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Probabilistic-Based Forecasting Method For Time Series Datasets

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ABSTRACT

In this paper, a new probabilistic technique (a variant of Multiple Model Particle Filter-MMPF) will be used to predict time-series datasets. At first, the reliable performance of our method is proved using a virtual random scenario containing sixty successive days; a large difference between the predicted states and the real corresponding values arises on the second, third, and fourth day. The predicted states that are determined by using our method converge rapidly towards the real values while a classical linear model exhibits a large amount of divergence if used alone here. Then, the performance of our approach is compared with some other techniques that were already applied to the same time-series datasets: IEX (Istanbul Stock Exchange Index), TAIEX (Taiwan Stock Exchange), and ABC (The Australian Beer Consumption). The performance evaluation metrics that are utilized here are the correlation coefficient, the mean absolute percentage error, and the root mean squared error.

Keywords: *Forecasting; Time series dataset; MMPF; Evaluation Metrics*

Zaman Serisi Veri Kümeleri İçin Olasılığa Dayalı Tahmin Yöntemi

ÖZ

Bu makalede, zaman serisi veri kümelerini tahmin etmek için Çoklu Model Parçacık Filtresinin (ÇMPF) bir çeşidi olarak düşünülebilecek yeni bir olasılık tabanlı teknik kullanılmaktadır. Yöntemimizin güvenilirlik performansı art arda altmış günden oluşan sanal bir rastgele senaryo kullanılarak kanıtlanmıştır. İkinci, üçüncü ve dördüncü günde tahmin edilen durumlar ile gerçekte karşılık gelen değerler arasında büyük bir fark ortaya çıkmaktadır; yöntemimiz kullanılarak tahmin edilen durumlar gerçek değerlere doğru hızla yakınsarken, tek başına klasik bir lineer model kullanıldığında büyük miktarda sapma göstermektedir. Makalede yaklaşımımızın performansı; Kök-Ortalama-Kare Hatası, Ortalama Mutlak Yüzde Hatası ve Korelasyon Katsayısı performans değerlendirme ölçütleri dikkate alınarak; BIST (Borsa İstanbul Endeksi), TAIEX (Tayvan Borsa Endeksi), ve ABC (Avustralya Bira Tüketimi) zaman serisi veri kümelerine halihazırda uygulanmış olan diğer bazı tekniklerle karşılaştırılmaktadır.

Anahtar Kelimeler: Tahmin; Zaman Serisi Veri Kümesi; ÇMPF; Değerlendirme Metrikleri

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I. INTRODUCTION

In recent years, time series forecasting has engaged a notable interest, especially in the economic and financial fields. In this context, different predictive approaches were proposed to achieve accurate and successful works; these approaches are divided into two main categories; non-probabilistic techniques including statistical methods, and probabilistic-based techniques which are similar to our method proposed in this paper.

In general, statistical-based methods try to fit a forecasting curve according to all the available historical data; the extension of the created curve represents the future prediction. In contrast, the probabilistic methods represent an adequate alternative that provides probabilistic distributions (densities) in the state space instead of that simple curve (suggested by statistical methods) in order to increase the predictive range at each moment to guarantee the best fit for all given data. Our objective is to find the probabilistic predictive approach that can take all the former data into account and understand the current behavior of the considered state.

Non-probabilistic methods include fuzzy logic-based systems [1], artificial neural networks-based systems [2], and hybrid techniques like adaptive neuro-fuzzy inference systems [3] that handle reasoning at a high level by using the linguistic information acquired from the environment. When they appeared, these lastly-cited methods were able to show some basic promising results. More advanced studies were recently carried out. For example, the Type-1 fuzzy sets were proposed as a classifier as in [4] and later used for dealing with forecasting problems [5]. All these new approaches seem competitive, but they mostly require to be combination with an additional optimization algorithm (Genetic algorithm or Grey wolf algorithm) as in [6] to determine the global solution in the state space. Such combined techniques may provoke some questions about their execution time, especially when they are applied to long-period datasets. Some probabilistic methods [7], [8], [9] already employed the Unscented Kalman Filter (UKF) and Extended Kalman Filter (EKF) to predict daily sales or stock markets. In general, UKF is always able to give more accurate results than (EKF), but unfortunately, it involves so expensive computational burden. Some other probabilistic techniques have also utilized the classical version of Particle Filter (PF) as in [10], [11], and [12]. But, they don't provide all the performance evaluation metrics which are indispensable to judge the efficiency of their approaches. In addition, they don't apply their methods to standard datasets to compare their results with the other existing methods. A Bayesian bilinear neural network was recently developed to predict financial markets and track the dynamism of their prices [13]. While different customized architectures of ANN-based predictors, including an adaptable design, were suggested to forecast the daily consumed electricity by a local industrial region [14]. In this paper, a mathematical model that considers three parameters describing three different effectors (economic, political, and natural) is adopted as a dynamic system of our Multiple Model Particle Filter which is in turn so adequate to give a reliable predictive approximation, i.e. future reading for fluctuated environments like stock markets. The RMSE, MAPE, and correlation coefficient are calculated here as performance metrics. Our approach is applied to some standard datasets and compared with a wide range of available existing techniques.

To the best of our knowledge, this is the first time where the Multiple Model Particle Filter is developed and utilized for handling forecasting problems as presented in this paper. The organization of the paper is as follows; in the next section, we explain in detail the theoretical basis of our probabilistic predictor. A lot of qualitative and quantitative analyses of our results that prove the efficiency of this approach, compared to many other techniques, are depicted in the third paragraph. Finally, we conclude with a summary describing the main features of our work.

II. THE PROBABILISTIC TECHNIQUE (MMPF)

The Particle Filter is an estimator that belongs to the family of recursive Bayes filters; it considers the current belief of the studied case and renews its state according to its suggested dynamic system and consecutive measurements. A Particle Filter can handle nonlinear functions with any probabilistic distribution by using a huge amount of particles representing all possible conditions of how the system could be represented at each moment in the state space. The PF contains two consecutive steps (Prediction and Correction). The estimated states for an upcoming moment are calculated and attributed to their corresponding particles to perform the prediction step. To satisfy real and practical considerations, this process is always supposed affected by some random noise.

In the correction step, successive readings will be used to calculate and assign weights to their corresponding particles; these weights describe to which extent each particle expresses the real considered state. The observation error of the given process should be considered and accurately modeled. The more the particle weight the more likely it follows the correct path in this estimating scenario.

Hence, a resampling phase that denotes a survival fit law is utilized to retain a predefined rate of the fittest particles that are able to repeat themselves according to their evaluated weights while the other lite particles will be removed. Thus, consecutive generations of the most qualified particles are developed to finally produce the best solution of the given system over all the studied periods.

The dynamic model suggested here is depicted as follows:

$$x_t = (B_1 + B_2 + B_3)x_{t-1} + \varepsilon \tag{1}$$

x_t is a state that is calculated at the current moment (t), while ε is the process noise.

The three parameters B_1, ,B_2 and B_3 take their corresponding values in the range [-1, 1]. The first parameter (B_1) is an economic factor representing the belief status (optimistic/pessimistic) of the considered stock markets or financial pointers. B_2 and B_3 represent natural and political factors that may impact the predicted state. The three parameters are daily decided depending on former relevant practices or analytic data of markets; these parameters determine if the estimated state x_t points up or down opposing with the last determined state x_{t-1} when the three parameters equal zero, the linear model will be converted to a random walk system:

$$x_t = x_{t-1} + \varepsilon \tag{2}$$

When the linear model shown in equation (1) is used alone, it appears as a naive approach that may be broken by improper manipulations. It is often unable to produce predictions that may converge to their real compatible states; like the real daily sales or market prices, as we report in the coming paragraphs. To defeat the poor properties of this linear model, we suggest using it as a dynamic model for an advanced probabilistic method that represents a variant of Multiple Models Particle Filter (MMPF) [15], which is in turn so adequate to be used when the estimated state has a maneuvering or switching behavior [8, 9], which is the case considered here.

Kalman Filter could be more adequate for modeling linear systems. But, its heavy computational burden that is required to deal with the big arrays and their associated calculation time has guided us to prefer the use of the (PF) which is able to model any dynamic system (linear or nonlinear) with any probabilistic distribution suggested in the state space. In this work, the (PF) adopts the dynamic system which is presented in equation (1) to predict a state at moment (t), then it creates and randomly distributes a cloud of (n) particles in its neighborhood. The particle's state is given as μ_t^i; $i = 1 \ldots n$. Whereas the weight assigned to each particle w_t^i describes its probability to represent the corresponding real state is determined as follows:

$$w_t^i = p\left(Z_t \middle| \mu_t^i(m_t)\right) * p(m_t^i|m_{t-1}) \tag{3}$$

Z_t is a real state, measured at moment (t).
m_t is a regime variable that can take the values {1, 2, 3}; they describe three different behaviors as shown in figure (1). The value 1 is to move straight between two successive moments, the value 2 is to move up, whereas the value 3 is to move down.
$\mu_t^i(m_t)$ is the particle estimated state customized by the regime variable m_t.
The probabilistic value $p\left(Z_t \middle| \mu_t^i(m_t)\right)$ which is determined for each particle (at each iteration) relies on evaluating the gap between the measurement Z_t and the particle estimated state $\mu_t^i(m_t)$ in accordance with the probabilistic distribution in equation (4).

$$p\left(Z_t \middle| \mu_t^i(m_t)\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{Z_t - \mu_t^i(m_t)}{\sigma}\right)^2\right) \tag{4}$$

$p(m_t^i|m_{t-1})$ is a particle transition probability that finds its value from the Transition Probability Matrix (TPM) that has the initial value in equation (5). The elements in this array represent the probability to alternate the particle state from one behavior to another, as three behaviors were already considered. The values of these elements are updated for each iteration; some of them could be increased when the others may decrease but the total sum of the elements sharing the same raw is always one.
The particle transition probability $p(m_t^i|m_{t-1})$ as given here retains a trace of all former changes of the predicted state to give an idea about its historic performance if it points to move up, down or retaining the same value.

$$TPM = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = 0.33 * \text{ones}(3,3) \tag{5}$$

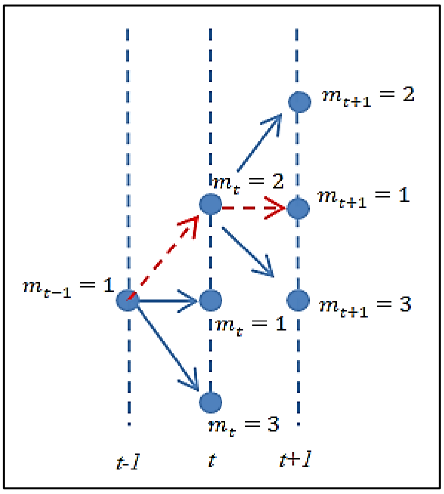


Figure 1. *The regime variable of the given particle was changed from 1 to 2 (moving up) between two successive moments. At (t+1) its regime variable becomes 1 (to retain its level).*

Then, the importance of the produced particles will be evaluated by calculating and assigning a weight to each particle. According to these weights, a phase will be applied to delete particles that have fewer weights. In this experiment, 35% of particles are held (as they are the most weight), and 65% are removed (they are the less weight).
The weights of the retained (k) particles are normalized as follows:

$$w_t^i = \frac{\widetilde{w}_t^i}{\sum_{j=1}^{k} \widetilde{w}_t^j} \tag{6}$$

Where; w_t^i is the normalized weight attributed to each particle.

A resampling phase [16] will be applied as shown in the following pseudocode, Figure (2).

Initialize the Cumulative Sum of Weights: $c_1 = w_t^1$
For $i = 2 : N$
  $c_i = c_{i-1} + w_t^i$
End for
For $j = 1 : N$
  $u_j = u_1 + \frac{(j-1)}{N}$
  While $u_j > c_i$
    $i = i + 1$
  End while
  $w_t^j = \frac{1}{N}$
End for

Figure 2. *Pseudocode describing the resampling phase which is used by the Particle Filter.*

In this case, and according to its normalized weight, each particle gives its corresponding successors at moment (t + 1). Hence, the more the particle weight the more successors it generates. At any iteration, the number of all particles has to meet a predefined constant (N). Consequently, a cloud of the fittest particles is created for each new moment, and a new estimation process to be fired.
Before applying the probabilistic method proposed here to some standard datasets in order to evaluate its performance compared with some other existing approaches, let's first prove its robustness when it is used with a randomly generated scenario (including the selection of the three parameters B1, B2, and B3), as it is shown in Figure (3). In this case, we suppose for example a prediction scheme for 60 successive days; the starting point was randomly selected. Between the second and fourth day, a sudden and vast change takes place between the predicted states, which are calculated according to the linear model explained in equation (1) and shown as small red circles on Figure (3), and the real values which are shown as small black diamonds on the same figure. The clouds (the blue stars), which are produced using our approach (MMPF) and composed of 30 particles per day, converge rapidly towards the real values while the linear model seems a naïve approach that accumulates a lot of errors over all the considered period. Therefore, it diverges away from the curve of real values.

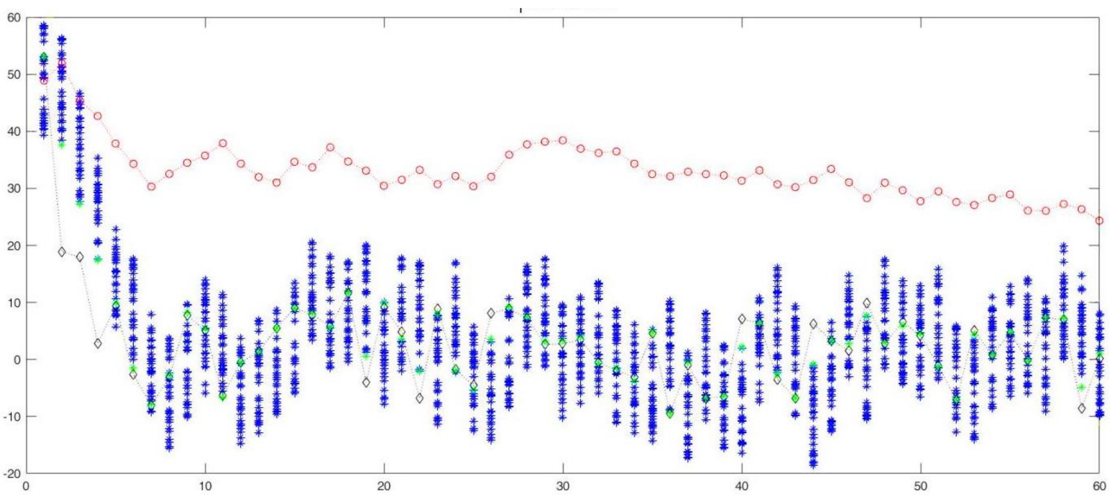


Figure 3. *Red circles are the calculated values according to the linear model explained in equation 1. Black small diamonds are assigned to the real values. Blue stars are the particles; the green stars are the most weighted particles, i.e. the nearest particles to their corresponding real values for each day.*

III. RESULTS AND ANALYSIS

To compare the performance of our approach, suggested in this paper (MMPF), with the performance of some other existing techniques like Exponential smoothing (ES), Multilayer perceptron ANN (MLP), Fuzzy function (FF), Fuzzy time series network (FTS-N), and Type -1 recurrent intuitionistic fuzzy functions (T1-R-IFF), we have to use the same time-series datasets already used by all of them. These datasets are:

- IEX (Istanbul Stock Exchange Index): Daily observed elements for the first six months between 2009 and 2013.
- TAIEX (Taiwan Stock Exchange): Daily observed elements between 1999 and 2004.
- ABC (The Australian Beer Consumption): Quarterly observed elements between 1956 and 1994.

The evaluation criterions considered here are the Root-Mean-Squared Error (RMSE), the Mean Absolute Percentage Error (MAPE), and the Correlation Coefficient (R).

1. Our probabilistic-based method (MMPF) is first tested with the first group of datasets (IEX). It gives smaller errors (RMSE and MAPE) and a better correlation coefficient (R) compared with the results determined by all the upper-mentioned existing methods which were calculated for only 15 test data from each dataset (these results were quoted from [17]), while in our study we test our approach for the total length of each dataset as it is shown in the three Tables (1, 2, and 3). If we consider just 15 test data, better results could be determined. All operations achieved here are carried out using MATLAB. In order to show an example illustrating how our approach is working, Figure (4) represents the forecasting for the total length of the dataset (IEX - 2009) which includes 103 days. This experiment requires at least 200 distributed particles per day. The same number of particles is considered to calculate the three performance criteria (RMSE, MAPE, and R) for all the other datasets (IEX – 2010, 2011, 2012, and 2013) as it is illustrated in Tables (1, 2, and 3). The bigger the number of particles, the better the performance is. This number should be bounded by an experimental limit to always keep an acceptable balance between reliable performance and minimal running time.

2. For the second dataset (TAIEX) which was daily observed between 1999 and 2004 and composed of six tables, one table per year, our probabilistic approach is compared with the results of a group of the best and newest existing techniques according to the performance criterion (RMSE) as it is illustrated in Table (4), this table was quoted from [17, 1, 3]. This comparison leads us to conclude that the performance of our probabilistic method is much better than all the other existing techniques. The Mean absolute percentage error (MAPE) is provided for our approach (Table 5) while it is unfortunately unavailable for the other techniques. The calculated correlation coefficient according to our approach doesn't break below the value of 0.99. Once again, 200 particles per day were used to give the highly reliable forecasting performance shown in Figure (6).

3. Finally, our probabilistic technique is going to be tested with the third dataset called ABC which is composed of 148 values observed quarterly between 1965 and 1994. Table (6) proves the high forecasting performance of our method when it is compared with some other existing methods. The performance criterions (RMSE, MAPE and R) are calculated for the last 16 elements from the dataset, similarly to the other existing methods, then for the total length of the dataset as it is shown in Table (6). 200 particles per day were used to give the forecasting diagram shown in Figure (7). For all the upper-mentioned tests, the MMPF was applied to each dataset for 100 successive iterations; the corresponding results illustrated in tables are the averages of all attempts.

Table 1. *For IEX; the RMSE is determined for all existing methods between 2009 and 2013 and compared with the corresponding values which are calculated according to the approach (MMPF).*

Year	Length of dataset	ES (15 test)	MLP (15 test)	FF (15 test)	FTS-N (15 test)	T1-R-IFF (15 test)	MMPF (Total length of data)
2009	103	540.21	525.73	534.13	514.56	450.185	207.8767
2010	104	1611.5	1603	1852	1357.4	1314.228	541.7540
2011	105	1129.7	1095.7	1145.6	916.54	872.1253	753.7474
2012	105	620.83	783.35	1037.6	581.71	510.6038	445.3562
2013	105	1268.7	1232.5	1278.6	1207.9	1016.646	747.9867
Mean		1034.19	1048.06	1169.59	915.62	832.76	539.3442

Table 2. *For IEX; the MAPE is determined for all existing methods between 2009 and 2013 and compared with the corresponding values which are calculated according to the approach (MMPF).*

Year	Length of dataset	ES (15 test)	MLP (15 test)	FF (15 test)	FTS-N (15 test)	T1-R-IFF (15 test)	MMPF (Total length of data)
2009	103	0.012	0.0114	0.0438	0.0112	0.0096	0.0044
2010	104	0.022	0.0220	0.0264	0.0202	0.0197	0.0071
2011	105	0.015	0.0146	0.0156	0.0121	0.0116	0.0092
2012	105	0.0088	0.0117	0.0161	0.0087	0.0074	0.0062
2013	105	0.109	0.0107	0.0108	0.0106	0.0091	0.0065
Mean		0.01374	0.014	0.02254	0.01256	0.01148	0.0067

Table 3. *For IEX; the correlation coefficient (R) is determined for all existing methods between 2009 and 2013. This coefficient doesn't break below 0.99 for all datasets (all years) when it is calculated according to our approach.*

Year	ES (15 test)	MLP (15 test)	FF (15 test)	FTS-N (15 test)	R-T1FF (15 test)
2009	0.881914	0.877104	0.885241	0.886303	0.902892
2010	0.493117	0.49661	0.428723	0.510228	0.683963
2011	0.786252	0.800986	0.766808	0.816094	0.81248
2012	0.912444	0.911624	0.904967	0.903832	0.918389
2013	0.779042	0.784258	0.790284	0.805135	0.87873

Table 4. *For TAIEX; the RMSE is determined for all existing methods between 1999 and 2004 and compared with the corresponding values which are calculated according to the approach (MMPF).*

Year	Chen et al. (2012)	Chen and Jian (2017)	Chen and Phuong (2017)	Tak et al. (2018)	Tak (2020)	MMPF
1999	99.87	101.82	99.97	98.33	97.81	72.54
2000	119.98	128.95	126.59	128.18	122.23	95.72
2001	114.47	110.66	110.17	106.48	106.81	72.37
2002	67.17	60.41	61.62	65.14	64.24	55.32
2003	52.49	50.65	53.01	52.38	51.5	41.14
2004	52.27	52.86	53.28	53.78	52.79	46.24
Mean	84.37	84.23	84.11	84.05	82.56	63.88

Table 5. *For TAEIX; the MAPE is calculated according to the approach (MMPF) between 1999 and 2004*

Year	MMPF
1999	0.0072
2000	0.0101
2001	0.0120
2002	0.0080
2003	0.0062
2004	0.0053
Mean	0.0082

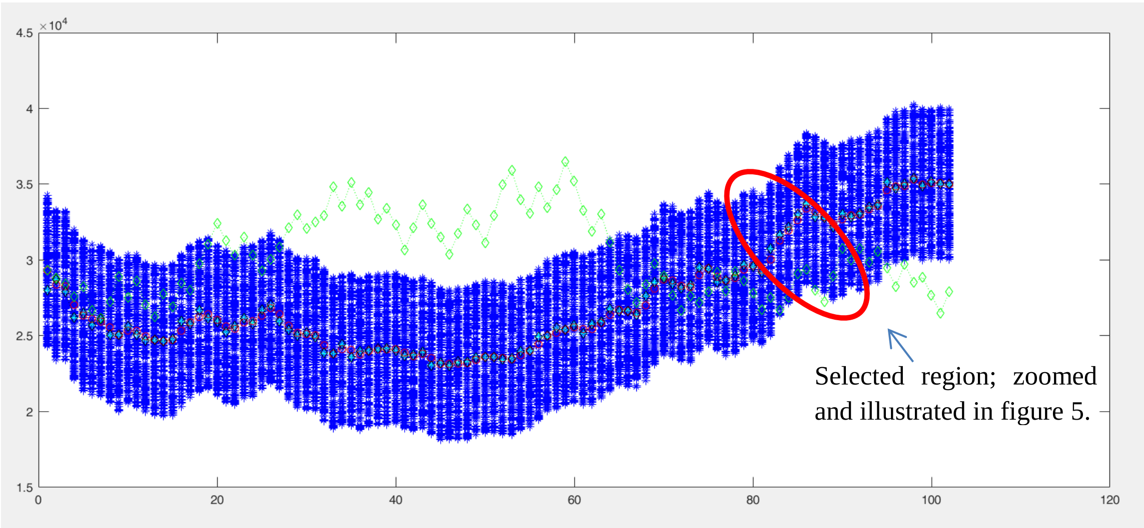


Figure 4. *The forecasting for 103 days for the dataset IEX-2009, 200 particles were used per day.*

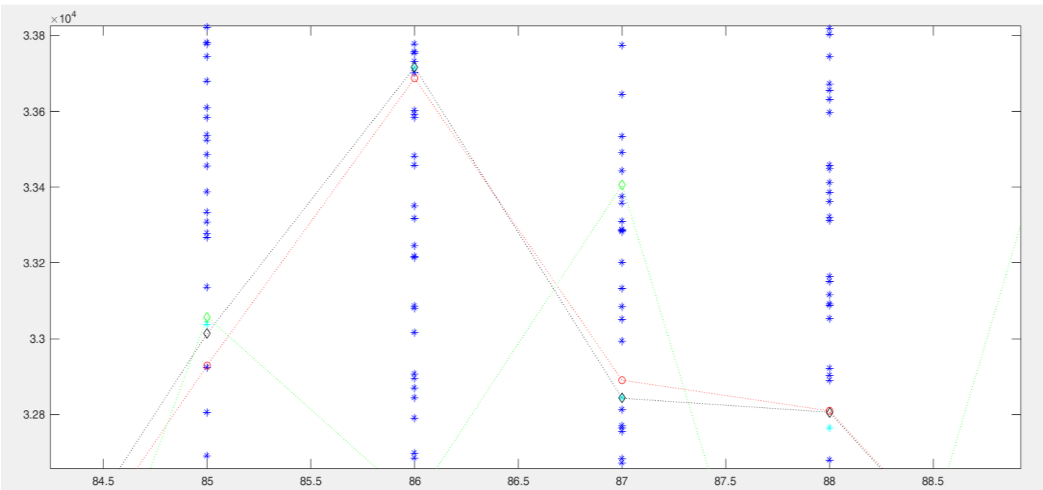


Figure 5. *A zoomed region from the figure (4), the blue stars are the particles, the small red circle is the main predict particle per day before diffusing the other particles around it. The green star is the most weighted particle per day. The black diamonds are the real values of the dataset. The green diamonds are produced by the simple linear model when used alone.*

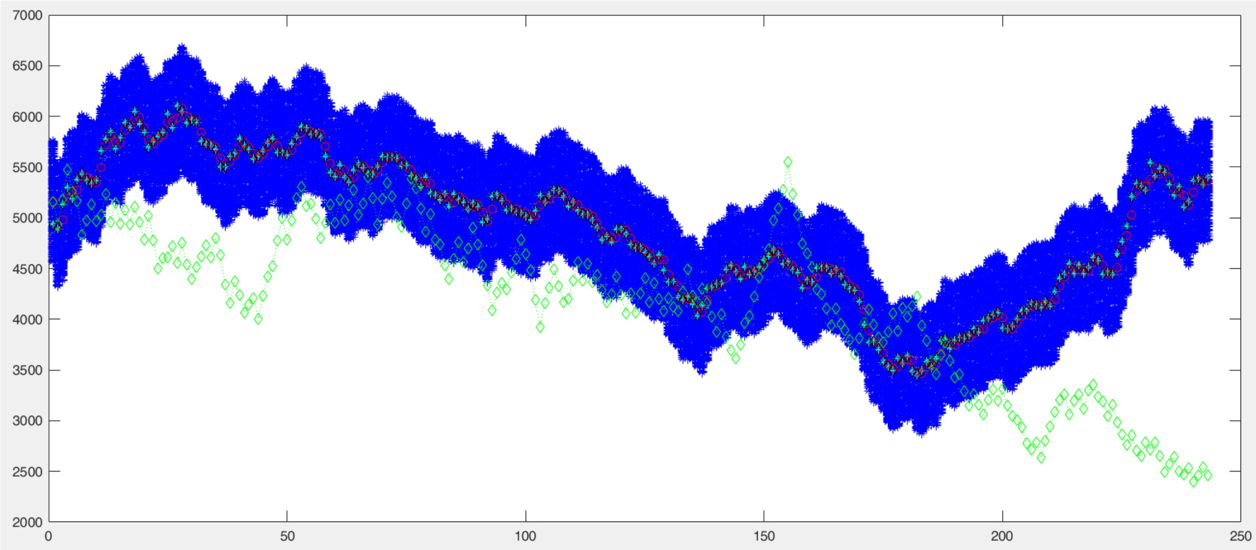
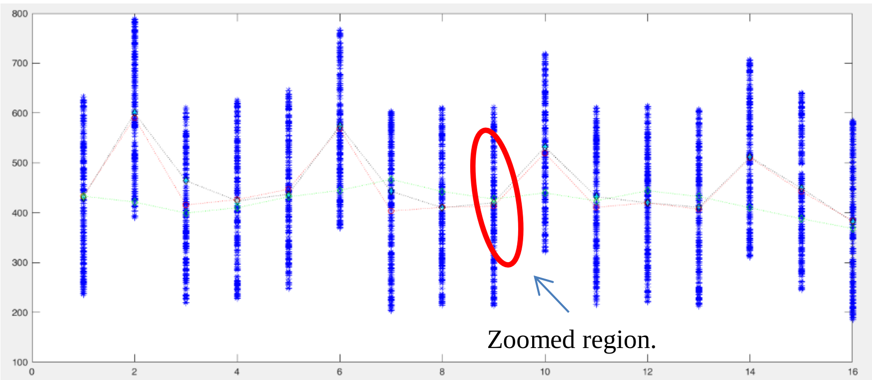


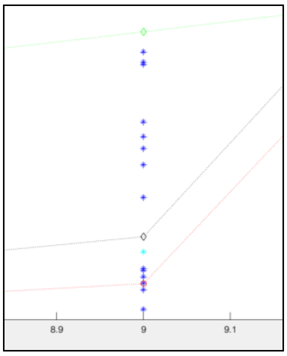
Figure6. *Forecasting for the total length of the dataset TAIEX-2001 that includes 245 days. In this experiment 200 particles were also used per day. The blue stars are the particles, the small red circle is the main predict particle per day before diffusing the other particles around it. The green star is the most weighted particle. The black diamonds are the real values of the dataset. The green diamonds are produced by the simple linear model when used alone.*

Table 6. *For ABC; the RMSE and MAPE are calculated for the last 16 elements from the dataset. Then, for the total length of the dataset. The vector of 16 tested elements is: [430.5000, 600.0000, 464.5000, 423.6000, 437.0000, 574.0000, 443.0000, 410.0000, 420.0000, 532.0000, 432.0000, 420.0000, 411.0000, 512.0000, 449.0000, 382.0000]*

	ARIMA	FANN	ANFIS	MANFIS	R-T1FF	T1-R-1FFs	MMPF 16 values	MMPF 148 values
RMSE	47.04	24.11	25.05	21.37	19.21	19.84	17.54	14.09
MAPE	0.0949	0.0476	0.0467	0.0401	0.0333	0.036	0.023727	0.026559
R	0.905	0.945	0.939	0.956	0.948	0.951	0.99	0.99



The forecasting diagram for just 16 days of ABC dataset, which is composed of 148 elements



A zoomed region

Fig.7. *In this experiment, 200 particles were used per day. The blue stars are the particles, the small red circle is the main predict particle per day before diffusing the other particles around it. The green star is the most weighted particle. The black diamonds are the real values of the dataset. The green diamonds are produced by the simple linear model when used alone.*

IV.CONCLUSION

To predict the fluctuations of stock markets which are mostly regarded as randomly changing environments, a variant of Multiple Model Particle Filter (MMPF) was proposed in this paper. The dynamic model of this filter takes into account three different parameters (economic, political, and natural) that decide the primitive prediction of the next state. Then, (MMPF) produces a cloud of (N) particles randomly distributed around it. To calculate the weights assigned at each moment to all particles we include a probabilistic value that considers all the historical story of the addressed state (whether it tends to move up, down, or keep its value), taking this historical behavior of the studied state into account makes our predictor more stable and always converging towards the real values. Our method was at first tested with a randomly generated virtual scenario that contains a sudden and wide deviation between supposed real values and their corresponding predicted values for some successive moments to prove the reliable performance of this probabilistic-based approach in such cases. Later, it was compared with different existing methods using the same datasets which were already used by all of them. In all cases, our probabilistic method has shown better performance according to the calculated evaluation metrics (RMSE, MAPE, and the Correlation Coefficient).

Even though the probabilistic approach presented here is able to model the uncertainty margin for a cloud of predicted values at the next given moment, more than predicting a single value, which is one of its main advantages, it could be considered a time-consuming method compared to statistical-based techniques. Performing this type of probabilistic-based forecasting in real-time requires utilizing more advanced parallel programming techniques; in this context, FPGA-based embedded systems may represent a good environment to develop many practical implementations in the near future.

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