ÖĞRETMEN ADAYLARININ 3 BOYUTLU GEOMETRİDE "GÖRME" DENEYİMLERİ

Zekeriya KARADAĞ[1]

Özet

Matematik Öğretimi dersi alan sınıf öğretmenliği öğrencilerinin katıldığı bu çalışmada, öğretmen adaylarına kağıt-kalem ve dijital ortamlarda soma küplerini içeren bir takım etkinlikler verildi. Araştırma katılımcıların başlangıçta oldukça düşük olan 3 boyutlu görselleştirme becerilerinin evrimleşmesini irdelemektedir. Katılımcılar, başlangıçta temel şekilleri bile çizmekte veya bilgisayar ortamında oluşturmakta zorlanmaktaydılar. Fakat, Google SketchUp ve StarBoard gibi teknolojik araçların devreye girmesiyle ve onların sağladığı esneklik ve kolaylıklara hakim olmaya başladıkça gelişmeye başladılar. Araştırma verileri öğretmen adaylarının bu etkinlikler sayesinde geometrik düşünme becerilerinin gelişebildiğini göstermektedir. Hatta, öğretmen adaylarının bu etkinlikler sayesinde gelecekteki öğrencileri için öğrenme stratejileri de geliştirdiklerini göstermektedir.

Anahtar kelimeler: *3B geometry, teknoloji, uzamsal görselleştirme, Dinamik ve Etkileşimli Matematik Öğrenme Ortamları (DEMO2).*

[1] *Assistant Professor, Bayburt University, Faculty of Education, zekeriya@bilelim.net, Bayburt*

PRE-SERVICE TEACHERS EXPERIENCING "TO SEE" IN 3d-GEOMETRY

Abstract

Pre-service teachers, taking a method course to teach mathematics at elementary classroom, were exposed to explore various tasks by using soma cubes both in digital environment and paper-and-pencil environment. The research explores the participants' evolving ability of spatial visualization, which was very low at the beginning. They were struggling while drawing or constructing even very basic constructions. However, the improvements were accelerated as they integrated technological tools, StarBoard and SketchUp because of the flexibility and usability provided by these tools. Data suggests that pre-service teachers may improve their geometrical thinking through these tasks. Moreover, they demonstrated they developed learning strategies for their prospective students.

Keywords: *3D geometry, technology, spatial visualization, Dynamic and Interactive Mathematics Learning Environments (DIMLE).*

INTRODUCTION

This paper briefly describes an exploratory research built on a previous one which is presented in an NCTM yearbook by Sack and van Niekerk (2009). The research, also repeated later by Sack and Vazquez (2013), is basically based on helping young students develop their spatial thinking. In their intervention, they used soma cubes and Geocadabra, to help their participants develop understanding of 3D objects' properties through various spatial development activities. These activities include interacting and describing 3D soma cubes verbally, drawing front, side and top views both in paper-and-pencil and digital environments. The software they employed is Geocadabra, as a great example of Dynamic and Interactive Mathematics Learning Environments (DIMLE) (Martinovic and Karadag, 2012).

Why 3D geometry? "Learning and teaching geometry should be much more than a rote memorization of the facts!" This quote could be stated by many mathematics educators and researchers, and many of readers could have heard similar phrases. One of the main reasons behind complaining of geometry is the struggle of geometry learners upon their failure on solving geometry problems. Students usually state that they fail because they cannot "see." What does "to see" mean? In some languages, including English, "to see" means "to understand." Does it mean that students cannot understand geometry? What could be the reasons leading non-understanding or, even worse, misunderstanding of some "geometric facts"? Besides the challenges in learning geometry, including 3D geometry, it is rather challenging to teach geometry. How should teachers teach geometry? In what ways would they deliver geometry curriculum even if they have failed to build a rigor geometry background?

Graduates of Primary Teacher departments in Turkey are supposed to teach a number courses including mathematics at Grade 1-4. These grades are significantly important for students to develop their mathematical and geometrical thinking abilities. However, the mathematics background of these teachers as well as teacher candidates is relatively low (Canturk-Gunhan, Turgut, & Yilmaz, 2009; Dursun, Isiksal, & Cakiroglu, 2010; Turgut, 2007; 2012).

Therefore, teacher education programs, both pre-service and in-service, should help these teachers to learn what geometrical thinking, as well as mathematical thinking, means and how they improve their own geometrical thinking abilities. Moreover, they should also learn how they can develop their prospective students' geometrical thinking. As researchers in mathematics education, we should spend more time to understand challenges of learning and teaching of geometry.

In order to find some preliminary answers to guide me deepen more into the topic, I focused on how pre-service teachers experience with 3D geometry through a series of tasks. As put forward by Rafi et al. (2005), "spatial ability has been recognized as an important human skill set to evaluate the effectiveness in learning, training, working, and even playing." (p. 707). This study aims to explore how elementary pre-service teachers improve their geometrical thinking and if they could see the pedagogical differences among some spatial tasks. Therefore, the following research questions were designed to set the stage for the study:

How do pre-service teachers improve their geometrical thinking through 3D tasks employing concrete materials as well as digital learning environments?

How are pre-service teachers aware of pedagogical reasoning leading these tasks?

The following section provides a brief description of the theoretical framework of the study. Then, the study will be explained and data will be presented. Finally, findings will be discussed.

THEORETICAL FRAMEWORK

The importance of 3D geometry is less appreciated comparing to 2D geometry. One reason for this under appreciation could be the misinterpretation of the relationship between 3D and 2D geometries. 2D geometry is much more abstract comparing to 3D geometry, because 3D objects are situated all over the world. However, 2D geometry objects are abstracted figures drawn from 3D artefacts. For example, we have cube or rectangular prism in the nature, and we call their faces as square or rectangle depending on the definitions we set. That is, cubes or prisms are concrete objects whereas their faces demonstrating certain features are separated from these figures and called 2D objects by themselves.

In almost all countries, including Turkey, geometry curriculum starts with 2D geometry and then proceeds into 3D geometry. However, research suggests that geometry curriculum in schools should start with 3D because every artefact in the nature is 3D objects, and that we abstract 2D figures from these 3D objects (Walter Whiteley, personal communication, 2007). Since artefacts situated in nature can be called daily life instruments, they could easily be perceived by novice students. Once these figures are properly perceived, moving further and focusing some specific parts such as faces and sides could be much more accessible by students.

This research builds its theoretical framework on Spatial Operational Capacity (SOC) theory as the former study does. In order to develop students' 3D visualization, the Spatial Operational Capacity (SOC) theory suggests including activities that learners can act with real 3D objects, their 2D representations, and other semiotic representations as well as that can transform information from one representation to another. The theory described by Sack and van Niekerk (2009) is explained as follows:

• full-scale models (or scaled-down models) of large objects that can be handled by the child;
• conventional-graphic models, two-dimensional graphic (2-D) representations that bear resemblance to the real, three-dimensional (3-D) objects; and
• semiotic models, which are abstract, symbolic representations that usually do not bear any resemblance to the actual objects. Examples include view and floor-plan diagrams. (p. 142)

For symbolic representations, the participants were engaged in using Dynamic and Interactive Mathematics Learning Environments (DIMLE) while constructing their samples (Karadag & Aktumen, 2013; Karadag, Martinovic, 2012; Karadag, Martinovic, and Freiman, 2011). In this research, Google SketchUp and StarBoard were used as DIMLE.

METHODOLOGY

The participants were pre-service teachers taken a method course at a university located in a small city in Turkey. Although more than 150 pre-service teachers took the course, the experience of only a small part of the group will be presented here because the themes present similar characteristics for the rest of the data. Data includes pictures, interview records, notes taken in the field, as well as reflection papers collected from participants.

Research Model

The study is an exploratory study to determine pre- service teachers' experience on the tasks to improve their ability "to see" through 3D geometry context and if they develop a pedagogical perspective on how to deliver geometry curriculum. A descriptive qualitative approach was followed to analyze the data.

Thick Description of the Study

Since the study is a qualitative research, it is my responsibility to provide enough data for the readers to help them understand how the research was conducted and how data was collected. Students taking a method course were asked if they were volunteers for inclusion of the research and also informed that no personal information will be shared and only pseudonyms will be used instead of their names. I, myself, acted as a participatory researcher in the study, meaning that I was participant of the study as the lecturer of the course.

More than 150 pre-service teachers participated in the study, and they were grouped into 5 to 7 depending on their preferences. They were asked to get themselves familiar with all the tasks, described before the study. The tasks included to describe a 3D figure composed by several some cubes to a friend, who did not see it, and to ask him or her draw on regular whiteboard, create in digital environment by employing either Google Sketch Up or Hitachi Starboard. Moreover, tasks included to draw top, front or side views of a given composite figure and to find possible orientations from one view only.

The emphasis was on the process of geometrical thinking rather than on drawing or constructing. This emphasis was achieved through asking questions during the presentation of their performances. They were asked pedagogical questions similar to the followings:

1. Could you please tell me which task demands more cognitive effort, drawing or describing?
2. Could you please tell me which task demands more cognitive effort, drawing or creating in Starboard?
3. Could you please tell me which task demands more cognitive effort, creating in Starboard or Google Sketch Up?
4. Could you please tell me which task demands more cognitive effort, drawing front view of a figure or finding the orientation of the figure based on front view only?
5. Why do you think creating in digital environment is much demanded comparing to drawing the side view of a 3D figure?

Data Collection Tools, Reliability and Validity Issues

In order to collect data addressing the research questions of the study, three purposefully selected students were interviewed and videotaped. The video recordings were transcribed and qualitatively analyzed. Furthermore, the observation and reflection note by the participant researcher was used to triangulate the results obtained from the interviews.

As suggested by qualitative research experts through a consensus, the study aimed to put the truth on the table rather than seeking for evidence for reliability and validity (i.e. Charmaz, 2006; Creswell, 2002, 2005, 2007; Creswell and Miller, 2000).

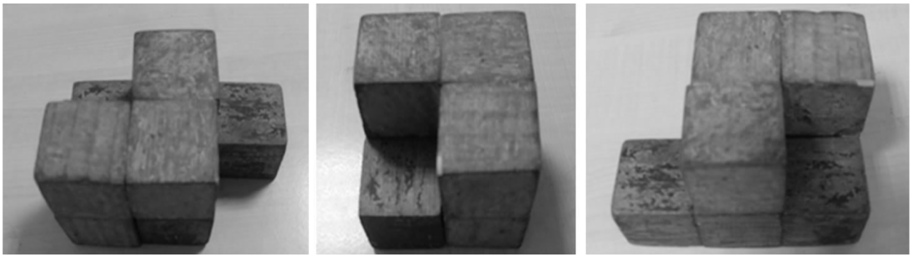
Tasks as data collection instruments

In the study, participants experienced how to describe a combination of soma cubes to a partner, how to draw what is described by the partner, how to create a digital version based on description by using two different software packages, how to draw certain views based on a short glance, and how to find possible orientations to place cubes based on a certain view. The following sections briefly describe the tasks:

Talk-and-draw

The figure 1 illustrates three different views of one of the combinations created by using two soma cubes. Two four-piece soma cubes, which are different both in shape and color, were used in the illustrated example. Although there are 8 cubes in total, one has to look at the construct through various perspectives to see all the cubes. Otherwise, some of the cubes remain unseen.

Figure 1. *Three different perspectives of a combined and complex construct*

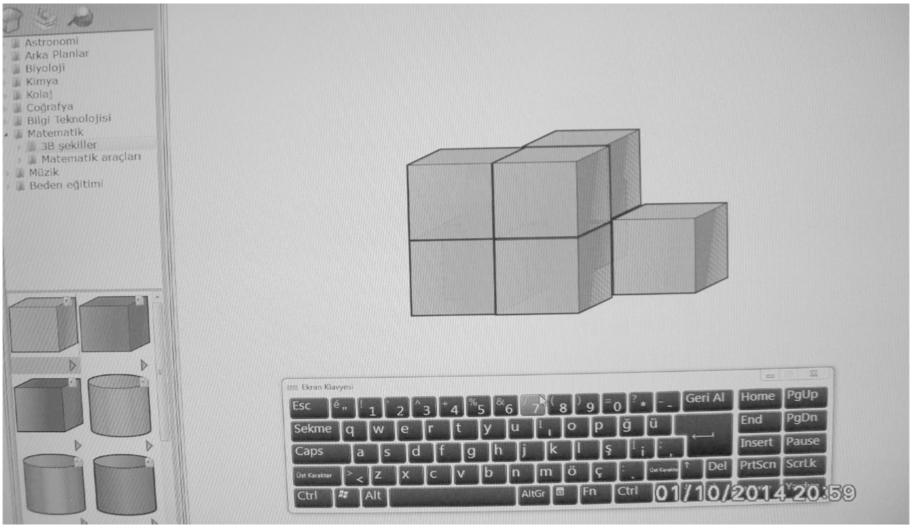


In this talk-and-draw task, participants were supposed to talk and draw the construct. One of the participants was allowed to see the construct and describe it to his or her friend whereas the other was not allowed to see it and asked to draw the construct based what was described.

Talk-and-construct

During data collection participants working in pairs, as opposed to preparing themselves for the study, experienced a describe-draw task as the first step. One of the pairs described the object to her partner, who was unable to see the original construct. Pairs performing this task were asked to create another construct by using StarBoard, a software common in the SmartBoards using in Turkey (Figure 2).

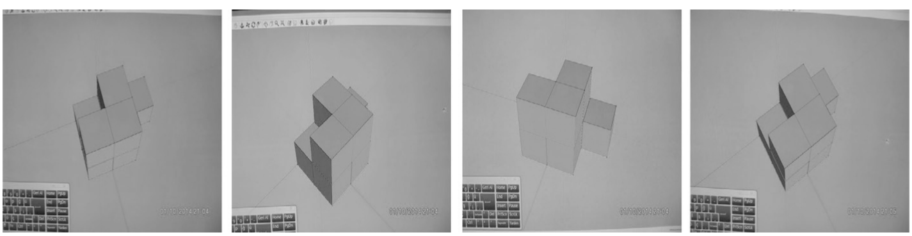
Figure 2. *A digitalized version of the construct illustrated in Figure 1*



Participants found using StarBoard comparatively easy after they got familiar in using the features of software. The reason for that was the opportunity of using ready-to-use figures seen on the left. The only problem they experienced was to place these cubes in the right order. However, almost all were successful in aligning cubes after spending some time.

In addition to using StarBoard to digitalize their mentally visual imagery, they also used Google SketchUp, which was found really full of fun by many participants (Figure 3). It was because they were able to rotate the construct and see various perspectives. Moreover, the opportunity to color the constructs made them more visible and made all faces identifiable.

Figure 3. *Another digitalized version of the construct illustrated in Figure 1*

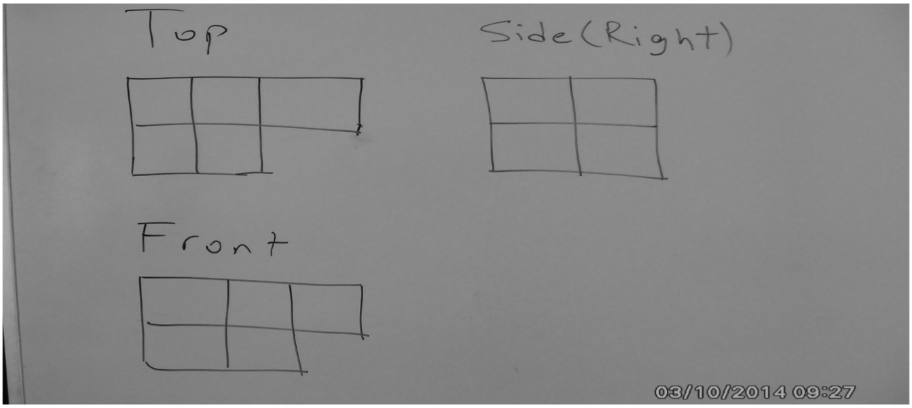


Challenging short-term memory

Participants were asked to draw front, top and side (right) views of the 3D objects in another task. The object was shown for a short period of time, and then, they were asked to draw a certain view based on what they have in their short-term memory. It challenged participants at the beginning; however, they became more successful while getting experienced.

Moreover, talented participants were challenged to draw or to construct the original figure stemming from these perspectives. Although the samples used in this study were not much complicated, still it was a high level task for many participants.

Figure 4. *Top, Front and Side views of the figure 1*

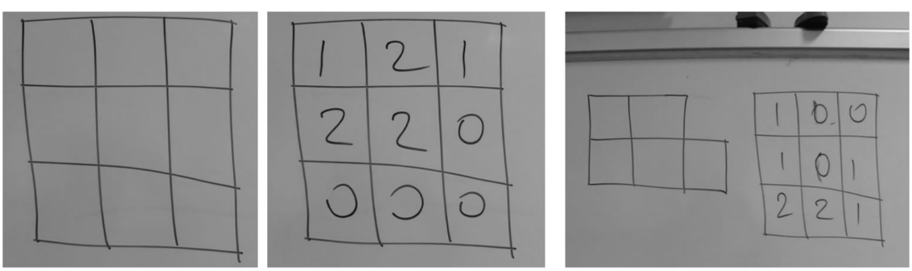


Probability in Geometry

At a further step of this task, some participants were asked to place numbers representing the number of cubes on a grid.

For example, the left part of the figure 4 represents a grid, which a participant was asked to place the number of cubes at a certain part. Considering the construct illustrated in figure 1, the right configuration should be as given in the middle part of the same figure. However, if the participant is exposed to only a certain view, i.e. front view is seen at the right part, it is rather difficult to find the exact placement if not impossible. A possible placement of cubes could be as seen in the right part of the figure.

Figure 5. *Placing cubes on a grid*



This final task was specifically important because it fostered participants' probabilistic thinking in addition to geometrical thinking. The middle arrangement as well as the right arrangement illustrated in figure 4 could be an answer for the front view given in the same figure. Participants were observed struggling at the beginning while accepting both configurations would be an answer for the same front view. Moreover, their main struggle was observed not only to accept both could be answers but to accept a mathematics, or geometry, problem would have more than one answer.

Focusing on the front view given on the right part of figure 5, the participant has to visualize possible constructions in her mind by putting 2 cubes at any place on the left column and middle column, and then one cube on any cell at the right column. Since 2+2+1 makes 5 cubes, the participant has to continue considering other possibilities to place 3 cubes. One constraint here should be not to place more than 2 cubes at any places back or front of the cells where 2 was written. Similarly, the participant should write 1 at most at the back cells of bottom-right cell.

FINDINGS AND INTERPRETATION

Qualitative research experts suggest analyzing qualitative data line by line, paragraph by paragraph or incident by incident. I followed the last procedure in order get answers to the research questions mentioned above because I wanted to deal with the data addressing directly my research questions.

Findings regarding the first research question

Having my first research question, "How do pre-service teachers improve their geometrical thinking through 3D tasks employing concrete materials as well as digital learning environments?" in mind, I looked for incidents to be able to conclude if they could or not could improve their geometrical thinking.

Data suggests that they could improve their performance in almost all tasks although many were complaining about their lack of abilities in drawing. However, some of them were still struggling while drawing on the board whereas almost all were successful in creating digital representations.

Regarding drawing top, front and side views, they were generally successful. When it comes to placing the number of cubes on the grid and drawing suggestions about the possible orientation of the figures, starting from numbers located on the grid, data suggested they were to extend their thinking.

Having said that they could successfully performed the tasks assıgned to them, let us talk about how they improved their geometrical thinking, or "to see" in 3D geometry. Three pre-service teachers selected for the interview helped me deepen my understanding through observation. The following excerpts may help the readers how the participants reflected on their understanding:

Researcher: “How do these tasks affect your understanding and “seeing” in 3D geometry?”

Participant A: “I have never had any experience of this kind throughout my schooling. Therefore, mathematics and geometry have been source of trouble. Now, I know how to see and work on a geometry object.”

Participant B: “Geometry has been a disaster for me! Although I memorized all the formulas, I could never solve a geometry problem, because I could not see the clues as my teacher asked.”

Participant C: “I was good at geometry if our teacher asked the similar questions at the exam. If the questions were different than we solved in the classroom, I was hardly successful.”

Findings regarding the second research question

Regarding the research problem, “How are pre-service teachers aware of pedagogical reasoning leading these tasks” I would present the following excerpt:

Researcher: “Do you think you want to use these activities or similar tasks with your prospective students?”

Participant B: “Absolutely! I would love to help them develop their own geometrical thinking.”

Researcher: “Do you really believe that they could benefit from these tasks?”

Participant B: “Well, I am sure they will. If I had experienced these tasks when I was at elementary school, I may have had more points at my high school entrance exams.”

DISCUSSION

Data suggests the following themes emerged from the study. How technology affects the participants’ visualization as well as transformation of their mental imagery in a digital environment seems one of the leading themes although it was not set as one of the goals of the research. Developmental visualization is emerged from analysis as expected. Moreover, data documents how participant gain new experiences in abstract thinking.

Technology

Most of the participants of this study had been exposed to use technology in their activities for the last three semesters. Still, a resistance had been observed in many activities. However, they preferred using technology rather than board-and-pen drawing when they were let free to decide. Similarly, when they interviewed, they were asked which method they would prefer if they are to decide. Almost all replies were on behalf of using technology. The reason behind this shift in decision is also questioned in interviews: “It is easier to use technology because I focus on my thinking if there is no need to think how to draw.” (Meltem)

I also questioned, “which method (drawing on board or constructing in the digital environment) challenges you more and which method demands more brain energy while performing the task?” Unsurprisingly, the responds were almost the same: “drawing made me think more because I am supposed to plan how to draw while thinking what to draw.”

This finding suggests that a good method to integrate technology in education would be to let users demand rather than to ask them to do so. By letting them face with a need to use technology, we may help them develop an intentional deserve to use and to integrate it their tasks. This method may also help them understand where and how to use more effectively in contrast to telling what and how to do.

Developmental visualization

A significant theme emerged from analysis, as expected, is the evidences on how participants develop their visualization abilities. Upon exposing a series of visualization tasks, many of the participants perform better comparing to their first experience. Although drawing or describing a simple figure was a challenge at the beginning, they got experienced in describing, drawing, constructing as well as finding a number of various alignments of cubes based on a given view.

I find documenting this finding really important for two main reasons: (1) they developed their visualization and creating mental images in their minds may be rather easier for their following life and (2) they should have learned a strategic approach on how to help their prospective students to develop a similar ability.

Abstract thinking

The participants were supposed to create mental images of the cubes to place the numbers on a given grid. This task is really challenging task because they start with a specific view, mostly front or side view because top view is found rather easier, and try to imagine what would be on the unseen part. For example, when they start with a front view they are supposed to consider possible arrangements for the back cells. The most common mistake they did at this point was to conclude that the number of cubes at the back cells cannot be more than the number of cubes set in the front cell.

To sum up, preliminary analysis of this study suggests that pre-service teachers learn how to develop their prospective students' visual ability while developing their own visualization. In addition, we may develop a better understanding of how to encourage our students to use technology in their learning tasks. Moreover, this series of tasks may open new windows to develop abstract thinking as well as visualization.

FURTHER IMPLICATIONS

In this study, I explored pre-service teachers' way of improving their geometrical thinking and of developing an understanding of pedagogical perspectives through a set of 3D tasks. My own observation and the personal informal communication between participants and me put forward that they enjoyed learning of 3D geometry through these tasks. Not only learning but they also developed an understanding of how they could help their prospective students develop their geometrical thinking.

When three selected students were interviewed, they also confirmed my observation notes and argued that they have learned through exploration. Moreover, they suggested working on these activities helped them improve their ability to look at the problems from various perspectives.

Regarding pedagogical benefits of the activities, they suggested drawing on the board and placing numbers on the grid demand cognitive effort. Interestingly, they all agreed on using technology if they have the option to choose.

Therefore, I would conclude that we should encourage students, pre-service teachers in this study, to use technology through various activities rather than explaining them why technology is useful in learning geometry. They would get familiar with technology and develop a better understanding of why, when, and where they should use technology.

In another conclusion, I would suggest including more activities in method courses. This inclusion may help them get more chance to communicate with each other and therefore collaborate to improve their understanding as well as to appreciate the importance of activities in mathematics and geometry.

Acknowledgement: Preliminary versions of this paper were presented as oral presentations at ICERI-2013 (International Conference of Education, Research, and Innovation) and ITTES-2014 (International Instructional Technologies and Teacher Education Symposium).

REFERENCES

Cantürk-Günhan, B., Turgut, M. & Yılmaz, S. (2009). Spatial Ability of a Mathematics Teacher: The Case of Oya, IBSU Scientific Journal, 3 (1), 151-158.

Charmaz, K. (2006). Constructing grounded theory: A practical guide through qualitative analysis. London: Sage Publications.

Creswell, J. W. (2007). Qualitative Inquiry and Research Design: Choosing Among Five Approaches (2nd Ed.) California: Sage Publications.

Creswell, J. W. (2005). Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research. NJ: Pearson Education.

Creswell, J. W. (2002). Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research. NJ: Pearson Education.

Creswell, J. W. & Miller, D. L. (2000). Determining validity in qualitative inquiry. Theory into Practice, 39(3), 124-130.

Dursun, Ö., Işıksal, M. & Çakıroğlu, E. (2010). İlköğretim Öğretmen Adaylarının Uzamsal Yeteneklerinin Cinsiyet ve Öğretmenlik Programlarına Göre İncelenmesi, IX. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi Proceedings, p. 11.

Karadag, Z., & Aktumen, M. (2013). Preface. Mevlana International Journal of Education. Vol. 3(3), pp. i-iv. http://dx.doi.org/10.13054/mije.si.2013.00

Karadag, Z., Martinovic, D., & Freiman, V. (2011). Dynamic and Interactive Mathematics Learning Environments (DIMLE). In L. R. Wiest, & T. Lamberg (Eds.), The Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, Nevada.

Martinovic, D., & Karadag, Z. (2012). Dynamic and interactive mathematics learning environments: the case of teaching limit concept. Teaching mathematics and its applications, doi:10.1093/teamat/hrr029

Rafi, A., Anuar, K., Samad, A., Hayati, M., & Mahadzir, M. (2005). Improving Spatial Ability Using a Web-based Virtual Environment. Automation in Construction, 14, 707-715.

Sack and van Niekerk (2009). Developing the spatial operational capacity of young children using wooden cubes and dynamic simulation software. In T. V. Craine & R. Rubenstein (Eds.) Understanding Geometry for a changing world. NCTM, US.

Sack, J., & Vazquez, I. (2013). Geocadabra construction box: A dynamic geometry interface within a 3D visualization teaching-learning trajectory for elementary learners. In Z. Karadag & M. Aktumen (Eds.) Dynamic and Interactive Mathematics Learning Environments, special issue on Mevlana International Journal of Education.

Turgut, M. (2007). İlköğretim II. Kademede Öğrencilerin Uzamsal Yeteneklerinin İncelenmesi, Unpublished Master Thesis, Dokuz Eylül University, İzmir.

Turgut, M. (2012). Matematik Ogretmeni Adaylarinin Uzamsal Gorsellestirme Becerileri. Journal of Research and Teaching, 1(2), 243-252.