Sound Radiation from a Floating Runway due to an Airplane Taking off Affected by Mean Flow

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Abstract
Sound radiation caused by taking off of an airplane from a floating runway is an unexplored area which has a serious but unstudied impact on marine life. For such a study, conventional means of using a three dimensional runway with time varying loading during takeoff is extremely difficult and time consuming. The analysis is made simpler by assuming the VLFS to be a simple, infinitely long beam supported by buoyancy. Sound radiation using Timoshenko-Mindlin beam model subjected to varying take off speeds and presence of mean flow is investigated.

Keywords:
Moving Load; Timoshenko-Mindlin beam; Floating runway; Sound radiation; Mean flow.

Nomenclature

ξ	Wave number variable
ζ	Non-dimensional wave number variable
$\gamma(=\dfrac{K_0}{K_B})$	Wave number ratio
ν	Poisson's ratio
ω	Angular frequency
$j\,(=\sqrt{(-1)}\,)$	Imaginary number
ρ_v	Mass density of the beam
ρ_0	Mass density of the acoustic medium
$\kappa^2\,(=\dfrac{\pi^2}{12})$	Cross sectional shape factor or the shear correction factor
$\alpha_0\,(=\dfrac{\rho_0 C_L}{\sqrt{12}\,\rho_v C_0})$	Fluid loading parameter
$\delta(x-Vt)$	Delta function
Π	Total acoustic power
h	Height of the beam
t	Time variable
x	Space variable in x direction
$p(x,y,t)_{y=0}$	Acoustic pressure acting on the surface of beam
$u(x,t)$	Transverse displacement of the beam
f_0	Strength of external force per unit width
$C_L\,(=\sqrt{\dfrac{E}{\rho_v}})$	Longitudinal wave speed

C_0	Speed of sound in acoustic medium
$D = \frac{Eh^3}{12(1-\nu^2)}$	Flexural rigidity of plate
E	Elastic modulus
$H(x)$	Heavyside step function
$\bar{I}$	The time averaged sound intensity
$I(=\frac{h^3}{12})$	Cross sectional moment of inertia per unit width
$K_0(=\omega/C_0)$	The acoustic wave number
$\bar{M}(=[V-U]/C_0)$	The modified Mach number
P	The sound pressure on the beam surface
$\dot{U}_s^*$	The beam surface velocity of conjugation
$\dot{U}_s = \frac{dU_s(\xi)}{dt}$	The beam surface velocity
U	The speed of mean flow of the fluid
W	Power per unit width

1. Introduction

Because of their relatively simple construction and ease of maintenance, pontoon-type very large floating structures (VLFS) are considered to be one of the most promising designs for a floating airport or runway, particularly in sheltered areas. The typical dimensions are 5 km long, 1 km wide, and only a few meters deep. Due to their dimensions, even when no incoming waves exist, the structure responds flexurally to moving loads like those from an airplane during landing or take-off. Hence study of transient responses of a VLFS to impulsive and moving loads is a must. Only a few studies of transient problems for VLFS have been reported to date, however these are limited to flexural deflections and sound generated by moving loads on such structures has not been addressed to date even though sound radiation from floating platforms has a serious and unstudied impact on marine life. However to study acoustic effects, a dynamic analysis of a three-dimensional runway with time varying loading during take-off is exceeding difficult. This analysis can be made simpler by assuming that the runway behaves as a simple, infinitely long beam supported by buoyancy. The model is assumed to be a simple beam, described by a Timoshenko-Mindlin beam equation.

Sound radiation due to moving loads on beams has been investigated earlier (Keltie and Peng, 1988) wherein results show that for beams under light fluid loading, the coincidence sound radiation peak for a stationary force is split into two coincidence peaks due to the effects of the Doppler shift, while for beams under heavy fluid loading there are no pronounced sound radiation peaks. Subsequently vibration response of periodically simply supported beam on the whole structure in wavenumber domain through Fourier transform was analyzed (Cheng and Chui, 1999). The result was an advance on traditional substructure methods. For an air-loaded beam subjected to a stationary line force, they showed that the radiated sound power exhibited peaks at certain wavenumber ratios. The wavenumber ratios at which radiation peaks occur nearly coincide with the lower bounding wavenumber ratios of the odd number of propagation zones. Cheng's formulation did not include the presence of numerous wavenumber components induced from the elastic supports and is subject to the restriction that the external force is located on one of the elastic supports. To discuss vibro-acoustic response of a fluid-loaded beam on periodic elastic supports subjected to a moving load "wavenumber harmonic series" was introduced (Cheng et al., 2000; 2001). Results show that the response of a beam on an elastic foundation can be approximated using a periodically, elastically supported beam when the support spacing is small compared with the flexural wavelength. For such beams when the force is stationary a single radiation peak occurs which splits into two peaks due to Doppler shift when the force becomes traveling. The authors undertook a number of studies to analyse the effect of moving loads on sound radiation from floating airports. These included effect on sound radiation by varying structural material, effect of damping factor on varying beam types and affect of inplane loading on sound radiation (Agarwala and Nair, 2012; 2013a; 2013b).

Acoustic analysis in the presence of a *mean flow* or *current* complicates the problem further by modifying the effect of the moving load. The available literature on study of acoustics in mean flow, however, is much smaller. The main reason for this is that the mean flow speed is often too low to have any real impact on acoustics that are of practical engineering relevance. Another reason is that the fluid-structure interaction problem usually becomes too complicated to be solved analytically when mean flow is considered. Consequently problems are often treated in the same way as those in a stationary fluid medium. The effect of mean flow on the response of a fluid-loaded structure thus remains mostly unexplored.

For short, sturdy beams the shear effect cannot be neglected as in conventional analysis using the Bernoulli-Euler's beam theory. The situation occurs when the cross section of the beam is relatively large in comparison with the beam span. Although the correction for the shear effect may yield results only a few percent more accurate in frequency prediction than those from classical beam theory for a moderately thick beam, the accuracy improvement may be quite profound when performing dynamic response analysis. It is with this reasoning that a Timoshenko-Mindlin plate is utilized for the present study.

To the best of the knowledge of the authors, no study of acoustic radiation from VLFS subjected to either a moving load or mean flow has been reported in the literature other than efforts by the authors (Agarwala and Nair, 2012; 2013a; 2013b). The aim of this study is to develop an expression for calculating sound radiation from floating structures subjected to mean flow and moving loads such as airplanes. In developing the expression, Fourier transform methodology for a Timoshenko-Mindlin plate is utilized (Keltie and Peng, 1988). Structural damping is ignored while effect of mean flow is included.

2. Formulation

Floating airports which are nearly 5000 m long can be considered to be infinitely long. Accordingly we can assume them to behave as a simple, infinitely long beam in contact with water surface. Structural damping is ignored for the floating airport as since there is no apparent resonant mechanism in this problem. Water is assumed to be inviscid, and the flow resulting from the airplane take-off is assumed to be irrotational. The x-axis is aligned with the length of the runway and the y-axis is directed vertically upwards, as seen in Figure 1. Because the floating runway is very narrow compared with its length, as a simplification, we will assume that the deformation and loading assumed not to vary across the runway. The structure is assumed to behave like a beam, described by the Timoshenko-Mindlin beam equation. An excitation force of length 2L moving at a subsonic speed V is assumed to be acting on the runway. The space $y>0$ is filled with an acoustic medium such as water. The other side of the plate is assumed to be vacuum. A subsonic mean flow of speed U, moving in the positive x direction is considered to be present in the water.

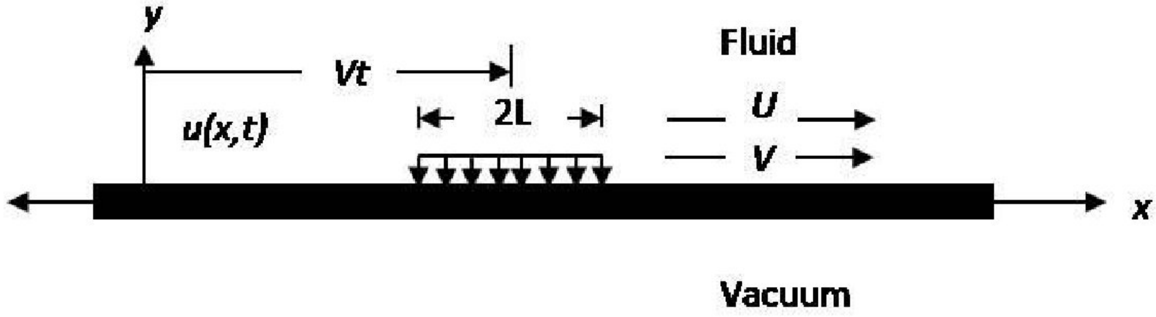


Fig. 1: Schematic representation of the problem geometry

We consider a uniformly distributed moving line force, given by

$$f(x,t) = \frac{f_0}{2L}[H(x\text{-}Vt+L)\text{-}H(x\text{-}Vt\text{-}L)]e^{j\omega t}$$

The vibration equation for an elastic plate, including rotational inertia and transverse shear effects, is given by the Timoshenko-Mindlin plate equation as

$$\left(\nabla^2 - \frac{\rho_v}{\kappa^2 G}\frac{\partial^2}{\partial t^2}\right)\left(D\nabla^2 - \frac{\rho_v h^3}{12}\frac{\partial^2}{\partial t^2}\right)u + \rho_v h\frac{\partial^2 u}{\partial t^2} = \left(1 - \frac{D}{\kappa^2 Gh}\nabla^2 + \frac{\rho_v h^2}{12\kappa^2 G}\frac{\partial^2}{\partial t^2}\right)[f(x,t) - p(x,y,t)_{y=0}] \quad (1)$$

Since a beam is considered as a one dimensional plate, $\nabla = \frac{\partial}{\partial x}$ making this substitution in Eq. (1) gives the Timoshenko-Mindlin beam equation as

$$D\frac{\partial^4 u(x,t)}{\partial x^4} + \rho_v h\frac{\partial^2 u(x,t)}{\partial t^2} - \rho_v I\left(1 + \frac{D\rho_v}{\kappa^2 G}\right)\frac{\partial^4 u(x,t)}{\partial x^2 \partial t^2} + \rho_v I\frac{\rho_v}{\kappa^2 G}\frac{\partial^4 u(x,t)}{\partial t^4}$$
$$= \left(1 - \frac{D}{\kappa^2 Gh}\frac{\partial^2}{\partial x^2} + \frac{\rho_v h^2}{12\kappa^2 G}\frac{\partial^2}{\partial t^2}\right)[f(x,t) - p(x,y,t)_{y=0}] \quad (2)$$

To account for the presence of current the operator, $\frac{\partial}{\partial t}$ is replaced by the operator $\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}$ in the expressions of pressure distribution and the boundary condition at $y = 0$. The pressure distribution induced by the vibrating beam in the acoustic medium thus satisfies the wave equation in two-dimensional space, given by

$$\left[\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} - \frac{1}{C_0^2}\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2\right]p(x,y,t) = 0 \quad (3)$$

Hence the boundary condition at $y = 0$ is modified as

$$\rho_0\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)^2 u = -\frac{\partial p}{\partial y}\Big|_{y=0} \quad (4)$$

By applying the spatial Fourier transformation $FT() = \int_{-\infty}^{\infty}()e^{i\xi x}dx$, the force function for a harmonic line force in wave number domain may be written as

$$\tilde{F}(\xi,t) = f_0 \frac{sin(\xi L)}{\xi L} e^{j(\omega+\xi V)t} \tag{5a}$$

the transformed displacement as

$$\tilde{U}_s(\xi,t) = U_s(\xi) e^{j(\omega+\xi V)t} \tag{5b}$$

and the transformed pressure as

$$\tilde{P}(\xi,y,t) = P(\xi,y) e^{j(\omega+\xi V)t} \tag{5c}$$

Upon substitution of Eq. (5a), (5b) and (5c) in the Eq. (2) and a combined Eq. (3) and Eq. (4), we get

$$U_s(\xi) = \frac{Z_F F(\xi)}{Z_m + Z_F Z_a} \tag{6}$$

and

$$P(\xi, y=0) = \frac{j\rho_0(\omega+\xi[V-U])^2}{K_y} U_s(\xi) \tag{7}$$

where the acoustic impedance operator (Z_a) is given by

$$Z_a = \frac{j\rho_0(\omega+\xi[V-U])^2}{K_y} \tag{8}$$

the beam impedance operator (Z_m) as

$$Z_m = D\xi^4 - \rho_v h(\omega+\xi[V-U])^2 - \xi^2 \rho_v I\left(1+\frac{D\rho_v}{\kappa^2 G}\right)(\omega+\xi[V-U])^2 + \rho_v I \frac{\rho_v}{\kappa^2 G}(\omega+\xi[V-U])^4 \tag{9}$$

the Z_F by

$$Z_F = 1 + \frac{D}{\kappa^2 G h}\xi^2 - \frac{\rho_v h^2}{12\kappa^2 G}(\omega+\xi[V-U])^2 \tag{10}$$

and K_y is given by

$$K_y = \begin{cases} -j\sqrt{\xi^2 - (K_0+\bar{M}\xi)^2} & for \quad \xi^2 > (K_0+\bar{M}\xi)^2 \\ \sqrt{(K_0+\bar{M}\xi)^2 - \xi^2} & for \quad \xi^2 < (K_0+\bar{M}\xi)^2 \end{cases} \tag{11}$$

where $\bar{M}$ *(= [V-U]/C₀)* is the Mach number and K_0*(= ω/C₀)* the acoustic wave number.

2.1 Total Acoustic Power

The time averaged sound intensity is given by (Morse and Ingrad, 1986) as

$$\bar{I} = \frac{1}{T}\int_0^T \overline{PV}dt \quad or \quad \bar{I} = \frac{1}{2} Re[P\dot{U}_s^*]$$

In order to find the total acoustic power (Π), the surface acoustic intensity distribution may be integrated over the infinite length of the beam as

$$\Pi = \int_{-\infty}^{\infty} \frac{1}{2} Re[P(x,y=0,t)\dot{U}_s^*(x,t)]dx$$

Upon substituting the sound pressure Eq. (7) and the surface velocity Eq. (6) of the beam in the total acoustic power and simplifying, the sound power radiated per unit width of the beam is given as

$$\Pi = \frac{\rho_0}{4\pi} Re\Big[\int_{-\infty}^{\infty} \frac{(\omega + \xi[V-U])^3}{K_y} |U_s(\xi)|^2 \, d\xi\Big] \quad (12)$$

Limiting the study to subsonic motion of the moving load, the limits within which K_y is real is given by

$$\xi_1 = \frac{-K_0}{1+\bar{M}} \le \xi \le \xi_2 = \frac{K_0}{1-\bar{M}}$$

This allows us to rewrite the expression for the total sound power as

$$\Pi = \frac{\rho_0}{4\pi} Re[\int_{\xi_1}^{\xi_2} \frac{(\omega + \xi[V-U])^3}{K_y} |U_s(\xi)|^2 \, d\xi] \quad (13)$$

Eq. (13) is the required expression for calculating the total sound power from a Timoshenko-Mindlin beam subjected to a moving load in the presence of a mean flow in the fluid.

2.2 Non-Dimensionalising

To be able to make the analysis of Eq. (13) simpler, the total sound power is expressed as a function of the wave number ratio, which is made dimensionless. Thus using the concept of non-dimensional parameters defined in [1] we get

$$\text{Wavenumber variable } (\zeta) = \frac{\text{Wavenumber variable } (\xi)}{\text{Acoustic wavenumber } (K_0)} \quad (14a)$$

$$K_B = \Big[\frac{\rho_v h \omega^2}{D}\Big]^{\frac{1}{4}} \quad (14b)$$

$$\gamma = \frac{K_0}{K_B} \quad (14c)$$

$$C_L = \sqrt{\frac{E}{\rho_v}} \quad (14d)$$

$$\alpha_0 = \frac{\rho_0 C_L}{\sqrt{12}\ \rho_v C_0} \quad (14e)$$

$$W = \frac{4\pi\omega(\rho_v h)^2}{\rho_0 f_0^2} \Pi \quad (14f)$$

Substituting Eq. (14) in Eq. (12) gives the dimensionless radiated sound power per unit width as

$$W = \int_{\zeta_1}^{\zeta_2} \alpha^3 \beta \left| Z_F \frac{sin(\zeta K_0 L)}{\zeta K_0 L} \right|^2 |D_w|^{-2} \, d\zeta \quad (15)$$

$$\text{where } \zeta_1 = \frac{-1}{1+\bar{M}} \le \zeta \le \zeta_2 = \frac{1}{1-\bar{M}}$$

$$\alpha = 1 + \bar{M}\zeta$$

$$\beta = \sqrt{\alpha^2 - \zeta^2}$$

$$D_w = \beta(D_1 - D_2 + D_3) + jD_4$$
$$Z_F = 1 + \frac{2(1+\nu)\gamma^4}{\kappa^2}\left(\frac{C_0}{C_L}\right)^2\left[\zeta^2 - \left(\frac{C_0}{C_L}\right)^2\alpha^2(1-\nu^2)\right]$$
$$D_1 = \gamma^4\zeta^4$$
$$D_2 = \alpha^2\left[1 + [1 + \frac{2(1+\nu)}{\kappa^2(1-\nu^2)}]\gamma^4\zeta^2\left(\frac{C_0}{C_L}\right)^2(1-\nu^2)\right]$$
$$D_3 = \frac{2(1+\nu)}{\kappa^2}\alpha^4\gamma^4\left(\frac{C_0}{C_L}\right)^4(1-\nu^2)$$
$$D_4 = Z_F\frac{\alpha_0\alpha^2}{\gamma^2}$$

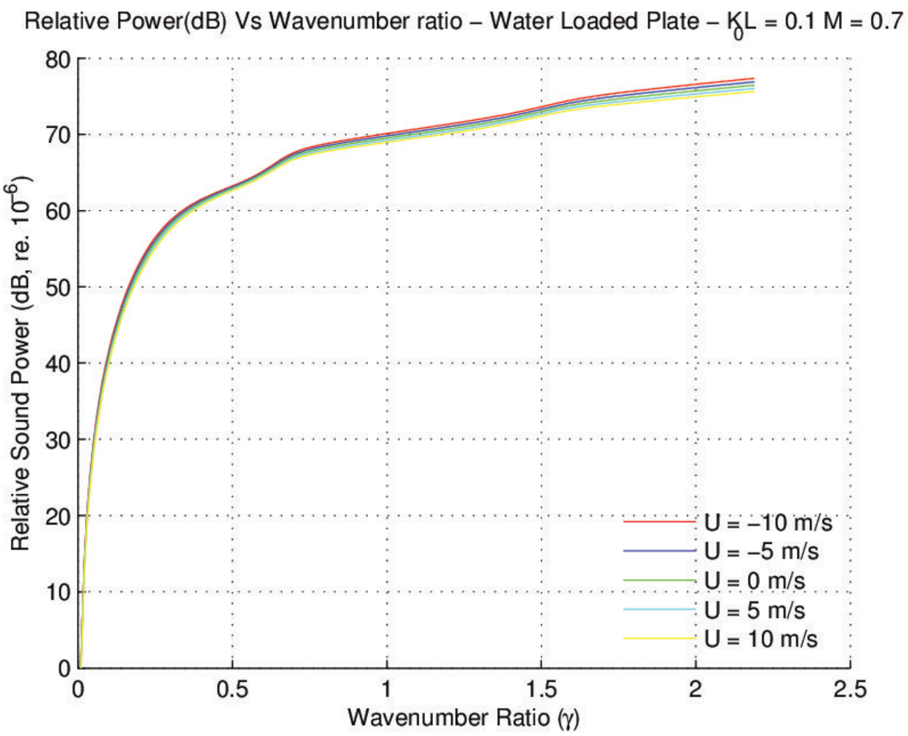
3. Analysis and Discussion

We investigate the effect of mean flow of the fluid on the total radiated sound power. In order to undertake the required investigation, Eq. (15) needs to be numerically evaluated for the case of a beam floating on water. Properties of the steel beam model analysed are $E = 20\times10^{10}\ N/m^2$, $\rho_v = 7800\,kg/m^3$ (i.e $D = 560\,KNm$), $h = 2.54\times10^{-2}\,m$, $\nu = 0.3$, $\kappa^2 = 0.85$, $C_0 = 1481\,m/s$ and $\rho_0 = 1000\,kg/m$. The external force (f_0) is assumed to be of unit magnitude. By varying the values of parameters M and K_0L, the sound power is computed and then plotted versus the wave number ratio (γ) or non-dimensional frequency. With the maximum surface current of Gulf Stream at *2.5 m/s* and accounting for discharges into the lagoon, the mean flow velocity is taken as varying between *-10 m/s* to *10 m/s*.

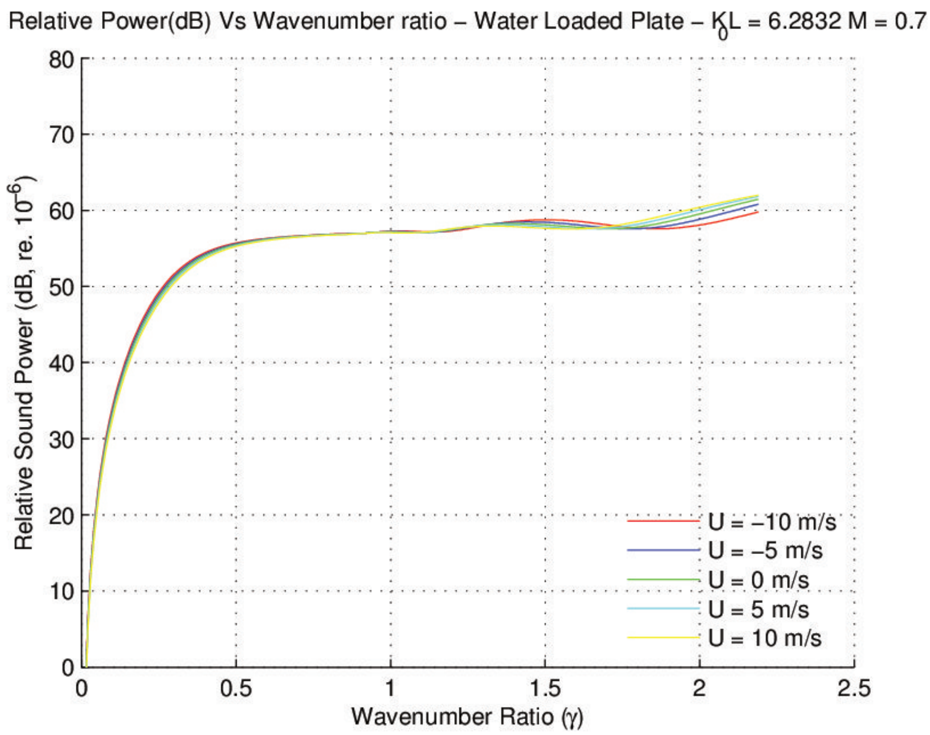
The sound power has been calculated for $K_0L = 0.1$ and 2π in the frequency range $0.01 < \gamma < 2.2$. Figure 2 shows the sound power generated by the moving load in presence of a mean flow on a Timoshenko-Mindlin beam for $M = 0.7$ and $K_0L = 0.1$ and 2π. All calculations have been undertaken using MATLAB.

With increased speed, the sound power generated increases, though marginally. However, it may be noted that the increased acoustic length K_0L reduces the sound power level over the entire range of the frequency range. This effect is expected since the total applied force strength is kept constant. No pronounced peak is noticed in the sound power curves. This is attributed to the fact that denser medium like water drain the energy faster from the structure disallowing the formation of the peak. One can see four distinct frequency ranges: the very low frequency region $(\gamma < 0.1)$; the low frequency region $(0.1 < \gamma < 1.0)$; the frequency region near coincidence $(\gamma \sim 1.0)$; and the frequency region above coincidence $(\gamma > 1.0)$. In the low frequency region and in the region above coincidence frequency, the sound powers radiated show no discernible difference. It is the low frequency region and the region near coincidence is significant.

Presence of current displays a proportional shift while nature of curves remains the same as that without current. The minus (-) current indicates direction of the current opposite to the direction of the subsonic moving force. The net effect is an increased Mach number and hence a shift of the curve upwards. The shift however is not very large. It may be noted that a high value of the current which makes the modified Mach number greater than 1 is not

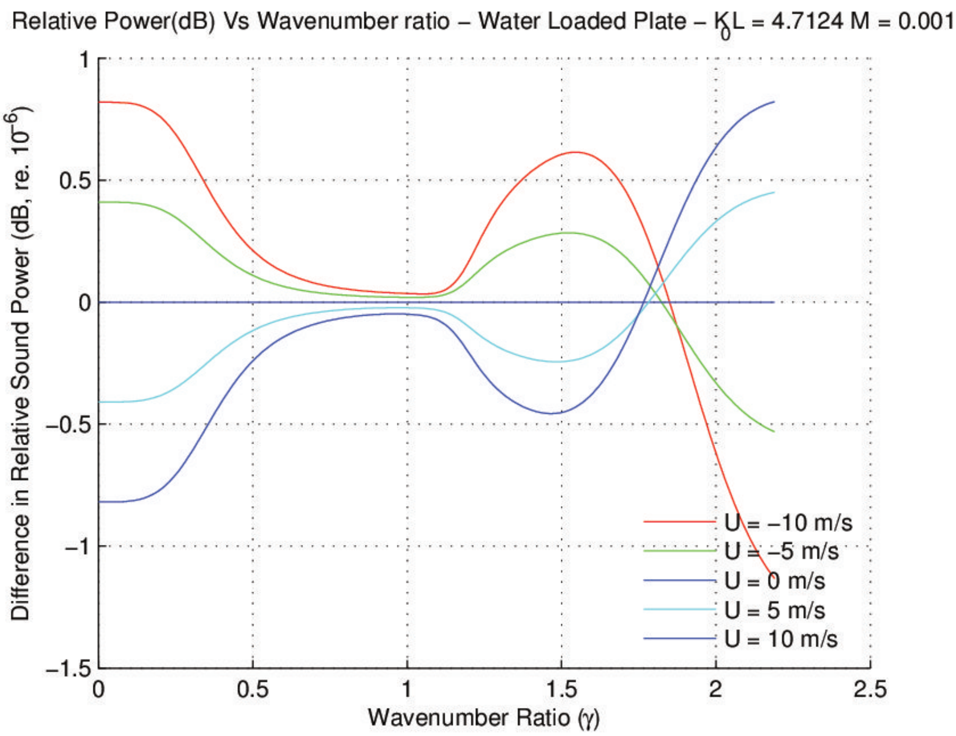


(a) $K_0L = 0.1$

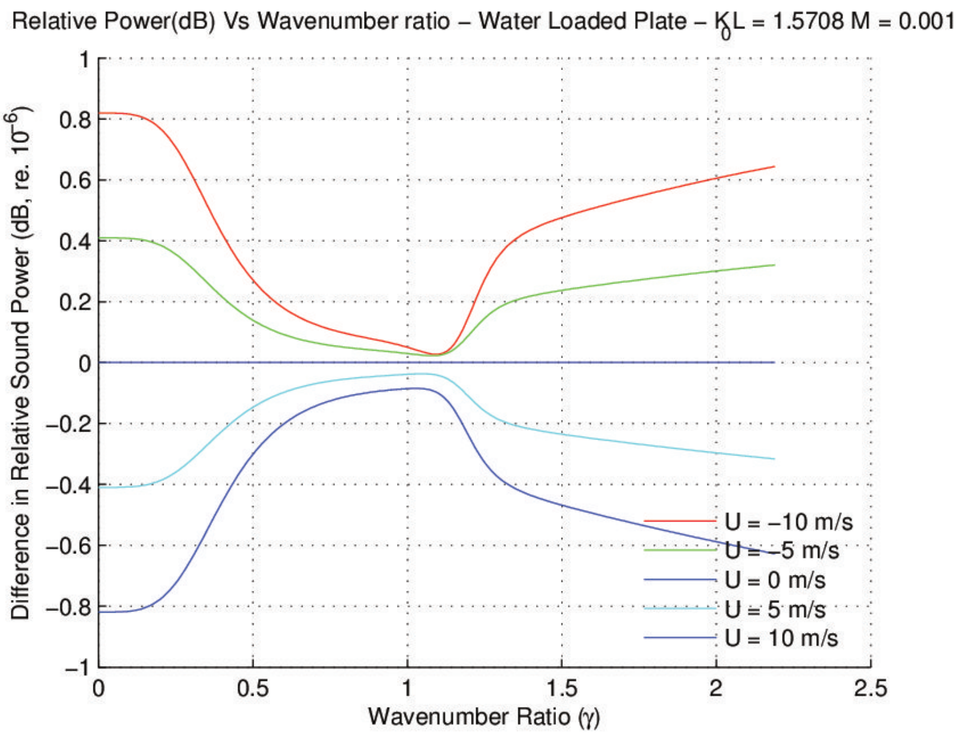


(b) $K_0L = 2\pi$

Fig. 2.: Relative sound power v/s wavenumber ratio **with current** for varying K_0L

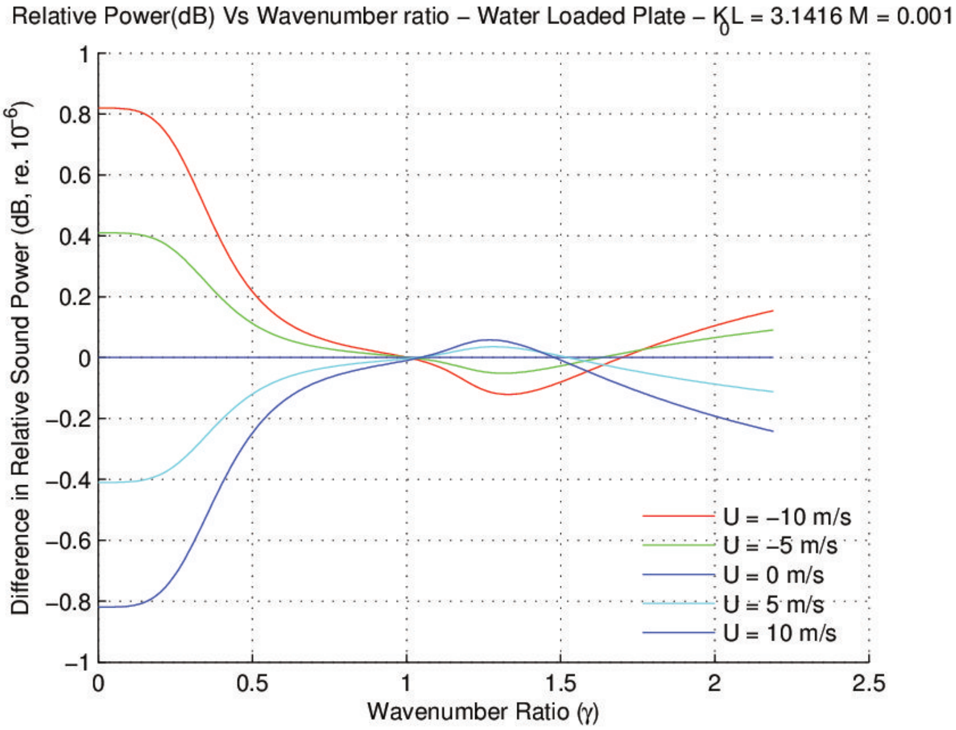


(b) $K_0L = 3\pi/2$

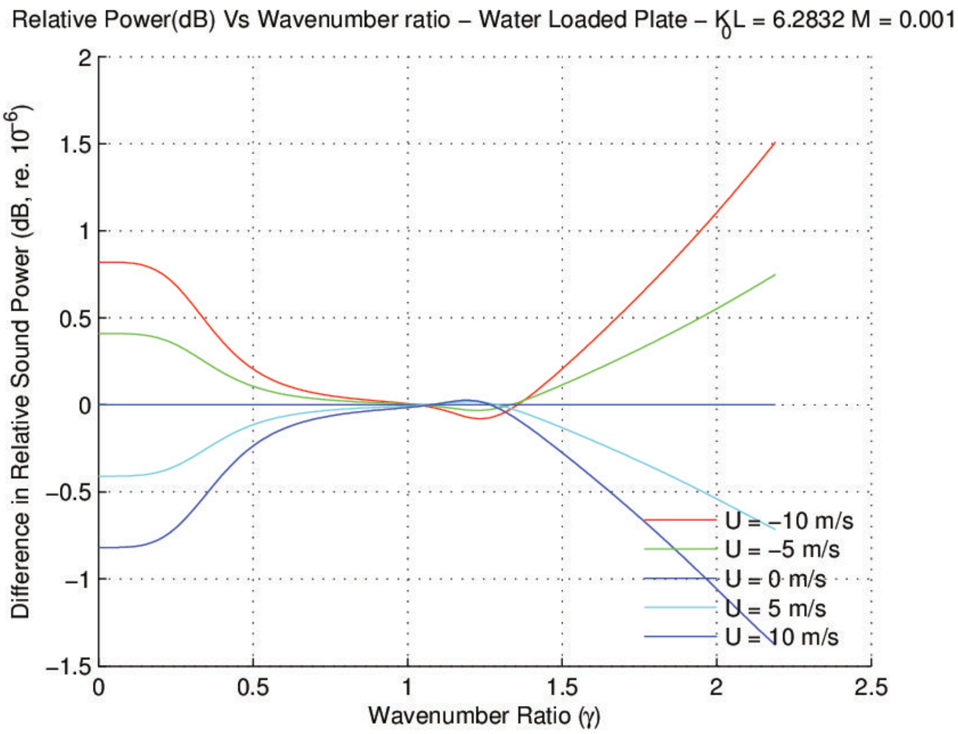


(a) $K_0L = \pi/2$

Fig. 3.: Difference in Relative sound power for non-integral multiples of π ; $M = 0.001$



(a) $K_0L = \pi$



(b) $K_0L = 2\pi$

Fig. 4.: Difference in Relative sound power for integral multiples of π; $M = 0.001$

permissible since the calculations are valid only for the subsonic speed domain. Since the variations due to the presence of current are not predominantly visible in Figure 4, we replot the figure as a difference curve with $U = 0$ as the reference to get Figure 3 and 4. Figure 3 is for fixed M with varying K_0L as non-integral multiples of π while Figure 4 is for integral multiples of π. It is interesting to note that the trend of curves of integral multiples and non-integral multiples is different, but consistent. The variation due to convective speed of loading is increased magnitudes for increased M while the curve trends remain to be the same. It is noted that for non-integral multiples of π, for every step increase of $\pi/2$, there is an added node with the magnitude of the previous nodes being reduced.

For integral multiples of π, for every step increase of π, there are two added nodes, again the previous nodes being reduced in magnitude. It may also be noted that the relative difference of sound power due to presence of mean flow is limited to 1 dB.

4. Conclusion

Sound produced by an airplane taking off from a floating runway in the presence of a mean flow has been analysed. The analysis is carried out for a one dimensional plate in lieu of a three dimensional runway with time varying loading. The sound generated at various speeds of convective loading has been calculated and as expected an increase in sound is observed with increasing Mach number. The overall sound generated reduces with an increased acoustic length K_0L over the entire frequency range. No pronounced peaks are observed in the sound power curves due to the denser medium of water wherein the energy drain is faster disallowing peak formation. The presence of current does not alter the sound produced prominently and the change is seen to be in the range of 1dB. Though the need to study effect of mean flow (current) may be considered irrelevant in light of the fact that such structures are set up in relatively calm waters behind islands or breakwater, however recent interests to have a floating airport in River Thames, UK and studies to widen range of potential setup sites for VLFS emphasizes this need. On analysing the difference of sound power with current a unique trend of curves is observed for acoustic lengths of integral and non-integral multiples of π. The *inter se* trend however remains consistent. It is thus concluded that the effect of sound produced by an aircraft takeoff on a floating runway needs to be catered for in the design of a VLFS for safer marine environment.

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