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Bending Analysis of Functionally Graded Nanobeam Using Chebyshev Pseudospectral Method

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Abstract

Static performance of functionally graded cantilever nanobeams exposed to lateral and axial loads from the end was examined by applying the Pseudospectral Chebyshev Method. A solution is given for bending analysis using Euler-Bernoulli beam theory. The nonlocal elasticity theory was first introduced by Eringen and is used to represent effect on a small scale. Using the aforementioned theory, the governing differential equations the phenomenon for functionally graded nanobeams are reproduced. It is supposed that the modulus of elasticity of the beam changes exponentially in the x-axis direction, except for the values taken as constant. The exponential change of material properties may not allow analytical problems to be solved with known methods. Therefore, numerical approach is inevitable for the solution of the problem.

Keywords: Nonlocal elasticity theory, nanobeam, Chebyshev pseudospectral method, bending analysis

1. Introduction

Functionally graded materials (FGMs) have gained wide application in different industrial areas due to varying toughness and other material properties in the form of graded functions along certain dimensions. In order to have two opposite properties such as high thermal conductivity and high thermal resistance, which are found in FGMs, in a material, a lot of research has been done in the literature for static, buckling and dynamic conditions in order to lightness, strength and durability [1,2,11–15,3–10].

With the rapid advancement of technology, FGMs have begun to be used in applied engineering fields in micro or nanoscale structures[14]. These materials, which have superior technical and physical properties, are of great importance in the pharmaceutical industry, especially with their use in electrically operated Micro-Electro-Mechanical Systems (MEMS)[16] and atomic force microscopes (AFMs) [17].

The size effect is extremely important in mechanical calculations of nano and micro-size constructions. Conventional elasticity technique cannot be used to explain the size effect since there is no material length size parameter. There are different non-traditional continuity assumptions to explain the size-dependent effect. Some of these theories include modified strain change, non-local elasticity, modified stress couple theories. Eringen [16] and Eringen and Edelen [17] first addressed the non-local elasticity principle in their research. This theory is frequently used in dissection mechanics, wave distribution, fracture mechanics, beam-type structures [18–20] and plate-type structures [21–23]. According to the non-local elasticity theory, many academic studies have been conducted in recent years. Some of these are [24–29] [8-13]. Wang et al. [30] addressed the vibration and bending problem in carbon nanotube using the beam-shell relationship and nonlocal elastic structure equations. A comprehensive analytical method for solving the fourth-order differential equation related to the bending analysis of bidirectional functional grade nanobeams obtained by Eringen's nonlocal elasticity theory is presented by Nazmul and Devnath[31]. Akgöz et al.[32] investigated the thermo-elastic vibrational behavior of thick microbeams based on elastic foundation with modified double stress theory. Dastjerdi and Akgöz[33] used 3D elasticity theory together with nonlocal theory to study the static and vibrational behavior of FG circular nanoplates. Akgöz and Civalek[34] utilized the double stress theory for the static and stability analysis of single-walled carbon nanotubes applying different theories. Different method was used for the analysis of micro- or nano-scale mechanical systems under different conditions in[35]. Civalek and Kiracioglu[36] applied the discrete singular convolution method for the numerical solution of the Timoshenko beam's equation of motion. Some studies[37,38] have established a nonlocal finite element method for thin beams.

The nonlocal parameter (e_0a) captures the small-scale effects on the response of structures in nano-size; e_0 is called the nonlocal material constant, and a is the internal scale parameter. In the previous studies related to nano functionally graded materials, the nonlocal parameter is generally assumed as a constant. A conservative estimation of the nonlocal parameter $0 < e_0a < 2$ nm for single-walled carbon nanotubes is suggested by Wang[39]. Therefore, in this study, the nonlocal parameter is taken as $e_0a= 1$ nm to investigate bending analysis on the responses of functionally graded nanobeam.

In this study, bending analysis of functionally graded cantilever nanobeams as well as extreme intense loads using modified stress couple and Pseudospectral Chebyshev Method (PCM) is discussed. A system of differential equalities is obtained with early and limit conditions. PCM with known initial conditions is used to these systems of differential equalities including non-local elasticity parameter. The precision of the proposed technique was confirmed using the literature, then the outcomes were graphically shown and discussed according to the calculations obtained.

2. Theory and Formulations

The geometric fitness condition, equilibrium equations and constitutive relations of the FG nano beam in the two-dimensional plane are as follows[40,41]

$$\frac{dw}{dx} = \varphi \tag{1}$$

$$\frac{d\varphi}{dx} = -\frac{M}{E(x)I} \tag{2}$$

$$\frac{dM}{dx} = P_1\varphi + T \tag{3}$$

$$\frac{dT}{dx} = 0 \tag{4}$$

where T and M are the shear force and the bending moment, φ and w are the slope of the FG nano beam and the lateral displacement. Also, I is moment of inertia, the elasticity modulus varies corresponding to the function given in the equation $E(x)=E_1e^{-\lambda x}$ [42], here λ is inhomogeneity parameter.

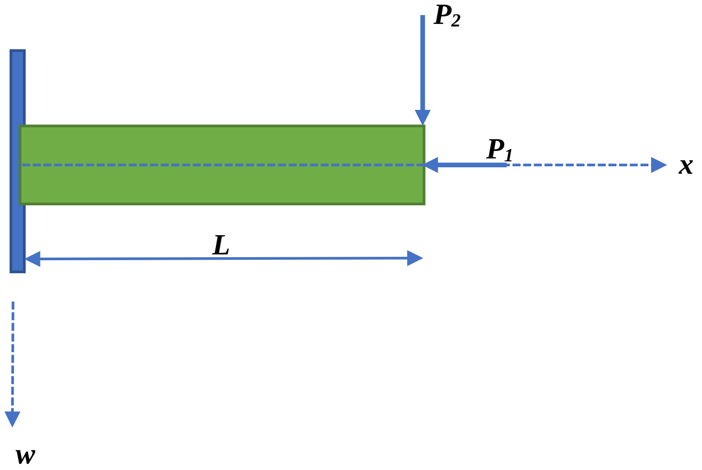

Fig. 1. Cantilever nano-beam with lateral and axial forces[32]

The final form of Eq.(2) according to the nonlocal elasticity theory takes the following form[43]

$$M - (e_0a)^2\frac{d^2M}{dx^2} = -E(x)I\frac{d^2w}{dx^2} \tag{5}$$

where a is the inner typical length, is a constant (e_0= 0.39, $a=4*10^{-8}$ cm). Using Eq. (2), above relation takes the following form

$$M = E(x)I[1-(e_0a)^2\frac{P_1}{E(x)I}]\frac{d\varphi}{dx} \tag{6}$$

The final form of the governing differential equations the situation is given as:[40,41]

$$\frac{d}{dx}\begin{bmatrix} w \\ \varphi \\ M \\ T \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-1}{E(x)I[1-(e_0a)^2\frac{P_1}{E(x)I}]} & 0 \\ 0 & P_1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} w \\ \varphi \\ M \\ T \end{bmatrix} \tag{7}$$

where P_1 the axial force, P_2 the side force, the inner typical length and e_0 is a constant. The primary conditions are given below;

$$w(0) = 0 \tag{8}$$
$$\varphi(0) = 0 \tag{9}$$

$$M(0) = -P_2\, L \tag{10}$$
$$T(0) = P_2 \tag{11}$$

From Eq. (7), the following expressions are obtained;

$$\frac{dw}{dx} = \varphi \tag{12}$$
$$\frac{d\varphi}{dx} = -\frac{M}{E(x)I[1-(e_0a)^2\frac{P_1}{E(x)I}]} \tag{13}$$
$$\frac{dM}{dx} = P_1\varphi + T \tag{14}$$
$$\frac{dT}{dx} = 0 \tag{15}$$

2. Solution Procedure

This section is where PCM is applied to discretize the main equalities and boundary conditions and transform them in algebraical equalities. This model is utilized to execute the evaluation of the static performance of the FG nanobeam by reference to the work of Trefethen[44], Fornberg[45] and Gottlieb[46]. It depends on a discretization of the main equalities with respect to the spatial variable. With regard to collocation points, the initial order *(n* + 1*)* × *(n* + 1*)* Chebyshev differentiation matrix

$$0 = x_0 < x_1 \ldots < x_n \ \text{ with } x_j = \frac{1}{2}[1 - \cos\left(\frac{j\pi}{n}\right)] \tag{16}$$

(j = 0, 1, . . . , *n)* will be represented by *D*. First-order Chebyshev differentiation matrix D offers a very precise approximation to $V'_n(x_j), V''_n(V_j)\ldots$, simply by multiplying differential matrix with vector data:

$$\begin{bmatrix} \frac{dV_n}{dx}(x_0) \\ \frac{dV_n}{dx}(x_1) \\ \vdots \\ \frac{dV_n}{dx}(x_n) \end{bmatrix} \approx D \begin{bmatrix} V_n(x_0) \\ V_n(x_1) \\ \vdots \\ V_n(x_n) \end{bmatrix}, \quad \begin{bmatrix} \frac{d^2V_n}{dx^2}(x_0) \\ \frac{d^2V_n}{dx^2}(x_1) \\ \vdots \\ \frac{d^2V_n}{dx^2}(x_n) \end{bmatrix} \approx D^2 \begin{bmatrix} V_n(x_0) \\ V_n(x_1) \\ \vdots \\ V_n(x_n) \end{bmatrix} \tag{17}$$

Details on the calculation of the Chebyshev differentiation matrix and codes as m-files can be discovered in remarkable references, e.g. Trefethen[44].
By using the Chebyshev differential matrix, derivatives of any order can be easily discretized. The final form of Eqs. (12-15) is written in the form below:

$$L_w w = 0 \tag{18}$$

where $L_w = D - \frac{\varphi}{w}$

$$L_\varphi \varphi = 0 \tag{19}$$

where $L_\varphi = D + \dfrac{M}{\varphi\, E(x)I[1-(e_0a)^2\frac{P_1}{E(x)I}]}$

$$L_M \varphi = 0 \tag{20}$$

$$L_M = D - \frac{P_1}{M}\varphi + \frac{T}{M}$$

$$L_T T = 0 \tag{21}$$

where $L_T = D$

To get the non-trivial solution, the boundary conditions (8-11) for the bending analysis are imposed on this linear system (18-21) with replacing just the initial and last row of the linear operators ($L_w; L_\varphi; L_M; L_T$) with the appropriate values and the values corresponding to the function on the right.

3. Validation of the Present Results

For the efficiency and reality of the numerical method, when the solutions are performed by taking zero instead of λ in the cantilever FG nano beam, which is taken as $E(x)=E_1 e^{-\lambda x}$[42], the cantilever nano beam, which has been solved in the literature[32]will be in a homogeneous state. E_1=1 nN/m^2, e_0a=1nm, I=1 nm^4 and P_2 =1.0 nN are taken in all calculations. In Table 1, the static deviation values of the homogeneous cantilever nano beam are given by using the material properties $E(x)=E_1 e^{-\lambda x}$ and taking λ=0. It has been observed that the displacement values obtained by PCM correspond exactly to the findings in the literature[32].

Table 1. Comparison of the static deflection (w) values with PCM solutions and the results in the literature [32] for a homogeneous cantilever beam

	P_2 =1.0 nN		P_2 =1.2 nN		P_2 =1.4 nN	
x	PCM	Ref. [32]	PCM	Ref. [32]	PCM	Ref. [32]
0.0	0.00017	0.00017	0.00020	0.00020	0.00024	0.00024
0.1	0.00467	0.00467	0.00561	0.00561	0.00655	0.00655
0.2	0.01846	0.01846	0.02215	0.02215	0.02585	0.02585
0.3	0.04013	0.04013	0.04816	0.04816	0.05618	0.05618
0.4	0.06828	0.06828	0.08194	0.08194	0.09560	0.09560
0.5	0.10153	0.10153	0.12183	0.12183	0.14214	0.14214
0.6	0.13846	0.13846	0.16615	0.16615	0.19384	0.19384
0.7	0.17768	0.17768	0.21322	0.21322	0.24876	0.24876
0.8	0.21780	0.21780	0.26136	0.26136	0.30492	0.30492
0.9	0.25741	0.25741	0.30890	0.30890	0.36038	0.36038
1.0	0.29513	0.29513	0.35415	0.35415	0.41318	0.41318

The tilt of the FG cantilever nanobeam and its effects on the lateral and axial forces applied to the beam are presented in Table 2. When Table 2 is analyzed, it is seen that the results in the literature are compatible with the current method.

Table 2. Comparison of the slope (φ) values for different concentrated forces with PCM solutions and the results in the literature[32]for a homogeneous cantilever beam

	P_2 =1.0 nN		P_2 =1.2 nN		P_2 =1.4 nN	
x	PCM	Ref. [32]	PCM	Ref. [32]	PCM	Ref. [32]
0.0	-0.00053	-0.00053	-0.00064	-0.00064	-0.00075	-0.00075
0.1	0.09525	0.09525	0.11431	0.11431	0.13336	0.13336
0.2	0.17895	0.17895	0.21474	0.21474	0.25052	0.25052
0.3	0.25008	0.25008	0.30010	0.30010	0.35012	0.35012
0.4	0.30822	0.30822	0.36987	0.36987	0.43151	0.43151
0.5	0.35292	0.35292	0.42350	0.42350	0.49408	0.49408

0.6	0.38372	0.38372	0.46047	0.46047	0.53721	0.53721
0.7	0.40020	0.40020	0.48023	0.48023	0.56027	0.56027
0.8	0.40189	0.40189	0.48227	0.48227	0.56264	0.56264
0.9	0.38835	0.38835	0.46603	0.46603	0.54370	0.54370
1.0	0.35915	0.35915	0.43098	0.43098	0.50281	0.50281

The bending moment of the FG cantilever nano beam and its effects on the lateral and axial forces applied to the beam are presented in Table 3. The effect of lateral and axial force on the bending moment is clearly seen when Table 3. is examined. The compatibility of the current results with the results in the literature is clearly displayed in Table 3.

Table 3. Comparison of the bending moment (M) values for different concentrated forces with PCM solutions and the results in the literature [32]for a homogeneous cantilever beam

	P_2 =1.0 nN		P_2 =1.2 nN		P_2 =1.4 nN	
x	PCM	Ref. [32]	PCM	Ref. [32]	PCM	Ref. [32]
0.0	-1,00E-05	-1,00E-05	-1,20E-05	-1,20E-05	-1,40E-05	-1,40E-05
0.1	-8,94E-06	-8,94E-06	-1,07E-05	-1,07E-05	-1,25E-05	-1,25E-05
0.2	-7,78E-06	-7,78E-06	-9,33E-06	-9,33E-06	-1,09E-06	-1,09E-06
0.3	-6,52E-06	-6,52E-06	-7,82E-06	-7,82E-06	-9,13E-06	-9,13E-06
0.4	-5,18E-06	-5,18E-06	-6,22E-06	-6,22E-06	-7,25E-06	-7,25E-06
0.5	-3,78E-06	-3,78E-06	-4,54E-07	-4,54E-07	-5,29E-06	-5,29E-06
0.6	-2,34E-06	-2,34E-06	-2,81E-06	-2,81E-06	-3,27E-06	-3,27E-06
0.7	-8,68E-08	-8,68E-08	-1,04E-06	-1,04E-06	-1,21E-06	-1,21E-06
0.8	6,14E-07	6,14E-07	7,36E-07	7,36E-07	8,59E-07	8,59E-07
0.9	2,09E-07	2,09E-07	2,51E-06	2,51E-06	2,92E-06	2,92E-06
1.0	3,54E-06	3,54E-06	4,25E-06	4,25E-06	4,96E-06	4,96E-06

4. Results and Discussion

In this study, static deflection, inclination and bending moment distributions in the beam were investigated in the static study of non-local FG nano beams subjected to end and lateral applied forces. It is assumed that the beam's modulus of elasticity changes exponentially. In addition, E_1=1 nN/m^2, e_0a=1nm, I=1 nm^4, P_1 =P_2 =1.2 nN in calculations. By applying extreme axial forces to the FG nano beam, it has been tried to obtain information about the effect of bending on its mechanical behavior. Figure 2 was created to see the effect of inhomogeneity parameter on deflection in cantilever FG nano beam subjected to end lateral and axial forces.

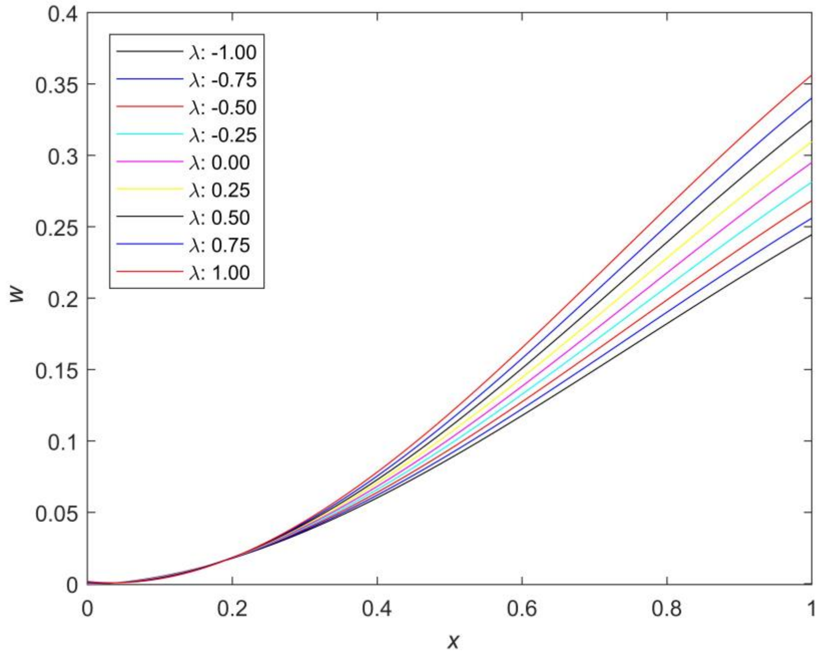

Fig. 2. Static deflection for end lateral and axial forces

When Figure 2 is examined, it is noted that the static deviation value rises with the rise in the distance from the fixed end and the static deviation rises with the rise in the inhomogeneity constraint. Figure 3 shows the impacts of end and lateral forces on the curvature of the cantilever FG nano-beam.

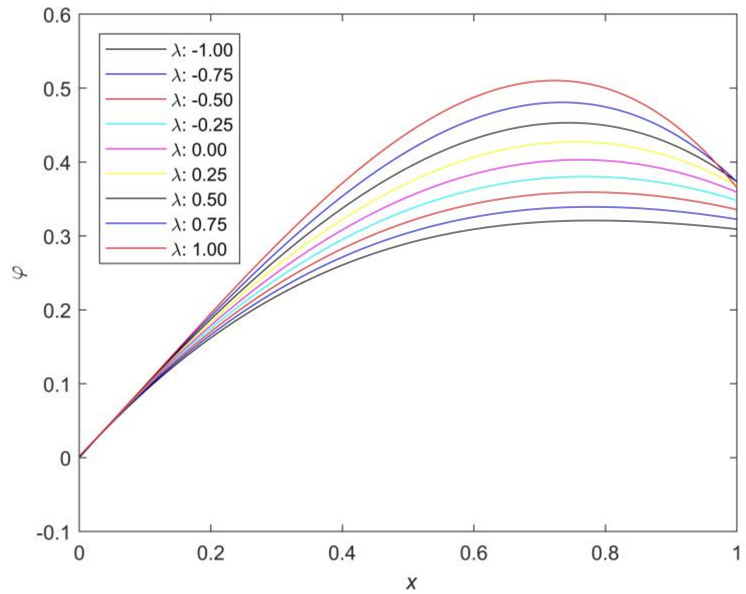

Fig. 3. Slope for end and lateral forces

Figure 3 shows the rise and decline in slope with increasing distance from the tip, emphasizing the importance of tip lateral concentrated forces. It can also be concluded that the higher values of the inhomogeneity constraint increase the slope considerably.

The impacts of end loads on the bending moment of FG cantilever nano-beams are presented in Figure 4. Again, in adding to the axial and lateral forces, the impacts of the inhomogeneity parameter on the bending moment are very clear.

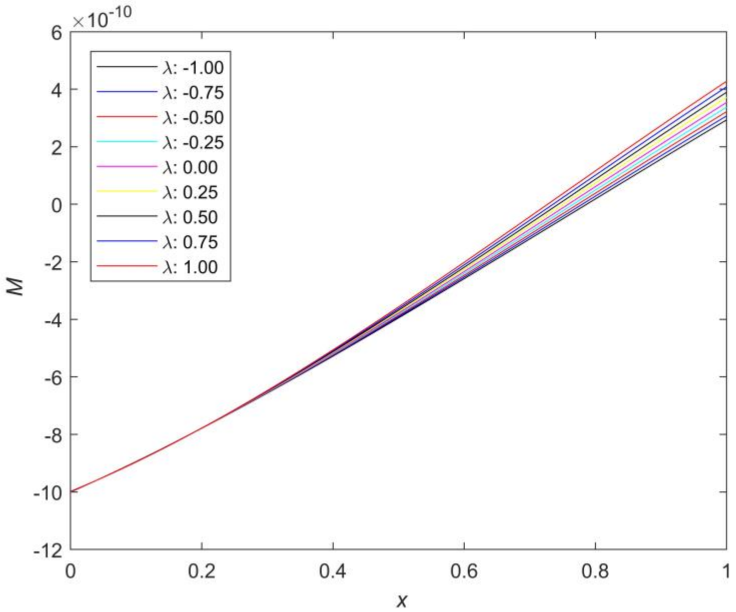

Fig. 4. Moment diagram for constant end and axial forces

5. Conclusion

In this study, it has been shown that PCM can be applied to the solution of the nonlocal primary worth problem involving homogeneous linear differential equalities. The FG nanotube was modeled as a beam with the Euler-Bernoulli theory. For the small-scale effect, the nonlocal elasticity theory is used. The results obtained and the methodology presented in this research can be used as an aid in the design and analysis of FG nanobeams. Therefore, it can be concluded:

- It has been determined that the homogeneity parameters have a strong effect on the static deflection, tilt and bending moment distribution.

- PCM has a high-level correctness rate with low computational cost due to its dense mesh structure close to the border and its rough structure towards the center points. Therefore, it can be used as a practical solution tool in solving such problems.

- It provides an effective performance when the inhomogeneity parameter is high when a functionally graded nano beam operates under the effect of end and lateral forces.

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