sine-cosine method [6], the Bernoulli sub-equation function method [7], the modified (G'/G^2)-expansion approach [8], the extended Sinh-Gordon equation expansion method [9], the extended rational sinh-cosh method [10], the improved modified extended tanh-function method [11], the modified F-expansion method [12], sine-Gordon expansion method and $\left(m+(G'/G)\right)$-expansion method [13], and so on.

The LGH equation which is one of the common values of mathematics and applied sciences is utilized for comprehending the notions in superconductivity and cyclotron waves which have many usage areas such as medicine, plasma physics, chemistry, biology, electricity-electronic, transportation, and so on. The interpenetration of such essential applications and the LGH equation has turned the focus of this study to analyze the exact wave solutions. This equation is given by

$$u_{tt}-u_{xx}-m^2u+n^2u^3=0, \tag{1}$$

where $u(x,t)$ is the electrostatic potential of the ion-cyclotron wave, m and n are real parameters, and x and t define the spatial and temporal coordinates [14]. The executed methods in the literature for observing the soliton solutions of the LGH equations are

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appeared in [14]-[20]. In this study, the MEFM is put to use which is not applied before. For this purpose the progress of the paper is as follows; in section2 the processes of the MEFM are given, the application of the method to the LGH equation comes immediately together with the graphical results in section3 and finally, the conclusion takes part at the end.

2. Materials and Method

The starting point of the MEFM is to convert an NLPDE into a nonlinear ordinary differential equation (NLODE) by the wave transform $\xi = k(x - ct)$ where k and c represent the wave height and the wave frequency. Let the following NLPDE

$$Q(U, U_x, U_t, U_{xx}, U_{xt}, U_{tt}, U_{xxtt}, \dots) = 0, \quad (2)$$

contains the highest order derivatives and the nonlinear terms. As mentioned above after the wave transform $\xi = k(x - ct)$, the related derivatives in equation (2) are evaluated and substituted into equation (2) to obtain the following NLODE

$$N(U, U^2, U', U'', \dots) = 0, \quad (3)$$

According to MEFM the solution of equation (3) is assumed to be

$$U(\xi) = \frac{\sum_{i=0}^{N} A_i[\exp(-\Omega(\xi))]^i}{\sum_{j=0}^{M} B_j[\exp(-\Omega(\xi))]^j}$$

$$= \frac{A_0 + A_1 \exp(-\Omega(\xi)) + \dots + A_n \exp(-n\Omega(\xi))}{B_0 + B_1 \exp(-\Omega(\xi)) + \dots + B_m \exp(-m\Omega(\xi))}, \quad (4)$$

where $A_i, B_j, (0 \le i \le N, 0 \le j \le M)$ are constants with $A_N \ne 0, B_M \ne 0$ and will be determined by using a ready-made package program. Besides the coefficients, the upper bounds $N, M,$ and the function $\Omega(\xi)$ are required to expand equation (4). N and M are discovered by the balancing rule which enables a relation between them taking into account the highest order derivative and the highest order nonlinear term in equation (3). Finally, the $\Omega(\xi)$ function is the solution of the ordinary differential equation given below

$$\Omega'(\xi) = exp(-\Omega(\xi)) + \mu exp(\Omega(\xi)) + \lambda. \quad (5)$$

The process goes on with the substitution of equation (4) into equation (3) taking into account equation (5) which leads to a system of algebraic equations. In this system, the coefficients are specified via Mathematica.

The MEFM offers five families for the solutions, hence after writing the stated coefficients in equation (4) five classes are obtained for each case. These families are given in [21] as in the following.

Family 1: When $\mu \ne 0, \lambda^2 - 4\mu > 0,$

$$\Omega(\xi) = \ln\left(\frac{-\sqrt{\lambda^2-4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \quad (6)$$

Family2: When $\mu \ne 0, \lambda^2 - 4\mu < 0,$

$$\Omega(\xi) = \ln\left(\frac{\sqrt{-\lambda^2+4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}(\xi + E)\right) - \frac{\lambda}{2\mu}\right). \quad (7)$$

Family3: When $\mu = 0,\ \lambda \ne 0$ and $\lambda^2 - 4\mu > 0,$

$$\Omega(\xi) = -\ln\left(\frac{\lambda}{exp(\lambda(\xi+E)) - 1}\right). \quad (8)$$

Family4: When $\mu \ne 0,\ \lambda \ne 0$ and $\lambda^2 - 4\mu = 0,$

$$\Omega(\xi) = \ln\left(-\frac{2\lambda(\xi+E)+4}{\lambda^2(\xi+E)}\right). \quad (9)$$

Family5: When $\mu = 0,\ \lambda = 0$ and $\lambda^2 - 4\mu = 0,$

$$\Omega(\xi) = \ln(\xi + E), \quad (10)$$

where $A_0, A_1, \dots, A_n, B_0, B_1, \dots, B_m, E, \lambda, \mu$ are constants.

2.1. Application of the Method to LGH Equation

Equation (1) is reduced to the following NLODE

$$(k^2c^2 - k^2)U'' - m^2U + n^2U^3 = 0, \quad (11)$$

by the wave transform $\xi = k(x - ct)$. The balancing rule reveals the relation between N and M as $M + 1 = N$. Therefore, it can be considered as $N = 2$ and $M = 1$. Thus, the assumed solution (4) of the equation (11) is in the form of

$$U(\xi) = \frac{A_0 + A_1 e^{-\Omega(\xi)} + A_2 e^{-2\Omega(\xi)}}{B_0 + B_1 e^{-\Omega(\xi)}}. \quad (12)$$

A system of algebraic equation is derived when the required derivative terms are obtained from equation (12) and substituted in equation (11). Then by solving this system with the help of the package program, it is encountered with many possibilities for the coefficients. We have just given some of them starting with case1 as follows,

$$A_0 = \frac{\lambda A_2 B_0}{2B_1},\ A_1 = \frac{1}{2},\ A_2 = \left(\lambda + \frac{2B_0}{B_1}\right),$$

$$n = -\frac{i\sqrt{2(-1+c^2)}kB_1}{A_2}, m = -\frac{ik\sqrt{(\lambda^2-4\mu)(-1+c^2)}}{\sqrt{2}}, \quad (13)$$

where $i = \sqrt{-1}$. These coefficients are substituted in equation (12) and it was confirmed that this traveling wave solution function provides equation (11), by getting support from Mathematica.

The coefficients in equation (13) and the families stated above in equations (6-10) are substituted in equation (12), respectively. Thus, the following situations are presented for the solution functions of equation (1). Besides, the graphs are shown under the related solutions by giving appropriate values to the variables in the resulting equations.

Family 1

The solution $U_{1,1}$ and the relevant graphs in Figure1 are obtained for case1/family1:

$$U_{1,1} = \frac{A_2(\lambda^2-4\mu+\lambda\omega)}{2B_1(\lambda+\omega)}, \tag{14}$$

where $\omega = \sqrt{\lambda^2 - 4\mu}\mathrm{Tanh}[\frac{1}{2}(\mathrm{EE} + \xi)\sqrt{\lambda^2 - 4\mu}]$ and $\xi = k(-ct + x)$.

Family 2

The solution $U_{1,2}$ and the relevant graphs in Figure2 are obtained for case1/family2:

$$U_{1,2} = \frac{A_2(\lambda^2-4\mu-\lambda\theta)}{2B_1(\lambda-\theta)}, \tag{15}$$

where $\theta = \sqrt{-\lambda^2 + 4\mu}\mathrm{Tan}[\frac{1}{2}(\mathrm{EE} + \xi)\sqrt{-\lambda^2 + 4\mu}]$ and $\xi = k(-ct + x)$.

Family 3

The solution $U_{1,3}$ and the relevant graphs in Figure3 are obtained for case1/family3:

$$U_{1,3} = \frac{\lambda Coth[\frac{1}{2}(EE+\xi)\lambda]A_2}{2B_1}. \tag{16}$$

Family4

The solution $U_{1,4}$ and the relevant graphs in Figure4 are obtained for case1/family4:

$$U_{1,4} = \frac{\lambda A_2}{(2+\mathrm{EE}\lambda+\xi\lambda)B_1}. \tag{17}$$

Family 5

The solution $U_{1,5}$ and the relevant graphs in Figure5 are obtained for case1/family5:

$$U_{1,5} = \frac{A_2}{(\mathrm{EE}+\xi)B_1} \tag{18}$$

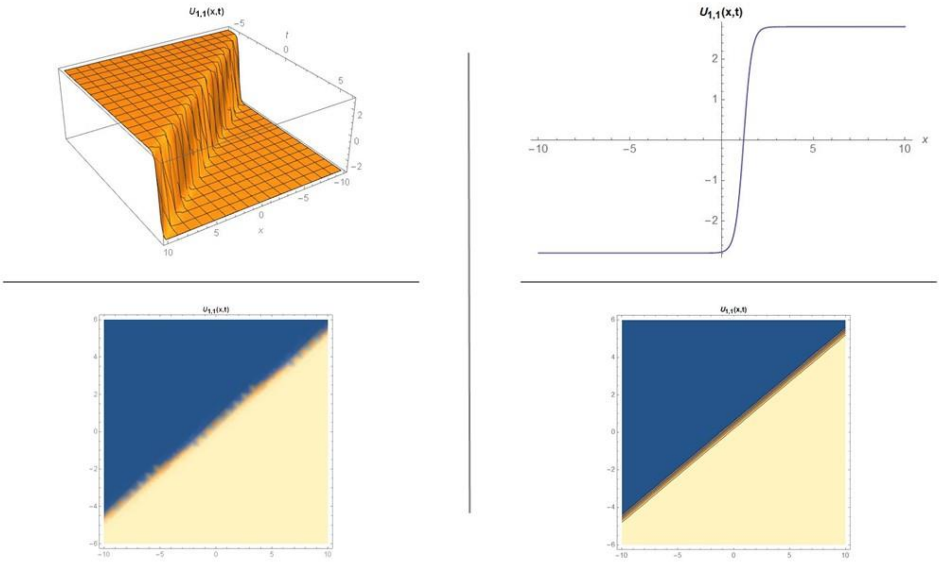

Figure 1. The three dimensional, density and contour graphs of solution (14) for the values $\lambda = 3, \mu = 1, A_2 = 0.5, B_1 = 0.2, k = 2, B_0 = 1, A_0 = 3.75, A_1 = 3.25, c = 2, EE = 0.75$ and two-dimensional graph for $t = 1$

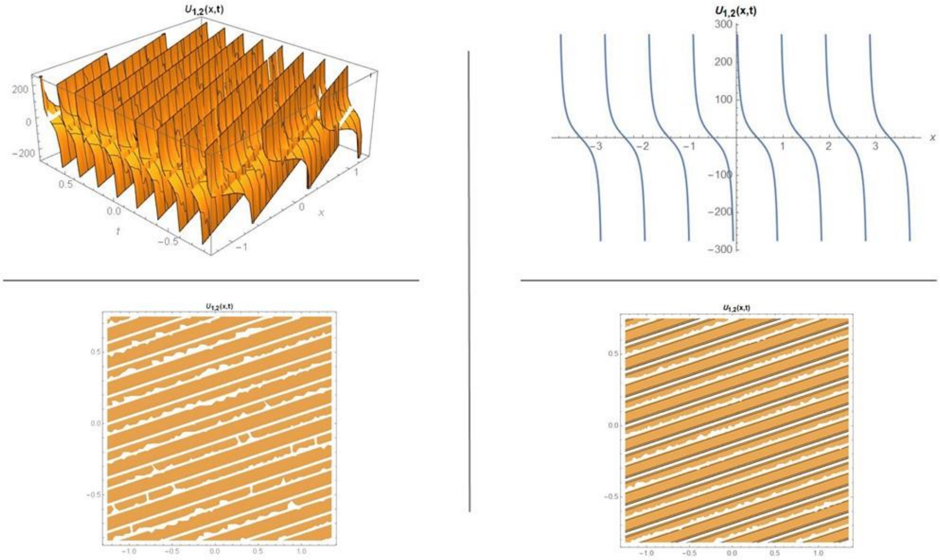

Figure 2. The three dimensional, density and contour graphs of solution (15) for the values $\lambda = 1, \mu = 3, A_2 = 0.5, B_1 = 0.02, k = 2, B_0 = 1, A_0 = 12.5, A_1 = 25.25, c = 5, EE = 0.75$ and two-dimensional graph for $t = 1$

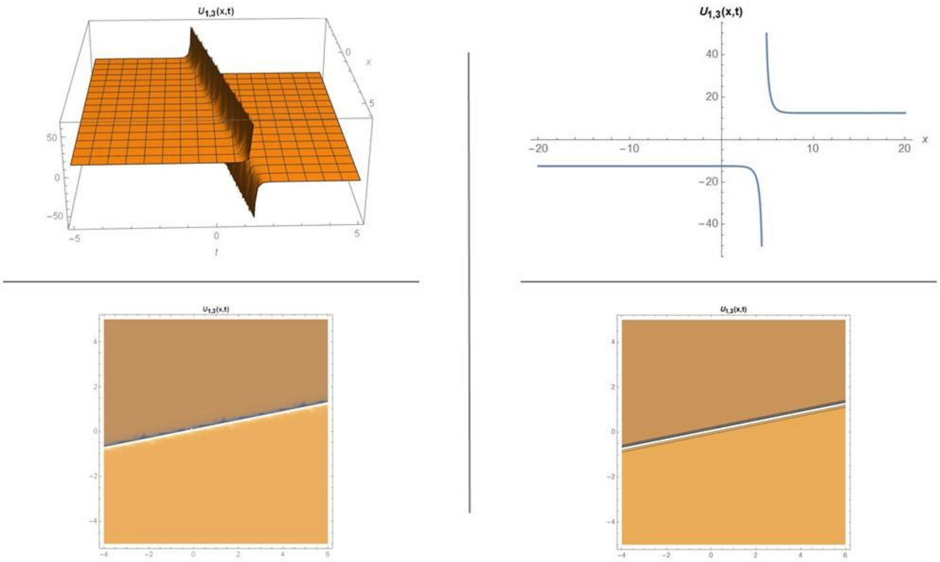

Figure 3. The three dimensional, density and contour graphs of solution (16) for the values $\lambda = 1, \mu = 0, A_2 = 0.5, B_1 = 0.02, k = 2, B_0 = 1, A_0 = 12.5, A_1 = 25.25, c = 5, EE = 0.75$ and two-dimensional graph for $t = 1$

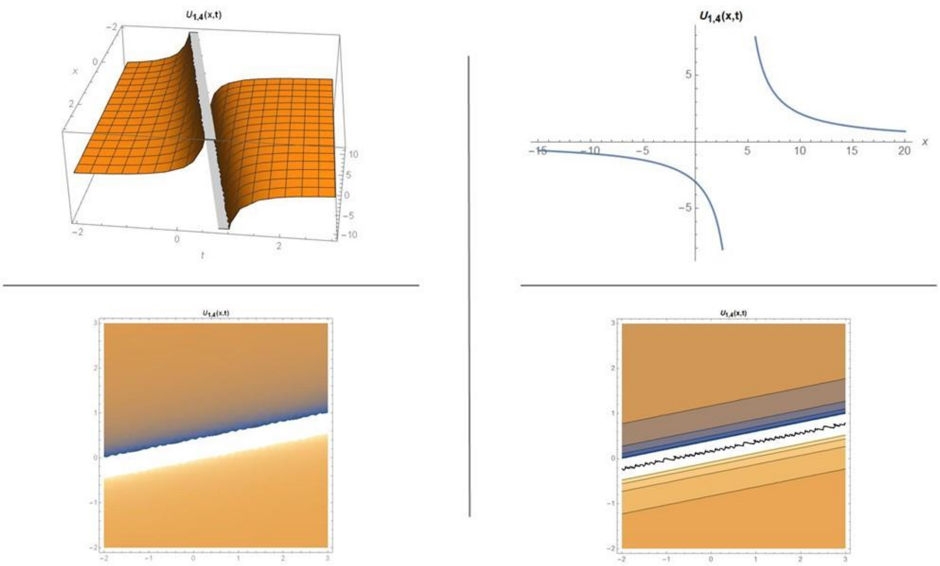

Figure 4. The three dimensional, density and contour graphs of solution (17) for the values $\lambda = 2, \mu = 1, A_2 = 0.5, B_1 = 0.02, k = 2, B_0 = 1, A_0 = 25, A_1 = 25.25, c = 5, EE = 0.75$ and two-dimensional graph for $t = 1$

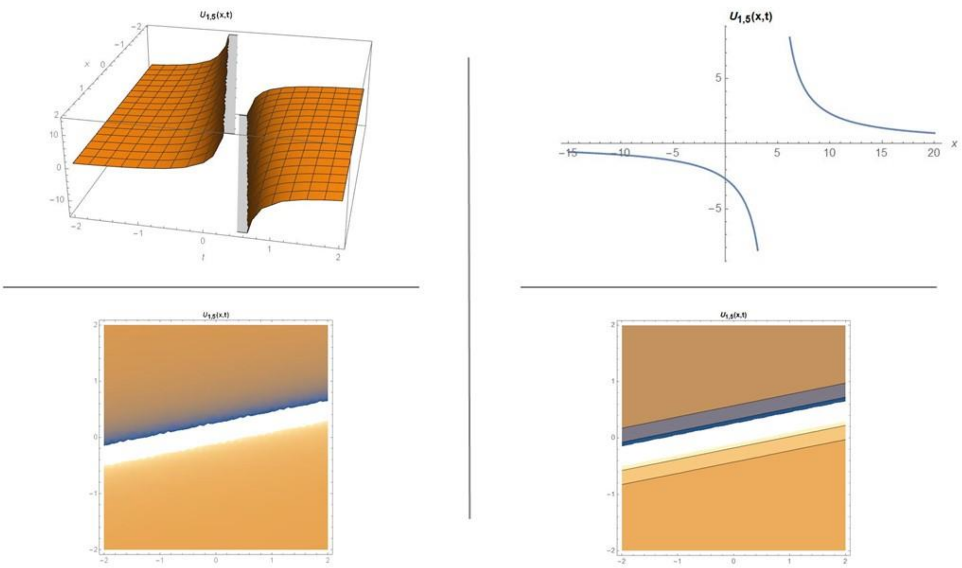

Figure 5. The three dimensional, density and contour graphs of solution (18) for the values $\lambda = 0, \mu = 0, A_2 = 0.5, B_1 = 0.02, k = 2, B_0 = 1, A_0 = 0, A_1 = 25, c = 5, EE = 0.75$ and two-dimensional graph for $t = 1$

We have discussed another possible coefficient group case2 below,

$$A_0 = -\frac{\sqrt{(1-c^2)k^2}\lambda B_0}{\sqrt{2}n}, A_1 = \frac{\lambda A_2}{2} - \frac{\sqrt{2(1-c^2)k^2}B_0}{n},$$
$$B_1 = -\frac{nA_2}{\sqrt{2(1-c^2)k^2}}, m = -\frac{\sqrt{(1-c^2)k^2(\lambda^2-4\mu)}}{\sqrt{2}}. \quad (19)$$

The parameters in equation (19) are substituted in equation (12) regarding the five $\boldsymbol{\Omega}(\boldsymbol{\xi})$ options as given in equations (6-10).

Family 1

The solution $U_{2,1}$ and the relevant graphs in Figure6 are obtained for case2/family1:

$U_{2,1} =$

$$-\frac{\frac{\sqrt{(1-c^2)k^2}\lambda B_0}{\sqrt{2}n}-\frac{4\mu^2 A_2}{(\lambda+\omega)^2}+\frac{\mu\left(n\lambda A_2-2\sqrt{2}\sqrt{(1-c^2)k^2}B_0\right)}{n(\lambda+\omega)}}{B_0+\frac{\sqrt{2}n\mu A_2}{\sqrt{(1-c^2)k^2}(\lambda+\omega)}}, \qquad (20)$$

where $\omega=\sqrt{\lambda^2-4\mu}\mathrm{Tanh}[\frac{1}{2}(\mathrm{EE}+\xi)\sqrt{\lambda^2-4\mu}]$ and $\xi=k(-ct+x)$.

Family 2

The solution $U_{2,2}$ and the relevant graphs in Figure7 are obtained for case2/family2:

$$U_{2,2}=-\frac{\frac{\sqrt{(1-c^2)k^2}\lambda B_0}{\sqrt{2}n}-\frac{4\mu^2 A_2}{(\lambda-\theta)^2}+\frac{\mu\left(n\lambda A_2-2\sqrt{2}\sqrt{(1-c^2)k^2}B_0\right)}{n(\lambda-\theta)}}{B_0+\frac{\sqrt{2}n\mu A_2}{\sqrt{(1-c^2)k^2}(\lambda-\theta)}}, \qquad (21)$$

where $\theta=\sqrt{-\lambda^2+4\mu}\mathrm{Tan}[\frac{1}{2}(\mathrm{EE}+\xi)\sqrt{-\lambda^2+4\mu}]$ and $\xi=k(-ct+x)$.

Family 3

The solution $U_{2,3}$ and the relevant graphs in Figure8 are obtained for case2/family3:

$$U_{2,3}=-\frac{\sqrt{(1-c^2)k^2}\lambda\mathrm{Coth}[\frac{1}{2}(\mathrm{EE}+\xi)\lambda]}{\sqrt{2}n}. \qquad (22)$$

Family 4

The solution $U_{2,4}$ and the relevant graphs in Figure9 are obtained for case2/family4:

$$U_{2,4}=-\frac{\sqrt{2(1-c^2)k^2}\lambda}{n(2+\mathrm{EE}\lambda+\xi\lambda)}. \qquad (23)$$

Family 5

The solution $U_{2,5}$ and the relevant graphs in Figure10 are obtained for case2/family5:

$$U_{2,5}=-\frac{\sqrt{2}\sqrt{2(1-c^2)k^2}}{\mathrm{EE}n+\xi n}. \qquad (24)$$

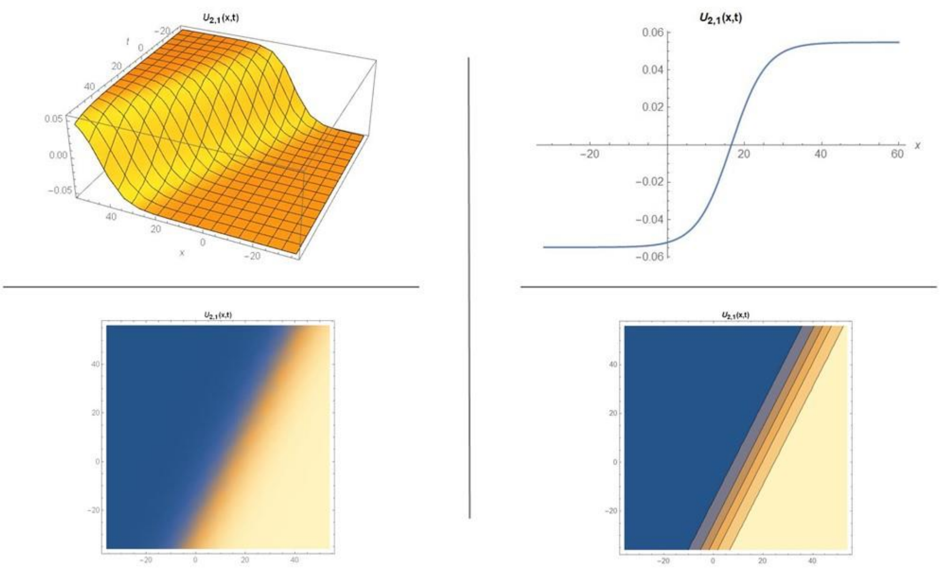

Figure 6. The three dimensional, density and contour graphs of solution (20) for the values $\lambda=3,\mu=1,n=2.5,A_2=0.25,B_1=-5.1031,k=-0.1,B_0=1,A_0=-0.0734847,A_1=0.32601,c=0.5,EE=0.75,m=-0.136931$ and two-dimensional graph for $t=1$

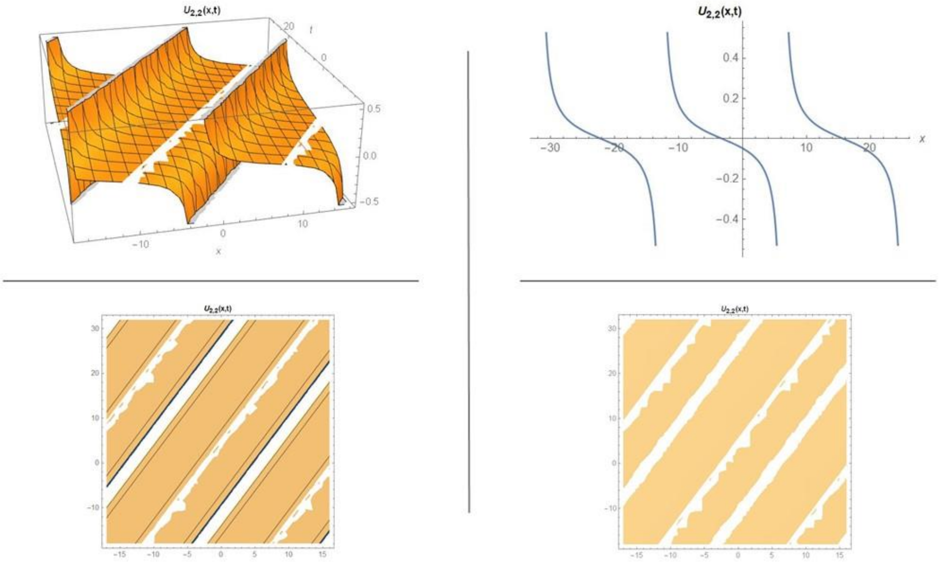

Figure 7. The three dimensional, density and contour graphs of solution (21) for the values $\lambda = 1, \mu = 3, n = 2.5, A_2 = 0.25, B_1 = -5.1031, k = -0.1, B_0 = -1, A_0 = 0.0244949, A_1 = 0.17399, c = 0.5, EE = 0.75, m = -0.203101i$ and two-dimensional graph for $t = 1$

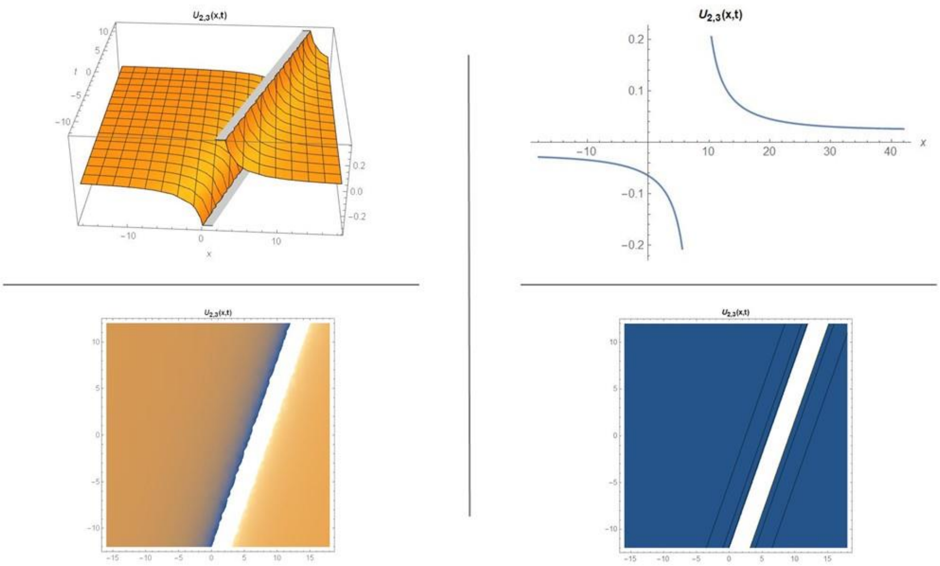

Figure 8. The three dimensional, density and contour graphs of solution (22) for the values $\lambda = 1, \mu = 0, n = 2.5, A_2 = 0.25, B_1 = -5.1031, k = -0.1, B_0 = -1, A_0 = 0.0244949, A_1 = 0.17399, c = 0.5, EE = 0.75, m = -0.0612372$ and two-dimensional graph for $t = 1$

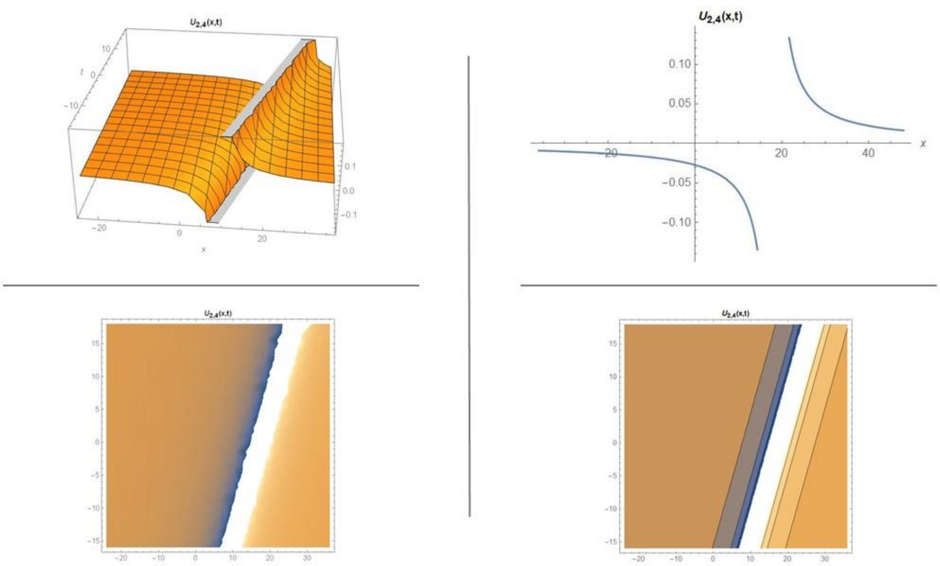

Figure 9. The three dimensional, density and contour graphs of solution (23) for the values $\lambda = 2, \mu = 1, n = 2.5, A_2 = 0.25, B_1 = -5.1031, k = -0.1, B_0 = -1, A_0 = 0.0489898, A_1 = 0.29899, c = 0.5, EE = 0.75, m = 0$ and two-dimensional graph for $t = 1$

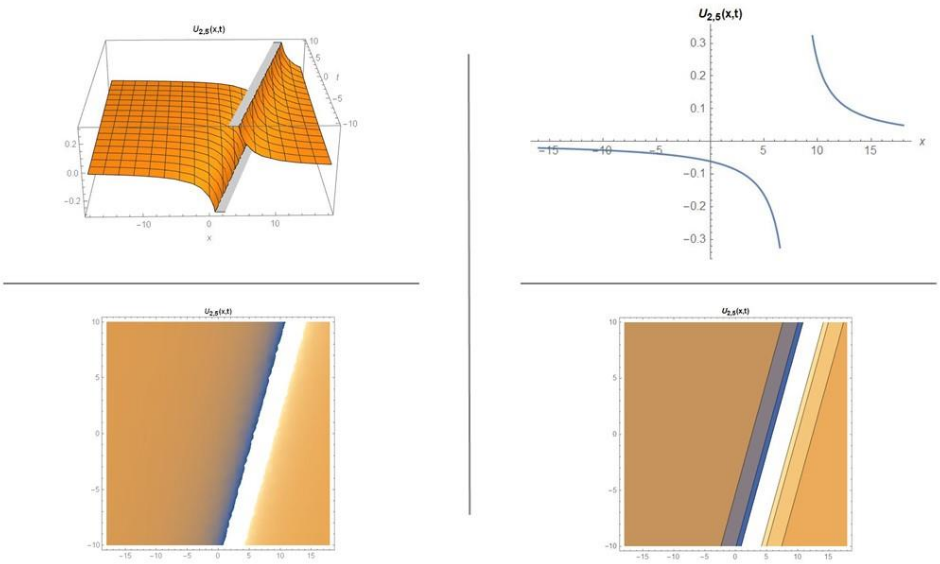

Figure 10. The three dimensional, density and contour graphs of solution (24) for the values $\lambda = 0, \mu = 0, n = 2.5, A_2 = 0.25, B_1 = -5.1031, k = -0.1, B_0 = -1, A_0 = 0, A_1 = 0.0489898, c = 0.5, EE = 0.75, m = 0$ and two-dimensional

3. Results and Discussions

The several solution forms of the LGH equation are illustrated above in Figures1-10. The kink type, singular kink type, and the periodic type solutions are observed. The solutions and the related graphics are different when compared to ([14]-[20]). Interpretation of these fresh solutions can put a new complexion on the applications of the LGH equation.

4. Conclusion

We have determined the new exact solution forms of the LGH equation as hyperbolic, trigonometric, and rational functions via the modified exponential function method, which is an effective and functioning method. The application of the MEFM method for this equation is not encountered in the literature. The process of plotting the graphs and the computations is overcome with the aid of Mathematica. The LGH equation is used in superconductivity, which has a wide application area, as mentioned in Section1. Therefore, the newly obtained wave solutions may be helpful to widen the knowledge in the related field and may develop new ideas.

Contributions of the authors

The authors confirm that the contribution is equally for this paper.

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

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