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FILTER PERFORMANCE COMPARISONS FOR ESTIMATING THE PROPAGATION OF FLEXURAL WAVE IN THIN PLATES

Tuğba Özge ONUR*

Bülent Ecevit University, Department of Electrical-Electronics Engineering, ZONGULDAK

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ABSTRACT

This paper demonstrates how flexural wave propagations in a thin plate can be modelled by estimating the combined effect of the excitation and the sensor. A theoretical model for flexural wave propagation in thin plates is derived and it is compared with measurements. In addition, the performances of used filters and ARX (autoregressive exogeneous) model are compared on estimating the wave propagation in a thin quartz glass plate. Results indicate that the most accurate estimation of wave propagation is obtained when a linear phase filter which attributes all dispersions to the wave is used.
Keywords: Flexural waves, filtering, ARX modeling, estimation of wave propagation.

İNCE PLAKALARDA ELASTİK DALGA YAYILIMININ KESTİRİMİNDE FİLTRE PERFORMANSLARININ KIYASLANMASI

ÖZET

Bu çalışmada uyartım ve sensörün birleştirilmiş etkisi kestirilerek ince bir plakadaki elastik dalga yayılımının nasıl modelleneceği gösterilmiştir. İnce düzlemlerde elastik dalga yayılımı için teorik model türetilerek ölçümlerle karşılaştırılmıştır. Ayrıca, ince kuartz cam düzlemde dalga yayılımı kestiriminde kullanılan filtrelerin ve ARX (Autoregressive Exogeneous) model performansları karşılaştırılmıştır. Elde edilen sonuçlar, en doğru dalga yayılımı kestiriminin dalga modeline bütün saçılmaların katkısının eklenmesini sağlayan doğrusal faz filtre kullanıldığında sağlandığını göstermektedir.
Anahtar Sözcükler: Elastik dalgalar, filtreleme, ARX modelleme, dalga yayılımı kestirimi.

1. INTRODUCTION

Flexural waves are surface waves that appear in thin media (thickness is small compared to the wavelength), for example plates and bars. A measurable property of a flexural wave is the local displacement which is perpendicular to the surface, depending on position and time. The investigation of flexural waves can be useful in structures. These waves travelling through plates have an important influence on the radiated sound field. Understanding how a wavefront propagates through a structure can give information about how this sound field is built up. For example, more efficient noiseless or vibration free systems can be designed if it is known exactly how these vibrations in plates behave. Also, the shape of wave carries information about the

* e-mail/e-ileti: ozgeozdinc@karaelmas.edu.tr, tel: (372) 257 40 10

material parameters of interest. For long-term stability and reliability of various devices, researchers should possess a deep understanding and knowledge of properties of materials and structures. In this regards, some methods have been investigated for modeling the flexural wave propagation and characterizing the material which has been used.

Integral Equation Methods for modeling the flexural waves are an outgrowth of several years of work. Integral equation methods have been around for several decades, and their introduction to flexural waves has been due to the seminal works of Archenbach in the 1970s [1]. There was a surge in the interest in this topic in the 1980s [2, 3] due to increased computing power. The interest in this area was on the wane when it was demonstrated that differential equation methods, with their sparse matrices, can solve many problems more efficiently than integral equation methods. Recently, due to the advent of fast algorithms, there has been a revival in integral equation methods in modeling of flexural waves. Chew et. al. has presented a new approach for flexural wave propagation [4]. However, some problems appear when a dispersive wave mode can be recorded for known distances between excitation and sensor. The most important one is that the recorded signal is affected by the excitation and by the sensor, both which have unknown transfer functions. Therefore, these transfer functions have to be modelled by using filters in order to estimate the most accurate wave shape.

Digital filters play an important role in signal processing applications. They are widely used in applications, such as digital signal filtering, noise filtering, signal frequency analysis, speech and audio compression, biomedical signal processing and image enhancement etc. A digital filter is a system which passes some desired signals more than others in order to reduce or enhance certain aspects of that signal. It can be used to pass the signals according to the specified frequency passband and reject the frequency other than the passband specification. The basic filter types can be classified into four categories such as lowpass, highpass, bandpass, and bandstop. On the basis of impulse response, digital filters are classified as Infinite Impulse Response (IIR) or Finite Impulse Response (FIR) filters depending upon whether the response of the filter is dependent on only the present or past inputs or on the present and past inputs as well as previous outputs, respectively [5]. Finite Impulse Response digital filter has strictly exact linear phase, relatively easy to design, highly stable, computationally intensive, less sensitive to finite wordlength effects, arbitrary amplitude frequency characteristic and realtime stable signal processing requirements etc. Thus, it is widely used in different digital signal processing applications [5-7].

FIR filter is described by differential equation. The output signal is a convolution of an input signal and the impulse response of the filter.

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \quad (1)$$

where *N-1* is the order of the filter, *N* is the length of filter (which is equal to the number of coefficients), $x(n)$ is the input signal, *h(n)* is the impulse response of FIR filter. The transfer function of a LTI (Linear time-invariant) FIR filter is obtained by taking the z transform of impulse response of FIR filter *h(n)*.

$$H(z) = \sum_{k=0}^{N-1} h(k)\, z^{-k} \quad (2)$$

There are many straightforward techniques for designing FIR digital filters to meet arbitrary frequency and phase response specifications, such as window design method or frequency sampling techniques [8]. The choice of the filters is based on four broad criterions. The filters should provide the following: as little distortion as possible to the signal; flat pass band and exhibit high attenuation characteristics with as low as possible stop band ripples [9] .

The difference equation for LTI IIR filter is defined as,

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k) \quad (3)$$

where $x(n)$ is the input signal, $y(n)$ is the output signal, $\{a_k, b_k\}$ are the coefficients of filter, *N* and *M* are equal to the numbers of coefficients of input and output signals, respectively

[5]. The transfer function of LTI IIR filter can be obtained by taking z transform of both sides of the difference equation,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} \tag{4}$$

where a_0 is assumed to be 1 and also the signs of a_k coefficients are assumed to change. Therefore IIR filters are commonly implemented using a feedback (recursive) structure, while FIR filters usually require no feedback (non-recursive).

This paper presents a new solution method for flexural wave propagation and demonstrates how an accurate wave shape can be estimated by using an appropriate filter. The received signal which is recorded by excitation on a thin quartz glass plate is modelled. In order to model the unknown combined effect of the excitation and the sensor, some filter types are investigated and applied.

2. THEORETICAL MODELING

In this work, the flexural wave propagation in a thin plate with excitation and transducer recording of transversal surface particle speed by a transducer is modelled theoretically. Also the transducer mechanical to electrical transition of an excited wave is modelled, that makes up a filter for the signal from the wave.

$$u[n, r;\ \theta] = h_{er}[n] * p_{SIR}[n, r; \theta] \tag{5}$$

$$h_{er}[n] = h_e[n] * h_r[n] \tag{6}$$

where n is time index (at sampling frequency $f_s=1/T_s$); r is the excitation position, distance from the sensor; θ is plate parameters; $u[n, r, \theta]$ is the measured signal; $h_e[n]$ is the excitation impulse response (source signal); $h_r[n]$ is the receiver electromechanical impulse response; $h_{er}[n]$ is collected filter; $p_{SIR}[n, r; \theta]$ is the spatial impulse response, i.e. propagation of the wave in the plane.

The plate thickness is small in comparison to the wavelength, and the plate is assumed to be infinitely large in the plane. To model the surface transversal speed, the derivative of the surface deflection [10] of an infinitely large plate is defined as

$$p_{SIR}(t, r, \theta) = \begin{Bmatrix} \frac{a}{4D\pi t} \sin\left(\frac{r^2}{4at}\right), & t > \frac{r}{\sqrt{2af_s}} \\ 0 & t \le \frac{r}{\sqrt{2af_s}} \end{Bmatrix} \tag{7}$$

where a and D are defined as given in Equation (8) [10],

$$a = \frac{D}{\rho h}, \qquad D = \frac{Eh^3}{12(1-v^2)} \tag{8}$$

In equations (7)-(8), r is defined as distance; t is time; D is stiffness of the plate; a is a plate parameter; v is Poisson's ratio; E is the modulus of elasticity (Pa) and ρ is the plate density (kg/m^3).

Instantaneous frequency bound is applied on the signal p in order to limit the frequency content. Thus, unphysical waves travelling faster than the current speed of sound, or frequencies of the measured signal larger than the Nyquist frequency haven't been taken into account. The instantaneous angular frequency is,

$$w(t) = \frac{d}{dt} \emptyset(t) \tag{9}$$

where $\phi(t)$ is as follows

$$\emptyset(t) = \frac{r^2\sqrt{\rho h}}{4t\sqrt{D}} \tag{10}$$

Frequency $f \in [0 \;\; f_s/2[$ is related to sampling frequency f_s (Hz), leads to the condition

$$\frac{2\pi f_s}{2} > w(t) \text{ and } t > \frac{r(\rho_0)^{1/4}}{2\sqrt{f_s}(D)^{1/4}} \tag{11}$$

The flexural wave surface speed is denoted with p_{SIR} and the transducer signal is the convolution as given in Equation (12),

$$u(t) = p_{SIR}(t) * h_{er}(t) \tag{12}$$

where $h_{er}(t)$ is the combined effect of the wave excitation and the transfer function of transducer. In frequency domain, the filter is estimated as,

$$\widetilde{H}_{er}(f) = \frac{\tilde{P}_{SIR}(f)}{U(f)} \tag{13}$$

The problem is that the received signal is a convolution of two unknown quantities. Since $p_{SIR}(t, r; \theta)$ is a solution of wave equation, $h_{er}(t)$ can be assumed to be subject to some bandwidth and phase constraints. It is possible to separate the two by implicitly obtaining an estimate of the wave shape.

3. ESTIMATION OF WAVE PROPAGATION

Butterworth filter, linear phase FIR filter designed by least squares and ARX model are applied for modeling the combined impulse response of the excitation and the sensor, $h_{er}(t)$ in order to estimate the most accurate wave shape with minimum error.

3.1. Butterwoth Filter

The Butterworth filter is one of the most popular analog filter design paradigms, first described in 1930 by Stephen Butterworth [11]. The basic philosophy of the conventional or integer order analog Butterworth filter is well practiced in various applications. It is designed to have a frequency response as flat as it is possible. The frequency response of these filters is monotonic and the sharpness of the roll-off from pass band to stop band is determined by the order of the filter. For conventional Butterworth filters the poles associated with the magnitude squared function are equally distributed in angle on a circle in the complex s-plane around the origin and having radius equal to the cut-off frequency (Ω_c). When the cut-off frequency and the filter order are specified, the poles can be obtained readily and from the pole position the transfer function of the filter can easily be obtained. Now, while designing a Butterworth filter we generally have four specifications, the pass band frequency (Ω_p), stop band frequency (Ω_s), maximum allowable pass band and stop band attenuation (α_p; α_s). Butterworth filters are well suited for many data analysis applications. The M-th order frequency response of an analog Butterworth filter can be expressed as,

$$|H(w)| = \frac{1}{\sqrt{1+w^{2M}}} \tag{14}$$

Design procedures can be found in [12].

3.2. Linear Phase Filter

The ability to have exact linear phase response is the one of the most important of FIR filters,

$$H(w) = |H(w)|e^{j\emptyset(w)} \qquad \text{where} \quad \emptyset(w) = -wn_0$$

A general FIR filter does not have a linear phase response but this property is satisfied when,

$$h(n) = \pm h(M-1-n),\; n = 0, 1, \dots, M-1$$

The design of linear phase filters constitutes an important class of problems in signal processing applications [13], due to the fact that such systems do not contribute to any group-delay distortion of the input signal. In this correspondence, author restricts her attention to the class of linear phase FIR filters of length *N*, with impulse response parameterized by the coefficient vector. It is well know that a sufficient condition for linear phase for real FIR filters is given by the even/odd symmetry condition of the coefficient vector.

There are several examples where linear phase filters arise naturally, often as a result of the solution to a design optimization problem. One such example is the design of narrow-band interference rejection filters for spread-spectrum systems which is known to have linear phase [14, 15]. Another recent instance arises in the design of L-filters (or linear combinations of order statistics of the input data) [16] which are well known to preserve signal edges while suppressing impulse-type noise.

In many practical (real-time) applications, such linear phase filters must be designed in a data-adaptive manner. Some of the earlier work in this area concentrated on the family of least squares (LS) algorithms, [17] that sought to utilize the symmetry property of the optimal filter to obtain fast, efficient implementations.

Linear-phase filter design by least squares has several advantages such as optimal with respect to square error criterion; simple, non-iterative method; allows the use of a frequency dependent weighting function [18]. Derivation algorithm for design of linear-phase FIR filters based on the square error criterion can be found in [17, 18].

3.3. ARX Model

The autoregressive with exogenous excitation (ARX) [19] is a parametric black-box time domain model that describes the system response at a time step *n* as a function of its response history, system output $\{y(0), y(1), ..., y(n-1)\}$, and of the excitation contents, system input $\{x(0),x(1), ..., x(n)\}$,

$$y(n) = \sum_{j=0}^{I} \alpha_j \, x(n-j) + \sum_{k=1}^{K} b_k \, y(n-k) \tag{15}$$

where $\{b_1, b_2, ..., b_K\}$ are the constants known as the autoregressive (AR) parameters and $\{a_0, a_1, ..., a_I\}$ are constants constituting the exogenous part of the model. Here, all parameters are supposed to be obtained by means of identification techniques. Once specified the model structure, its order (parsimony), the number of parameters (I+K+1) which, in the context of vibrating structures is connected to the number of degrees of freedom, still remains to be determined.

FIR filters can be interpreted as a particular ARX model in which linear system output at a time step *n* only depends on the input sequence excitation $\{x(0), x(1), ..., x(n)\}$. Thus, they are obtained setting $b_k = 0$ in Equation (16),

$$y(n) = \sum_{j=0}^{I} \alpha_j \, x(n-j) \tag{16}$$

and in the Z-domain Equation (16) casts as,

$$\frac{Y(z)}{X(z)} = H(z) = \sum_{j=0}^{I} a_j z^{-j} \tag{17}$$

According to Equation (17), FIR filters do not possess poles [19].

4. EXPERIMENTAL SETUP

A single low-cost passive sensor 7BB-20-6 with frequency specification 6.3±0.6kHz is mounted on a quartz glass plate whose dimensions are 0.002×0.5675×0.49 (m×m×m) in order to perform experimental studies. Excitations are done by means of a sharp metal object. The sensor is connected to a USB Ucontrol UCA202 Behringer device. A computer is used for acquisition of the data from the measurements and recording the wave propagation in the plate. For the used plate parameters, thickness *h* is 0.002 m and density *ρ* is $2.5x10^3$ kg/m^3. The signals are sampled with 48 kHz while they are recorded. Excitations are done on various points on the plate and the *r* distances from the sensor to the excitation point are varied between 0.2 m and 1 m. The experimental setup is shown in Figure 1.

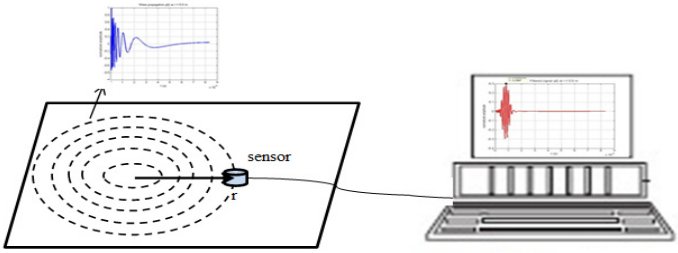

Figure 1. Measurement setup. Excitation in the center of the dashed circles where r is the distance from the excitation to the sensor

5. RESULTS

The signals obtained from the setup composed of a quartz glass plate with 2 mm thickness have been evaluated in simulations. The exact value ranges of used quartz glass plate in the measurements are as, E= [$50x10^9$ - $90x10^9$] (Pa) for elasticity modulus and v= [0.16 - 0.27] for Poisson's ratio [20]. These give the limits for the plate parameter as D = [34.2091 - 64.7179] according to the Equation (8) and it is chosen as 49 Nm in the simulations. The principle is then verified with experiments on a 2 mm thick quartz glass plate with known density by exciting with a sharp material at different distances from the sensor. Then the recorded signals are sent to the computer and processed with MATLAB. Figure 2 shows the surface particle speeds obtained from two different excitation points.

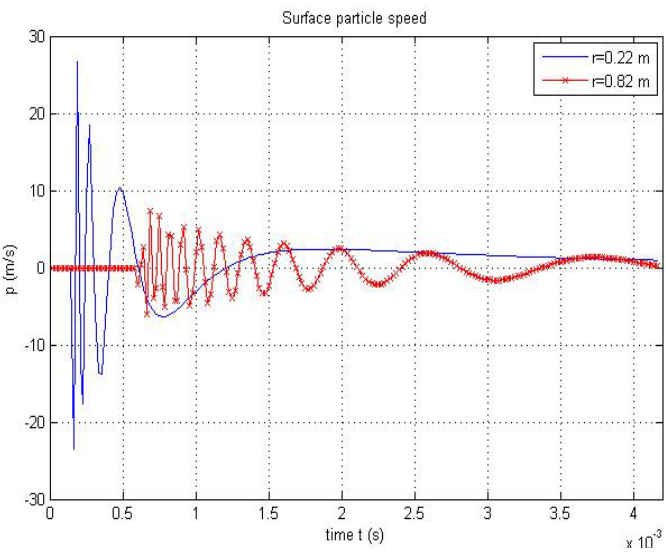

Figure 2. Surface particle speeds at r=0.22 (m) and r = 0.82 (m)

As shown in Figure 2, when the excitation distance from the sensor is reduced, surface particle speed gets much less delayed and oscillated.

In order to model the combined impulse response of the excitation and the sensor a 8th order Butterworth filter, 10th order linear phase FIR filter by LS and ARX model with a_j coefficients are designed. The values of a_j coefficients in ARX model are designed as {-0.0487, -0.0014, -0.0453, -0.2748, 0.0761, 0.5068, 0.0761, -0.2748, -0.0453, -0.0014, -0.0487, 0, 0, 0, 0} from a_1 to a_{15}. The magnitude and phase responses of the designed Butterworth and Firls filters are shown in Figure 3 and 4, respectively.

As shown in Figure 3, designed Butterworth filter doesn't have a linear phase response in the frequency range. On the other hand in Figure 4, FIR filter designed by LS has linear phase response in the frequency ranges between about 3 kHz and 20 kHz.

Figure 5 shows a measured pulse and the final modelled pulses with Butterworth filter, linear phase FIR filter by LS and ARX model after the joint estimation of $h_{er}(t)$ has completed. The pulse was excited by tapping the plate with a sharp object 0.82 cm away from the sensor. The signals received from the transducer is recorded during about 1 ms. Since some parts of the measured amplitude is very small or close to zero, the filters are band limited around the transducer center frequency. Thus, according to equation (12), dividing by small numbers is avoided.

As shown in Figure 5, signal obtained from the transducer can be estimated with FIR filter by LS and ARX model, exactly. Nevertheless designed Butterworth filter fails to estimate, properly. The reason for success of Firls filter is linear phase response in the desired frequency ranges. Also, since FIR filter can be interpreted as a particular ARX model, the designed ARX model is able to estimate the observed signal properly, too. Because, flexural waves are highly dispersive waves and therefore in order to obtain an accurate estimation, all dispersions have to be attributed to the wave. According to the filter magnitude and phase responses given in Figure 3 and 4, linear phase is necessary for this.

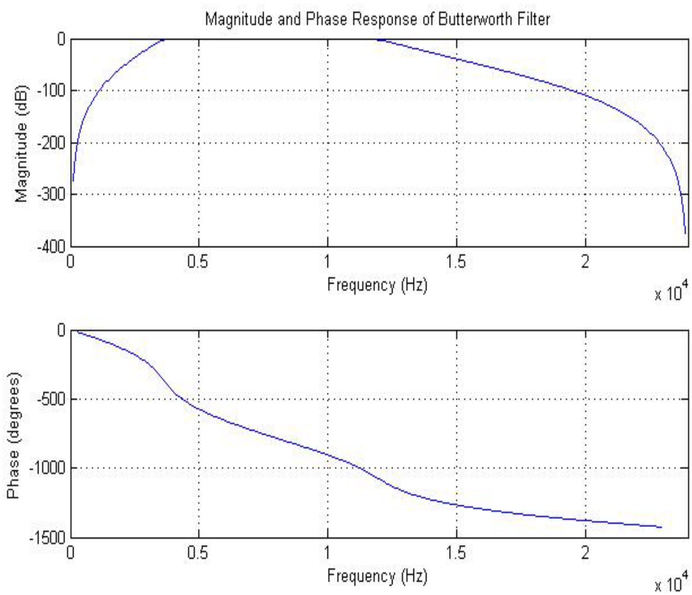

Figure 3. Magnitude and phase response of designed 8th order Butterworth filter

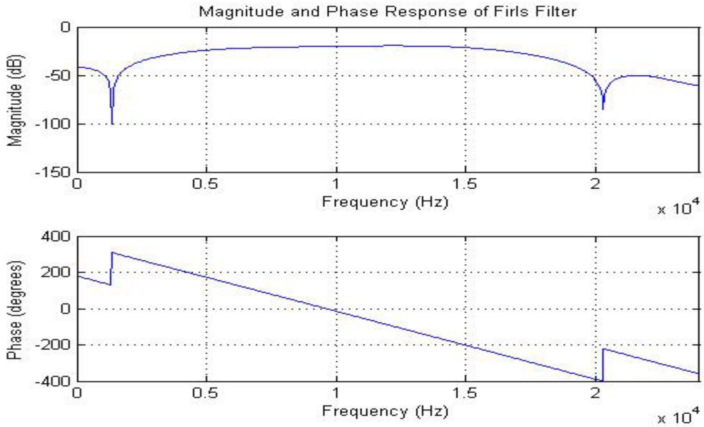

Figure 4. Magnitude and phase response of designed 10th order Firls filter

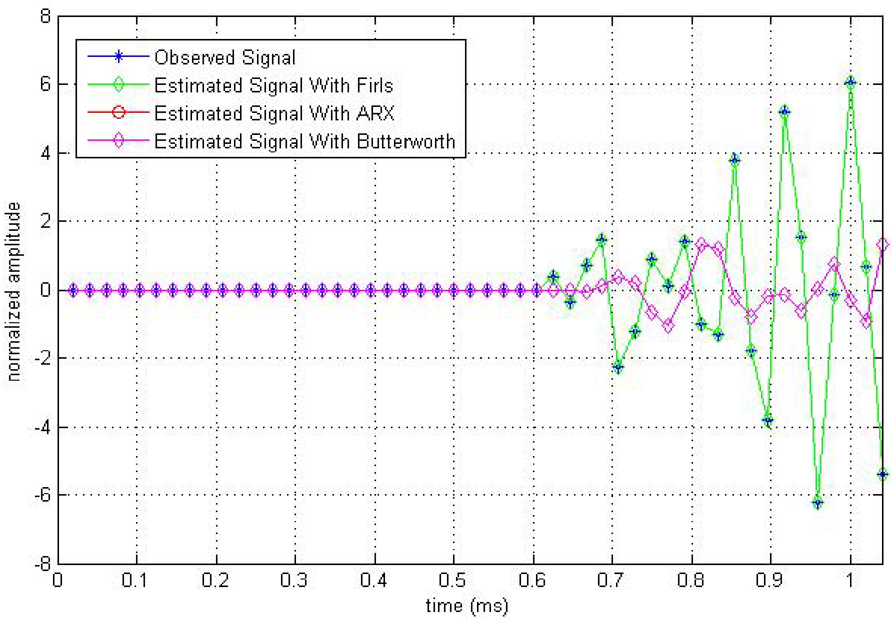

Figure 5. Observed and estimated signals with designed filters and ARX model

6. CONCLUSISONS

In this paper, the performances of 2 different filter types and ARX model have been investigated in order to model propagation of a flexural wave in thin plates. A dispersive wave mode has been recorded for known distances between excitation and sensor and the shape of the received wave has been obtained. Since the recorded signal is affected by both the excitation and the sensor, the combined effect of the excitation and the sensor has been modelled via designing FIR filter by LS, Butterworth filter and ARX model. Results indicate that propagation of flexural waves in thin plates can be modelled by means of estimating the combined effect of the excitation and the sensor instead of the methods given in [1, 2, 4]. It also demonstrates that, since wave components with different frequencies travel at different speeds, a linear phase filter attributes all dispersion into the wave propagation. Therefore, the most accurate estimation of observed signal has been obtained when linear phase FIR filter by LS and ARX model are used in order to model the joint estimation of sensor and the excitation.

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