The Role Of Representation Types In Pattern Activities

Örüntü Etkinliklerinde Temsil Türlerinin Rolü

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Abstract

In that study the type of patterns, the role representation types in mathematics, especially in pattern activities were discussed and some activity examples are given for students which have potential to develop their algebraic thinking skill requiring translation between representations such as symbolic to figural.

Keywords: *pattern, pattern types, represantation types in pattern*

Özet

Bu çalışmada, örüntü türleri, matematikte özellikle örüntü etkinliklerinde temsillerin rolü tartışılmış ve öğrencilerin cebirsel düşünmelerinin gelişimine potansiyel katkısı olan temsiller arası geçiş gerektiren (örneğin sembolik temsilden şekil temsiline) etkinlik örneklerine yer verilmiştir.

Anahtar kelimeler: *örüntü, örüntü türleri, örüntüde temsil tüleri*

1. Introduction

Pattern is a crucial topic of mathematics education. Patterns are found in arithmetic and geometric sequences problems, as well as in various other real situations. Souviney (1994) indicated that patterns can be based on geometric attributes (shapes, regions, and angle), measurement attributes (color, texture, length, weight, volume, number), relational attributes (proportion, sequence, functions), and affective attributes (values, likes, dislikes, familiarity, heritage, culture). There are many explanations of patterns in the literature. McGarvey (2012) defined a pattern as an act of perceiving or imposing structural regularities on physical, behavioural, visual, or symbolic phenomena. Souviney (1994), for example, defined a pattern as a systematic configuration of geometric figures, sounds, symbols, and actions. Patterns can also be seen in spoken and written words, musical forms and video images, ornamental designs, natural geometry, traffic, and the objects that we create (Reys et al., 1998). Radford (2006) indicated a semiotic perspective for algebraic thinking and the generalization of patterns. He asserted that a semiotic system provides us with specific ways to signify or to say certain things, while another semiotic system provides us with other ways of signification. The system includes spoken words, gestures, drawings, formulas, etc.

Previous studies have shown many different kinds of pattern types refering to the type of representation system within which the terms in a sequence are expressed as numerical or figural/geometric forms an labelled number patterns, pictorial/geometric patterns, patterns in computational procedures, linear and quadratic patterns, repeating patterns, etc. Stacey (1989) classified patterns as linear and quadratic patterns according to their *n*th terms expressed as *an+b* and *an²+bn+c*, respectively, while Orton&Orton (1999) indicated that for number patterns if constant differences first appear in the first difference row, then the rule is linear an+b, if it is the second differences where constants appear the rule is quadratic *an²+bn+c*. For example 1,4,7,10,13 linear sequences produce constant first differences, and can be built up using that property, the difference is constant and 1,4,9,16, quadratic sequence are more complicated and differencing does not produce constants until the second differences (Orton & Orton, 1999). Smith (1997) indicated that patterns can be numerical (involving numbers) or non-numerical (involving shapes, sounds, or other attributes such as colour and position). Numerical or figural/geometric patterns refer to the type of representation system within which the terms in a sequence are expressed.

Patterns are the heart and soul of mathematics (Zazkis & Liljedahl, 2002). Pattern activities allow students to learn and engage in algebraic activity (Lee & Freiman, 2006) and make generalisations (Threlfall, 1999; Cathcart et al., 2003; Rivera & Becker, 2007). Van De Walle (2004) indicated that identifying and extending patterns is an important process in algebraic thinking and patterns are tool for students' development of algebraic thinking (Zazkis & Liljedahl, 2002; Lin et al., 2004; Van De Walle, 2004; Nathan & Kim, 2007; Steele, 2008; Rivera, 2013). Moreover, it is

emphasised that reasoning on and proving number patterns are important for developing algebraic thinking (Lin et al., 2004). Patterns can also contribute to the development of functional thinking (Souviney, 1994; Van De Walle, 2004; Warren & Cooper, 2006), in terms of seeing relationships and making connections (Cathcart et al., 2003), problem solving (Reys et al., 1998; Cathcart et al., 2003; Bassarear, 2008), counting (Frobisher & Threlfall, 1999; Bassarear, 2008), and using number systems (Frobisher & Threlfall, 1999) and arithmetical operations (Frobisher & Threlfall, 1999; Bassarear, 2008). Cathcart et al. (2003) asserted that through the study of patterns, children can learn to see relationships and make connections, generalizations, and predictions about the world around them. Fox (2005) asserted that studying patterns is closely connected to mathematical content areas such as numbers, geometry, measurement, and data.

In some studies, patterns are classified as repeating or growing (Reys et al., 1998; Cathcart et al., 2003; Van De Walle, 2004; Warren & Cooper, 2006). *Repeating patterns* have a recognisable repeating cycle of elements, referred to as the 'unit of repeat' (Zazkis & Liljedahl, 2002). This kind of pattern can have one attribute such as the colour, size, shape, or orientation of objects (Threlfall, 1999). The following are examples of repeating patterns: alphabetic letters such as A-B-A-B-A-B, geometric shapes such as ▼●▼●▼●, and actions such as stand, sit, stand, sit, stand, sit, stand, sit (Warren & Cooper, 2006). *Growing patterns* change over time (Cathcart et al., 2003) and involve a progression from step to step. Moreover those patterns are called sequences (Van De Walle, 2004). Growing patterns may be linear such as Y B B Y B B B B Y B B B B B B (in this example, only B's are growing) (Reys et al., 1998) and 2,4,6,... (Cathcart et al., 2003) or quadratic such as 1,4,9,16,...For example, Zazkis and Liljedahl (2002) classified patterns into numerical patterns, pictorial/geometric patterns, patterns in computational procedures, linear and quadratic patterns, and repeating patterns. Hence, repeating and growing (linear and non-linear) patterns are systematic configurations that include any types of representations such as shapes, symbols, and real-world objects according to their cognitive structures or schemas.

In Turkey, primary teachers follow the standard national mathematics curriculum set by the Ministry of National Education. Within this curriculum, there are learning areas, sub-learning areas, objectives, samples of activities, and explanations of activities. The Turkish primary school mathematics curriculum has four learning areas: numbers, data, geometry, and measurement. Patterns have been incorporated into the numbers and geometry learning areas since 2005. In these learning areas, students are expected to recognise, identify, describe, and extend repeating and growing patterns and find algebraic rules/formulas (MEB, 2009).

Representations in pattern activities

The role of representations in the teaching and learning of mathematics has evolved considerably in the past two decades. Representations are typically a sign or a

configuration of signs, characters, or objects. They stand for (symbolise, depict, enco-de, or represent) something other than themselves (Goldin & Shteingold, 2001). There are many different representation forms including diagrams, graphs, tables, sketches, equations, and words (Bassarear, 2008). In the well-known classification of Lesh, Post, and Behr (1987), representations are pictures, manipulative models, oral langu-age, written symbols, and real-world situations.
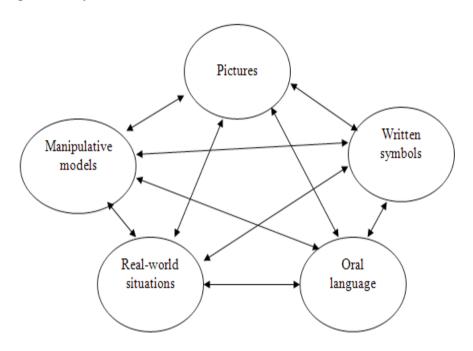

Figure 1. Representation types

Pictures, or diagrams-static figural models, can be internalised as images (Lesh et al., 1987), while manipulative models are objects that students can touch, move, and stack. According to Ainsworth (2006), learners should understand the form of representation and the representation–domain relationship as well as how to select and construct an appropriate representation. Representations improve students' un-derstanding of mathematical relations and develop their problem-solving ability (Cai & Lester, 2005; Fennell & Rowan, 2001). Furthermore, as shown in the classification in Figure 1, representations are interrelated. Translation refers to the psychological process of going from one form of representation to another (e.g. going from an equ-ation to a graph or vice versa; Janvier & Bednarz, 1987). Translations among repre-sentations assist mathematical learning and problem solving (Lesh et al., 1987) as well as help students understand mathematical concepts and relations more in depth. NCTM's (2000) standards and principles emphasised that selecting, applying, and

translating mathematical representations are very important. Steele (2008) indicated that "recognizing and understanding the relationship between different external representations of algebraic situations helps students identify and generalise patterns that lead toward understanding variables and functions" (p. 98).

As mentioned earlier, representation types are very important in pattern activities. Patterns can be represented in many ways (Van De Walle, 2004) such as

- *physical materials* (e.g. beads, coloured buttons, cubes),

- *oral* (e.g. the "do, mi, mi, do" pattern),

- *numbers or symbolic rules* (e.g. 2, 6, 12, 20 or $y=x^2+3x$), and

- *figures* (geometric or not).

Bassarear (2008) indicated that connecting geometric and numerical representations such as 1, 10, 100, and 1000 by using base ten blocks is very important for understanding number systems. Indeed, forming connections among representations is a central goal of algebra instruction (Brenner et al., 1997). Furthermore, representation is very important in pattern activities such as generalisation (Nathan & Kim, 2007; Rivera & Becker, 2008; Steele, 2008). Nathan and Kim (2007) indicated that different ways of presenting patterns affected the pattern generalization with graphs and words performance of students. Rivera & Becker (2008) found that generalizations involving linear figural patterns changed students' representational skills and fluency, that is, from being verbal (situated) to symbolic (formal). In that current study representation types in pattern activity was considered such as *pictures (figures), manipulative models (physical materials), oral language, written symbols (numbers or symbolic rules) and real-world situations* as indicated in the studies of Van De Walle (2004) and (Lesh, Post & Behr, 1987).

2. Discussion And Conclusion

Pattern is one of the very important topics in mathematics education. Patterns have many contributions for students like making generalisations (Threlfall, 1999; Cathcart et al., 2003; Rivera & Becker, 2007), developing of algebraic thinking (Zazkis & Liljedahl, 2002; Lin et al., 2004; Van De Walle, 2004; Nathan & Kim, 2007; Steele, 2008; Rivera, 2013), functional thinking (Souviney, 1994; Van De Walle, 2004; Warren & Cooper, 2006), in terms of seeing relationships and making connections (Cathcart et al., 2003), problem solving (Reys et al., 1998; Cathcart et al., 2003; Bassarear, 2008), counting (Frobisher & Threlfall, 1999; Bassarear, 2008). During those pattern activities students need to use many representation types such as physical materials, oral, numbers or symbolic rules and, figures.

As seen from the literature representation types are very important for pattern activities (Van De Walle, 2004; Nathan & Kim, 2007; Rivera & Becker, 2008; Steele,

2008). Considering using different type of representations and translation among representations is very important for algebraic thinking, it is necessary to apply that kind of activities in mathematics education. During the patterning activities like being repeating pattern such as 1,2,3,1,2,3,1,2,3,... can be given and asked to create different repeating pattern representing the same rule but using different representations such as using figures- geometric shapes like ○△▽ ○△▽ ○ △ ▽ or etc. Moreover, pattern translation ability of students can be assessed using different pattern representation types. Hence, translation performance of students between representation types (e.g. from symbolic to figural representation) can be examined. For example, a growing number pattern like 2,4,6,8,10 can be presented to the students and asked them to produce growing figural pattern based on those numbers or a similar example for that activity like a 2,6,12,20,.. number pattern can be given and asked them to create a growing figural pattern considering that number pattern. Furthermore, $4n+1$ linear and $n.(n+1)$ non-linear (quadratic) algebraic formulas can be presented to students and asked them to produce figural patterns based on those formulas. As seen from the suggestions those activities have potential to improve students' algebraic thinking, to put forward their mathematical content knowledge and also to reveal creativity level of the students. Consequently, the importance of translation between representations in pattern activity should be emphasized in the mathematics curriculums and more examples apply in the classrooms.

3. References

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