Regional Frequency Analysis of 6 hours Maximum Rainfall over the Upper Euphrates–Tigris basins, Turkey

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Abstract: Accurately estimating design rainfall or in other words, probable maximum rainfall has a crucial efficiency on fulfilling the expected benefit from any hydraulic structure since the value predicted as design criteria directly influences its planning, management and cost. What amount of a hydro-climatic variable which takes place under the influence of many environmental factors in a given region would be in future time is estimated based on its statistical behaviour. In many efforts related to water resources, the curve (IDF) representing the relationship among intensity, duration frequency of rainfall is basis for design rainfall amount required in the construction of any water-related structure. The availability of information extracted the curve of IDF substantially depends upon the frequency analysis and reliability of the current data. In this sense, the reliability of the data is very important as well as frequency analysis. The 6 hours maximum rainfall amounts from 18 sites in Upper Euphrates–Tigris basins were used as a material for regional frequency analysis based on L-moments approach. The existence of discordant stations was checked with discordancy measure for whole sites in the study area. First, single homogeneous region was tried to be formed. However, due to the irregularity value of Mardin station being greater than the critical value, no single homogeneous region could be obtained. Clustering analysis method was applied to obtain sub homogeneous regions. According to the dentograms of clustering analysis, the basin was divided into two sub homogeneous regions. Mardin station has been ignored in the study due to its discordant in the sub-regions. The generalized extreme value distribution was selected as the most appropriate regional distribution for the sub-regions.

Keywords: Euphrates–Tigris basin, Maxima rainfall, L-moments

Maksimum 6 saatlik Yağışlara Göre Yukarı Fırat-Dicle Havzalarının Bölgesel Frekans Analizi

Öz: Tasarım kriteri olarak öngörülen değer doğrudan planlama yönetim ve maliyeti etkilediği için tasarım yağışlarını veya diğer bir ifadeyle olası maksimum yağış miktarını doğru tahmin etmek, herhangi bir hidrolojik yapıdan beklenen faydaya ulaşmada önemli bir etkiye sahiptir. Gelecekte oluşabilecek birçok çevresel faktörün etkisi altında yer alan belirli bölgedeki hidro-klimatik değişken miktarı istatistiksel davranışlara bağlı olarak tahmin edilir. Su kaynakları ile ilgili olarak birçok çalışmada yağışın şiddet-süre-frekansı (IDF) arasındaki ilişkiyi gösteren eğri su ile ilgili herhangi bir yapının inşasında gerekli olan tasarım yağışlarının temelidir. Bu durum IDF eğrisinden çıkarılan bilginin kullanılabilirliğinin önemli ölçüde mevcut verilerin frekans analizi ve güvenilirliği üzerine bağlı olduğunu vurgulamaktadır. Bu anlamda, frekans analizinin yanı sıra verilerin güvenilirliği çok önemlidir. L-moment yaklaşımına dayanan bölgesel frekans analizi için materyal olarak Yukarı Fırat-Dicle havzalarındaki 18 istasyonun 6 saatlik maksimum yağış miktarları kullanılmıştır. Uyumsuz istasyonların varlığı, çalışma alanındaki tüm istasyonlar için düzensizlik ölçüsü ile kontrol edilmiştir. İlk olarak tek homojen bölge oluşturulmaya çalışılmıştır. Ancak Mardin istasyonunun düzensizlik ölçüsü değeri kritik değerden büyük çıkmasından dolayı tek homojen bölge elde edilememiştir. Alt homojen bölgeler elde edebilmek için kümeleme analizi yöntemi uygulanmıştır. Kümeleme analizden elde edilen dentograma göre havza iki alt homojen bölgeye ayrılmıştır. Mardin istasyonunun her iki bölgede düzensizliğinden dolayı çalışmada gözardı edilmiştir. Alt bölgeler için en uygun dağılım olarak genelleştirilmiş ektrem değer dağılımı seçilmiştir.

Anahtar Kelimeler: Fırat-Dicle havzası, Maksimum yağış, L-moment

1. Introduction

Planning and management of water resources and designing suitable projects to be used in the design of any hydrological structures are very important. Often due to insufficient hydrological measurements, it is difficult to predict appropriate project criteria. Nowadays, statistical approaches are widely used to overcome this problem. These methods are able to estimate the expected values on both a point and regional basis. Point frequency analysis is easier than regional one. The results obtained by point frequency analysis are less reliable. Rainfall data from the raingauge stations with similar characteristics are used to solve the problems mentioned above. Thus, the reliability of the obtained data is increased. Rain gauge stations having long-term data are used in regional frequency analysis. The regional frequency analysis is carried out based on all of the data obtained from the different stations, the observations of which have similar frequencies. Recently, researchers focusing on frequency analysis of hydrologic data have started to use the L-moment method developed by Hosking in regional frequency analysis. The L-moments being a linear combinations of the probability weighted moments (PWM) is widely used method. The method includes regionalization, discordancy measure, heterogeneity measure and L-moment algorithm. The L-moment method was prefered used in many countries. In Turkey, Yürekli et al. (2009) applied the L-moment method to the 7-day minimum flow series in the Çekerek Stream. The Generalized Pareto distribution (GPA) was selected as the most suitable regioanal probability distriution. Anlı et al. (2011) used 44 rain gauge stations in Konya Closed Basin for their study. The basin were divided into three regions with L-moment approach. Pearson Type 3 (P3) distribution for Region 1, Generalized Extreme Value (GEV) distribution for Region 2 and Generalized Normal distribution for Region 3 were selected as regional distribution.

In the study, it was aimed to apply the regional frequency analysis to the 6 hour maximum rainfall data measured in the Euphrates- Tigris basins using the L-moment method.

2. Materıal and Method

The 6 hour of maximum rainfall data from 18 rain gauging stations with recording lengths ranging from 43 to 73 years in the Upper Euphrates-Tigris basins were used. Some characteristics related to the rain gauging stations are given in Table 1.

Table 1. Some Characteristic of Rain Gauging Stations
Çizelge 1. Yağmur ölçer istasyonlarının bazı özellikleri

Gauging Station	Observation Period (year)	Latitude	Longitude	Elevation (m)	Annual Maxima Rainfall (mm)
Adıyaman	50	37° 44'	38° 13'	711	62.9
Ağrı	48	39° 43'	43° 3'	1672	47.6
Batman	46	37° 56'	41° 15'	650	36.9
Bingöl	49	38° 53'	40° 30'	1183	56
Bitlis	49	38° 24	42° 6'	1545	51
Diyarbakır	73	37° 55'	40° 13'	773	43.9
Elazığ	58	38° 40'	39° 14'	1027	39.1
Erzincan	57	39° 44'	39° 27'	1238	34.5
Erzurum	56	39° 32'	41° 9'	1915	38.6
Gaziantep	58	37° 2'	37° 13'	886	69.9
Hakkari	43	37° 34'	43° 46'	1758	40.5
Kilis	49	36° 42'	37° 6'	681	60.2
Malatya	54	38° 21'	38° 18'	985	39.5
Mardin	48	37° 19'	40° 42'	1064	76.3
Muş	49	38° 44'	41° 29'	1287	52.9
Siirt	56	37° 55'	41° 55'	916	57.4
Şanlıurfa	56	37° 10'	38° 45'	680	72.9
Tunceli	47	39° 6'	39° 32'	1019	44.6

2.1. The method of L-moments

L moments, which describes the shapes of frequency distributions, have little sensitivity with respect to normal product moments in a long recorded data. L-moments approach estimates the characteristics and parameters related to a given hydrologic data set in a simple and effective way. The L-moment summarizes the statistically probability dispersion of similar data (Hosking, 1990; Eslamian and Feizi, 2007).

L-moment of the x series is expressed as the function of probability weighted moment (Anlı, 2011). The probability weighted moments obtained from the sequenced observations as neutral sample estimate is defined in Equation 1 by Greenwood et al. (1979).

$$b_r = n^{-1} \sum_{j=1}^{n} x_{(j)} \frac{(j-1)(j-2)...(j-1)}{(n-1)(n-2)...(n-i)} \tag{1}$$

Where the initial four of b_r value for (r=0,1,2,3) is found probability weighted moments (b_0, b_1, b_2, b_3). For whatever distribution the initial four L-moments are handily computed from PWM using;

$\ell_1 = b_0,$
$\ell_2 = 2b_1 - b_0,$
$\ell_3 = 6b_2 - 6b_1 + b_0,$
$\ell_4 = 20b_3 - 30b_2 + 12b_1 - b_0. \tag{2}$

The dimensionless L moment ratios including L-coefficient of variation, L-skewness and L-kurtosis are estimated by using Equation 3, respectively.

$t = \ell_2 / \ell_1$ (L-CV)
$t_3 = \ell_3 / \ell_2$ (L-S)
$t_4 = \ell_4 / \ell_2$ (L-K) $\tag{3}$

2.2. Discordancy measure

This measure provides the detection of discordant stations as a whole from a group of stations. Discordant site (s) are determined with Equation 4.

$$D_i = \frac{1}{3} N (u_i - \bar{u})^T K^{-1} (u_i - \bar{u}) \tag{4}$$

Where u_i is vector of the L-moment ratios for any station, K is covariance matrix of vector. $\bar{u}$ is average of vector. The discordancy in a region is determined according to the critical value corresponding to the number of stations belonging to a region. The critical values is shown in Table 2.

2.3. Heterogeneity measure

To test whether the selected region is homogeneous after discordancy test, the homogeneity of the region is evaluated using homogeneity measures. Homogeneity measures are depent on the simulation of 500 homogeneous regions with population parameters equal to the regional average sample l-moment ratios (Hosking and Wallis, 1997). The value of H statistic is given in Equation 5.

$$H = \frac{(V - \mu_v)}{\sigma_v} \tag{5}$$

The value of H-statistic states that the region under consideration is acceptably homogeneous when H < 1, probably heterogeneous when $1 \leq H \leq 2$, and certainly heterogeneous when $H \geq 2$ (Hosking and Wallis, 1997).

Table 2. Critical values for Discordancy Measure
Çizelge 2. Düzensizlik ölçüsü için kritik değerler

Number of station	Critical Value	Number of station	Critical Value
5	1.333	10	2.491
6	1.648	11	2.632
7	1.917	12	2.757
8	2.140	13	2.869
9	2.329	14	2.971
		≥15	3

2.4. Goodness of fit test

In regional frequency analysis, the best fit shows a single probability distribution for the data which obtained from the stations in the selected homogeneous region. The most appropriare regional frequency distribution is

chosen according to the goodness-of-fit-test (Z^{DIST}). This statistic are written as:

$$Z^{DIST} = \left(\tau_4^{DIST} - t_4^R + B_4\right)/\sigma_4 \qquad (6)$$

$$B_4 = N_{sim}^{-1} \sum_{m=1}^{N_{sim}} \left(t_4^{(m)} - t_4^R\right) \qquad (7)$$

In Equations, τ_4^{DIST} is the population l-kurtosis of selected distribution, t_4^R is regional average L-kurtosis ratio of sample, B_4 is the bias of regional average sample l-kurtosis, and σ_4 is the standard deviation of regional average sample L-kurtosis. N_{sim} is the number of simulations performed with the help of Kappa distribution, *m* is the number of region performed simulation.

In this study, general logistic (GLO), general extreme value (GEV), general normal (GNO), pearson type 3 (PE3) and general pareto (GPA) distributions has been used. The $|Z^{DIST}| \leq 1.64$ should be for an appropriate regional distribution. But, the distribution giving the minimum $|Z^{DIST}|$ is considered as the best-fit distribution for the region.

2.5. Regional L-moment algorithm

The well-known approach, called as index-flood method in a streamflow analysis and index-storm in a precipitation analysis has been commonly used in regional quantile estimates belonging to hydro-meteorologic data. Due to the use of rainfall data in the study, the approach will be hereafter referred to as index-storm method. Mathematically, the quantile estimates at site i for a region with N sites are calculated by

$$Q_i(F) = \mu_i q(F) \qquad (9)$$

Where μ_i is index rainfall (a site-specific scaling factor) value for site i, F is non-exceedance probability, and q is dimensionless distribution function (growth curve). The regional frequency analysis of 6 hour maximum precipitation measured in the Upper Euphrates-Tigris basin was achieved by using the FORTRAN routines developed by Hosking (1996).

3. Results and Discussion

To realize the regional frequency analysis, some basic L-moment ratios, which are L-mean (λ1), L-coefficient of variation (τ2), L-skewness (τ3) and L-kurtosis (τ4) of the basin stations have been calculated (Table 3). Hosking (1990) imply that L-moment ratios of a series are bounded.

L-coefficient of variation (L-CV), L-skewness and L-kurtosis are $0 < \tau_2 < 1$, $-1 < \tau_3 < 1$ ve $1/4(5\tau_3^2 - 1) \leq \tau_4 < 1$, respectively (Yürekli, 2005). These conditions have been ensured in Table 3.

The discordancy measure (D_i) is determined by using the calculated L-moment ratios to form homogeneous regions (Table 3). In the first stage, all stations were considered as whole region. The cluster analysis was performed because of the discordant stations in the whole region. In the Minitab-17 package program, clustering analysis was performed according to Ward connectivity and Euclidean distance measure. The stations (sites) based on the analysis were divided into two regions. The clustering analysis was carried out using latitude, longitude, elevation values and the long-term averages of annual maximum precipitation amounts of the stations given in Table 1 (Kysely ve ark. 2005; Anlı ve Öztürk, 2011). Mardin station was neglected because it caused discordancy in both regions.

Another stage, Bitlis site in the region I and Bingöl and Gaziantep sites in the region II had heterogeneity. Therefore, Bingöl and Gaziantep sites included in the region I and Bitlis site in the region II, respectively. Analysis of discordancy and heterogeneity on the region I and II showed that the discordancy and heterogeneity conditions were satisfied.

The L-moment group averages, the kappa distribution parameters and the heterogeneity measure results for the two regions are given in Table 4.

Table 3. L-moment Ratios and Discordancy Measures of Maximum 6-hour Rainfall for Stations
Çizelge 3. 6 Saatlik maksimum yağışların L-moment oranları ve düzensizlik ölçüsü

Region	Gauging Station	L-1	L-CV	L-CS	L-CK	D(İ)
I	Adıyaman Ağrı Batman Erzurum Hakkari Kilis Şanlıurfa Diyarbakır Bingöl Gaziantep	27.872	0.2116	0.2198	0.1996	0.33
		18.094	0.2163	0.2468	0.2511	1.03
		18.596	0.2311	0.1395	0.1171	0.43
		16.550	0.1794	0.1027	0.1881	1.74
		18.826	0.2793	0.1604	0.1193	2.29
		23.700	0.2511	0.2345	0.1203	1.58
		24.020	0.2675	0.3545	0.2335	1.55
		21.258	0.2364	0.1503	0.0908	0.76
		25.184	0.2363	0.2301	0.1612	0.07
		24.586	0.2471	0.2684	0.1796	0.22
II	Bitlis Elazığ Erzincan Malatya Muş Siirt Tunceli	25.720	0.1637	0.1404	0.0855	1.81
		20.083	0.2059	0.1095	0.1257	0.48
		17.244	0.1891	0.1181	0.1278	0.18
		20.861	0.1985	0.0800	0.1161	0.80
		19.737	0.2159	0.2331	0.2086	1.07
		23.979	0.2016	0.1607	0.2077	1.44
		21.853	0.2230	0.1908	0.1559	1.21

The heterogeneity measure is obtained by applying the Kappa distribution. Standard test statistic values (H1) for the region I and for Region II are 0.89 and -0.07, respectively (Table 4). Since the values provided are between -1 < H <1, the two regions formed in the Upper Euphrates-Tigris basin may be accepted as homogeneous regions.

The determination of the appropriate distribution for both regions is depent on the ZDIST measure. The ZDIST values for the probability distributions used in the study for the formed regions are shown in Table 5.

able 4. The results related to heterogeneity of the selected regions
Çizelge 4. Seçilen bölgelerin heterojenlik sonuçları

Region	Group Average L-Moments			Parameters of Regional Kappa Distribution				H-Statistic
	τ_2^R	τ_3^R	τ_4^R	ξ	α	k	h	
I	0.2335	0.2119	0.1648	0.7870	0.3253	-0.0521	0.0482	0.89
II	0.1995	0.1451	0.1465	0.8506	0.2852	0.0187	-0.0690	-0.07
ξ. position parameter α. scale parameter k and h, figure parameters								

The appropriate distribution(s) for the Region I and II are GEV, GNO and P3, respectively. In regions where more than one distribution is suitable, regional frequency analysis can be performed for each of these distributions. However, it is more accurate to perform the frequency analysis with respect to

the distribution with the smallest ZDIST value. According to this, the most suitable distribution in both regions is determined as Generalized Extreme Value Type I (GEV) and, the amount of precipitation obtained for different re-occurrence probability by using the distribution is given in Table 6.

Table 5. Z-statistic Values for the simulation results
Çizelge 5. Simülasyon sonuçları için Z-İstatistik Değerleri

Region	Statatistical Distribution	Z-value
I	GLO	1.95
	GEV	0.04*
	GNO	-0.49
	P3	-1.55
	GPA	-4.38
II	GLO	1.84
	GEV	-0.33*
	GNO	-0.41
	P3	-0.91
	GPA	-4.92

Table 6. Rainfall quantities obtained at different re-occurrance probability based on generalized extreme value (gev) distribution

Çizelge 6. Genelleştirilmiş ekstrem değer (GEV) dağılımına bağlı olarak farklı tekrarlama olasılıklarında elde edilen yağış miktarları

Region	Re-occurrance Probability P%									
	0.010	0.020	0.050	0.100	0.200	0.500	0.900	0.950	0.990	0.999
I	7.36	8.39	10.10	11.81	14.20	20.01	34.23	40.13	54.55	77.72
II	7.87	8.98	10.74	12.47	14.81	20.17	31.51	35.64	44.55	56.23

4. Conclusions

Many studies in the literature reported advantage of the use of regional frequency analysis. Based on the analyzes described for this study, the following conclusions were drawn.

- First, all the stations were taken as one region but, with regard to the discordancy measure, the whole region condition was not carried out because Mardin site had discordancy.

- Clustering analysis was performed using Ward correlation method, Euclidean distance measure and, the study area was divided into two sub-regions.

- The best fit distributions for teh sub-regions were found as GEV, GNO and P3, respectively. However, the ZDIST measure showed that the generalized extreme value distribution (GEV) was the most appropriate distribution to predict the quintiles at the different re-occurrance probability.

References

Anlı AS and Öztürk F (2011). Ankara'da Ölçülen Yıllık Maksimum Yağışların Bölgesel Frekans Analizi. Tokat Gaziosmanpaşa Üniversitesi Ziraat Fakültesi Dergisi, 28(1): 61-71.

Anlı AS, Yurekli K, Hınıs MA and Apaydın H (2011). Regional Frequency Analysis for Annual Rainfall Totals of Konya Closed Basin in Turkey. 5th Atmospheric Science Symposium - ATMOS 2011 Proceedings, 27-29 April 2011, 301-312, Istanbul – Turkey.

Eslemian SS and Feizi H (2007). Maximum Monthly Rainfall Analysis Using L-moments for an Arid Region in Isfahan Province, Iran. Journal of Applied Meteorology and Climatology, 46: 494-503.

Greenwood JA, Landwehr JM, Matalas NC and Wallis JR (1979). Probability Weighted Moments: Definition and Relation to Parameters of Several Distributions Expressable in Inverse Form. Water Resources Research, 15: 1049-1054.

Hosking JRM (1994). The four-parameter kappa distribution. IBM Journal of Research and Development, 38: 251-258.

Hosking JRM and Wallis JR (1988). The effect of intersite dependence on regional flood frequency analysis. Water Resources Research, 24: 588-600.

Hosking JRM and Wallis JR (1993). Some Statistics Useful in Regional Frequency Analysis. Water Resources Research, 29: 271-281.

Hosking JRM (1990). L-Moments: Analysis and Estimation of Distributions Using Linear Combinations of Order Statistics. Journal of The Royal Statistical Society Series B, 52: 105-124.

Hosking JRM (1996). Research report: Fortran routines for use with the method of L-moments,

Version 3.03. Yorkstown Heights, NY: IBM Research Division: 39 pp.

Hosking JRM and Wallis JR (1997). Regional frequency analysis: An approach based on L-moments. Cambridge University Press, Cambridge, UK. 224 p.

Kysely J, Huth R and Picek J (2005). Regional Analysis of Extreme Precipitation Events in The Czech Republic. Geophysical Research Abstracts, Vol. 7, 01867.

Lettenmaier DP and Potter KW (1985). Testing flood frequency estimation methods using a regional flood generation model. Water Resources Research, 21: 1903-1914.

Okman C (1994). Hydrology. University of Ankara Faculty of Agriculture, Publication No:1388, 359p, Ankara (in Turkish).

Sankarasubramanian A and Sirinivasan K (1999). Investigation and Comparison of Sampling Properties of L-Moments and Conventional Moments. Journal of Hydrology, 218: 13-34.

Yurekli K (2005). Regional Analysis of Montly Rainfalls over Amasya Province via L- moment. Tokat Gaziosmanpaşa Üniversitesi Ziraat Fakültesi Dergisi, 22(2): 51-56.

Yurekli K, Modarres R and Öztürk F (2009). Regional daily maximum rainfall estimation for Cekerek Watershed by L-moments. Meteorological Applications, 16(4): 435-444.