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Nearness Γ-Near Rings

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Abstract: The aim of this study is to introduce nearness Γ-near ring, nearness Γ-subnear ring and nearness Γ-ideal. Moreover, some properties of these structures are investigated.

Keywords: Nearness ring, nearness Γ-near ring, nearness Γ-ideal.

1. Introduction

A generalization of rough sets, near sets and near approximation spaces were introduced in 2007 [12, 20]. The selection of probe functions that provide a basis for defining and distinguishing affinities between objects is the first step in near set theory. A probe function is a real-valued function representing a feature of objects such as images.

Instead of abstract points, the sets in the nearness approximation space are mainly composed of perceptual objects (non-abstract points). Perceptual objects are featured points. Feature vectors can be used to describe these points [12]. The upper approximation of a set is determined by matching descriptions of objects in the set of perceptual objects. The consideration of upper approximations of perceptual object subsets is a fundamental method in algebraic structures built on nearness approximation space. In a nearness groupoid, the binary operation has the closeness property in upper approximation of set instead of set.

In 1936, Zassenhaus defined the near-ring as a generalization of ring [21]. The most basic source in near ring theory is Pilz's book titled *Near Rings* [15].

Nobusawa defined the idea of a Γ-ring that is more general than a ring [9]. Barnes weakened the axioms in Nobusawa's description of the Γ-ring [1]. Barnes, Kyuno [6] and Luh [7] investigated the structure of Γ-rings and discovered a number of generalizations that are analogous to ring theory.

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Satyanarayana defined the Γ-near ring as a generalization of near-ring and Γ-ring [16].

In 2012, İnan and Öztürk [3, 4] investigated the nearness groups. In 2013, nearness group of weak cosets was introduced [11]. In 2015, İnan et al. [5] also investigated the nearness semigroups. In 2019, nearness ring was introduced as well [10].

The aim of this study is to introduce nearness Γ-near ring, nearness Γ-subnear ring and nearness Γ-ideal. Moreover, some properties of these structures are investigated.

2. Preliminaries

Perceptual objects are points that are describable with feature vectors. Let $\mathcal{O}$ be a set of perceptual objects, $X \subseteq \mathcal{O}$, $\mathcal{F}$ be a set of probe functions and $\Phi : \mathcal{O} \longrightarrow \mathbb{R}^L$ be a mapping, where the description length is $|\Phi| = L$.

$\Phi(x) = (\varphi_1(x), \varphi_2(x), \varphi_3(x), \cdots, \varphi_i(x), \cdots, \varphi_L(x))$ is an object description of $x \in X$ such that each $\varphi_i \in B \subseteq \mathcal{F}$ $(\varphi_i : \mathcal{O} \longrightarrow \mathbb{R})$ is a probe function that represents features of sample objects $X \subseteq \mathcal{O}$ [12].

Sample objects are near each other if and only if the objects have similar descriptions. Recall that each φ_i defines a description of an object. Δ_{φ_i} is defined by $\Delta_{\varphi_i} = |\varphi_i(x') - \varphi_i(x)|$, where $x, x' \in \mathcal{O}$.

Let $x, x' \in \mathcal{O}$ and $B \subseteq \mathcal{F}$.

$$\sim_B = \{(x, x') \in \mathcal{O} \times \mathcal{O} \mid \Delta_{\varphi_i} = 0 \text{ for all } \varphi_i \in B\}$$

is called the indiscernibility relation on $\mathcal{O}$, where description length is $i \leq |\Phi|$ [12].

Definition 2.1 *[8] Let $\mathcal{O}$ be a set of perceptual objects, Φ be an object description and $A \subseteq \mathcal{O}$. Then the set description of A is defined as*

$$Q(A) = \{\Phi(a) \mid a \in A\}.$$

Definition 2.2 *[8, 14] Let $\mathcal{O}$ be a set of perceptual objects and A, $B \subseteq \mathcal{O}$. Then the descriptive (set) intersection of A and B is defined as*

$$A \underset{\Phi}{\cap} B = \{x \in A \cup B \mid \Phi(x) \in \mathcal{Q}(A) \text{ and } \Phi(x) \in \mathcal{Q}(B)\}.$$

If $Q(A) \cap Q(B) \neq \varnothing$, then A is called descriptively near B and denoted by $A\delta_\Phi B$. Also, $\xi_\Phi(A) = \{B \in \mathcal{P}(\mathcal{O}) \mid A\delta_\Phi B\}$ is a descriptive nearness collection [13].

Definition 2.3 *[12] Let $X \subseteq \mathcal{O}$ and $x \in X$.*

$$[x]_{B_r} = \{x' \in \mathcal{O} \mid x \sim_{B_r} x'\}$$

is called nearness class of $x \in X$.

Definition 2.4 *[12] Let* $X \subseteq \mathcal{O}$.

$$N_r(B)^* X = \bigcup_{[x]_{B_r} \cap X \neq \varnothing} [x]_{B_r}$$

is called upper approximation of X.

A nearness approximation space is $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B), \nu_{N_r})$, where $\mathcal{O}$ is a set of perceptual objects, $\mathcal{F}$ is a set of probe functions, "$\sim_{B_r}$" is an indiscernibility relation relative to $B_r \subseteq B \subseteq \mathcal{F}$, $N_r(B)$ is a collection of partitions and $\nu_{N_r} : \wp(\mathcal{O}) \times \wp(\mathcal{O}) \longrightarrow [0,1]$ is an overlap function that maps a pair of sets to $[0,1]$ representing the degree of nearness between sets. The subscript r denotes the cardinality of the restricted subset B_r.

Definition 2.5 *[3] Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B), \nu_{N_r})$ *be a nearness approximation space and "*$\cdot$*" be a binary operation defined on* $\mathcal{O}$. $G \subseteq \mathcal{O}$ *is called a nearness group if the following properties are satisfied:*

(NG_1) *For all* $x, y \in G$, $x \cdot y \in N_r(B)^* G$,

(NG_2) *For all* $x, y, z \in G$, $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ *property holds in* $N_r(B)^* G$,

(NG_3) *There exists* $e_G \in N_r(B)^* G$ *such that* $x \cdot e_G = e_G \cdot x = x$ *for all* $x \in G$ *(*e_G *is called the near identity element of* G*),*

(NG_4) *There exists* $y \in G$ *such that* $x \cdot y = y \cdot x = e_G$ *for all* $x \in G$ *(*y *is called the inverse of* x *in* G *and denoted as* x^{-1}*).*

Additionally, if the property $x \cdot y = y \cdot x$ is satisfied in $N_r(B)^* G$ for all $x, y \in G$, then G is said to be a commutative nearness group.

Also, $S \subseteq \mathcal{O}$ is called a nearness semigroup if $x \cdot y \in N_r(B)^* S$ for all $x, y \in S$ and $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ property is satisfied in $N_r(B)^* (S)$ for all $x, y, z \in S$.

Theorem 2.6 *[4] Let* G *be a nearness group,* H *be a nonempty subset of* G *and* $N_r(B)^* H$ *be a groupoid. Then* $H \subseteq G$ *is a subnearness group of* G *if and only if* $x^{-1} \in H$ *for all* $x \in H$.

Definition 2.7 *[10] Let* $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B), \nu_{N_r})$ *be a nearness approximation space and "*$+$*" and "*$\cdot$*" be binary operations defined on* $\mathcal{O}$. $R \subseteq \mathcal{O}$ *is called a nearness ring if the following properties are satisfied:*

(NR_1) R *is a commutative nearness group with binary operation "*$+$*",*

(NR_2) R *is a nearness semigroup with binary operation "*$\cdot$*",*

(NR_3) *For all* $x, y, z \in R$,

$$x \cdot (y+z) = (x \cdot y) + (x \cdot z) \text{ and } (x+y) \cdot z = (x \cdot z) + (y \cdot z)$$

properties hold in $N_r(B)^* R$.

In addition,

(NR_4) *R is said to be a commutative nearness ring if* $x \cdot y = y \cdot x$ *for all* $x, y \in R$,

(NR_5) *R is said to be a nearness ring with identity if* $N_r(B)^* R$ *contains an element* 1_R *such that* $1_R \cdot x = x \cdot 1_R = x$ *for all* $x \in R$.

Definition 2.8 *[15, 21] Let N be a nonempty set and "$+$" and "$\cdot$" be binary operations defined on N. N is called a (right) near ring if the following properties are satisfied:*

(N_1) *N is a group with binary operation "$+$" (It does not need to be commutative),*

(N_2) *N is a semigroup with binary operation "$\cdot$",*

(N_3) *For all* $x, y, z \in N$, $(x+y) \cdot z = (x \cdot z) + (y \cdot z)$ *properties hold in* $N_r(B)^* N$.

Definition 2.9 *[1] A Γ-ring (in the sense of Barnes) is a pair (M, Γ), where M and Γ are (additive) commutative groups for which exists a _ : $M \times \Gamma \times M \to M$, the image of (a, α, b) being denoted by $a\alpha b$ for $a, b \in M$ and $\alpha \in \Gamma$, satisfying for all $a, b, c \in M$ and all $\alpha, \beta \in \Gamma$:*

- $(a+b)\alpha c = a\alpha c + b\alpha c$,
- $a(\alpha+\beta)b = a\alpha b + a\beta b$,
- $a\alpha(b+c) = a\alpha b + a\alpha c$,
- $(a\alpha b)\beta c = a\alpha(b\beta c)$.

Definition 2.10 *[1] Let M be a Γ-ring. A left (right) ideal of M is an additive subgroup U of M such that $M\Gamma U \subseteq U$ ($U\Gamma M \subseteq U$). If U is both a left and a right ideal, then we say that U is an ideal of M.*

Definition 2.11 *[16] A Γ-near ring is a triple $(M, +, \Gamma)$, where*

(ΓN_1) *$(M, +)$ is a group (need not be commutative),*

(ΓN_2) *Γ is a non-empty set of binary operators on M such that $(M, +, \gamma)$ is a (right) near ring for each $\gamma \in \Gamma$,*

(ΓN_3) $(a\alpha b)\beta c = a\alpha(b\beta c)$ *for all* $a, b, c \in M$ *and* $\alpha, \beta \in \Gamma$.

Definition 2.12 *[19]Let $(\mathcal{O}, \mathcal{F}, \sim_{B_r}, N_r(B), \nu_{N_r})$ be a nearness approximation space and $M, \Gamma \subseteq \mathcal{O}$ be an additive commutative nearness groups in $\mathcal{O}$. $M \subseteq \mathcal{O}$ is named an Γ-ring in nearness approximation space or shortly, nearness Γ-ring if the followings are provided:*

$(N\Gamma_1)$ $a\alpha b \in N_r(B)^* M$,

$(N\Gamma_2)$ $(a\alpha b)\beta c = a\alpha(b\beta c)$ *property verify on* $N_r(B)^* M$,

$(N\Gamma_3)$ $(a+b)\alpha c = a\alpha c + b\alpha c$, $a(\alpha+\beta)b = a\alpha b + a\beta b$, $a\alpha(b+c) = a\alpha b + a\alpha c$ *properties verify on* $N_r(B)^* M$ *for all* $a,b,c \in M$ *and all* $\alpha,\beta \in \Gamma$.

In addition, M *is called a commutative nearness* Γ*-ring if* $a\alpha b = b\alpha a$ *for all* $a,b \in M$ *and all* $\alpha \in \Gamma$.

M *is called a nearness* Γ*-ring with identity if* $N_r(B)^* M$ *contains* 1_M *such that* $1_M\alpha a = a\alpha 1_M = a$ *for all* $a \in M$ *and all* $\alpha \in \Gamma$.

3. Nearness Γ-near rings

Throughout this section, $\mathcal{O}$ considered as a set of perceptual objects in nearness approximation space unless otherwise stated.

Definition 3.1 *Let* $N,\Gamma \subseteq \mathcal{O}$ *be additive nearness groups. If for all* $k,\ell,m \in N$ *and all* $\beta,\gamma \in \Gamma$ *the conditions*

$(\mathcal{N}\Gamma N_1)$ $k\beta\ell \in N_r(B)^* N$,

$(\mathcal{N}\Gamma N_2)$ $(k+\ell)\beta m = k\beta m + \ell\beta m$ *property provides on* $N_r(B)^* N$,

$(\mathcal{N}\Gamma N_3)$ $(k\beta\ell)\gamma m = k\beta(\ell\gamma m)$ *property provides on* $N_r(B)^* N$

are satisfied, then N *is called an* Γ*-near ring in nearness approximation space or shortly nearness* Γ*-near ring.*

In addition, if $k\beta\ell = \ell\beta k$ *for all* $k,\ell \in N$ *and all* $\beta \in \Gamma$, *then* N *is called a commutative nearness* Γ*-near ring.*

Example 3.2 $\mathcal{O} = \{k_{ij} \mid 0 \le i,j \le 4\}$ *be a set of perceptual objects and* $B = \{\varphi\} \subseteq \mathcal{F}$ *be a set of probe function. Probe function*

$$\varphi : \mathcal{O} \longrightarrow V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$$

is given in Table 1.

Table 1

	k_{00}	k_{01}	k_{02}	k_{03}	k_{04}	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}
φ	v_1	v_2	v_3	v_5	v_5	v_4	v_4	v_5	v_6	v_7

	k_{20}	k_{21}	k_{22}	k_{23}	k_{24}	k_{30}	k_{31}	k_{32}	k_{33}	k_{34}
φ	v_7	v_6	v_6	v_8	v_7	v_8	v_6	v_7	v_8	v_8

	k_{40}	k_{41}	k_{42}	k_{43}	k_{44}
φ	v_8	v_5	v_7	v_8	v_3

Thus

$$\begin{aligned}[k_{00}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{00}) = v_1\} \\ &= \{k_{00}\},\end{aligned}$$

$$\begin{aligned}[k_{01}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{01}) = v_2\} \\ &= \{k_{01}\},\end{aligned}$$

$$\begin{aligned}[k_{02}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{02}) = v_3\} \\ &= \{k_{02}, k_{44}\} = [k_{44}]_\varphi,\end{aligned}$$

$$\begin{aligned}[k_{03}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{03}) = v_5\} \\ &= \{k_{03}, k_{04}, k_{12}, k_{41}\} \\ &= [k_{04}]_\varphi = [k_{12}]_\varphi = [k_{41}]_\varphi,\end{aligned}$$

$$\begin{aligned}[k_{10}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{10}) = v_4\} \\ &= \{k_{10}, k_{11}\} = [k_{11}]_\varphi,\end{aligned}$$

$$\begin{aligned}[k_{13}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{13}) = v_6\} \\ &= \{k_{13}, k_{21}, k_{22}, k_{31}\} \\ &= [k_{21}]_\varphi = [k_{22}]_\varphi = [k_{31}]_\varphi,\end{aligned}$$

$$\begin{aligned}[k_{14}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{14}) = v_7\} \\ &= \{k_{14}, k_{20}, k_{24}, k_{32}, k_{42}\} \\ &= [k_{20}]_\varphi = [k_{24}]_\varphi = [k_{32}]_\varphi = [k_{42}]_\varphi,\end{aligned}$$

$$\begin{aligned}[k_{23}]_\varphi &= \{k \in \mathcal{O} \,|\varphi(k) = \varphi(k_{23}) = v_8\} \\ &= \{k_{23}, k_{30}, k_{33}, k_{34}, k_{40}, k_{43}\} \\ &= [k_{30}]_\varphi = [k_{33}]_\varphi = [k_{34}]_\varphi = [k_{40}]_\varphi = [k_{43}]_\varphi.\end{aligned}$$

Therefore

$$\xi_\varphi = \left\{[k_{00}]_\varphi, [k_{01}]_\varphi, [k_{02}]_\varphi, [k_{03}]_\varphi, [k_{10}]_\varphi, [k_{13}]_\varphi, [k_{14}]_\varphi, [k_{23}]_\varphi\right\}.$$

Hence, a set of partitions of $\mathcal{O}$ *is* $N_1(B) = \{\xi_\varphi\}$ *for* $r = 1$. *Thus*

$$\begin{aligned}N_1(B)^* N &= \bigcup_{[k]_\varphi \cap N \neq \emptyset} [k]_\varphi \\ &= \{k_{00}\} \cup \{k_{01}\} \cup \{k_{10}, k_{11}\} \\ &= \{k_{00}, k_{01}, k_{10}, k_{11}\}\end{aligned}$$

and

$$\begin{aligned}N_1(B)^* \Gamma &= \bigcup_{[k]_\varphi \cap \Gamma \neq \emptyset} [k]_\varphi \\ &= \{k_{00}, k_{02}, k_{44}\},\end{aligned}$$

where $N = \{k_{00}, k_{01}, k_{10}\}$, $\Gamma = \{k_{00}, k_{02}\} \subseteq \mathcal{O}$.

Let

$$\begin{aligned}+_1 \quad : \quad & \mathcal{O} \times \mathcal{O} && \longrightarrow \mathcal{O} \\ & (k_{ij}, k_{mn}) && \longmapsto k_{ij} +_1 k_{mn}\end{aligned}$$

be a binary operation (first addition) on $\mathcal{O}$ *such that*

$$k_{ij} +_1 k_{mn} = k_{pr}, \quad i + m \equiv p \pmod 2 \text{ ve } j + n \equiv r \pmod 2.$$

Then $(N, +_1)$ *is a nearness group.*

Furthermore, let

$$+_2 \quad : \quad \begin{array}{ll} \mathcal{O} \times \mathcal{O} & \longrightarrow \mathcal{O} \\ (k_{ij}, k_{mn}) & \longmapsto k_{ij} +_2 k_{mn} \end{array}$$

be a binary operation (second addition) on $\mathcal{O}$ *such that*

$$k_{ij} +_2 k_{mn} = k_{st}, \quad i + m \equiv s \pmod 4 \text{ ve } j + n \equiv t \pmod 4 \quad .$$

Then $(\Gamma, +_2)$ *is a nearness group.*

Since $k_{01} + k_{10} = k_{11} \notin N$, $N \subseteq \mathcal{O}$ *is not a group with binary operation "*$+_1$*" and so* N *is not a* Γ*-near ring.*

Let

$$\begin{array}{ll} \mathcal{O} \times \Gamma \times \mathcal{O} & \longrightarrow \mathcal{O} \\ (k_{ij}, k_{uv}, k_{mn}) & \longmapsto k_{ij} k_{uv} k_{mn} = k_{ij} \end{array}$$

be an operation on $\mathcal{O}$.

From Definition 3.1, it is easily shown that

$(\mathcal{N}\Gamma N_1)$ $k\beta\ell \in N_r(B)^* N$,

$(\mathcal{N}\Gamma N_2)$ $(k +_1 \ell)\beta m = k\beta m +_1 \ell\beta m$ *property provides on* $N_r(B)^* N$,

$(\mathcal{N}\Gamma N_3)$ $(k\beta\ell)\gamma m = k\beta(\ell\gamma m)$ *property provides on* $N_r(B)^* N$

for all $k, \ell, m \in N$ *and all* $\beta, \gamma \in \Gamma$.

Consequently, N *is a nearness* Γ*-near ring.*

Lemma 3.3 is obvious since $N \subseteq N_r(B)^* N$.

Lemma 3.3 *Every* Γ*-near ring is a nearness* Γ*-near ring.*

From definition of nearness Γ-ring, it is clear that Lemma 3.4 is true.

Lemma 3.4 *Every nearness* Γ*-ring is a nearness* Γ*-near ring.*

A nearness Γ-near ring is not always a Γ-near ring, and also a nearness Γ-near ring is not always a nearness Γ-ring.

Examples 3.5 and 3.6 are show that the opposites of the Lemma 3.3 and Lemma 3.4 are not true.

Example 3.5 *From Example 3.2 N is a nearness Γ-near ring. But N is not a Γ-near ring because of $k_{01} +_1 k_{10} = k_{11} \notin N$ for $k_{01}, k_{10} \in N$.*

Example 3.6 *From Example 3.2 N is a nearness Γ-near ring. But N is not a nearness Γ-ring because of $k_{10}k_{02}(k_{01} +_1 k_{10}) \neq (k_{10}k_{02}k_{01}) +_1 (k_{10}k_{02}k_{10})$ for $k_{10}, k_{01} \in N$ and $k_{02} \in \Gamma$.*

Lemma 3.7 *Let $N \subseteq \mathcal{O}$ be a nearness Γ-near ring and $0_N \in N$. If $0_N\gamma k \in N$ then*

(i) $0_N\gamma k = 0_N$,

(ii) $(-k)\gamma\ell = -(k\gamma\ell)$ for all $k, \ell \in N$ and all $\gamma \in \Gamma$.

Proof (i) For all $k \in N$ and all $\gamma \in \Gamma$,

$$0_N\gamma k = (0_N + 0_N)\gamma k = 0_N\gamma k + 0_N\gamma k.$$

Since the near identity element is unique, $0_N\gamma k = 0_N$.

(ii) From (i), $0_N\gamma\ell = 0_N$ for all $k, \ell \in N$ and all $\gamma \in \Gamma$. Then

$$0_N = 0_N\gamma\ell = ((-k) + k)\gamma\ell = (-k)\gamma\ell + k\gamma\ell.$$

Since the inverse element is unique, $(-k)\gamma\ell = -(k\gamma\ell)$. □

For all $k, \ell \in N$ and all $\gamma \in \Gamma$, the equalities $k\gamma 0_N = 0_N$ and $k\gamma(-\ell) = -(k\gamma\ell)$ may not be provided.

Definition 3.8 *Let N be a nearness Γ-near ring. The set*

$$N_0 = \{k \in N \mid k\gamma 0_N = 0_N,\ \gamma \in \Gamma\}$$

is called a zero symmetric part of N and the set

$$N_c = \{k \in N \mid k\gamma 0_N = k,\ \gamma \in \Gamma\}$$

is called a constant part of N.

If $N = N_0$, then N is called a zero symmetric nearness Γ-near ring. If $N = N_c$, then N is called a constant nearness Γ-near ring. The set of all zero symmetric nearness Γ-near rings is denoted by $\mathcal{N}_0$ and the set of all constant nearness Γ-near rings is denoted by $\mathcal{N}_c$.

If the condition $d\gamma(k+\ell) = d\gamma k + d\gamma\ell$ holds in $N_r(B)^* N$ for all $k, \ell \in N$ and all $\gamma \in \Gamma$ then d is called a distributive element. Also, the set of all nearness Γ-near ring with the identity is represented as $\mathcal{N}_1$ and the set of all distributive elements in N is represented as N_d. If $N = N_d$, then N is called a distributive nearness Γ-near ring.

Definition 3.9 *Let N be a nearness Γ-near ring and $(S,+)$ be a subnearness group of $(N,+)$. S is called a nearness Γ-subnear ring of N if $S\Gamma S \subseteq N_r(B)^* S$.*

Example 3.10 *Let N be a nearness Γ-near ring. Then N_0 and N_c are nearness Γ-subnear rings of N.*

Theorem 3.11 *Let $N,\Gamma \subseteq \mathcal{O}$, N be a nearness Γ-near ring, $S \subseteq N$ and $N_r(B)^* S$ be an additive groupoid and Γ-groupoid. Then S is a nearness Γ-subnear ring of N iff $-s \in S$ for all $s \in S$.*

Proof ($\Rightarrow$) Let S be a nearness Γ-subnear ring of N. Then $(S,+)$ is a nearness group and hence $-s \in S$ for all $s \in S$.

($\Leftarrow$) Let $-s \in S$ for all $s \in S$. Since $N_r(B)^* S$ is an additive groupoid, $(S,+)$ is a nearness group from Theorem 2.6. Therefore, since $N_r(B)^* S$ is a Γ-groupoid and $S \subseteq N$, $p\beta r, r\gamma s \in N_r(B)^* S$ and $(p\beta r)\gamma s = p\beta(r\gamma s)$ property holds in $N_r(B)^* S$ for all $p,r,s \in S$ and all $\beta,\gamma \in \Gamma$.

Furthermore, since $N_r(B)^* S$ is an additive groupoid, Γ-groupoid and N is a nearness Γ-near ring, $(p+r)\beta s = (p\beta s)+(r\beta s)$ property holds in $N_r(B)^* S$ for all $p,r,s \in S$ and all $\beta \in \Gamma$.

Consequently, S is a nearness Γ-subnear ring of N. □

Definition 3.12 *Let N be a nearness Γ-near ring and J be a subnearness group of $(N,+)$. Let $N_r(B)^* S$ be an additive groupoid and Γ-groupoid. Then J is called a nearness Γ-ideal of N if the following properties are satisfied:*

(1) $J\Gamma N = \{x\gamma k \mid x \in J, \gamma \in \Gamma, k \in N\} \subseteq N_r(B)^* J$,

(2) $k\gamma(l+x) - k\gamma l \in N_r(B)^* J$ for all $k,l \in N$, all $x \in J$ and all $\gamma \in \Gamma$.

Furthermore, J is called a right nearness Γ-ideal of N if only the condition (1) is satisfied. Also, J is called a left nearness Γ-ideal of N if only the condition (2) is satisfied.

Example 3.13 *From Example 3.2, let we consider nearness Γ-near ring $N = \{k_{00}, k_{01}, k_{10}\}$ and $J = N$. Since J is an additive nearness subgroup of N, $J\Gamma N = J$ by definition of the operation $\mathcal{O} \times \Gamma \times \mathcal{O} \longrightarrow \mathcal{O}$ from Example 3.2 and $J \subseteq N_r(B)^* J$, J is a right nearness Γ-ideal of N. Also, since $k\gamma(l+x) - k\gamma l \in N_r(B)^* J$ for all $k,l \in N$, all $x \in J$ and all $\gamma \in \Gamma$, J is a left nearness Γ-ideal of N. Hence J is a nearness Γ-ideal of N.*

Remark 3.14 *Every nearness Γ-ideal of N is also a nearness Γ-subnear ring of N.*

Theorem 3.15 *Let $N \subseteq \mathcal{O}$ be a nearness Γ-near ring, $I, J \subseteq N$ and $N_r(B)^*I$, $N_r(B)^*J$ be additive groupoids and Γ-groupoids. If I, J are both right (left) nearness Γ-ideals of N and $(N_r(B)^*I) \cap (N_r(B)^*J) = N_r(B)^*(I \cap J)$, then $I \cap J$ is also a right (left) nearness Γ-ideal of N.*

Proof Since I and J are both right nearness Γ-ideals of N, $I\Gamma N \subseteq N_r(B)^*I$ and $J\Gamma N \subseteq N_r(B)^*J$,

$$\begin{aligned}(I \cap J)\Gamma N &= \{x\gamma k \mid k \in N, \gamma \in \Gamma, x \in I \cap J\} \\ &= \{x\gamma k \mid k \in N, \gamma \in \Gamma, x \in I \text{ and } x \in J\} \\ &= \{x\gamma k \mid k \in N, \gamma \in \Gamma, x \in I\} \cap \{x\gamma k \mid k \in N, \gamma \in \Gamma, x \in J\} \\ &= I\Gamma N \cap J\Gamma N \\ &\subseteq (N_r(B)^*I) \cap (N_r(B)^*J) \\ &= N_r(B)^*(I \cap J).\end{aligned}$$

Therefore $(I \cap J)\Gamma N \subseteq N_r(B)^*(I \cap J)$, that is, $I \cap J$ is a right nearness Γ-ideal of N.

Let $x \in I \cap J$. Since I and J are both left nearness Γ-ideals of N, then $k\gamma(l + x) - k\gamma l \in N_r(B)^* I$ and $k\gamma(l + x) - k\gamma l \in N_r(B)^* J$ for all $k, l \in N$ and all $\gamma \in \Gamma$. Therefore $k\gamma(l + x) - k\gamma l \in (N_r(B)^*I) \cap (N_r(B)^*J)$ and so $k\gamma(l + x) - k\gamma l \in N_r(B)^*(I \cap J)$ from the hypothesis. Hence $I \cap J$ is a left nearness Γ-ideal of N. □

Corollary 3.16 *Let $N \subseteq \mathcal{O}$ be a nearness Γ-near ring, $J_i \subseteq N$ $(1 \leq i \leq n,\ n \geqslant 2)$ and $N_r(B)^*J_i$ be additive groupoids and Γ-groupoids. If J_i are right (left) nearness Γ-ideals of N and $\bigcap_{1 \leq i \leq n} N_r(B)^*J_i = N_r(B)^*(\bigcap_{1 \leq i \leq n} J_i)$, then $\bigcap_{1 \leq i \leq n} J_i$ is also a right (left) nearness Γ-ideal of N.*

Declaration of Ethical Standards

The authors declare that the materials and methods used in their study do not require ethical committee and/or legal special permission.

Authors' Contributions

Author [Baki Çokakoğlu]: Collected the data, contributed to research method or evaluation of data (%40).

Author [Mustafa Uçkun]: Thought and designed the research/problem, contributed to research method or evaluation of data, wrote the manuscript (%60).

Conflict of Interest

The authors declare no conflicts of interest.

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