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# The Two-Dimensional Strip Cutting Problem: Improved Results on Real-World Instances

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**Abstract**: Cutting and packing problems arise in various industrial settings such as production of metal, glass sheets, papers, etc. The demand of items should be met while minimizing loss of waste material. One of the most known as a contemporary problem in field of operations research is the two-dimensional strip cutting problem. A set of $m$ rectangular items is to be cut from a two-dimensional strip of width $W$ and infinite height. Each item $i$ ($i=1,2,\dots,m$) has a width $w_i$, a height $h_i$, and a demand $d_i$. The objective is to determine how to cut the demanded items using the minimum height of strip and meet all the demands, while respecting the two stages of guillotine cuts. We address the arc-flow formulation for this NP-hard problem. A graph compression method is proposed and it is shown that substantially better results are achieved in obtaining optimal or near-optimal solutions of real-world instances.

**Keywords:** Integer programming, Arc-flow formulation, Strip cutting problem, Graph compression.

## Introduction

The considered problem consists of a set of two dimensional items (square/rectangular) that are required to be cut from a raw material (strip). This NP-hard problem still attracts the researchers and motivate them to develop highly efficient algorithms to provide optimal/good solutions for both benchmark instances and real world problems. A set of factors might be considered by the researchers to solve the two-dimensional cutting problems, such as guillotine cut constraints, types of in-stock raw material, and the possibility of item rotation. According to Lodi et al. (1999), the two dimensional-cutting problems could be classified as follows:

- items could be rotated and guillotine cut is not considered (RF)
- items could not be rotated and guillotine cut is not considered (OF)
- items could be rotated and guillotine cut is considered (RG)

- items could not be rotated and guillotine cut is not considered (OG)

More classified and detailed typology of the cutting and packing problems is stated in Dyckhoff (1990) and Wäscher et al., (2007).

To solve the two-dimensional cutting problems either exactly or heuristically, a lot of approaches have been suggested in the literature. The reader is referred to the very interesting and recent papers of Delorme et al. (2016) and Iori et al. (2021) where all aspects of cutting and packing problems (variants, complexity, formulations, open problems) are presented.

Various methods have been proposed in the literature for the exact solution of cutting and packing problems, including branch-and-bound algorithms (Hifi 1998; Lodi and Monaci 2003; Mrad et al. 2013), arc-flow based formulations (Macedo et al. 2010), and one-cut formulation (Martinovic et al. 2018). The reason behind this diversity is the flexibility of adapting one formulation to fit many variants of cutting or packing problems. Among the exact methods related to the two-dimensional strip cutting problem, the arc-flow based mathematical formulation with pre-processing graph construction algorithm achieved a good results in case of the one-dimensional for bin packing problem (Brandão & Pedroso (2016)). The mentioned graph construction algorithm can reduce the initial size of a graph corresponded to the two-dimensional strip cutting problem. Therefore, we presents here the impact of using arc-flow formulation with compressed graph to solve forty-three real world instances from the literature (Macedo et al. (2010)). In the remaining of this paper, we present the arc-flow mathematical model in section II. The graph compression steps are described in section III together with an illustrative example. The computational results are presented in section IV.

## Proposed Methodology

### The arc flow formulation of the two-dimensional strip cutting problem

Consider a two-dimensional strip of width $W$ and infinite height, and a set of $m$ rectangular items. Each item $i$ ($i=1,2,\ldots,m$) has a width $w_i$, a height $h_i$, and a demand $d_i$. The objective is to determine how to cut the demanded items using the minimum height of strip and meet all the demands, while respecting the two stages of guillotine cuts. In guillotine cuts, the first stage is to cut strip vertically to produce levels $\pi_k, k \in \{1,\ldots,n\}$, as shown in figure 1. While in the second stage, each level from stage 1 is cut horizontally to produce the demanded items as shown in Figure 2.

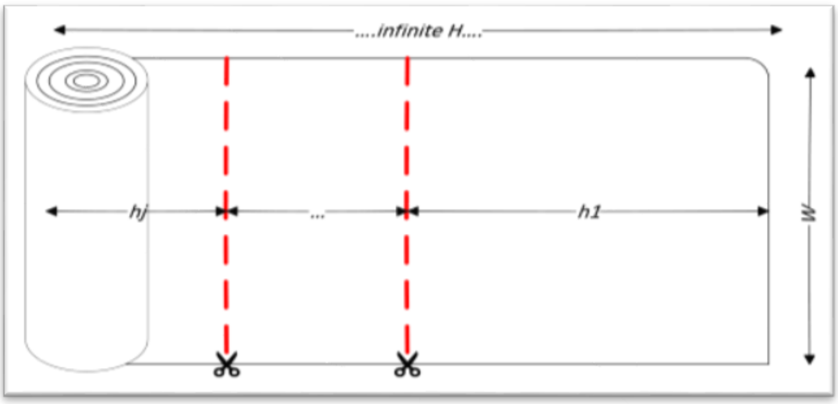


Figure 1. First stage of guillotine cut

The problem can be formulated using an arc-flow network as follows. A graph $G_k = (V_k, A_k)$ is built for each level $\pi_k, k \in \{1,\ldots,n\}$, where $V_k = \{0,\ldots,W\}$ is the set of vertices. Each level of type $k$ has height $h_k$ and can be composed only by items from set $S_k=\{i|w_i \leqslant h_k\}$. The set of arcs is $A_k = \{(i,j):0 \leqslant i < j \leqslant W \text{ and } j - i = w_t\ \forall t \in S_k \text{ or } j - i = 1\}$. Interestingly, the following rules prevent the occurrence of symmetric paths in any graph corresponding to any cutting pattern:

- The items are ordered in decreasing order of their widths.
- On each shelf, any path must include an item with the same height as the shelf.
- The number of occurrences of an item in a path cannot exceed its own demand.

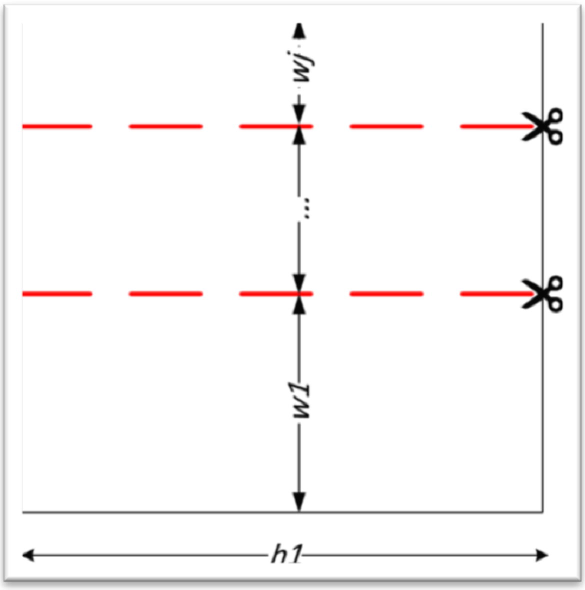


Figure 2. Second stage of guillotine cut

For the sake of clarity, let us consider the following illustrative example. A set of four items has to be cut from a strip with width $W$ = 11. The items' data are provided in Table 1. In the first cutting stage, we obtain four shelves with dimensions (9, 11), (7, 11), (6, 11) and (4, 11). In the second stage, each shelf is represented by a graph as shown in figure 3 (note that most of the dotted arcs have been removed for the sake of clearness).

Table 1. Data of the illustrative example

| Item | Height ($h_i$) | Width ($w_i$) | Demand ($d_i$) |
|---|---|---|---|
| $I_1$ | 9 | 7 | 1 |
| $I_2$ | 7 | 6 | 1 |
| $I_3$ | 6 | 6 | 1 |
| $I_4$ | 4 | 4 | 1 |

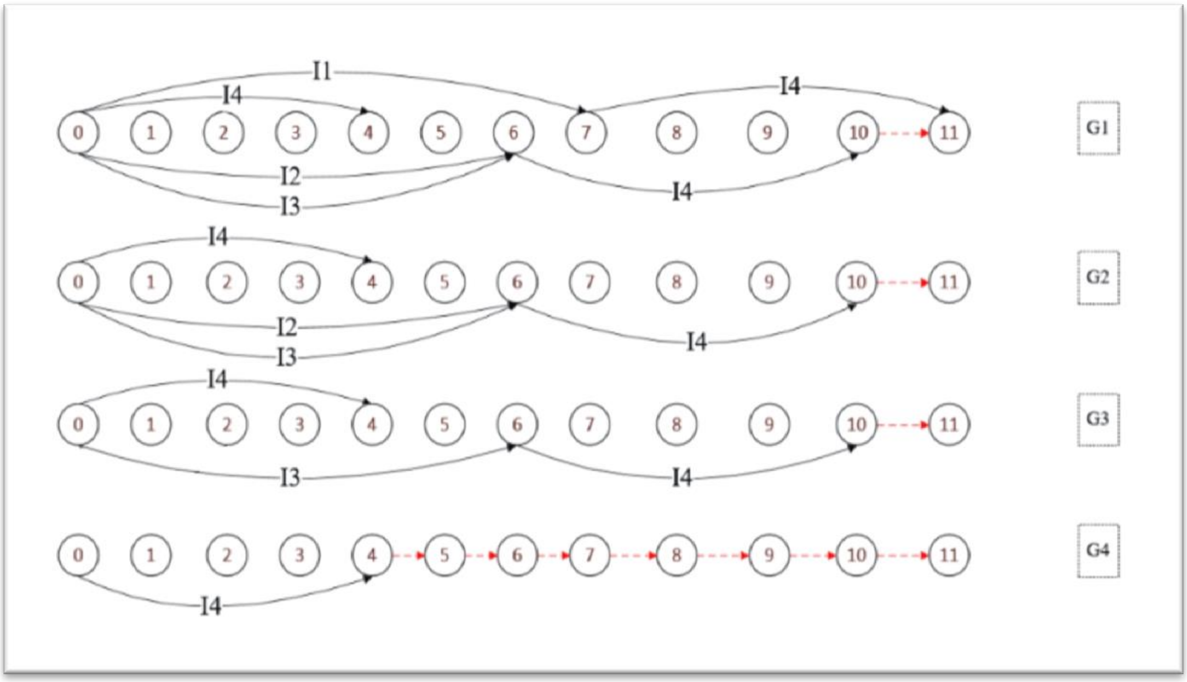


Figure 3. Arc-flow network of the illustrative example

After generating all the graphs, the addressed problem is formulated by the following mathematical model:

- Decision variables:

$\boldsymbol{x_{abh}^k}$: an integer variable associated with each arc ($a$,$b$) of $A_k$ that takes the number of items of width ($b$ - $a$) and height $h \in H_k$ ($H_k$is the set of different item heights in $S_k$) placed at position $a$ from the beginning of a shelf $k$. This variable represents the flow of the arc $(a, b)$ associated with the item of height $h$ in the graph $G_k$.
$\boldsymbol{z_k}$: the total flow of the graph corresponding to $\pi_k$.

- The objective function:

$$Min \sum_{k=1}^{n} h_k z_k \quad (1)$$

Subject to:

$$\sum_{(a,b)\in A^k, h\in H_k} x_{abh}^k - \sum_{(b,c)\in A^k, h\in H_k} x_{bch}^k = \begin{cases} z_k \text{ } if \text{ } b = 0 \\ 0 \text{ } if \text{ } b = 1, \dots, W-1 \text{ }, k = 1, .., n \\ -z_k \text{ } if \text{ } b = W \end{cases} \quad (2)$$

$$\sum_{k, h_k \geq h_j} \sum_{(a,a+w_i)\in A^k} x_{a,a+w_j,h_j}^k \geq d_j \text{ }, \forall j \in \{1, \dots, n\} \quad (3)$$

$$x_{abh}^k \geq 0 \text{ } and \text{ } integer, \forall (a, b) \in A_k, \forall h \in H_k, \forall k \in \{1, \dots, n\} \quad (4)$$

$$\mathrm{z_k} \geq 0, \forall \mathrm{k} \in \{1, \dots, \mathrm{n}\} \quad (5)$$

In objective function (1), we minimize the total used height from strip. The flow conservation is balanced in (2), where the entering flow is equal to the leaving flow for each node. In (3) the total number of cut items for each item should be larger than or equal to its demand $d_j$ $\forall j \in \{1, \dots, n\}$. Equations (4) and (5) identify the domain of variables.

## The Compressed Graph

The graph compression plays an important role in reducing the number of nodes/arcs of the initial graphs (Brandão and Pedroso 2016). Using compressed graphs helps in constructing a smaller model size in terms of number of variables, which in turns has an impact on the solution time.

*Step 1 Level Graph*

Consider the following example where the width of the strip is equal to 10. Three items are to be cut with widths 8, 6 and 4, respectively. The initial graph has a total of 6 arcs and 5 nodes (see figure 4). In the level graph, each node $u$ has two labels. The first one is *x(u)* which denotes the position of the node. The second one is *l(u)* which denotes the item of the node. The level graph of our example is displayed in Figure 5.

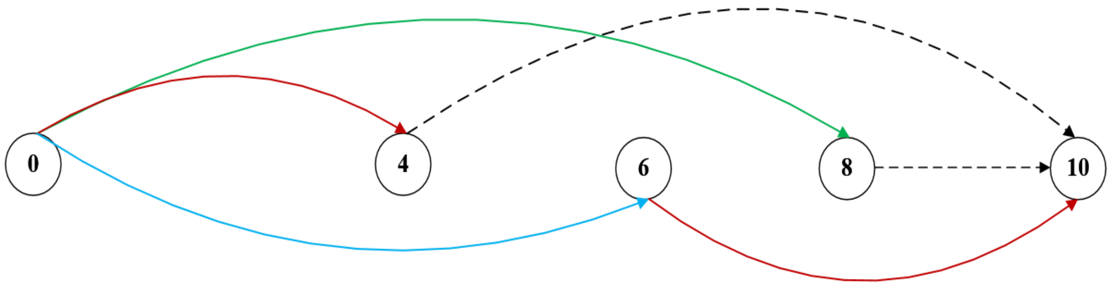


Figure 4. The initial graph

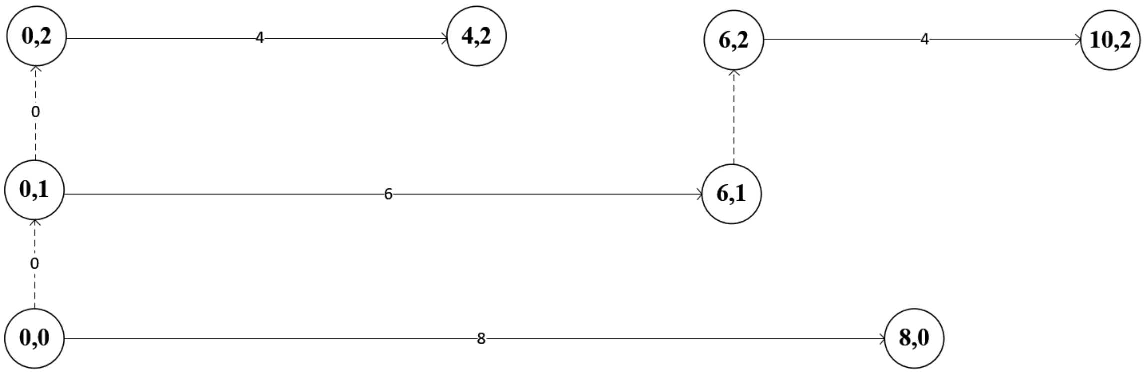


Figure 5. The level graph

*Step 2 Reduced graph*

In this step, we apply a reversed topological sorting to the set of arcs belonging to the level graph. Let set *A* contain all nodes of level graph and sorted in topological order reversely. The topological order is a linear ordering of vertices such that for every directed edge (*u*,*v*), node *u* comes before *v* in the ordering. Finally, each node *u* belongs to the set sorting is labeled $\emptyset^d(u)$ based on equation (6). The resulting final compressed graph of G1 is shown in Figure 6.

$$\emptyset(u) = \begin{cases} 0 & if\ u = S, \\ W & if\ u = T, \\ min_{(u',v,i)\in A:u'=u}\{\emptyset(v) - w_i\} & otherwise \end{cases} \quad (6)$$

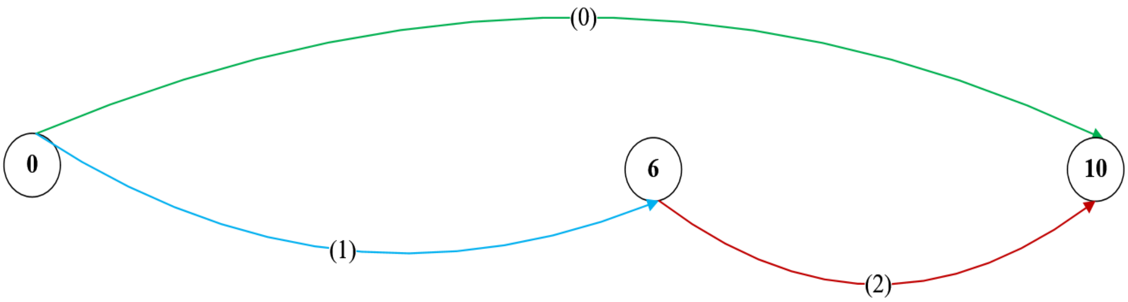


Figure 6. The compressed graph

## Results and Discussion

To evaluate the performance of the compressed arc-flow mathematical model, the method was coded in CPLEX IMB ILOG software version 12.6 and run on a PC with Windows10 operating system, 32 GB RAM and intel i7 CPU 3.4 GHz processor. The experiments were conducted on a set of 43 real-world instances A1-A43 used by Macedo et al. (2010).

Tables 2 and 3 depict the results of the arc-flow model without and with compressed graph, respectively. In these tables, we display for each instance:

- *D*: the total number of items demands
- *Optimal*: the optimal value of the objective function
- *Number of arcs*: the number of arcs before/after graph compression and the reduction percentage of the graph size
- *Computation time*: the required time to find the optimal solution before/after compression and the ratio of the former over the latter

Table 2. Results of the arc-flow formulation without graph compression

| Instance | D | Optimal | Number of arcs | Computation Time |
|---|---|---|---|---|
| A1 | 24 | 3678 | 9 | 0.04 |
| A2 | 38 | 32380 | 11 | 0.01 |
| A3 | 17 | 7986 | 4 | - |
| A4 | 16 | 3044 | 22 | - |
| A5 | 138 | 24870 | 451 | 1.00 |
| A6 | 17 | 1860 | 10 | 0.01 |
| A7 | 58 | 14039 | 43 | 0.05 |
| A8 | 16 | 1240 | 6 | - |
| A9 | 770 | 125823 | 28718 | 3.19 |
| A10 | 44 | 4815 | 31 | 0.01 |
| A11 | 724 | 94873 | 12867 | 2329.35 |
| A12 | 44 | 26928 | 16 | 0.01 |
| A13 | 304 | 27550 | 1104 | 3.08 |
| A14 | 809 | 139959 | 18684 | 14.20 |
| A15 | 339 | 80646 | 336 | 0.69 |
| A16 | 744 | 172244 | 7969 | 0.55 |
| A17 | 135 | 9902 | 102 | 0.04 |
| A18 | 559 | 134003 | 5802 | 0.50 |
| A19 | 507 | 119006 | 6550 | 0.12 |
| A20 | 215 | 53181 | 534 | 0.02 |
| A21 | 450 | 57348 | 3052 | 0.80 |
| A22 | 24 | 5005 | 6 | - |
| A23 | 248 | 27365 | 916 | 0.43 |
| A24 | 217 | 72231 | 49209 | 4.62 |
| A25 | 156 | 36591 | 50050 | 32.98 |
| A26 | 61 | 14791 | 4107 | 0.10 |
| A27 | 180 | 40438 | 109583 | 543.12 |
| A28 | 106 | 23529 | 30664 | 2.79 |

| | | | | |
|---|---|---|---|---|
| A29 | 218 | 57364 | 66787 | 5.31 |
| A30 | 39 | 7743 | 3357 | 0.12 |
| A31 | 64 | 16175 | 3230 | 5.22 |
| A32 | 184 | 56264 | 48971 | 1.77 |
| A33 | 309 | 72981 | 123658 | 159.24 |
| A34 | 46 | 11142 | 1238 | 0.35 |
| A35 | 144 | 34554 | 18114 | 18.92 |
| A36 | 52 | 14738 | 808 | 0.07 |
| A37 | 78 | 9743 | 200 | 0.04 |
| A38 | 160 | 46057 | 3006 | 0.29 |
| A39 | 22 | 6355 | 196 | 0.02 |
| A40 | 163 | 20510 | 3736 | 54.07 |
| A41 | 71 | 22522 | 4426 | 0.11 |
| A42 | 13 | 6953 | 37 | 0.01 |
| A43 | 22 | 7568 | 119 | 0.02 |

Table 3. Results of the arc-flow formulation with graph compression

| Instance | D | Optimal | Number of arcs | Computation Time |
|---|---|---|---|---|
| **A1** | 24 | 3678 | 7 | 0.02 |
| **A2** | 38 | 32380 | 7 | 0.01 |
| **A3** | 17 | 7986 | 2 | - |
| **A4** | 16 | 3044 | 10 | - |
| **A5** | 138 | 24870 | 143 | 0.56 |
| **A6** | 17 | 1860 | 5 | 0.01 |
| **A7** | 58 | 14039 | 30 | 0.01 |
| **A8** | 16 | 1240 | 5 | - |
| **A9** | 770 | 125823 | 5604 | 1.08 |
| **A10** | 44 | 4815 | 17 | 0.01 |
| **A11** | 724 | 94873 | 2378 | 22.63 |
| **A12** | 44 | 26928 | 11 | 0.01 |
| **A13** | 304 | 27550 | 163 | 0.38 |

| | | | | |
|---|---|---|---|---|
| **A14** | 809 | 139959 | 3537 | 3.79 |
| **A15** | 339 | 80646 | 113 | 0.29 |
| **A16** | 744 | 172244 | 1485 | 0.53 |
| **A17** | 135 | 9902 | 43 | 0.04 |
| **A18** | 559 | 134003 | 988 | 0.77 |
| **A19** | 507 | 119006 | 1487 | 0.05 |
| **A20** | 215 | 53181 | 149 | 0.02 |
| **A21** | 450 | 57348 | 709 | 0.93 |
| **A22** | 24 | 5005 | 5 | - |
| **A23** | 248 | 27365 | 234 | 0.35 |
| **A24** | 217 | 72231 | 15066 | 5.09 |
| **A25** | 156 | 36591 | 12362 | 10.99 |
| **A26** | 61 | 14791 | 979 | 0.10 |
| **A27** | 180 | 40438 | 25366 | 1682.88 |
| **A28** | 106 | 23529 | 6111 | 0.84 |
| **A29** | 218 | 57364 | 13814 | 0.89 |
| **A30** | 39 | 7743 | 807 | 0.10 |
| **A31** | 64 | 16175 | 803 | 4.42 |
| **A32** | 184 | 56264 | 11690 | 0.66 |
| **A33** | 309 | 72981 | 44162 | 17.48 |
| **A34** | 46 | 11142 | 357 | 1.26 |
| **A35** | 144 | 34554 | 5091 | 17.49 |
| **A36** | 52 | 14738 | 205 | 0.07 |
| **A37** | 78 | 9743 | 82 | 0.04 |
| **A38** | 160 | 46057 | 827 | 0.56 |
| **A39** | 22 | 6355 | 68 | 0.01 |
| **A40** | 163 | 20510 | 1726 | 96.10 |
| **A41** | 71 | 22522 | 858 | 0.07 |
| **A42** | 13 | 6953 | 18 | 0.01 |
| **A43** | 22 | 7568 | 48 | 0.02 |

The obtained results provide strong evidence of the positive impact of the graph compression in terms of graph size and computation time. Indeed, the number of arcs of the generated graph is reduced, on average, by 63.89% after using the proposed graph compression technique. This size reduction exceeds 80% in many of the considered instances. Moreover, the required computation time of the uncompressed version is about 4.61 times that of the compressed one.

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## Scientific Ethics Declaration

The authors declare that the scientific ethical and legal responsibility of this article published in EPESS journal belongs to the authors.

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