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Experimental Evaluation of Sliding-Mode Control Techniques

Murat FURAT*[1] ve İlyas EKER[1]
[1]*Çukurova University, Electrical and Electronics Eng. Dept., Çukurova University, Adana, TURKEY*

Abstract

Sliding-mode control (SMC) is one of the robust and nonlinear control methods. Systematic design procedure of the method provides a straightforward solution for the control input. The method has several advantages such as robustness against matched external disturbances and unpredictable parameter variations. On the other hand, the chattering is a common problem for the method. Some approaches have been proposed in the literature to overcome the chattering problem.

In the present paper, an experimental evaluation and practical applicability of conventional (first-order) sliding-mode control techniques are investigated. Experimental applications are performed using an electromechanical system for speed tracking control and disturbance regulation problems. The graphical results are illustrated and the performance measurements are tabulated based on time-domain analysis. The experimental results indicate the fact that the sliding-mode control is applicable to practical control systems with the cost of some disadvantages.

Key Words: Conventional sliding-mode control, SMC, chattering, real system application

Kaymalı-Kutup Kontrol Tekniklerinin Deneysel Değerlendirilmesi

Özet

Klasik kayan kipli-kontrol (KKK), doğrusal olmayan ve dayanıklı kontrol yöntemlerinden biridir. Metodun düzenli tasarım işlemi, kontrol girişi için kolay çözüm sağlar. Bu yöntemin eşleşen dış bozuculara ve belirsiz parametre değişimlerine karşı dayanıklılığı gibi çeşitli avantajları vardır. Diğer yandan, çıtırdama bu yöntemin ortak problemidir. Literatürde, çıtırdama problemini aşmak için bazı yaklaşımlar önerilmiştir.

Bu çalışmada, klasik (birinci derece) KKK yöntemlerin deneysel değerlendirmesi ve pratik uygulanabilirliği araştırılmıştır. Deneysel uygulamalar, bir elektromekanik sistemin hız kontrolü ve bozucu ayarlaması üzerine yapılmıştır. Zamana bağlı grafiksel sonuçlar gösterilmiş ve performans ölçümleri tablolar halinde sunulmuştur. Deneysel sonuçlar, bazı dezavantajlarına rağmen kayan kipli kontrolün pratik kontrol sistemlerine uygulanabilirliğini göstermiştir.

Anahtar Kelimeler: Klasik kayan-kipli kontrol, KKK, çıtırdama, gerçek sistem uygulaması

* Yazışmaların yapılacağı yazar: Murat FURAT, *Çukurova University, Electrical and Electronics Eng. Dept., Çukurova University, Adana, TURKEY.* mfurat@cu.edu.tr

1. Introduction

The control methods, in general, can be classified into model-based and non-model based methods. Model-based control methods are systematic and can be applied to general cases, and specifications in terms of robustness and tracking accuracy can be priory assigned, as well as various optimally criteria can be fulfilled. However, in real systems, there are many parameters, external disturbances and uncertainties which are unpredictable and difficult to model. Therefore, non-model based control methods are largely applied in practice since they do not require knowledge of complex mathematical models. The methods can guarantee limited performances and robustness of control system and the design procedure is not systematic hence in complex systems it can be difficult to apply. The sliding-mode control method, theoretically, is able to reject the matched disturbances and it is able to provide robust control signal under uncertainties [1].

Sliding-mode control (SMC) is a particular type of variable structure control which has been studied extensively for over 50 years [1-4]. The first publication in the literature may be found in 1977 [1] and sliding-mode control has been one of the significant interests in the control research community worldwide [4]. Since it has systematic design procedure, it is one of the most powerful solutions for many practical control designs [3]. It is a robust control scheme based on the concept of changing the structure of the controller in response to changing the state of the system in order to obtain desired output [5]. Therefore SMC is a successful control method for nonlinear systems.

There are many solution techniques in the literature for the conventional sliding-mode control [2,4]. Controller design procedure, number of control tuning parameters and properties of subsystems are of interest in these techniques [6]. The method has been applied to several problems such as automotive, induction motors, automotive climate control, diesel engine, alternator [3], DC motors [5], chemical processes [7,12], SMA actuator [8], submarines [9], electrical drives [10]. Also, numerous SMC techniques were proposed in the literature to increase the performance of SMC by combining SMC with soft computing control methods, i.e. neural networks, fuzzy logic, genetic algorithm [4].

The contribution of the paper is that the SMC techniques presented in [1,5,7,12] are experimented on a real system to demonstrate applicability of the techniques to practical systems. The results are presented graphically and comparison measures based on time-domain analysis are tabulated.

The present paper is organized as follows. In Section 2, fundamentals of SMC are outlined. The experimental setup and experimented SMC techniques are introduced in Section 3. The applications of selected SMC techniques to the real system and graphical results with discussion are given in Section 4. In the last section, the results of the experiments are concluded.

2. Fundamentals of Sliding-Mode Control

Robustness and systematic design procedure are well-known advantages of sliding-mode controllers [1,3]. Traditionally, the conventional sliding-mode control method has been designed for the systems with relative degree of one. Since the control input appears in the first derivative of sliding function, its relative degree with respect to control is one. This type of sliding-mode control method is called as first-order SMC.

A first-order sliding-mode controller consists of two distinct control laws: switching control and equivalent control [1,3,5]. The most important task is to design the switching control law which enforces the system to the sliding surface, $s(t)$, defined by the user, and to maintain the system state trajectory on this surface as shown in Fig. 1 in which e and $\dot{e}$ denote the tracking error and first-time derivative of the tracking error, respectively. 't' in $s(t)$ is the independent variable time. The dynamic performance of the system is directly dependent choosing an appropriate switching control law. Lyapunov technique is generally used to determine the stability of the closed-loop systems [1].

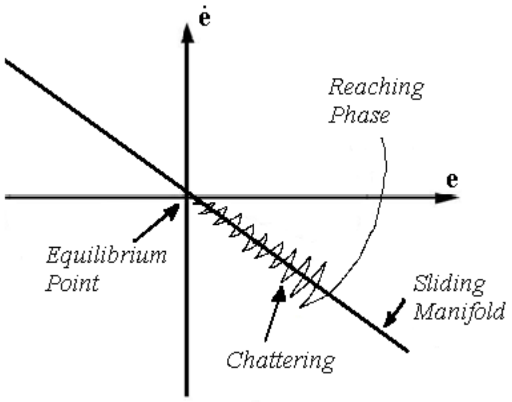

Figure 1. Graphical representation of sliding-mode control

Ideal sliding-mode can be found by equivalent control approach [1]. First time derivative of $s(t)$ along the system trajectory is set equal to zero and the resulting algebraic system is solved for the control law. If the equivalent control exists, it is substituted into $s(t)$ and the resulting equations are the ideal sliding-mode [1].

The first step in the sliding-mode control solution is to determine a sliding manifold which is also called sliding surface or sliding function, $s(t)$ being a function of the tracking error, $e(t)$, $e(t) \in R$ that is the difference between set point and output measurement, as:

$$s(t) = \left(\lambda + \frac{d}{dt} \right)^{n-1} e(t) \qquad (1)$$

where n denotes the order of uncontrolled system, λ is a positive constant, $\lambda \in R^{+}$ where R and R^{+} denotes set of real and positive real numbers, respectively. λ is the tuning parameter which determines the slope of sliding manifold. When the system is in the sliding-mode, both $s(t)$ and $\dot{s}(t)$ are equal to zero, $s(t) = \dot{s}(t) = 0$.

The objective is to determine a control law $u(t)$, so that the tracking error and its derivative should converge to zero from any initial state to the equilibrium point in a finite time. $u(t)$ consists of two additive signals switching (discontinuous) signal, $u_{sw}(t)$, and equivalent (continuous) signal, $u_{eq}(t)$, determined separately [1,5]:

$$u(t) = u_{eq}(t) + u_{sw}(t) \qquad (2)$$

If the initial trajectory is not on the sliding surface, the switching control, $u_{sw}(t)$, enforces the error toward the origin of the sliding surface and this is called the reaching phase. The equivalent control may not be able to move the system state toward sliding surface. Therefore, the switching control is designed on the basis of relay-like function because it allows changes between the structures infinitely fast. The equivalent control is found by equating derivative of sliding function to zero. On the other hand, the switching control can be selected directly as [1]:

$$u_{sw}(t) = -k sign(s(t)) \qquad (3)$$

where k is a positive constant that should be large enough to suppress all matching uncertainties and unpredictable system dynamics and $sign(\cdot)$ is a signum function [3].

The block diagram of conventional sliding-mode control method is illustrated in Fig. 2.

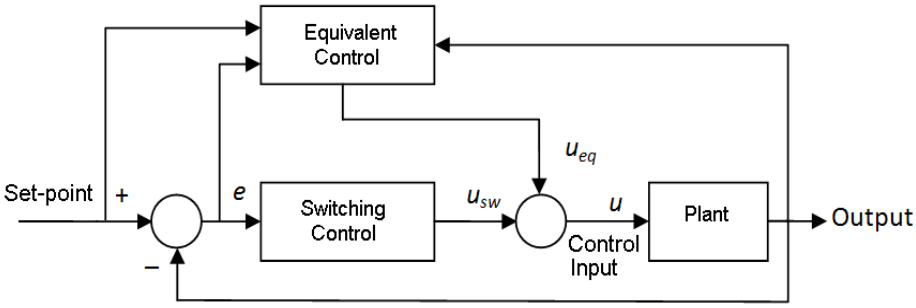

Figure 2. Conventional sliding-mode control for a plant

2.1. Chattering Phenomenon

In the design of classical sliding-mode controllers for practical applications, it is required to determine a proper sliding surface so that the tracking errors and output deviations can be reduced to a satisfactory level. The classical sliding-mode control, despite the advantages of simplicity and robustness, generally suffers from the well-known problem, called chattering, which is a very high-frequency oscillation of the sliding variable around the sliding manifold [2,4]. The chattering is highly undesirable for the real systems and actuator since it may lead to actuator failure and unnecessarily large control signal as shown in Fig. 1. In practice, the presence of time delays in many industrial processes and the actuators, such as time lag, transportation lag, time delay, dead time and physical limitations, cannot switch at an infinite frequency along the sliding surface as demanded by the theory of classical sliding-mode control algorithms [8].

Numerous approaches have been proposed to overcome chattering phenomenon. It may be reduced by smoothing out the control discontinuity with achieving exponential stability. Commonly preferred approaches are to replace the relay control by a saturation function, sigmoid functions, hyperbolic functions, and hysteresis saturation functions [1,5,7,12]. In the literature, the chattering problem is a current research topic [14-16].

2.2. Stability

Stability of sliding-mode control has been shown by Lyapunov stability theorem [1,3,5]. The control design task then becomes to find a suitable discontinuous control that satisfies the condition $\dot{V}(t) < 0,\ s(t) \neq 0$ where $V(t)$ denotes Lyapunov function. This is accomplished by choosing a positive-definite Lyapunov function, $V(t)$, ($V(t) > 0,$ for $t > 0,\ s(t) \neq 0$) based on the error dynamics of the system, then its derivative is negative definite. Since $V(t)$ is clearly positive-definite, $\dot{V}(t)$ is negative semi-definite and $V(t)$ tends to infinity as $s(t)$ tends to infinity then the equilibrium point at the origin is globally stable. Therefore, $s(t)$ tends to zero as the time t tends to infinity. Moreover, all trajectories starting off the sliding surface $s(t) = 0$ must reach it asymptotically and then will remain on this surface. Readers are directed to the reference [17] for more information about the stability of SMC.

3. Experimental Setup and Preliminaries

A direct-current motor system, dc motor, is selected as an electromechanical system for the experiments which is mounted on a DIGIAC 1750 Process Control Training Set as shown in Fig. 3. The motor drives a shaft that carries disks which operate various transducers and a tachogenerator. A computer having 2.0 GHz microprocessor and 1 GB RAM is used to perform experiments. A slotted opto transducer is used to measure shaft speed. The tachogenerator which produces voltage proportional to shaft speed is used as speed transducer for the feedback. The measurements are transmitted from experimental set to the computer via a data acquisition card (DAQ). Matlab/Simulink is used for all calculations and controller design.

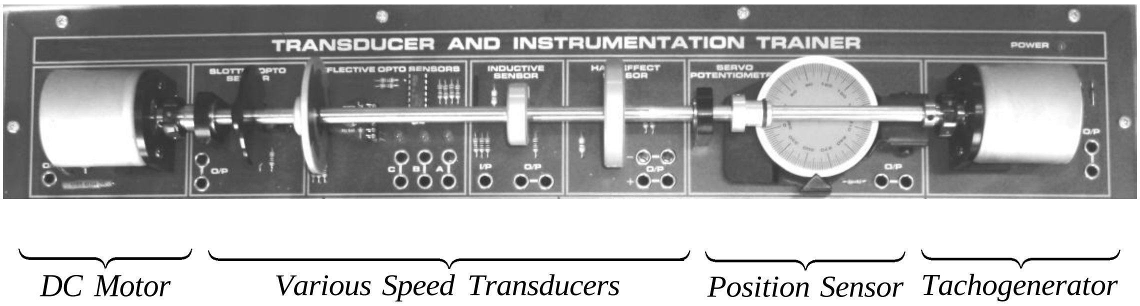

Figure 3. A view of the electromechanical system

The mathematical description of the electromechanical system is given in appendix [19]. Since the parameters of the mathematical model cannot be obtained properly, first-order plus dead-time model is used to approximate the model of the electromechanical system. For this purpose, a step input, $u(t)$, of 5.12 Volts in magnitude, in open-loop conditions, is applied to the armature of the system and output shaft speed is measured in rpm. The output steady-state voltage produced by the tachogenerator is measured to be 4.43 Volts that corresponds to 1200 rpm output shaft speed. A second-order system model can be used to approximate the model of the present system as [11]:

$$G(s) \cong \frac{Ke^{-t_d s}}{\tau_p s+1} \cong \frac{K}{(\tau_p s+1)(t_d s+1)} \tag{4}$$

where K is the gain, t_d is the time delay and τ_p is the time constant.

Using the input-output plots of the system, the plant coefficients are found to be $t_d = 0.0035,\ \tau_p = 0.145,\ K = 0.86$. The modeling error is illustrated in Fig. 4 in which +30 rpm and -18 rpm of speed deviations corresponding to 2.5% of the modeling error (modeling error= measured output – model output) including transient and steady-state outputs.

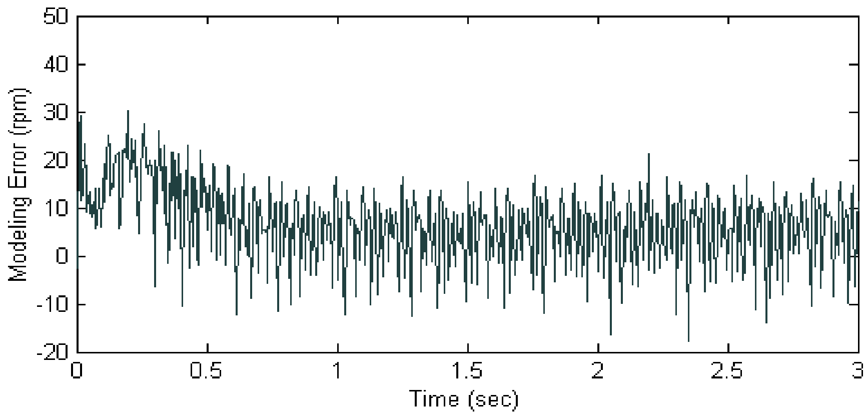

Figure 4. Modeling error

The experimental results are presented to demonstrate the performance of selected SMC techniques which mainly characterize the classical SMC. The parameters of the controller are tuned during the experiments, avoiding complicated calculations which may cause large chattering that is dangerous for the actuator. As a design requirement, overshoot at the output is not desired.

Technique I:

The first technique was proposed by Vadim I. Utkin [1] in 1977. The related sliding surface is given in Eq. (1). The control signal is sum of the equivalent signal and switching signal. The switching signal is selected to be a signum function with a constant gain of magnitude 1.5 (k=1.5) and the tuning parameter λ is chosen to be 13.75.

Technique II:

The second SMC technique is based on PID sliding surface as [5]:

$$s(t) = k_1 e(t) + k_2 \int_0^{\tau} e(\tau)d\tau + k_3 \dot{e}(t) \qquad (5)$$

where the gain parameters are adjusted to be $k_1 = 30,\ k_2 = 1,\ k_3 = 1.1.$ The switching control is selected to be a saturation function whose gain is 3.5 (k=3.5) and $\Omega = 20,$ where Ω is the thickness of boundary layer.

$$u_{sw} = ksat\left(\frac{s(t)}{\Omega}\right) \qquad (6)$$

The saturation function in Eq. (6) is defined as [5]:

$$sat\left(\frac{s(t)}{\Omega}\right) = \begin{cases} \dfrac{s(t)}{\Omega} & \text{if } \left|\dfrac{s(t)}{\Omega}\right| \le 1 \\ sign\left(\dfrac{s(t)}{\Omega}\right) & \text{if } \left|\dfrac{s(t)}{\Omega}\right| > 1 \end{cases} \qquad (7)$$

Technique III:

The third technique is proposed for stable processes [7]. The structural limitations of PI and PID controllers may not give satisfactory responses with large time constant or poorly located complex poles. Because of this reason, a PI – PD based sliding surface is selected as:

$$s(t) = ae(t) + b\int e(t)dt - cy(t) - d\dot{y}(t) \qquad (8)$$

where the parameters are adjusted to be a=19.5, b=100, c=9.76, d=0.1. Tangent hyperbolic function is selected as switching control whose gain is 3.5 (k=3.5) and $\Omega = 20.$

Technique IV:

The last technique is presented to regulate the nonlinear chemical processes [12]. The sliding surface is given by

$$s(t) = \left(\frac{d}{dt} + \lambda\right)^n \int e(t)dt \qquad (9)$$

where the tuning parameter λ is adjusted to be 60. The proposed switching control is as [12]:

$$u_{sw} = K_d \frac{s(t)}{|s(t)| + \phi} \qquad (10)$$

where ϕ is chosen to be 3 and the switching gain, K_d , is 5.2.

4. Experimental Study and Discussion

Controllers of the techniques are implemented in Matlab/Simulink environment and sample time is selected to be 5 ms. The set point is adjusted to be 4.43 Volts corresponding to 1200 rpm shaft speed. The step responses are illustrated in Fig. 5.

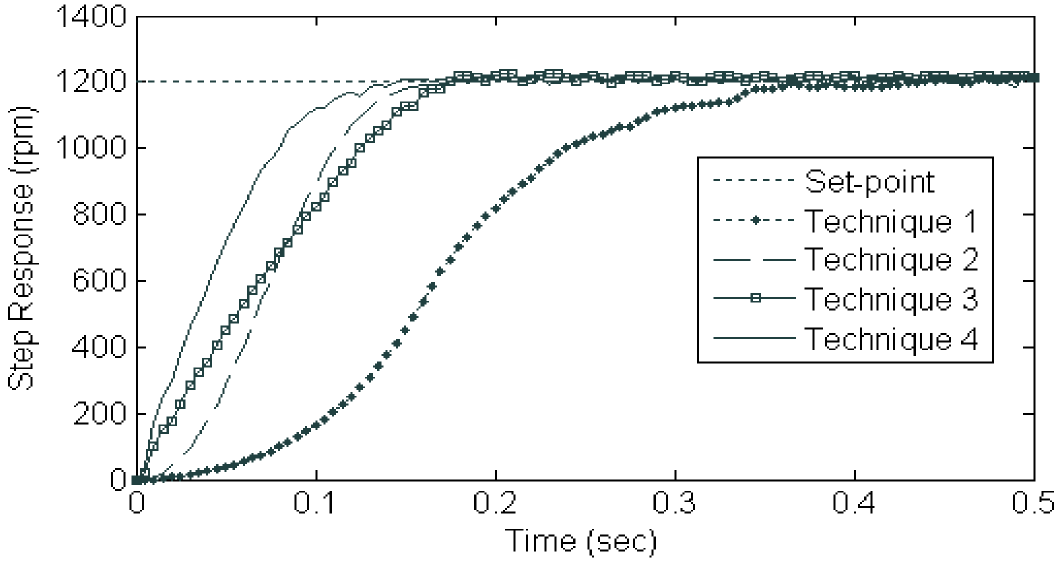

Figure 5. Step responses of experimented SMC techniques, Technique 1: Classical SMC [1], Technique 2: SMC with PID sliding surface [5], Technique 3: SMC for stable systems [7], Technique 4: SMC to regulate nonlinear chemical systems [12]

The rise time, settling time, output speed deviation and delay time of the responses are compared and tabulated in Table 1.

Table 1. Time domain step response specifications

SMC Techniques	Rise Time (ms)	Settling Time (ms) (5%)	Output Speed Deviation (rpm)	Delay Time (ms)
Technique 1 [1]	194	340	±25	170
Technique 2 [5]	90	140	±15	75
Technique 3 [7]	130	160	±20	69
Technique 4 [12]	85	110	±21	42

The control signals produced by the controllers are illustrated in Fig. 6. Since the output of the data acquisition card is limited to ±10 Volts, the output voltage is cropped when it exceeds the limit. Large variation of control signal is not desired for the real systems. The technique, proposed in [1], has large variations while the other techniques have relatively small variations.

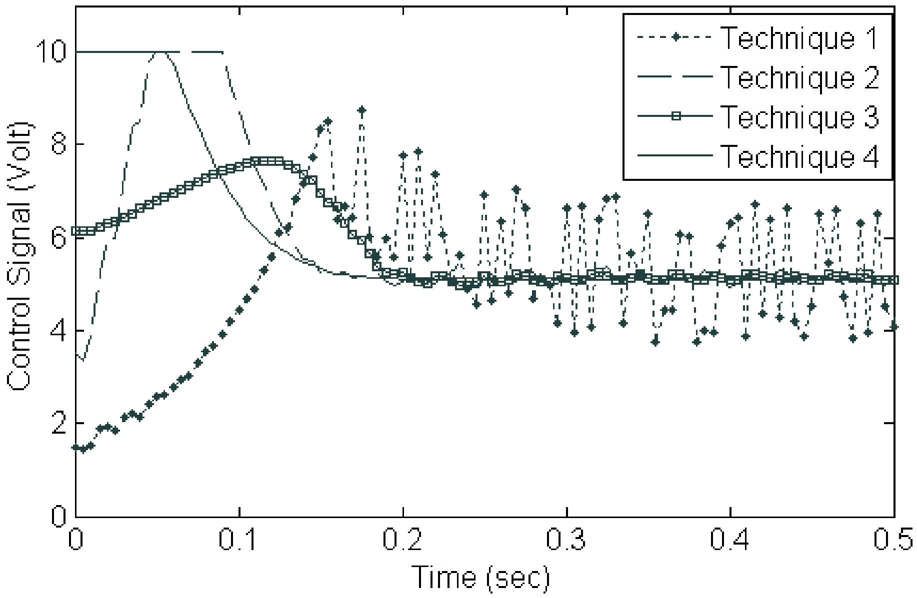

Figure 6. Control signals of experimented SMC techniques, Technique 1: Classical SMC [1], Technique 2: SMC with PID sliding surface [5], Technique 3: SMC for stable systems [7], Technique 4: SMC to regulate nonlinear chemical systems [12]

The objective of sliding-mode control is to force both error and derivative of error to the equilibrium point. Then the selected sliding surface, $s(t)$, tends to zero in a finite time and the system states should remain on the surface. In Fig. 7, Fig. 8 and Fig. 9, the selected sliding surfaces are illustrated. The variations in $s(t)$ of [1] in Fig. 7 are larger than that of the other techniques. The sliding surfaces of the techniques in [5,7] converge faster than others. The slowest one is in [12] as illustrated in Fig. 9.

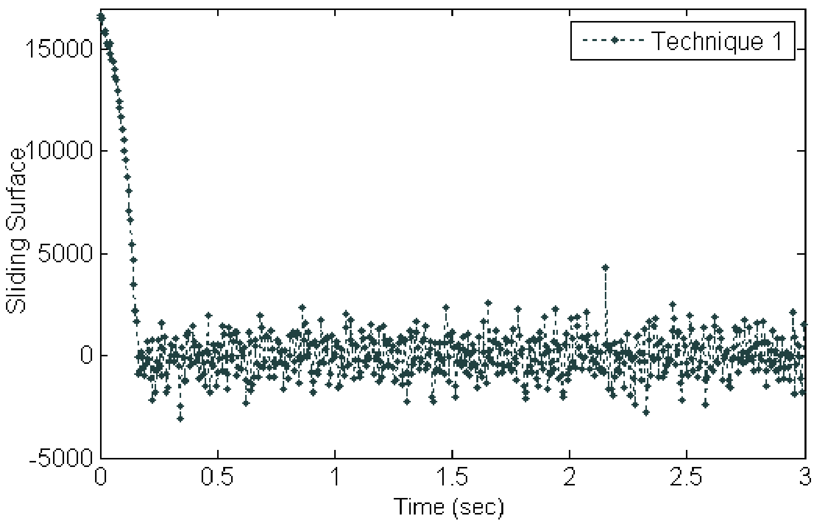

Figure 7. Sliding surfaces of experimented SMC techniques, Technique 1: Classical SMC [1]

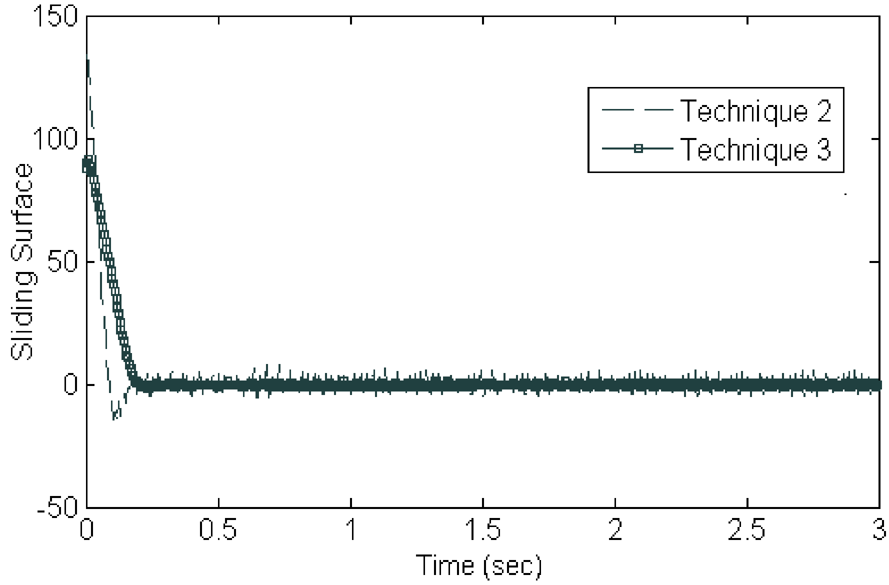

Figure 8. Sliding surfaces of experimented SMC techniques, Technique 2: SMC with PID sliding surface [5], Technique 3: SMC for stable systems [7]

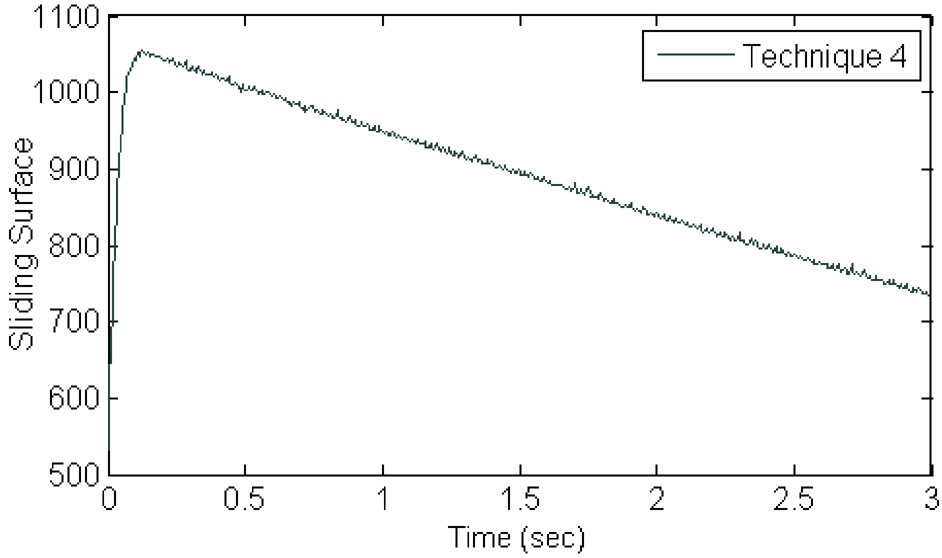

Figure 9. Sliding surfaces of experimented SMC techniques, Technique 4: SMC to regulate nonlinear chemical systems [12]

The error versus derivative of error for each technique is illustrated in Fig. 10 and Fig. 11. Since the range (numerical magnitudes) of derivative of the error in technique 1 [1] is larger than the other techniques, it is illustrated in different figure, Fig. 10. The others are illustrated in Fig. 11.

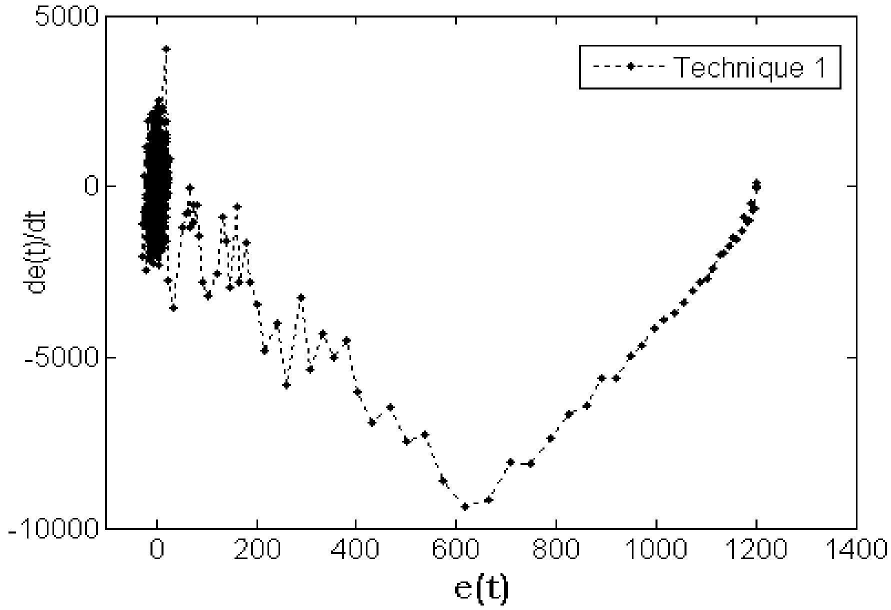

Figure 10. Error versus derivative of error change of experimented SMC techniques, Technique 1: Classical SMC [1]

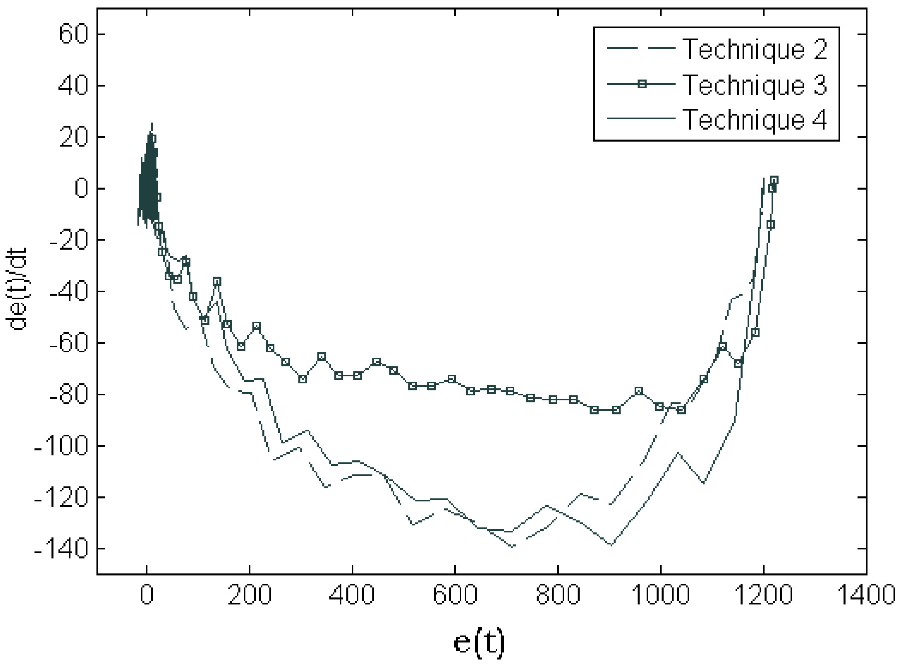

Figure 11. Error versus derivative of error change of experimented SMC techniques, Technique 2: SMC with PID sliding surface [5], Technique 3: SMC for stable systems [7], Technique 4: SMC to regulate nonlinear chemical systems [12]

The switching signals of all experimented techniques are illustrated in Fig. 12. Technique 1 proposed in [1] produced strong switching signal compared with the others. Since the switching function is selected as *signum* function with its gain 1.5, the switching signal produces ±1.5 Volts due to disturbances and uncertainties of the real system. In technique 2 proposed in [5], the switching signal has smaller variations with smoothness. The tangent hyperbolic function used in technique 3 in [7] produces a reasonable signal compared with the signal produced by the technique 2. In contrast, the switching signal of technique 4 in [12] has a constant magnitude but smallest variation. As a result, using smoothed switching signals such as saturation function in technique 2 [5], tangent hyperbolic in technique 3 [7] and a smooth function in technique 4 [12], defined by the Eq. (10), produces less chattering whereas it may increase the time required to reach equilibrium point.

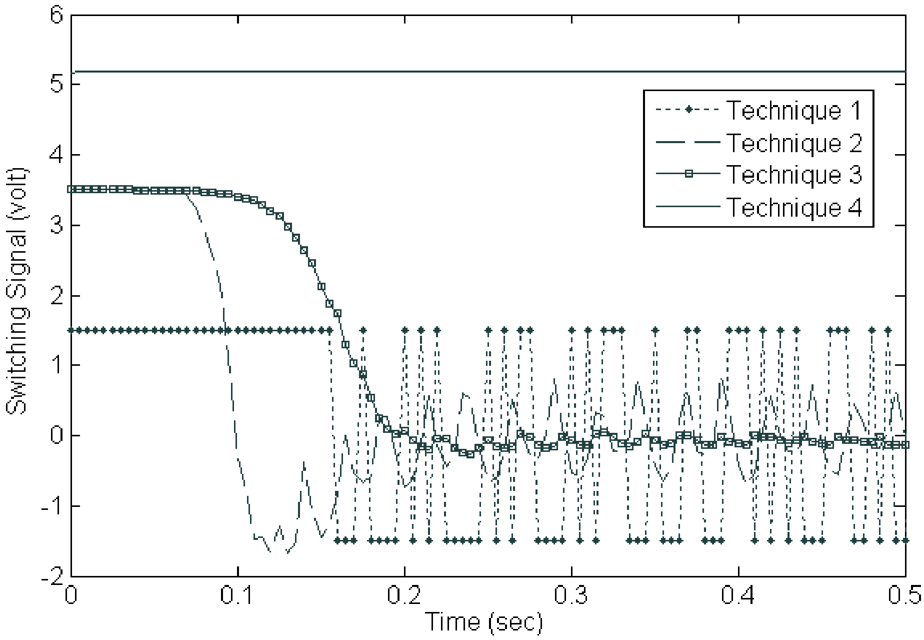

Figure 12. Switching signal of experimented SMC techniques, Technique 1: Classical SMC [1], Technique 2: SMC with PID sliding surface [5], Technique 3: SMC for stable systems [7], Technique 4: SMC to regulate nonlinear chemical systems [12]

Performance of the experimented techniques can be also measured with the index of total variation of the control signal [18]:

$$\Delta_{TV} = \sum_{i=1}^{\infty} \left| u(i+1) - u(i) \right| \tag{11}$$

where u is the control variable and subscript i refers to sampled values.

The variation at the controller output, both in transient-state and steady-state, affects the energy consumption of the actuator. Also, the magnitude of the total variance shows the smoothness of the control signal. Therefore, the controllers should be designed to generate control actions with Δ_{TV} as small as possible [18]. In other words, minimization of Δ_{TV} can be performed by means of reducing the chattering in the control signal. In an ideal controller, the control signal at different times in the steady-state is equal for the same set-point [18].

Further analysis on the performance of the techniques can be performed by measuring the standard deviation of the tracking error, σ, as follows:

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(e(i)-\mu)} \qquad (12)$$

where N is the sample number, e denotes the tracking error and μ is the mean value of tracking error.

The standard deviation is a useful property to understand the variation of the tracking error. In Table 2, the total variance and standard deviation measurements are given. The smallest total variance in magnitude was measured in the technique [12] since the proposed controller produced saturated control signal in the transient-state due to the output limitation of the DAQ (±10 Volts). In fact, the output of the controller in [12] produces high control signal resulting high Δ_{TV}. The minimum deviation of the tracking error was also measured in the technique [12] that results in most accurate control signal was produced by the controller.

Table 2. Performance specifications

SMC Techniques	Total Variance of Control Signal	Standard Deviation of Tracking Error
Technique 1 [1]	647.39	0.8434
Technique 2 [5]	1086.20	0.6390
Technique 3 [7]	40.37	0.6581
Technique 4 [12]	16.43	0.5403

The performance of the techniques were also compared with the performance indices, such as [20,21]:

- IAE (Integral Absolute Error) : $\int |e(t)|\,dt$
- ISCI (Integral Squared Control Input) : $\int u^2(t)dt$
- ISE (Integral Squared Error) : $\int e^2(t)dt$

Table 3. Results of performance indices

SMC Techniques	IAE	ISCI	ISE
Technique 1 [1]	0.7892	85.225	2.2854
Technique 2 [5]	0.4876	100.426	1.2578
Technique 3 [7]	0.4591	87.259	1.3622
Technique 4 [12]	0.5241	85.834	1.2382

The maximum integral absolute error and integral squared error were measured in [1] which means that that the magnitude of error deviation was maximum. On the other hand, the integral squared of control input was measured nearly the same as in [1,7,12].

5. Conclusions

In this study, selected first-order sliding-mode control techniques have been applied to an electromechanical system experimentally to investigate the applicability of the proposed techniques. A second-order model is approximated to use in the experiments since most of real systems can be represented by a second-order model. Step response, control signal and switching signal variations, error versus derivative of error graphs were obtained to compare the performances of the techniques. During the experiments, the parameters were tuned manually since the presence of chattering may cause a harmful effect on the system components.

Based on the experimental results and time-domain analysis tabulated in Table 1 and Table 2, it is clear that the techniques presented in [5,7,12] have produced better results than the technique presented in [1]. The maximum magnitude of chattering is observed in the Utkin's technique [1] that may be harmful for fast actuators if the switching gain is not adjusted properly. However, the other techniques have less chattering in the control signal which can be acceptable for the real systems. Since the tracking error converges exponentially to zero under uncertainties, the SMC techniques presented in [5,7,12] can be candidate to use in industrial applications as alternative to commonly used PID controller. In addition, the first-order sliding-mode control algorithm has systematic solution. Therefore, it is easy to understand and apply to real systems. Several works on SMC techniques were summarized in [2,4,22]

6. Acknowledgements

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8. APPENDIX

Mathematical Model of the Electromechanical System

The electromechanical system consists of a DC motor and a tachogenerator connected via a shaft. There are various speed transducers and a position sensor on the shaft which can be designated as the load on the shaft. The electrical and mechanical equations of the system are as follows [19]:

$$v_a(t) = L_a \frac{d}{dt} i_a(t) + R_a i_a(t) + K_m \omega_m(t) \tag{13}$$

$$J_m \left(\frac{d}{dt} \omega_m(t) \right) = T_m(t) - T_s(t) - R_m \omega_m(t) - T_f(\omega_m) \tag{14}$$

$$J_L \left(\frac{d}{dt} \omega_L(t) \right) = T_s(t) - R_L \omega_L(t) - T_d(t) - T_f(\omega_L) \tag{15}$$

$$T_s(t) = k_s(\theta_m(t) - \theta_L(t)) - B_s(\omega_m(t) - \omega_L(t)) \tag{16}$$

$$\frac{d}{dt}\theta_m(t) = \omega_m(t), \;\; \frac{d}{dt}\theta_L(t) = \omega_L(t) \tag{17}$$

where

- v_a : armature voltage of the motor,
- R_a : armature resistance,
- L_a : armature inductance,
- i_a : armature current,
- $\omega_{m,}\ \omega_L$: rotational speeds of the motor,
- $J_{m,}\ J_L$: moments of inertia,
- $R_{m,}\ R_L$: coefficients of the viscous friction.
- K_m : torque coefficient,
- T_m : generated motor torque,
- T_d : external load disturbance,
- T_f : nonlinear friction,
- T_s : transmitted shaft torque,

Model of the nonlinear friction T_f can be obtained by an asymmetrical characteristic as:

$$T_f(\omega) = \left(\alpha_0 + \alpha_1 e^{-\alpha_2|\omega|}\right)\operatorname{sgn}1(\omega) + \left(\alpha_3 + \alpha_4 e^{-\alpha_5|\omega|}\right)\operatorname{sgn}2(\omega) \quad (18)$$

where $\alpha_1 - \alpha_5$ are positive constants and $\alpha_0 \neq \alpha_3$, $\alpha_1 \neq \alpha_4$, $\alpha_2 \neq \alpha_5$ and the functions sgn1 and sgn2 are defined as:

$$\left.\begin{aligned} \operatorname{sgn}1(\omega) &= \begin{cases} 1 \text{ if } \omega \geq 0 \\ 0 \text{ if } \omega < 0 \end{cases} \\ \operatorname{sgn}2(\omega) &= \begin{cases} 0 \text{ if } \omega \geq 0 \\ \text{-1 if } \omega < 0 \end{cases} \end{aligned}\right\} \quad (19)$$

Block diagram of the electromechanical system is illustrated in Fig.13.

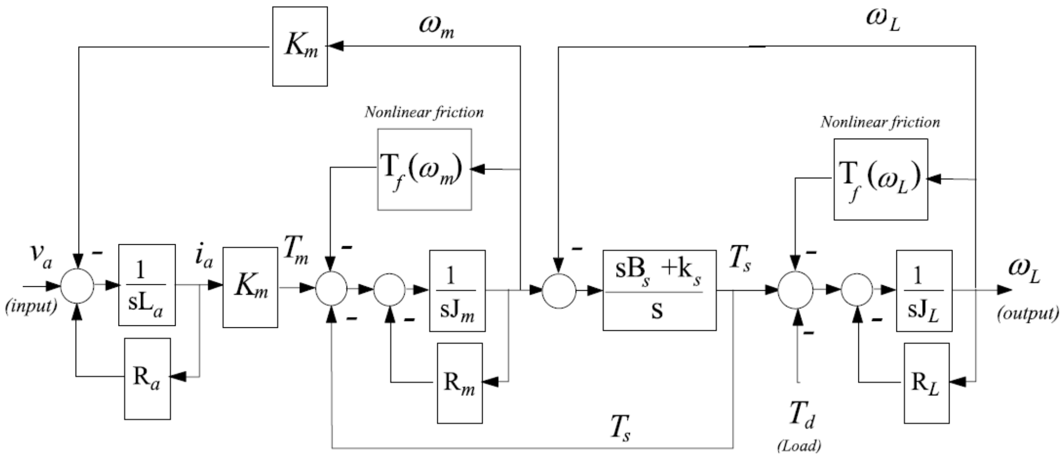

Figure 13. Block diagram of the electromechanical system