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RESEARCH ARTICLE

PID CONTROL IMPLEMENTATION of AN INVERTED PENDULUM SYSTEM

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ABSTRACT

Inverted pendulum which consists of a straight pole and a horizontally moving cart has been one of the most widely researched systems and is considered a classical problem in the area of control engineering. Main objective of the system is to control and stabilize both the angle of the pendulum and position of the cart. In this study, an example of such type of system was designed, implemented and controlled. Stabilization and control objective of the system was successfully achieved by using a Proportional-integral-derivative (PID) controller and graphs of input and outputs of the system was obtained in MATLAB environment.

Keywords: *Nonlinear systems, PID controller, Inverted pendulum*

1. INTRODUCTION

Inverted pendulums are non-linear systems which have a stable, and an unstable equilibrium states. They are regarded as underactuated systems because pendulum motion is provided by cart motion, not by the pendulum shaft itself. One of the two main objectives of the system is to bring the pendulum to the upper unstable position from the lower/stable position. Other objective is to keep the system stable in that state by controlling the pendulum angle and the cart position within the desired reference points.

Literature shows that many researches have shown great interest in inverted pendulum and its varieties and suggested various linear and non-linear control algorithms for the non-linear inverted pendulum systems. According to studies, PID as a linear algorithm proved to be useful and effective on controlling non-linear under-actuated systems such as inverted pendulum despite its simplicity.

Lozano et al. [1] proposed a control system design that takes the displacement of the vehicle underneath, while bringing the pendulum to upper unstable equilibrium point. The control strategy they suggested is based on the control of the total energy of the system. In the study, a Lyapunov function was obtained using the total energy of the system and the movement of the vehicle was realized using LaSalle's invariance principle. They have shown that the control system they have obtained can be used in practice by testing in a real pendulum system.

Huang and Huang [2] aimed to solve the problem of inverted pendulum using gray theory analysis in their work. Dynamic model of underactuated mechanical system was obtained by using Lagrange equation. Based on the dynamic model and the sliding mode control, the speed and braking system of the vehicle were controlled. Simulation and experimental results had shown that the system works effectively.

Gani et al. [3] proposed a fuzzy logic-based control algorithm for the control of the inverted pendulum system. The researchers simulated the control algorithms via MATLAB/Simulink and observed that it takes 25 seconds for the system to be stable.

Elibol [4] used an energy-based control method to study the problem of lifting the pendulum to the upper equilibrium position and then keeping it in that point by using Linear Quadratic Regulator (LQR).

Kajita et al. [5] have studied a three-dimensional inverse pendulum problem for controlling a plane of arbitrary motion of a two-legged robot motion. In the study, they proposed a 3D linear inverted pendulum mode for the control of robot walk. The walking motion of the humanoid robot was analyzed in a dynamic simulator and a stable walking on a circle motion was obtained.

Sugihara et al. [6] proposed a real-time motion generator for the movement of humanoid robots in their work. They have reduced the computation time for real-time applications by using the dynamics of the inverted pendulum in their work.

Pathak et al. [7] examined the dynamic model of a wheeled inverted pendulum in terms of feedback linearity and controllability. First, they derived a dynamic model that takes the moment of the motors as the input value and then compared the results with the models proposed in previous studies.

Gun [8] designed a practical application of a non-linear and underactuated inverted pendulum system. He used Adaptive Neuro Fuzzy Inference System (ANFIS) and Proportional-Integral-Derivative (PID) controller to control the two outputs against a single input. The results of the system which performed successfully were interpreted graphically and the video visual of the experimental work was also presented.

Grasser et al. [9] built a two-wheeled vehicle named JOE with the solution of the inverse pendulum problem. The vehicle consisting of two coaxial wheels controlled by DC motors was an example of mobile inverted pendulum which showed a successful performance against external disturbances.

Sharif [10] preferred to use the sliding mode control in the analysis of the non-linear inverted pendulum and designed the sliding mode control algorithm using the Ackerman formula and linear matrix equations to stabilize the system.

Kharola et al. [11] approached the inverse pendulum problem in a different way and aimed to control the system on a sloping surface. The proposed system was controlled by PID and fuzzy logic controllers and simulated in MATLAB/Simulink environment.

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Studies show that the inverted pendulum system is an important test environment in the analysis of underactuated non-linear systems. Many algorithms have been proposed and their performances have been examined both in simulations and experimental setups.

In this study, a two degree-of-freedom inverted pendulum control system with single input and two outputs was designed and implemented. While the input of the system is the speed of the cart carrying the pendulum in the horizontal axis, its outputs are the pendulum angle and the cart position which are changed by the cart movements. Movement of the cart on a 2m long track is provided by a brushless servomotor connected to the track, while the pendulum angle and cart position are measured by incremental optical encoders. Digital signal processor (DSP) used in the system computes the current error by processing angle and position information provided by the encoders. The speed of the cart which stabilizes the system by correcting the calculated errors is determined by the PWM signals generated by the DSP. The PID controller used in the system calculates at what speed and in which direction the cart must change its position on track in relation to angle and position errors. Thus, both outputs of the system are being controlled simultaneously.

2. MECHANICAL AND ELECTRICAL HARDWARE

Components of the main mechanical hardware of the system as illustrated in Fig. 1 consists of a 2m long track, a pendulum with a length of 0.5m and weight of 180g and a cart with a weight of 230g which carries the pendulum along the track.

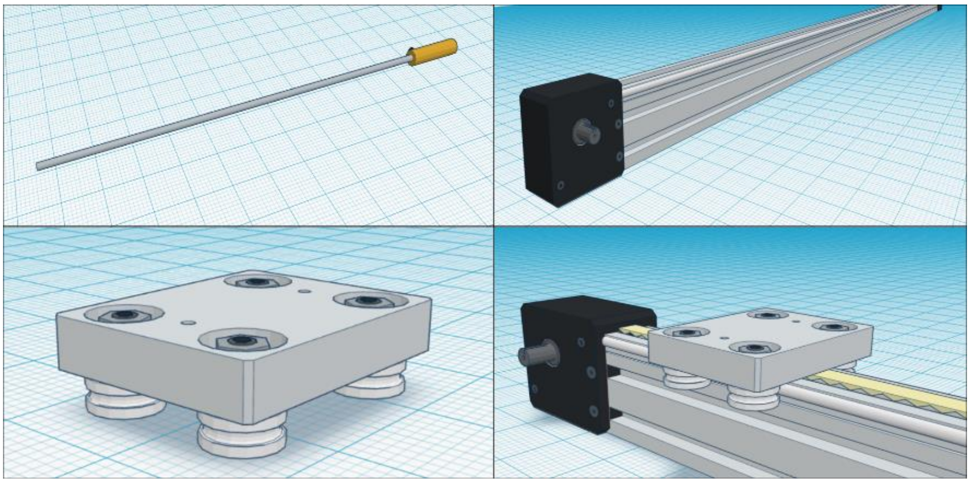

Figure 1. Mechanical hardware of the system.

The mechanical hardware used in the physical implementation of the system were chosen to allow the system to operate efficiently, allow experimental work, application of different control methods and algorithms, and to minimize exposure to physical factors that could negatively affect the system such as friction, wear and oxidation. In addition, limit switches are connected to both ends of the track as shown in Fig. 2. With the use of limit switches, the rail length available for vehicle movement has been reduced to 1.86 m.

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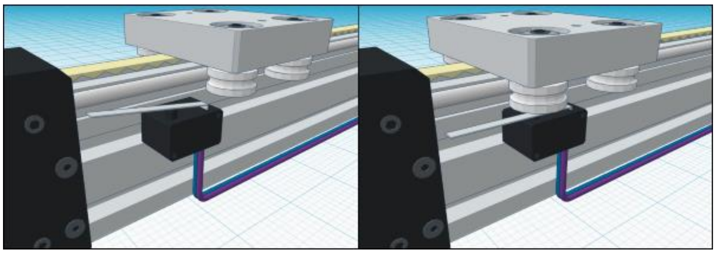

Figure 2. Limit switches.

Actuator of the system which provides cart movement is a 3-phase brushless servomotor. Its nameplate ratings are as given in Table 1. A compatible motor drive was also used to control the motor efficiently. Connections of the motor to the track and the drive is as shown in Fig.3.

Table 1. Nameplate ratings of the motor.

Nominal			
Torque	**Speed**	**Current**	**Voltage**
2 Nm	3000 rpm	4A	220V

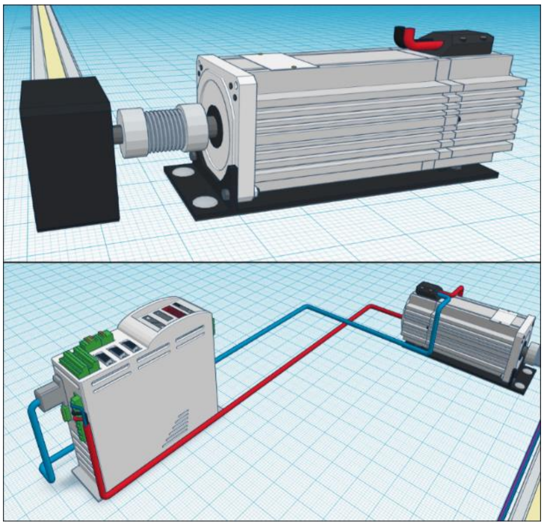

Figure 3. Connections of the motor with the track and the motor drive.

3. ELECTRONIC HARDWARE

Electronic hardware of the system consists of two rotary encoders which are used for measurement of pendulum angle and cart position, and a DSP. While the pendulum is connected to an optical and incremental encoder for angle measurement, cart position is measured by the internal encoder of the

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motor. Pendulum encoder and its connections between the pendulum and the cart are as shown in Fig. 4.

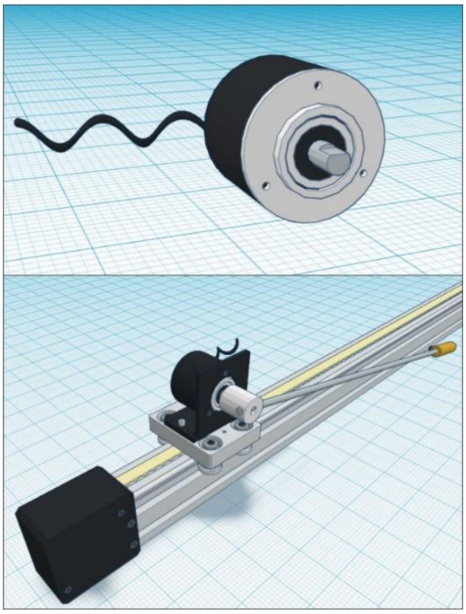

Figure 4. Connections between the pendulum, pendulum encoder and the cart.

Incremental encoders used in the system generate sequential two-bit logic signals using two channels separated by 90 degrees of phase shift. Direction and displacement information is obtained by interpreting the signal sequences composed of these signals as in Fig. 5.

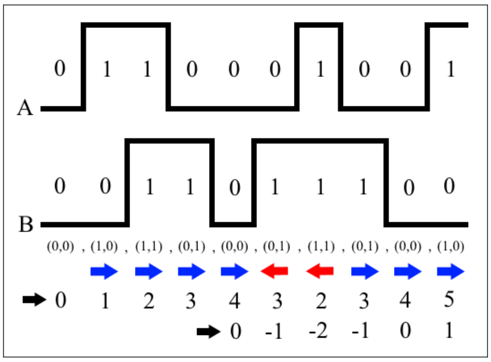

Figure 5. Counting of encoder signal sequences.

As seen in Fig. 5, obtaining the direction and displacement information from the encoder signals is not sufficient for detecting the position of the encoder shaft. Because counting of the signals generated by the encoder starts at zero whatever the position of the encoder shaft is when the system is started. For the cart, regardless of its position on the track when the system is started, the two-bit signal sequence generated by the motor encoder will not be sufficient to determine the position information. As for the pendulum, regardless of its angle at the start of the system, counting of the signals generated by the pendulum encoder starts at zero. Therefore, if the pendulum is already in motion when the system is started, it will cause a shift in angle information. If all these are taken into consideration, it is clear that reference points are required for both encoders. In this study, preferred reference points are the direction of gravity for the pendulum angle and the midpoint of the track for the cart position.

Digital signal processor which is used to control the system in this study is a TMDSDOCK28335 Experimenter Kit (manufactured by Texas Instruments) which has a TMS320F28335 microprocessor. It has two Enhanced Quadratic Encoder Pulse (eQEP) modules which are both used for reading and processing the encoder signals. Pulse Width Modulation (PWM) required to control the motor speed are generated by the ePWM module. All DSP inputs and outputs used in the system are as shown in Fig. 6 below.

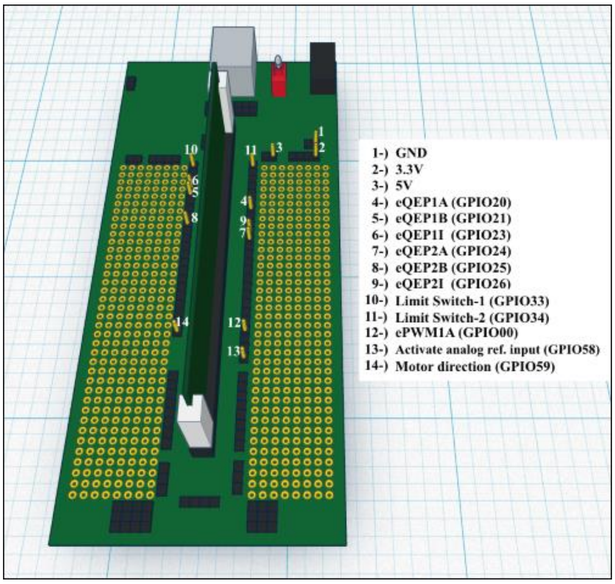

Figure 6. DSP inputs and outputs used in the system.

Motion control of the servomotor which is the only actuator of the system is provided by a motor drive [12]. Used analogue and logic inputs of the motor drive are as seen in Fig. 7.

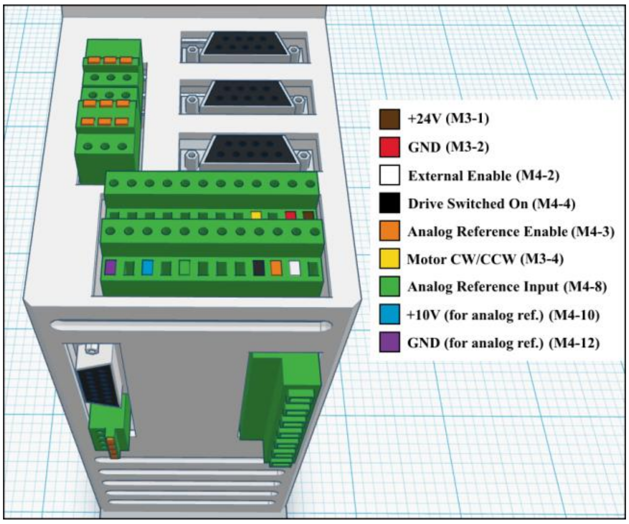

Figure 7. Motor drive inputs and outputs used in the system.

For a more compact connection between system components, another board was designed to be placed under the DSP as a dock, as shown in Fig. 8. It has two 5V/3.3V logic level converters for dropping voltage levels of encoder channels (5V) to DSP's input/output voltage level (3.3V).

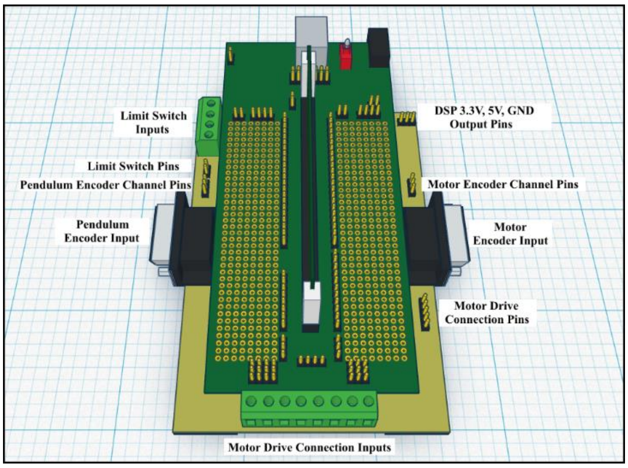

Figure 8. Dock design and its connectors.

Designed dock also includes optocoupler circuits for controlling the motor drive inputs which have 10V and 24V voltage levels with DSP output voltage level, which is 3.3V. Connection between analog reference input (M4-8) and +10V reference output (M4-10) of motor drive is being switched using the PWM output of DSP (GPIO00) via a high speed optocoupler. Two more optocouplers are used for controlling logic inputs of the motor drive (M4-3, M3-4). Using optocouplers between DSP and motor drive also provides electrical isolation. Diagram of the mentioned optocoupler circuits is given in Fig. 9.

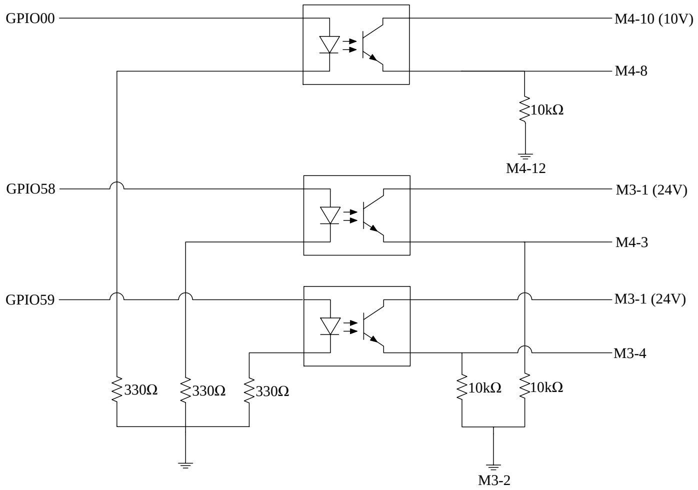

Figure 9. Diagram of optocoupler circuits used between DSP and motor drive.

An overview of the system used in the study with all its mechanical, electrical and electronic hardware is shown in Fig. 10.

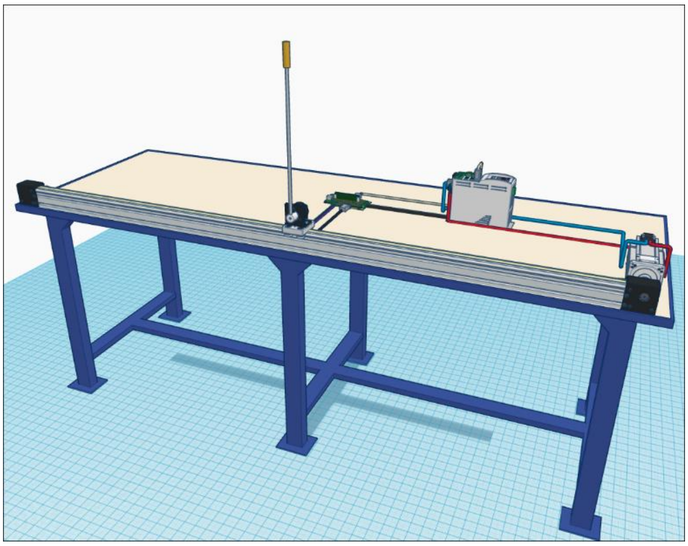

Figure 10. An overview of the inverted pendulum system.

4. READING AND CONVERSION OF ENCODER SIGNALS

Two outputs of the system, which are pendulum angle and cart position, are measured by incremental, optical rotary encoders. PPR (pulses per revolution) values are 5000 for pendulum and 2500 for motor

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encoder, which means total counts per revolution are 20000 and 10000 for pendulum and motor encoders respectively.

Encoder channel outputs are connected to the DSP's eQEP1 and eQEP2 module inputs. Since these modules count the encoder signals one by one, these counts must be converted to units of angle and length, which are radians and centimetres for this study. For conversion to radians, signal count of pendulum encoder is multiplied by the angle ratio, which is calculated as seen in Eq. 1.

$$r_\theta = \frac{RPR}{CPR} = \frac{2\pi \; rad}{20000} = 0{,}000314 \; rad \tag{1}$$

r_θ : *Angle ratio*

RPR : *Radians per revolution*

CPR : *Counts per revolution*

In order to convert the motor encoder counts to centimetres, it is examined how many encoder signals are generated in the 1 meter movement of the cart on the rail. The length ratio is then calculated as shown in Eq. 2 using the number of counts which is found to be about 27400.

$$r_x = \frac{LPM}{CPM} = \frac{100 \; cm}{27400} \cong 0{,}00365 \; cm \tag{2}$$

r_x : *Length ratio*

LPM : *Length per meter*

CPM : *Counts per meter*

5. ADJUSTMENT OF ANGLE AND POSITION REFERENCES

Since incremental encoders are used in angle and position measurement in the system, the pendulum angle and cart position at the start of the system take the value of 0. This leads to offsets in the angle and position references, for example, if the pendulum is swinging at the start or the cart is not at the midpoint of the track. For correction of these offsets, a number of conditions have been used.

For the pendulum, the encoder value is desired to be zero at the lower/stable equilibrium point. Since the pendulum is naturally held at this point by gravity while it is still, no reference adjustment is required. However, if the pendulum is already in motion when the system is started, a shift in the angle reference is inevitable. In order to prevent this, first thing the system does as soon as it is started is to check if the pendulum speed is zero or not. If it is zero, then the pendulum is stationary at the lower equilibrium point with the angle value of zero as desired. If not, the system waits for the pendulum to stop at the lower equilibrium point by losing its energy due to frictions and then resets the encoder signal count. Thus, the angle value is set to zero, as seen on Fig. 11.

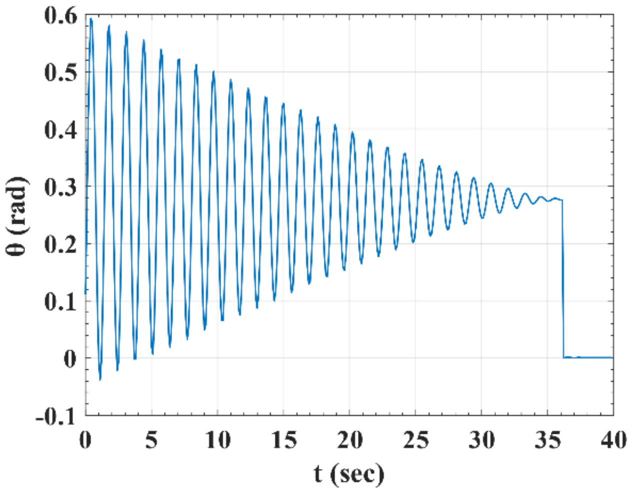

Figure 11. Adjustment of angle reference.

For adjustment of position reference, the limit switch on the right end of the track is used. At the start of the system, the cart always goes to that limit switch, no matter where it was in the beginning. When the limit switch gets in contact with the cart and closes, it is obtained that the cart is at the right end of the track. After that, the position value is set to zero and then the cart immediately turns to the other direction and starts moving to the left. When it travels exactly half the length of the track, which is 0.93 m, it immediately stops, and then the position value is set to zero again. At this point, it is ensured that the cart is at the midpoint of the track, with the position value of zero as desired. Fig. 12 shows the adjustment of the position reference after the system is started with the cart is at the left end, midpoint and right end of the track.

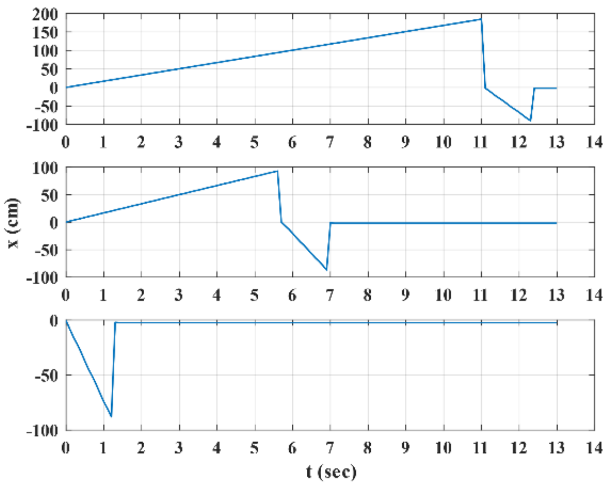

Figure 12. Adjustment of position reference.

6. SWING-UP METHOD FOR THE PENDULUM

When the angle and position references of the system are adjusted, the cart is in the middle of the track while the pendulum is in the lower/stable equilibrium point. In this state, the position and angle values are both zero as desired. This can be considered as the default state of the system in stable equilibrium.

However, the system is desired to be kept in the state of unstable equilibrium. For this to happen, the pendulum must first be lifted to the upper/unstable equilibrium point. This can be achieved by transferring energy to the pendulum with cart movements. When left-to-right and right-to-left movements of the cart are made at the right times, the pendulum can be given a little more height each time, until it finally reaches the upper/unstable equilibrium point. This is achieved by moving the cart in the opposite direction to the direction from which the swinging pendulum comes, as shown in Fig. 13.

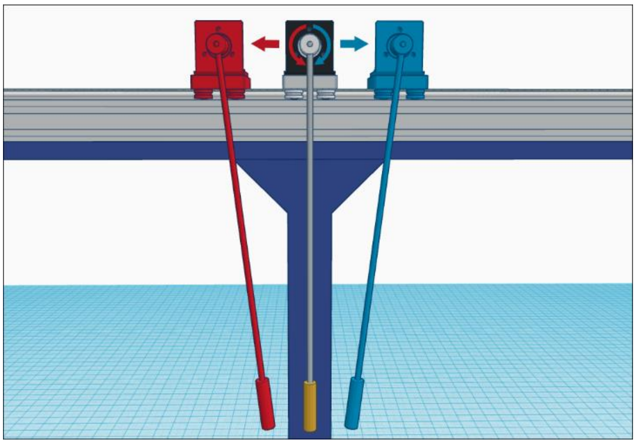

Figure 13. Directions of motion during the swing-up process.

For maximum energy transfer from the cart to the pendulum during a swing, left-to-right and right-to-left movements of the cart starts at the moment the pendulum has maximum kinetic energy, which is the lower equilibrium point. Cart movement continues until the pendulum has maximum potential energy, which is the moment the pendulum comes to a stop before it changes direction. Cart motions follow this rule until the angle value of the pendulum is $\pm\pi/2$ rad where it is parallel with the ground. When the pendulum hits those limits during a swing, the cart stops its motion and waits for the next swing. To summarize, the cart moves left-to-right when the pendulum is swinging clockwise and its angle is between the [$-\pi/2$ rad, 0 rad] range. Similarly, it moves right-to-left for counterclockwise swing in the range of [0 rad, $\pi/2$ rad].

Following the method described above, the pendulum gains a little more height with each swing, eventually reaching the upper equilibrium point. However, when the pendulum reaches its peak, it is desired to have a reasonable speed so that it can be easily caught and stabilized by the controller. For this purpose, speed of the cart is gradually decreased in relation to the maximum height the pendulum has reached. In this way, it can be said that the swing-up process starts fast and ends slowly. The pendulum angle and cart position during the swing-up process with gradually decreasing cart speed can be seen in Fig. 14.

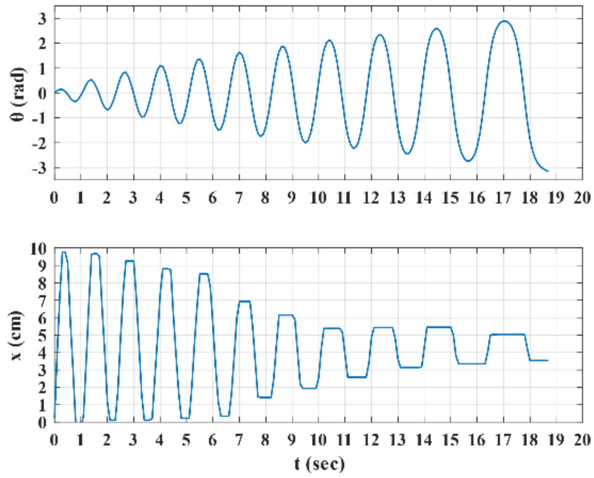

Figure 14. The pendulum angle and cart position during the swing-up process.

At the end of the swing-up process, the angle of the pendulum becomes π or -π at the upper equilibrium point, depending on the direction of the last swing. Therefore, at the end of the swing-up process, the angle reference is set to either of those, right before the control process begins and catches the pendulum at the top.

7. PID CONTROL OF THE SYSTEM

For the stabilizing of the system at the upper/unstable equilibrium point, a classical PID controller is used. As soon as the swing-up process raises the pendulum to the top, it is caught by the controller and held there, while keeping the cart in the midpoint of the track. As a single input and two outputs control system, angle error of the pendulum and position error of the cart are both eliminated by the algorithm. As seen in Fig. 15, two separate PID controllers are used for pendulum angle and cart position.

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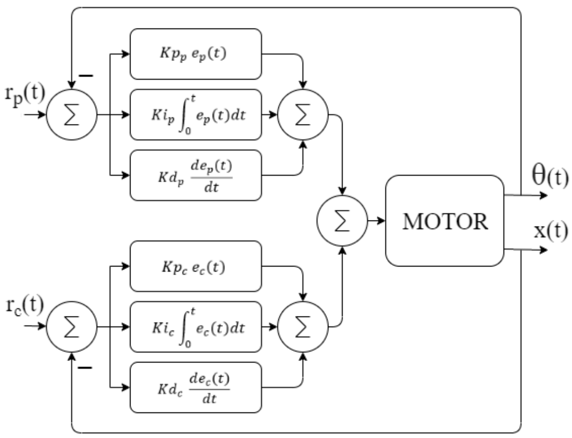

Figure 15. Block diagram of the control system.

As digital signal processors work in discrete time, approximate values of the integral and derivative terms are calculated with a discrete approach. Discrete time equivalents of the analog expressions in the controller blocks in Fig. 15 are calculated as follows [8]. T_s terms seen in Eq. 4 and Eq. 5 represent the sampling period, which is 10 ms.

$$K_p e(t) = K_p e[\mathrm{n}] \tag{3}$$

$$K_i \int_0^t e(t)dt \cong K_i T_s \sum_0^n e[n] \tag{4}$$

$$K_d \frac{de(t)}{dt} \cong K_d \frac{e[n]-e[n-1]}{T_s} \tag{5}$$

After the angle and position errors are processed in their dedicated controllers and two separate controller outputs are produced, they are added together to become a single control output which is used as the duty cycle value of the PWM signal generated by the DSP. Since the output voltage level of the DSP is 3.3V, it gets converted to 10V using the optocoupler circuit shown in Fig. 9. This PWM signal with the 0-10V voltage range can be seen as the ultimate controller output, since it is applied to the analog reference input of the motor drive, which determines the motor speed. A flowchart of the system with all its processes is given in Fig.16 below.

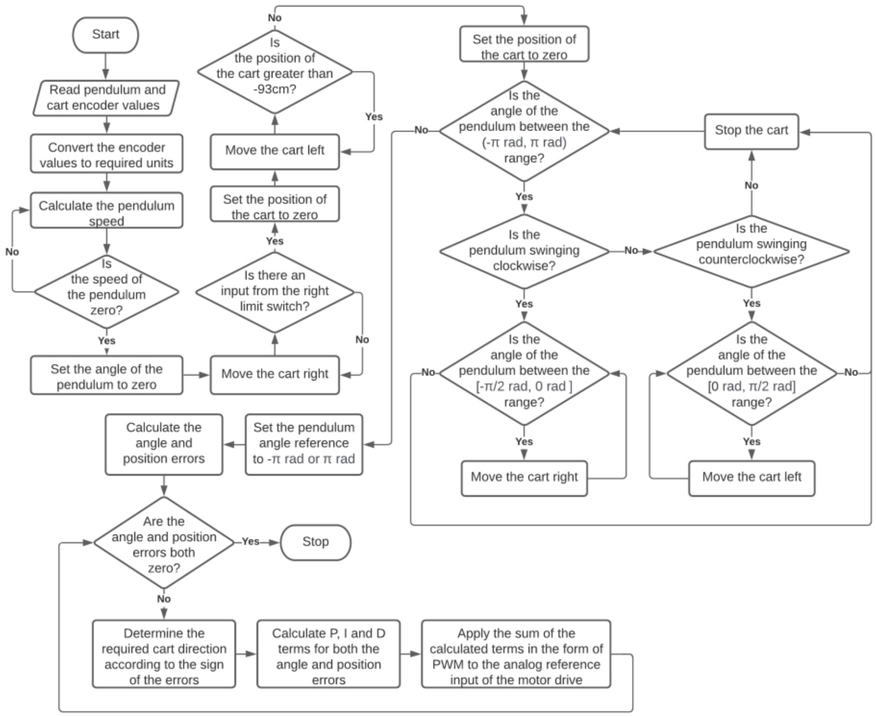

Fig. 16 A flowchart of the system with all its processes.

K_p, K_i and K_d parameters for the pendulum and the cart are determined heuristically by making small changes and observing the system behavior. By eventually setting the appropriate values, system could be stabilized as shown in Fig. 17. From the figure, it can be deduced that when the system was started, the cart was already around the midpoint of the track. However, the system did not yet have this information. Therefore, it adjusted the position reference by moving the cart to the limit switch at the right end of the track, and then bringing it back to the midpoint again. After immediately starting and ending the swing-up process, it is seen that the system is stabilized around 35th second.

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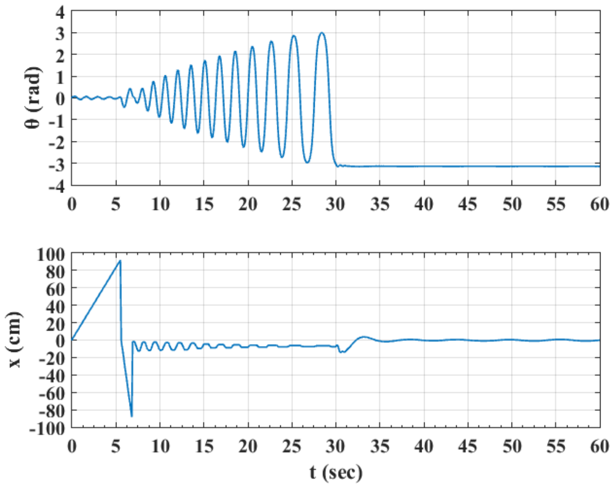

Figure 17. PID control of the inverted pendulum system.

After stabilizing the system with corrected angle and position references, a new feature has been added to the system by making the position reference variable. By pressing the limit switches at both ends of the track, position reference of the cart can be increased or decreased. Therefore, the cart can be moved along the track while still keeping the pendulum at the equilibrium point. In Fig. 18, it is seen that the position reference was increased up to around 50 cm by pressing the limit switch at the right end of the track at around 50th second. Then it was decreased down to around -50 cm using the other limit switch. Finally it was brought back to the midpoint of the track. It is seen that the stability of the system was maintained throughout these processes. Also, at around 40th second, a disturbance was applied to the system by poking the tip of the pendulum. This shows that the system resists external disturbances by stabilizing itself again.

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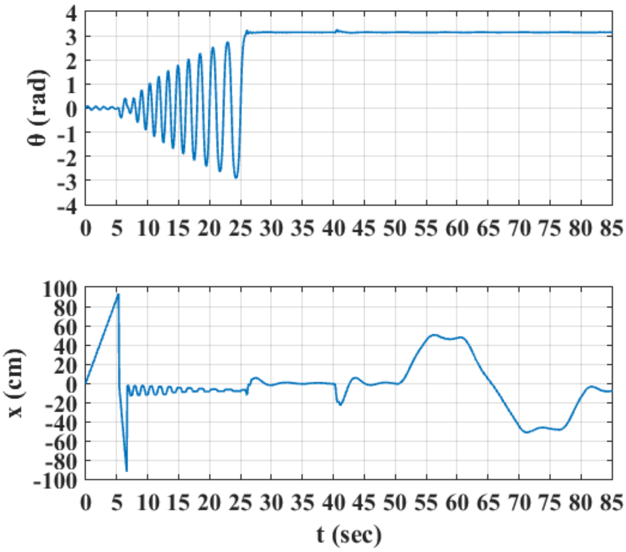

Figure 18. PID control of the system with variable position reference.

8. SUMMARY AND CONCLUSION

In this study, a PID controlled inverted pendulum system shown in Fig. 19 was designed and implemented. Controller parameters were tuned by hand heuristically. It was acknowledged that even for highly unstable, underactuated, single input and multiple output systems, the PID algorithm still proves its efficiency, reliability and convenience. It was seen that in the worst case, where the initial conditions of the system are pendulum being dropped from the highest point and the cart being at the left end of the track, it takes approximately 78 seconds for the system to reach steady state. In further studies, performance, stability and robustness of the system can be further improved by fine tuning the control parameters.

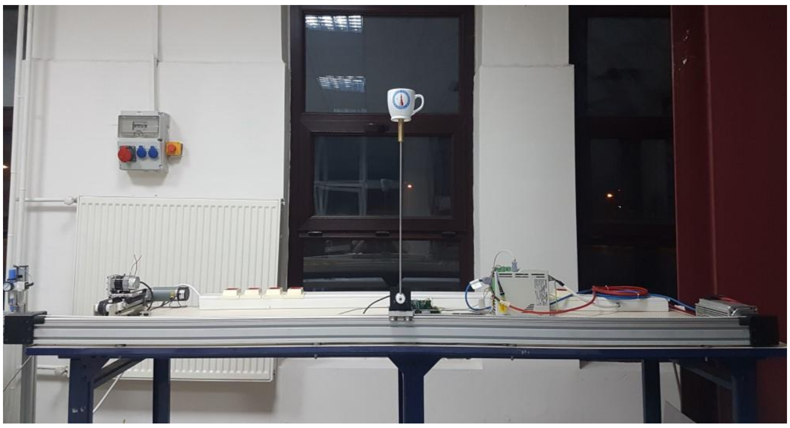

Figure 19. Inverted pendulum system.

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