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Development of New Loan Payment Models with Piecewise Geometric Gradient Series

Erdal AYDEMİR[1] **Ramazan EROĞLU**[2]

Abstract

Engineering economics plays an important role in decision making. Also, the cash flows, time value of money and interest rates are the most important research fields in mathematical finance. Generalized formulae obtained from a variety of models with the time value of money and cash flows are inadequate to solve some problems. In this study, a new generalized formulae is considered for the first time and derived from a loan payment model which is a certain number of payment amount determined by customer at the beginning of payment period and the other repayments with piecewise linear gradient series. As a result, some numerical examples with solutions are given for the developed models.

Keywords: *Annuity, Piecewise Geometric Gradient Series, Payments, Time Value of Money*

JEL Classification Codes: *C0, C02, G21, G12*

Parçalı Geometrik Değişimli Seriler ile Yeni Borç Ödeme Modellerinin Geliştirilmesi

Öz

Karar vermede, mühendislik ekonomisi önemli rol oynamaktadır. Bununla birlikte, finans matematiği alanında en önemli konular arasında paranın nakit akışı, zaman değeri ve faiz oranları yer almaktadır. Paranın zaman değeri ve nakit akışı problemlerinden elde edilen formüller bilimsel yazında bulunmasına rağmen bazı problemlerin çözümünde bu formüller yetersiz kalmaktadır. Bu çalışmada, başlangıçta belirli sayıda taksit miktarını müşterinin belirlediği, sonraki taksit miktarlarının parçalı aritmetik (miktarsal) değişim gösterdiği bir borç ödeme modeli ilk olarak ele alınmakta ve çözüm için genel formülleri elde edilmektedir. Sonuçta, geliştirilen modeller sayısal örneklerle uygulamalı olarak gösterilmiştir.

Anahtar Kelimeler: *Anüite, Parçalı Aritmetik Değişimli Seriler, Amortisman, Paranın Zaman Değeri*

JEL Sınıflandırma Kodları: *C0, C02, G21, G12*

[1] Yrd. Doç. Dr., Süleyman Demirel Üniversitesi, Mühendislik Fakültesi Endüstri Mühendisliği Bölümü, erdalaydemir@sdu.edu.tr

[2] Süleyman Demirel Üniversitesi, İİBF İşletme Bölümü, ramazan3290@gmail.com

1. INTRODUCTION

The problems with payments of a debt stand with equality of the debt's present value and the sum of the present values (Shao and Shao, 1998: 502). For the economic analysis, we know the engineering economics also plays an important role in many decision making problems (Blank and Tarquin, 2005, Parvez, 2006). The repayment models of the series (instalments of the series) are fixed, geometric and arithmetic gradients that are located in the literature for the general formulae (Park, 1997: 55).

Another way to describe types of loans from scientific literature, some papers have been presented about its contracting (Graham et al., 2008; Hertzel and Officer, 2012), security, credit risk and insurance (Hatchett and Kühn, 2009; Mahayni and Schenieder, 2012), pricing options (Gündüz and Uhrig-Homburg, 2011; Peng et al., 2012; Dai and Chiu, 2013), payment instruments and transactions (Hancock and Humphrey, 1998; Parlour and Rajan, 2003; Maskara and Mullineaux, 2011; Goel and Mehrota, 2012). All of this study, there is no contribution about repayment plans to identify in generalized formulae.

The all of above payment models has an assumption that the payments are made at the end of each period. When we look at the scientific literature in chronological just last twenty years, some models were developed. According to Formato (1992), the new payment series was developed as skipped instalments that have addressed the problem of repayment of a debt in equal instalments rounds without instalments (back payments) in a certain time period at some randomly selected periods in some cases (for example holiday expenditures). Then, instead of the cyclic-equal instalments, Moon (1994) developed a new model that was called geometric gradients skip payment. In 2002, an arithmetic gradients skip payment model was developed in loan payment models by Eroglu and Karaoz.

Firstly, the generalized formulae which has included regular/irregular piecewise and geometric/arithmetic gradients are developed by Eroglu (2000). In addition, advanced models that are piecewise payment models with arithmetic and geometric gradients with arbitrary skips in randomly determined are also developed with the generalized formulae by Eroglu (2001) and Eroglu and Ozdemir (2012a). Then, the other developments were about rhythmic skip models for the loan payments model by Eroglu and Ozdemir (2012b) and Eroglu et al. (2013).

However, the costumers can be requested with equal instalments, increasing/decreasing instalments, and non-payments periods in a payment model jointly and/or separately within some cases which are interest rates, solvency levels, concurrent debt payments etc. In order to provide these situations, a new mathematical model is developed loan amortization payment models with piecewise geometric gradients in the case of the periods which have a certain number of instalments to be determined by the customers and the others have equal annuities with including non-payment periods in this study. The main purpose of this study is to reply the differences of the customer requests. So, the payment models developed by obtaining new two mathematical models are derived from the general formula and these are explained with numerical examples in current convenient financial payment problems.

2. MATHEMATICAL MODEL

Fixed payment models are mostly used model for the loan amortization payments that are given from credit institutions. In other words, the amount of payment is equal to each other. In fact, after a certain period from the payments was started, the customer's payment ability could be involved from the acquisition of the variability (cases of increasing or decreasing). In this case, the customers want to change the instalment level in the amount of a certain number of beginning periods itself for considering under changes in income for the coming periods. In this study, a new mathematical model is developed to pay a debt that is taken from a credit institution in repayments (instalments) circuits (monthly, quarterly etc.). The basic assumption of the model is determined to instalment of the pieces that is assumed to be equal to each other for the periods u in the beginning by the customer payment ability, the remaining pieces ($s + 1$ periods) of the debt are paid in the period. In here, s is defined as non-payment periods. A payment period has the number of f periods and also a non-payment period has the number of h

periods. The payment and non-payment periods are followed each other. In addition, another assumption is the amounts equal instalments during the period which change geometric changes for following periods.

The notations are given as follows for the mathematical model:

s : number of non-payment/term periods

f : payment numbers/term numbers for an in-payment period

h : non-payment numbers/term numbers for a non-payment period,

p : loan (debt) amount

g : relative geometric gradients in payment amount, $(G = 1 + g)$

r : interest rate for period/term $(R = 1 + r)$

b : payment amount by customer identified

u : payment numbers/terms by customer identified

d_k : payment amount for kth in-payment period

n : number of all terms for all loan repayments (in-payment and non-payment periods)

The number of all terms for all loan repayments is defined as Eq.(1).

$$n = u + s\left(f + h\right) + f \qquad (1)$$

The present value for a payment amount b in i^{th} period with r can be calculated as

$$\frac{b}{(1+r)^i} = bR^{-i} \qquad (2)$$

and it is mentioned that the first u number of payments are is equal by customer identified and the rest of the payments have geometric gradients. This case can be defined as follows in Eq.(3).

$$d_k = dG^k; \quad k = 0,1,...,s \qquad (3)$$

The repayment of a loan taken from banks and/or credit institutions, using the pre-determined and fixed interest rate of a period is r, is based to equality with present value of loan and sum of the present value of all payments. Here, we can obtain some equations that are given as follows:

$$p=\sum_{i=1}^{u}\frac{b}{R^i}+\sum_{k=0}^{s}\sum_{i=u+k(f+h)+1}^{u+k(f+h)+f}\frac{d_k}{R^i} \tag{4}$$

$$=\begin{cases}\dfrac{b\left(GR^{-(f+h)}-1\right)(R^u-1)+d\left(1-R^{-f}\right)\left[\left(GR^{-(f+h)}\right)^{s+1}-1\right]}{rR^u\left(GR^{-(f+h)}-1\right)}; & G\neq R^{f+h} \quad (4a)\\[2ex] \dfrac{b\left(R^u-1\right)+d\left(s+1\right)\left(1-R^{-f}\right)}{rR^u}; & G=R^{f+h} \quad (4b)\end{cases}$$

and

$$d=\begin{cases}\dfrac{prR^u\left(GR^{-(f+h)}-1\right)+b(1-R^u)\left(GR^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left[\left(GR^{-(f+h)}\right)^{s+1}-1\right]}; & G\neq R^{f+h} \quad (5a)\\[2ex] \dfrac{rpR^u-b\left(R^u-1\right)}{\left(s+1\right)\left(1-R^{-f}\right)}; & G=R^{f+h} \quad (5b)\end{cases}$$

If we use $g=0$, we can obtain as $G=1+g=1$. In addition, if we use Eq.(4a) and Eq.(5a) in the case of $g=0$, we can also obtain the first u number of payments are is equal. These equations are given as follows in Eq.(6) and Eq.(7) respectively for determining payment amount.

$$p=\frac{b\left(R^{-(f+h)}-1\right)(R^u-1)+d\left(1-R^{-f}\right)\left[R^{-(f+h)(s+1)}-1\right]}{rR^u\left(R^{-(f+h)}-1\right)} \tag{6}$$

$$d=\frac{prR^u\left(R^{-(f+h)}-1\right)+b(1-R^u)\left(R^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left[R^{-(f+h)(s+1)}-1\right]} \tag{7}$$

Then, if we use $b=0$ in Eq.(6) and Eq.(7), we can also obtain generalized formulae for the first u number of non-payments periods and the following periods with equal amount.

$$p = \frac{d\left(1-R^{-f}\right)\left[R^{-(f+h)(s+1)}-1\right]}{rR^{u}\left(R^{-(f+h)}-1\right)} \tag{8}$$

$$d = \frac{prR^{u}\left(R^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left[R^{-(f+h)(s+1)}-1\right]} \tag{9}$$

In addition, if we use $u=0$ in Eq.(8) and Eq.(9), we can obtain generalized formulae for rhythmic skips with equal amount of payments.

$$p = \frac{d\left(1-R^{-f}\right)\left[R^{-(f+h)(s+1)}-1\right]}{r\left(R^{-(f+h)}-1\right)} \tag{10}$$

$$d = \frac{pr\left(R^{-(f+h)}-1\right)}{\left(1-R^{-f}\right)\left[R^{-(f+h)(s+1)}-1\right]} \tag{11}$$

3. QUANTITIVE ANALYSIS

3.1. Numerical Example 1

A customer wants to apply for loan amount of 16000 TL (Turkish Liras) from a bank and/or credit institution. About the payment plan, the amount of the first three instalments are as 650 TL that are determined by the customer, the rest of the instalments will include three payments periods with two payment months and one non-payments month is totally six instalments. In payment periods, there is geometric gradient series with 3.5% from one payment period to following payment period. In addition, monthly interest rate is determined by 1.2%. Thus, the model parameters are defined as $p=16000$, $u=3$, $s=2$, $f=2$, $h=1$, $b=650$, $r=0.012$, $R=1.012$, $g=0.035$ and also $G=1.035$. Then, if Eq.(5a) is used, the payment amount is $d=2482.255$ TL in a payment period. Consequently, this payment plan is demonstrated with Table 1 by using Eq.(5).

Table 1. The Payment Model for Example 1

Months	Payment Amount (TL)	Rest of Loan Amount (TL)
0		16000.000
1	650.000	16000.000 * 1.012 - 650.000 = 15542.000
2	650.000	15542.000 * 1.012 - 650.000 = 15078.504
3	650.000	15078.504 * 1.012 - 650.000 = 14609.446
4	2482.255	14609.446 * 1.012 - 650.000 = 12302.504
5	2482.255	12302.504 * 1.012 - 2482.255 = 9967.879
6	0.000	9967.879 * 1.012 = 10087.494
7	2569.134	10087.494 * 1.012 - 2569.134 = 7639.410
8	2569.134	7639.410 * 1.012 - 2569.134 = 5161.949
9	0.000	5161.949 * 1.012 = 5223.892
10	2659.054	5223.892 * 1.012 - 2659.054 = 2627.525
11	2659.054	2627.525 * 1.012 - 2659.054 = 0.000

3.2. Numerical Example 2

According to numerical example 2, a customer wants to apply for loan amount of 16000 TL from a bank and/or credit institution. About the payment plan, the amount of the first two instalments are as 650 TL that are determined by the customer, the rest of the instalments will include three payments periods with two payment months and one non-payments month is totally six instalments. In payment periods, there is geometric gradient series with 3.0301% from one payment period to following payment period. In addition, monthly interest rate is determined by 1%. Thus, the model parameters are defined as $p = 16000$, $u = 2$, $s = 2$, $f = 2$, $h = 1$, $b = 650$, $r = 0.01$, $R = 1.01$, $g = 0.030301$ and also $G = R^{f+h} = 1.01^{2+1} = 1.030301$. Then, if Eq.(5b) is used, the payment amount is $d = 2540.117$ TL in a payment period. Consequently, this payment plan is demonstrated with Table 2 by using Eq.(5).

Table 2. The Payment Model for Example 2

Months	Payment Amount (TL)	Rest of Loan Amount (TL)
0		16000.000
1	650.000	16000.000 * 1.01 - 650.000 = 15510.000
2	650.000	15510.000 * 1.01 - 650.000 = 15015.100
3	2540.117	15015.100 * 1.01 - 2540.117 = 12625.134
4	2540.117	12625.134 * 1.01 - 2540.117 = 10211.268
5	0.000	10211.268 * 1.01 = 10313.381
6	2617.085	10313.381 * 1.01 - 2617.085 = 7799.430
7	2617.085	7799.430 * 1.01 - 2617.085 = 5260.339
8	0.000	5260.339 * 1.01 = 5312.943
9	2696.385	5312.943 * 1.01 - 2696.385 = 2669.687
10	2696.385	2669.687 * 1.01 - 2696.385 = 0.000

3.3. Numerical Example 3

As a numerical example 3, a customer wants to apply for loan amount of 12000 TL from a bank and/or credit institution. About the payment plan, the first two instalments are non-payment periods that are determined by the customer, the rest of the instalments will include three payments periods with one payment month is totally six instalments. In addition, monthly interest rate is determined by 2%.

Thus, the model parameters are defined as $p = 12000$, $u = 2$, $s = 1$, $f = 3$, $h = 1$, $r = 0.02$ and also $R = 1.02$. Then, if Eq.(9) is used, the payment amount is $d = 2250.265$ TL in a payment period.

Table 3. The Payment Model with Non-Payments Start to Loan for Example 3

Months	**Payment Amount (TL)**	**Rest of Loan Amount (TL)**		
0		12000.000		
1	0.000	12000.000 * 1.02	=	12240.000
2	0.000	12240.000 * 1.02	=	12484.800
3	2250.265	12484.800 * 1.02 - 2250.265	=	10484.231
4	2250.265	10484.231 * 1.02 - 2250.265	=	8443.651
5	2250.265	8443.651 * 1.02 - 2250.265	=	6362.259
6	0.000	6362.259 * 1.02	=	6489.504
7	2250.265	6489.504 * 1.02 - 2250.265	=	4369.029
8	2250.265	4369.029 * 1.02 - 2250.265	=	2206.144
9	2250.265	2206.144 * 1.02 - 2250.265	=	0.000

4. CONCLUSIONS and DISCUSSIONS

In fact, we know the fixed re-payment models is not satisfied to the customers in variable interest rates at the current economies. Today economies are designed with higher variability in the cases from globalism. So, the customer types and the debt volumes are also changed in this process. The customers are requested the new re-payment financial models for their variable debts levels in globally market conditions. As a result of these changes, the new re-payment models must be derived in alternatively for loan. Hereafter, the loan financial companies and banks must be presented more alternatives re-payments models to the customers under market conditions which are varieties of the interest rates, currencies, revenues, solvency levels, debt volumes etc.

If the variations of the payment series are increased, the number of customer type is also increased in simultaneously. For this reason, the derivation of new financing (payment) models both in terms of financial institutions both the customers is great importance. In addition, the variability of revenues from customers in the event of time sometimes the payment and/or non-payments terms may be useful to differentiate customers.

In this study, in the beginning periods (months) a certain number of repayments (instalments) the amount specified by the customer the next equal instalments circuits formed in the period of repayments from one period to the piecewise geometric change (gradients) demonstrate that arise when solving problems used in the new loan payment models and the general formulae are derived and current examples of the model by functioning is shown. This study was the determination of the amount of payment for customers and pay the appropriate balance is important in forming. The results are obtained with the numerical samples with a repayment schedule is shown in clearly.

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