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Research Article

Vibration suppression of a micro beam subjected to magneto-electric load

Kenan YILDIRIM[1,*]

[1]*Mus Alparslan University, Mus, Turkey*

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ABSTRACT

Control of vibrations, induced by magneto-electric load, by using piezoelectric actuator in a magneto-electroelastic micro beam is taken into account. Wellposedness and controllability properties of the system is discussed. Performance index functional to be minimized on control duration is chosen as a modified kinetic energy of the micro beam. By means of a maximum principle, optimal vibration control problem is transformed to a solving of a system of partial differential equations linked by terminal-initial-boundary conditions. Solutions are obtained by means of MATLAB and for indicating the effectiveness and robustness of the applied control actuation, results are presented in tables and graphical forms.

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INTRODUCTION

The definition, magneto-electro-elastic solid, is widely used to address to a kind of smart materials have capacity to transform reversibly their properties to respond external excitation such as temperature, moisture, stress, electric or magnetic fields. Since last two decades, magneto-electro-elastic (MEE) composites have gained great importance due to their ability of transforming one form of energy to another, having simple geometry and economic design and being useful in smart or intelligent structure applications. Much studies are done for examining on several properties of Magneto-electro-elastic structures. In [2], the state-vector method is applied to examine the free vibration of MEE laminate plates. The natural frequencies and corresponding mode shapes are calculated and compared with results existing in the literature. [3] aimed computationally investigatation for effects of thermal and mechanical loads on dynamic characteristics of intelligent composite structures. In [4], the numerical calculations are made for investigating the nonlinear vibration, nonlinear bending, and magneto-electric potential distributions through the thickness of the beam in different thermal environmental conditions. In [5], the non-local theory solution to a three-dimensional rectangular permeable crack in MEE materials is showed by using the generalized Almansi's theorem and the Schmidt approach. The problems are modeled through

***Corresponding author.**
*E-mail address: k.yildirim@alparslan.edu.tr

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Fourier transform as three pairs of dual integral equations, in which the unknown variables are the jumps of elastic displacement, electric and magnetic potential jumps across the crack surfaces. In [6], the effect of hygrothermal environment on the free vibration characteristics of (MEE) plates is presented by using finite element method. For this aim, higher order shear deformation theory is employed to evaluate the displacement fields. In [7], In order to estimate the first natural frequency and the critical angular velocity of a thermo-electro-magneto-elastic single-layer cylindrical nano-shell resting on a Winkler foundation, the governing equations of motion are derived based on the Hamiltonian Principle and by using first shear deformation theories in conjunction with modified couple stress theory. The effects of the centrifugal acceleration are considered in the formulation. In [8], Exact solutions are obtained for three-dimensional, anisotropic, linearly magneto-electroelastic, simply-supported, and multilayered rectangular plates subjecting to static loadings. While the homogeneous solutions are obtained in terms of a new and simple formalism that resemble the Stroh formalism, solutions for multilayered plates are presented in terms of the propagator matrix. In [9], analytical solutions are presented for free vibrations of three-dimensional, linear an isotropic, MEE multilayered rectangular plates under simply supported edge condition and the dispersion equation that characterizes the relationship between the natural frequency and wave number can be obtained in a simple form. In [10], Free vibration studies of multiphase and layered MEE beam is carried out. In-plane plate finite-element analysis is used to obtain the behavior of MEE beam. In [11], a partial mixed layerwise finite element model for adaptive plate structures is presented. Static analysis of MEE laminated plate structures is considered. The mixed finite element formulation is presented by considering a Reissner mixed variational principle. In [12], authors considered size-dependent geometrically nonlinear free vibration of magneto-electro-thermo elastic (METE) nanoplates using the nonlocal elasticity theory. The mathematical formulation is developed based on the first-order shear deformation plate theory, von Krmn-type of kinematic nonlinearity and nonlocal elasticity theory. In [16], Thermo-electro-magneto-mechanical bending analysis of a sandwich nanoplate is presented by Kirchhoffs plate theory and nonlocal theory. The sandwich nanoplate includes an elastic nano-core and two piezomagnetic face-sheets actuated by applied electric and magnetic potentials. The governing equations for the electro-magneto-mechanical bending are derived in terms of the displacement components and electric and magnetic potentials. Then, the problem is solved analytically by using Naviers method. In [17], Thermo-electro-mechanical transient analysis of a sandwich nanoplate is studied in this paper. The sandwich nanoplate consists of a KelvinVoigt viscoelastic nanoplate and two integrated piezoelectric face sheets resting on a visco-Pasternak foundation. The sandwich nanoplate is subjected to thermal and mechanical loads, and the piezoelectric face sheets are subjected to an applied electric potential. Twovariable sinusoidal shear deformation plate theory is used for the description of the displacement components. The governing equations of motion are derived using Hamiltons principle by calculation of strain and kinetic energies and energy due to external forces. In [15], authors proposed the cell-based smoothed finite element method (CS-FEM) for superior calculations, in which the strain smoothing technique is introduced into FEM. Also, it is shown that CSFEM possesses high accuracy, low mesh restriction, much less computational-cost than FEM, and stronger handling ability when encountering strong mesh distortions and large deformations. In [18], Wave propagation analysis of a nanobeam made of functionally graded MEE materials with rectangular cross section rest on Visco-Pasternak foundation is studied in this paper. For modeling the axial, rotation and transverse deformations, Timoshenko beam model is used. Fundamental MEE equations of the model are derived for a general functionally graded beam excited to electric and magnetic potentials. Surface elasticity is employed for more confident modeling the behavior of nanobeam. Using Hamilton principle and calculation of kinetic and strain energies, the equations of motion are derived. In [19], A dynamic solution is presented for the propagation of harmonic waves in inhomogeneous (functionally graded) MEE composite plates. The materials properties are assumed to vary in the direction of the thickness according to a known variation law. The Legendre orthogonal polynomial series expansion approach is employed to determine the wave propagating characteristics in the plates. In [13], authors presented for the first time the thermal effect on dynamic characteristics of MEE intelligent structures by combining the coupled multiphysic (CP) CS-FEM with the modified Newmark method. The precision and converging of CPCS-FEM were proved by the comparing with FEM. In [20], The general solution of three-dimensional problems in transversely isotropic MEE media is obtained through five newly introduced potential functions. The displacements, electric potential, magnetic potential, stresses, electric displacements and magnetic inductions can all be expressed concisely in terms of the five potential functions, all of which are harmonic. The derived general solution is then applied to find the fundamental solution for a generalized dislocation and also to derive Green's functions for a half-space MEE solid. In [14], authors aimed to conduct the research on studying the hygrothermal effect on the MEE-based structure using CS-FEM.

In [21], Two independent state equations are established for transversely isotropic magneto-electro-elastic media by introducing proper stress and displacement functions. The free vibration problem of simply supported rectangular plates with general inhomogeneous (functionally graded) material properties along the thickness direction is then considered. An approximate laminate model is employed to transform the state equations with variable coefficients to the ones with constant coefficients. As understood from literature review, there are very few and limited studies interested on vibration control of MEE micro beam. The original contribution of this paper and the reason making this paper is important is that vibrating control of MEE micro beam by using piezoelectric patch actuator is firstly considered in this paper and maximum principle is employed for determining the control voltage function optimally. In particularly, in the present paper, active vibration control of a MEE micro beam is taken into account and vibrations based on magneto-electric load are suppressed by means of piezoelectric patch actuators bonded on the surface of the micro beam. Before control actuation of the system, wellposedness and controllability properties of the System are stated and proved by a lemma and a theorem. The main aim of this study is to minimize the performance index functional of the system, which is defined a modified kinetic energy of the system, by using minimum control voltage function to be applied to piezoelectric patch. For determining the control voltage function, maximum principle is employed due to existence of Heaviside function in the equation of motion. By means of maximum principle, optimal control problem is transformed to a solving of system of partial differential equations including state and adjoint variables and they are linked to each other via terminal-boundary-initial conditions. The solution of the system is obtained by means of MATLAB and obtained results are presented in tables and graphical forms for indicating the effectiveness of the introduced control actuation.

DEFINITION OF THE PROBLEM

Consider a five-layer structure which consists of a central host layer, which is a MEE microbeam, and both side of the MEE micro beam is partially covered by elastic but non conductive rubber and two piezoelectric path actuators are perfectly deployed on these rubbers on the both side of a MEE micro beam as it is seen in Fig. 1. Initially, it is assumed that micro beam subjecting to magneto-electric load is undeformed and at rest position. Piezoelectric patch actuators are put on the same points of the micro beam for effective control actuation. The vibrations in a magneto-electro-elastic micro beam subjecting to magneto-electric load is given by as follows [1];

$$DIu_{XXXX} - \zeta u_{XX} + \rho A u_{TT} = q(X) + C(T,X) \quad (1)$$

where state variable u transverse displacement at $(T,X) \in \bar{\Omega} = \{T \in (0,T_f),\ X \in (0,l)\}$, T is time variable, T_f is predetermined terminal time, X is space variable, l is the length of the MME microbeam, $D = B + ea + fb$ in which B is the elasticity coefficient, e is the piezoelectric coefficient, a is height of MEE micro beam, f is piezo magnetic coefficient, b is the width of MEE microbeam, $\zeta = e\delta_1 + f\delta_2$, in which δ_1 and δ_2 are electric potential and magnetic potential, respectively, between lower and upper surfaces of MEE microbeam, $q(X)$ is external magneto-electric load function, $C(T,X) = \kappa V(T)(H''(X - X_1) - H''(X_1 - X_2))$ in which κ is the elasticity coefficient of the rubber, V is the control voltage function to be applied to piezoelectric actuator, H is Heaviside function, $(X - X_2)$ is the location points of piezoelectric actuators. Eq. (1) is subject to the following initial conditions

$$u(T,X) = u_0(X),\ u_T(T,X) = u_1(X) \quad \text{at} \quad T = 0, \quad (2)$$

and boundary moment conditions

$$U(T,X) = 0,\ U_{XX}(T,X) = 0 \qquad \text{at } X = 0,l. \quad (3)$$

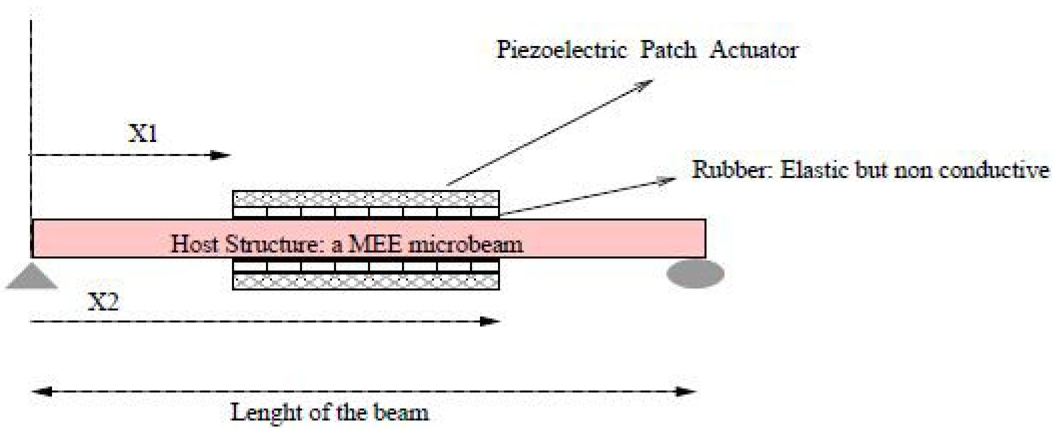

Figure 1. A MEE microbeam diagram with patches in a thermal environment.

Let solution of the system satisfies followings:

$$u, \frac{\partial^{i+j} u}{\partial T^j \partial X^i} \in L^2(\bar{\Omega}), \quad j = 0,1,2, \quad i = 0,1,\ldots,4 \quad (4a)$$

$$u_0(X) \in H^1(0,l) = \left\{ u_0(X) \in L^2(0,l) : \frac{\partial u_0(X)}{\partial X} \in L^2(0,l) \right\}, \quad (4b)$$
$$u_1(X) \in L^2\ (0,l)$$

where $L^2(\bar{\Omega})$ denote the Hilbert space of real-valued square-integrable functions defined in the domain $\bar{\Omega}$ in the Lebesque sense with norm and usual inner product defined by

$$\|\eta^2\| = \langle \eta, \eta \rangle, \quad \langle \rho, \eta \rangle \Omega = \int_{\bar{\Omega}} \rho \eta d\bar{\Omega}.$$

Then, the system under consideration has a solution [23].

Lemma 1. *With Eq.4, the system defined by Eqs.* (1)–(3) *has a unique solution.*

Proof. Let us assume that in the same physical conditions, u_1 and u_2 are two different solutions to the system under consideration. Then, the difference

$$W(T,X) = u_1(T,X) - u_2(T,X)$$

satisfies the following homogeneous equation

$$DIW_{XXXX} - \zeta W_{XX} + \rho A W_{TT} = 0 \quad (5)$$

and following homogeneous boundary and initial conditions, respectively.

$$W(T,X) = W_{xx}(T,X) = 0 \quad \text{at} \quad X = 0,l \quad (6)$$
$$W(T,X) = W_T(T,X) = 0 \quad \text{at} \quad T = 0.$$

Let us show that $W(T,X)$ is identically equal to zero in $\bar{\Omega}(T,X)$. Then, consider the energy integral as follows;

$$\varepsilon(T) = \frac{1}{2}\int_0^l \left\{ DI \frac{\partial^4}{\partial X^4}(W)^2 - \zeta \frac{\partial^2}{\partial X^2}(W)^2 + \rho A \left(\frac{\partial W}{\partial T} \right)^2 \right\} dX \quad (8)$$

and let us show that $\varepsilon(T)$ is independent of T. Differentiating $\varepsilon(T)$ with respect to T, we obtain

$$\frac{d\varepsilon(T)}{dt} = \int_0^l \left\{ DI \frac{\partial^4}{\partial X^4}\left(W \frac{\partial W}{\partial T} \right) - \zeta \frac{\partial^2}{\partial X^2}\left(W \frac{\partial W}{\partial T} \right) + \rho A \frac{\partial^2 W}{\partial T^2}\frac{\partial W}{\partial T} \right\} dX \quad (9)$$

Applying integration by parts and using boundary conditions given by Eq. (6), one observes Eq. (9) as follows;

$$\frac{d\varepsilon(T)}{dt} = \int_0^l \left\{ DI \frac{\partial^4 W}{\partial X^4} - \zeta \frac{\partial^2 W}{\partial X^2} + \rho A \frac{\partial^2 W}{\partial T^2} \right\} \frac{\partial W}{\partial T} dX.$$

Because of the right hand-side of Eq. (5) is zero, we observe

$$\frac{d\varepsilon(T)}{dt} = 0 \text{ , That is } \varepsilon(T) = \text{constant}$$

Taking the initial conditions defined by Eq. (7) into consideration, we obtain that

$$\varepsilon(0) = \frac{1}{2}\int_0^l \left\{ DI \frac{\partial^4}{\partial X^4}(W)^2 - \zeta \frac{\partial^2}{\partial X^2}(W)^2 + \rho A \left(\frac{\partial W}{\partial T} \right)^2 \right\} \Big|_{T=0} dX = 0. \quad (10)$$

Then, it follows from Eqs. (7)–(10) that $W(T,X)$ is identically equal to zero in $\bar{\Omega}$ that is $u_1 = u_2$. Hence, proof is completed.

For convenience, let us introduce nondimensional parameters as follows;

$$x = \frac{X}{l},\ w = \frac{u}{l},\ t = \frac{1}{l^2}\sqrt{\frac{DI}{\rho A}}T,$$
$$\xi = \frac{\zeta l^2}{DI}, \quad f(x) = \frac{l^4}{DI} q(x),\ \mathrm{V}(T) = \frac{l^4}{DI} V(T). \quad (11)$$

In the light of Eq. (11), nondimensional equation of motion is stated as follows;

$$w_{xxxx} - \xi w_{xx} + w_{tt} = f(x) + C(t,x) \quad (12)$$

in which w is the transversal displacement at $(t,x) \in \Omega = (0,t_f) \times (0,1)$ and $C(t,x) = \kappa \mathrm{V}(t)(H''(x - x_1) - H''(x - x_2))$. Eq. (12) is subjected to following boundary conditions

$$w(t,x) = 0, \quad w_{xx}(t,x) = 0 \ \text{ at } \ x = 0,1 \quad (13)$$

and initial conditions

$$w(t,x) = w_0(x), \quad w_t(t,x) = w_1(x) \quad \text{at} \quad t = 0. \quad (14)$$

OPTIMAL CONTROL PROBLEM

The main goal of this study is to determine voltage function optimally to be applied to piezoelectric actuator which minimizes a given performance index at a predetermined terminal time t_f with a minimum expenditure of control voltage energy. Before defining the performance index functional of the system, let us define the admissible control voltage function set as follows;

$$V_{ad} = \{V(t)\ |V \in L^2(\Omega), \quad |\mathrm{V}(t)| \le \mathrm{V}_0 < \infty\}. \quad (15)$$

Then, the performance index functional of the system is given by as follows:

$$\mathcal{I}(\mathrm{V}(t))=\int_0^l\left[\vartheta_1 w^2(t_f,x)+\vartheta_2 w_t^2(t_f,x)\right]dx+\vartheta_3\int_0^{t_f}\mathrm{V}^2(t)dt \quad (16)$$

in which ϑ_1, $\vartheta_2 \geq 0$, $\vartheta_1 + \vartheta_2 \neq 0$ and $\vartheta_3 > 0$ are weighting constants. First integral on the left-hand side in Eq. (16) represents the modified dynamics response of the magneto-electro micro beam. First and second terms in this integral are quadratic functional of the displacement and velocity of the beam, respectively. Second term on the left-hand side in Eq. (16) is the measure of the total control voltage energy on the $(0,t_f)$ Then, optimal control problem is stated as follows;

$$\mathcal{I}\left(\mathrm{V}^o(t)\right)=\min_{\mathrm{V}\in\mathrm{V}_{ad}}\mathcal{I}(\mathrm{V}) \quad (17)$$

subject to the Eqs. (12)–(14). In order to achieve the Maximum principle for obtaining optimal control voltage function, let us introduce an adjoint variable $\nu \in L^*$, L^* is the dual to $L^2(\Omega)$ and has the same norm and inner product like in $L^2(\Omega)$ Adjoint system corresponding to Eqs. (12)–(14) is expressed as follows;

$$\nu_{xxxx} - \xi\nu_{xx} + \nu_{tt} = 0 \quad (18)$$

subjected to following boundary and terminal conditions, respectively;

$$\nu(t,x) = 0, \quad \nu_{xx}(t,x) = 0 \quad \text{at} \quad x = 0 \quad (19)$$

$$2\vartheta_1 w(t,x) = \nu_t(t,x), \quad -2\vartheta_2 w_t(t,x) = \nu(t,x) \quad t = t_f. \quad (20)$$

The existence and uniqueness of the solutions to adjoint system defined by Eqs. (18)–(20) is shown similar to Eqs. (1)–(2). Note that is the unique solution to system given by Eqs. (12)–(14) and also convexity properties of the performance index functional guarantees the uniqueness of the control voltage function $V(t)$. Hence, it is easy to observe that the system under consideration has a unique state and control function. Then, the system is called as observable, which equal to controllable [22]. Namely, the system Eqs. (12)-(14) is controllable. Maximum principle is derived a necessary condition for the optimal control function in terms of Hamiltonian functional. In case of some convexity assumptions, satisfied by Eq. (16), on performance index functional of the system, maximum principle is also sufficient condition for optimal control function. Let us derive the maximum principle as follows;

Theorem 1. *(Maximum principle) The maximization problem is presented as follows;*

$$\text{If } \mathcal{H}\left[t;w^o,\nu^o,V^o(t)\right]=\max_{\mathrm{V}(t)\in\mathrm{V}_{ad}}\mathcal{H}\left[t;w,\nu,V\right] \quad (21)$$

in which, Hamiltonian function is presented by

$$\mathcal{H}[t;w,\nu,V] = \kappa V[\nu_x(t,x_1) - \nu_x(t,x_2)] - \vartheta_3 V^2(t), \quad (22)$$

then

$$\mathcal{I}[V^o(t)] \leq \mathcal{I}[V(t)], \ \forall \mathrm{V} \in V_{ad} \quad (23)$$

where $V^o(t)$ is the optimal control voltage function.

Proof. Before starting to proof, let us define following operator

$$\varphi(w) = w_{xxxx} - \xi w_{xx} + w_{tt} \quad (24)$$

and its adjoint operator as follows;

$$\varphi^*(\nu) = \nu_{xxxx} - \xi\nu_{xx} + \nu_{tt} \quad (25)$$

The deflections in the state variable and its derivatives with respect to the time variable are defined by

$$\Delta w = w - w^o, \qquad \Delta w_t = w_t - w_t^o.$$

The operator

$$\varphi(\Delta w) = \Delta C(t,x)$$

is subject to the following homogeneous boundary and initial conditions, respectively;

$$\Delta w(t,x) = \Delta w_{xx}(t,x) = 0 \quad \text{at} \quad x = 0,1 \quad (26)$$

and initial conditions

$$\Delta w(t,x) = \Delta w_t(t,x) = 0 \quad \text{at} \quad t = 0. \quad (27)$$

Now, note that the following functional

$$\iint_\Omega\{\nu\varphi(\Delta w)-\Delta w\varphi^*\}d\Omega=\iint_\Omega\{I_1+I_2\}d\Omega=\iint_\Omega\{\nu\Delta C(t,x)\}d\Omega \quad (28)$$

in which

$$I_1=\iint_\Omega\{\nu(\Delta w_{tt})-\Delta w(\nu_{tt})\}d\Omega \quad (29)$$

$$I_2=\iint_\Omega\{\nu(\Delta w_{xxxx}-\xi\Delta w_{xxx})-\Delta w(\nu_{xxxx}-\xi\nu xx)\}d\Omega \quad (30)$$

Applying the integration by parts to each term in the I_1 and I_2 in Eq. (28) and employing boundary and terminal conditions, respectively, given by Eq. (19) and Eq. (20), Eq. (28) becomes as follows;

$$\begin{aligned}&\iint_\Omega \nu\varphi(\Delta w)-\Delta w\varphi^*(\nu)d\Omega=I_1+I_2\\&=2\int_0^1\left\{\vartheta_1 w(t_f,x)\Delta w(t_f,x)+\vartheta_2 w_t(t_f,x)\Delta w_t(t_f,x)\right\}dx \quad (31)\\&=\iint_\Omega \nu\Delta C(t,x)d\Omega\end{aligned}$$

For the right-hand side of Eq. (31), employe the properties of dirac-delta function

$$H''(x-x_1)=\delta'(x-x_1),\quad \int_0^1 \delta'(x-x_1)g(x)dx = -g'(x_1),\quad x_1\in(0,1) \tag{32}$$

in which prime denotes spatial derivative. In the light of Eq. (32), the right hand-side of Eq. (31) is observed

$$\begin{aligned}&\iint_\Omega \nu\Delta C(t,x)d\Omega)\\ &=\iint_\Omega \kappa\nu(t,x)\,\Delta \mathrm{V}(t)\left[H''(x-x_1)-H''(x-x_2)\right]d\Omega \\ &=\int_0^{t_f}\kappa\Delta \mathrm{V}(t)\left(\nu_x(x_2,t)-\nu_x(x_1,t)\right)dt\end{aligned} \tag{33}$$

Focus on the difference of the performance index functional

$$\begin{aligned}\Delta\mathcal{I}\left[\mathrm{V}(t)\right]&=\mathcal{I}\left[\mathrm{V}(t)\right]-\mathcal{I}\left[\mathrm{V}^o(t)\right]\\ &=\int_0^l\Big\{\vartheta_1\left[w^2(t_f,x)-w^{o2}(t_f,x)\right]\\ &\quad+\vartheta_2\left[w_t^{\,2}(t_f,x)-w_t^{\,o2}(t_f,x)\right]\Big\}dx\\ &\quad+\int_0^{t_f}\vartheta_3\left[\mathrm{V}^2(t)-\mathrm{V}^{o2}(t)\right]dt\end{aligned} \tag{34}$$

Expanding $w^2(t_f,x)$ and $w_t^2(t_f,x)$ in Taylor series around $w^{o^2}(t_f,x)$ and $w_t^{o^2}(t_f,x)$, yields

$$w^2(t_f,x) - w^{o^2}(t_f,x) = 2w^o(t_f,x)\Delta w(t_f,x) + r, \tag{35a}$$

$$w_t^2(t_f,x) - w_t^{o^2}(t_f,x) = 2w_t^o(t_f,x)\Delta w_t(t_f,x) + r_t \tag{35b}$$

where $r = 2(\Delta w)^2$ + higher order and positive terms > 0 and $r_t = 2(\Delta w_t)^2$ + higher order and positive terms > 0. Substituting Eq. (35) into Eq. (34) gives

$$\begin{aligned}\Delta\mathcal{I}\left[\mathrm{V}(t)\right]&=\int_0^1\Big\{\vartheta_1\left[2w^o(t_f,x)\Delta w(t_f,x)+r\right]\\ &\quad+\vartheta_2\left[2w_t^{\,o}(t_f,x)\Delta w_t(t_f,x)+r_t\right]\Big\}dx\\ &\quad+\int_0^{t_f}\vartheta_3\left[\mathrm{V}^2(t)-\mathrm{V}^{o2}(t)\right]dt.\end{aligned} \tag{36}$$

From Eq. (31) and due to $\vartheta_1 r + \vartheta_2 r_t > 0$, one obtains

$$\begin{aligned}\Delta\mathcal{I}\left[\mathrm{V}(t)\right]&\ge\int_0^{t_f}\kappa\Delta \mathrm{V}(t)\left(\nu_x(x_2,t)-\nu_x(x_1,t)\right)dt\\ &\quad+\int_0^{t_f}\vartheta_3\left[\mathrm{V}^2(t)-\mathrm{V}^{o2}(t)\right]dt\ge 0\end{aligned} \tag{37}$$

which leads to

$$\begin{aligned}&\kappa V(t)(\nu_x(x_2,t) - \nu_x(x_1,t)) + \vartheta_3 V^2(t)\\ &\ge \kappa V^o(t)(\nu_x(x_2,t) - \nu_x(x_1,t)) + \vartheta_3 V^{o2}(t)\end{aligned} \tag{38}$$

that is,

$$H[t;\, w^o,\nu^o,V^o] \ge H[t;\, w,\nu,V].$$

Hence, we obtain V

$$\mathcal{I}[V] \ge \mathcal{I}[V^o], \qquad \forall \mathrm{V} \in V_{ad}.$$

Therefore, the optimal control voltage function is given by

$$\mathrm{V}(t)=\kappa\frac{\nu_x(t,x_1)-\nu_x(t,x_2)}{2\vartheta_3}. \tag{39}$$

NUMERICAL RESULTS AND DISCUSSIONS

In this section, in order to indicate the effectiveness and robustness of the introduced control algorithm for damping excessive vibrations in a magneto-electro-elastic micro beam by optimally determined control voltage function to be applied to piezoelectric patch actuator, obtained theoretical results are presented in tables and graphical forms by solving following system of equations linked by terminal-initial-boundary conditions via MATLAB.

$$w_{xxxx} - \xi w_{xx} + w_{tt} = f(x) + C(t,x), \tag{40a}$$

$$C(t,x)=\kappa \mathrm{V}(t)\left(H''(x-x_1)-H''(x-x_2)\right),\quad \mathrm{V}(t)=\kappa\frac{\nu_x(t,x_1)-\nu_x(t,x_2)}{2\vartheta_3} \tag{40b}$$

$$w(t,x) = 0,\ w_{xx}(t,x) = 0 \quad \text{at} \quad x = 0,1 \tag{40c}$$

$$w(t,x) = w_0(x),\quad w_t(t,x) = w_1(x) \quad \text{at} \quad t = 0. \tag{40d}$$

$$\nu_{xxxx} - \xi\nu_{xx} + \nu_{tt} = 0, \tag{41a}$$

$$\nu(t,x) = 0,\ \nu_{xx}(t,x) = 0, \quad \text{at} \quad t = 0,1, \tag{41b}$$

$$2\vartheta_1 w(t,x) = \nu_t(t,x) \tag{41c}$$

$$-2\vartheta_2 w_t(t,x) = \nu(t,x) \quad \text{at} \quad t = t_f. \tag{41d}$$

Before expounding the results in graphs and tables, consider the optimal control voltage function given by Eq. (39), in which, it is easy to see that as the value of ϑ_3 is decreasing, the value of the control voltage function is increasing. As a conclusion of this status, dynamic response of the beam given by first integral on the left side of the Eq. (16) is minimized by using minimum control voltage energy. Effectiveness of the introduced control actuation is examined in two cases. Both of two cases, t_f is taken into account as 5. Weighted coefficients are taken into account as $\vartheta_{1,2} = 1$ and $\vartheta_3 = 10^3$ and $\vartheta_3 = 10^{-3}$ for uncontrolled and controlled case, respectively. All figures are plotted in the middle point of the MEE micro beam, x = 0.5. In the first case, called a, followings are considered;

$$\xi = 0.001, \quad x_1 = 0.45, \quad x_2 = 0.55, \quad f(x) = \exp(-x),$$
$$w_0(x) = \sqrt{2}\ \sin(\pi x), \quad w_1(x) = \sqrt{2}\ \sin(\pi x).$$

For case a, controlled and uncontrolled displacements of the beam at the midpoint of the beam are given by Fig. 2 and it can be clearly observed that excessive vibrations induced by external magneto-electric load in the MEE micro beam system is suppressed successfully. Same observation is also valid that the velocities of excessive vibrations is also effectively suppressed by using minimum control voltage energy via introduced control actuation. In the second case, called b, control actuation is applied by considering followings;

$$\xi = 1, \quad x_1 = 0.35, \quad x_2 = 0.65, \quad f(x) = \exp(-x),$$
$$w_0(x) = \sqrt{2}\ \sin(\pi x), \quad w_1(x) = \sqrt{2}\ \sin(\pi x).$$

For case b, controlled and uncontrolled deflections of the MEE beam is given in Fig. 4. It is easy to see that excessive vibrations, due to magneto-electric load, in the MEE micro beam system are effectively suppressed by means of optimal control voltage function. In Fig. 5, the velocities of the MEE micro beam corresponding to case b are plotted and it can be easily seen that decreasing of velocities for case b is successfully obtained. In cases c and d, negative magneto electric potential and different locations of piezoelectric patches are taken into account as follows, respectively;

$$\xi = -0.5, \quad x_1 = 0.35, \quad x_2 = 0.65, \quad f(x) = \exp(-x),$$
$$w_0(x) = \sqrt{2}\ \sin(\pi x), \quad w_1(x) = 0.$$

$$\xi = -0.0005, \quad x_1 = 0.25, \quad x_2 = 0.75, \quad f(x) = \exp(-x),$$
$$w_0(x) = 0, \quad w_1(x) = \sqrt{2}\ \cos(\pi x).$$

For cases c and d, un/controlled displacements and velocities are plotted in Figures 6–7 and Figures 8-9, respectively. It is observed that introduced control actuation is very effective and suppress the undesirable vibrations in the micro beam. Let us give the dynamic response of the MEE micro beam as follows;

$$\mathcal{I}(w) = \int_0^1 \left[w^2(t_f, x) + w_t^{\ 2}(t_f, x) \right] dx \tag{42}$$

and used voltage energy over $(0, t_f)$

$$\mathcal{I}(\mathrm{V}) = \int_0^{t_f} \mathrm{V}^2(t)\, dt. \tag{43}$$

The dynamic response of the MEE micro beam system is given by table forms and it seemed from tables that as weighted coefficient ϑ_3 in optimal control voltage function decreases, dynamic response of the micro beam decreases due to an increasing in the value of control voltage function. These observations show that introduced control actuation for undesirable vibrations in a MEE micro beam system is successful and effective.

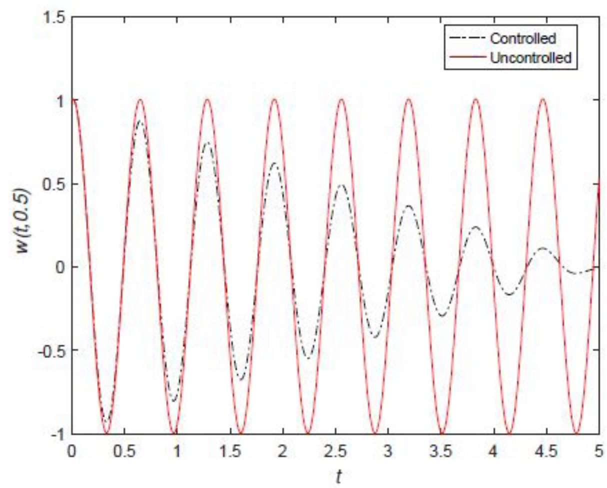

Figure 2. Uncontrolled and controlled displacements for case a.

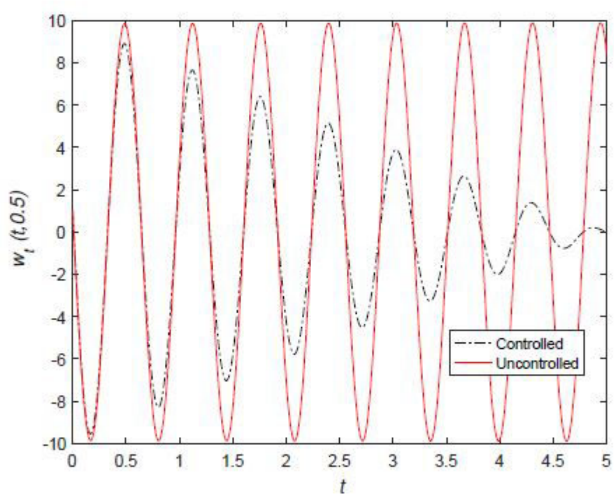

Figure 3. Uncontrolled and controlled velocities for case a.

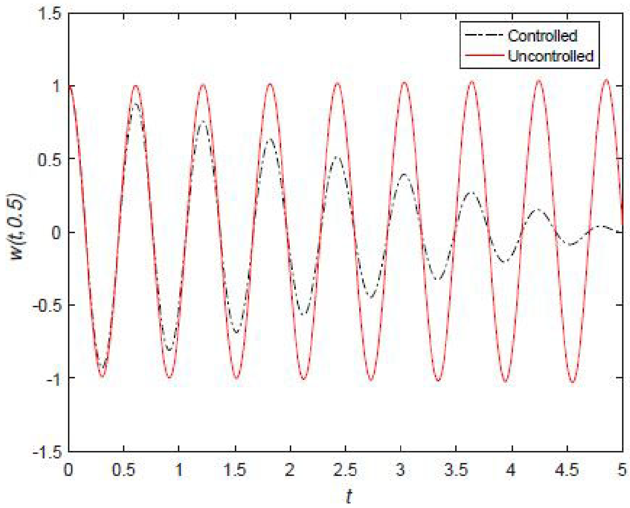

Figure 4. Uncontrolled and controlled displacements for case b.

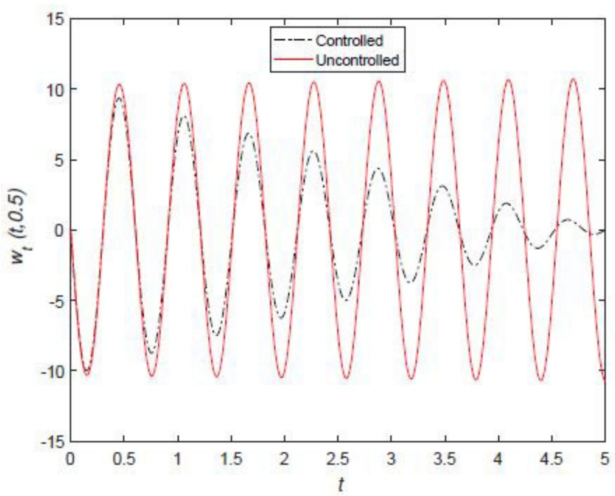

Figure 5. Uncontrolled and controlled velocities for case b.

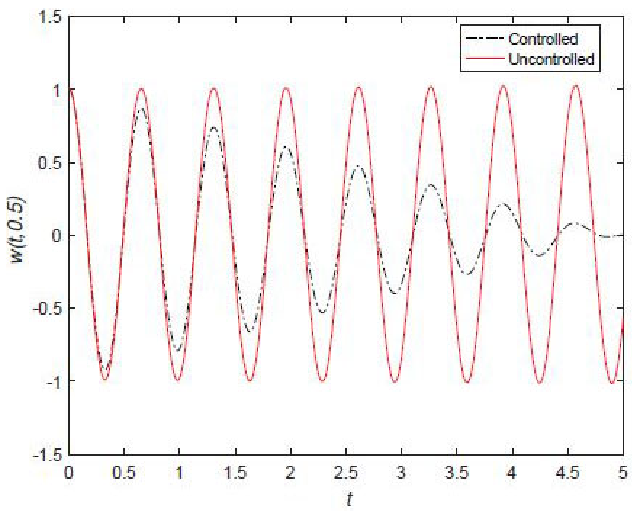

Figure 6. Uncontrolled and controlled displacements for case c.

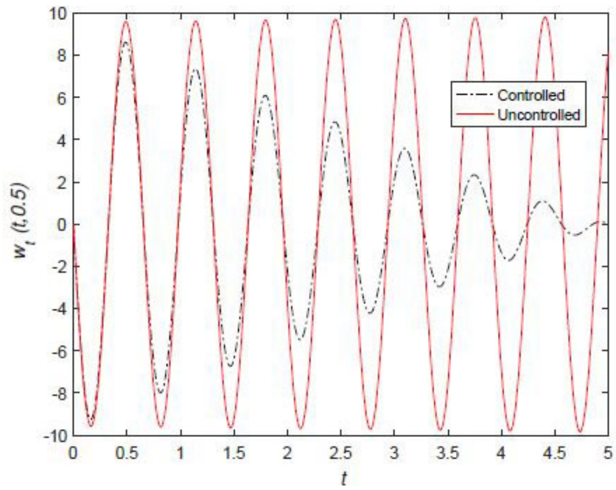

Figure 7. Uncontrolled and controlled velocities for case c.

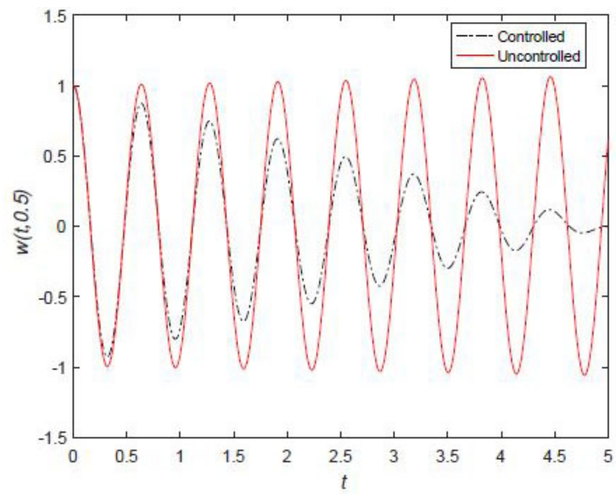

Figure 8. Uncontrolled and controlled displacements for case d.

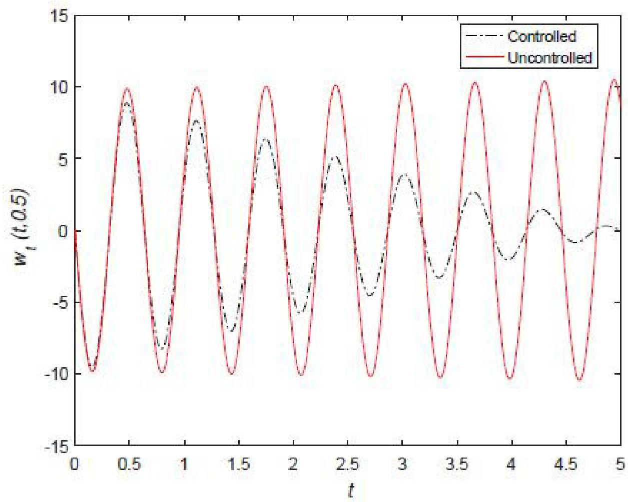

Figure 9. Uncontrolled and controlled velocities for case d.

Table 1. The values of $\mathcal{I}(w)$ and $\mathcal{I}(V)$ for different values of ϑ_3 in case a

ϑ_3	$\mathcal{I}(w)$	$\mathcal{I}(V)$
10^3	71.4	$3.4\ e^{-5}$
10^0	5.2	2.06
10^{-3}	$1.1\ e^{-4}$	2.4

Table 2. The values of $\mathcal{I}(w)$ and $\mathcal{I}(V)$ for different values of ϑ_3 in case b

ϑ_3	$\mathcal{I}(w)$	$\mathcal{I}(V)$
10^3	115	$4.7\ e^{-4}$
10^0	0.08	0.26
10^{-3}	$1.1\ e^{-4}$	0.28

CONCLUSION

In this study, wellposedness and controllability results of the Magneto-electro-elastic beam system is discussed and results are presented by a lemma. Damping out of undesirable vibrations based on magneto-electric load in a magneto-electro-elastic micro beam system is taken into account and control voltage function to be applied to piezoelectric patch actuator is optimally determined by means of maximum principle, which transforms the optimal control problem to solving a system of partial differential equations, including state and adjoint variable, linked by terminal-boundary-initial conditions. The optimal solution of this system of partial differential equations are obtained by means of MATLAB and for indicating the effectiveness and robustness of the introduced control actuation, obtained results are presented in graphical and tables forms.

AUTHOR CONTRIBUTION

KY completed this study and wrote the manuscript. KY read and approved the final manuscript.

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DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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