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On The Concept A.C.H. Function

Şaziye YÜKSEL[1] Ahu AÇIKGÖZ[1]

Abstract: *In this note,the concept of almost continuity in the sense of Husain (a.c.H.) and the main Theorem 1 was given. In addition, the concept of strong localy compact was studied. We gave the proof in Theorem 1.2. which regardless of previous proof, if $f : X \to Y$ is mapping from a topological space X to a topological space Y which is a strong localy compact (where f denotes mapping which the function of the close graphicness and a.c.H.), then the function is continuous. (This theorem is the generalization of Theorem 1 given in.*

Keywords: **Almost-continous, completely closed, strong locally compact.**

Özet: Bu makalede Husain anlamında hemen hemen süreklilik kavramı ve temel teorem 1 [1] verildi. Ayrıca, kuvvetli lokal kompakt kavramı [2], çalışıldı. "Eğer X topolojik uzayından Y kuvvetli lokal kompakt uzayına giden $f : X \to Y$ fonksiyonu, kapalı grafikli ve a.c.H. ise bu takdirde f fonksiyonu süreklidir." şeklinde verilen Teorem 1.2 [2]'nin ispatını, [2]'de verilenden farklı şekilde ispatladık (Bu teorem, [1]'de verilen Teorem 1'in genelleştirilmesidir.).

Anahtar Kelimeler: **Hemen hemen süreklilik, tam kapalı, kuvvetli lokal kompakt.**

Introductıon

The function $f : X \to Y$ is almost continuous at $x \in X$ if and only if for each open $V \subset Y$ containing f(x), $(f^{-1}(V))^{-} \subset X$ is a neighbourhood of x [1].From here, if $(f^{-1}(V))^{-}$ is a neighbourhood of x, by definition of the neigbourhood, there exists $U \subset X$ open subset such that $x \in U$ and $U \subset (f^{-1}(V))^{-}$. From here, if there is $x \in U \subset X$ open set such that $U \subset (f^{-1}(V))^{-}$, by definition of the interior point, x is a interior point of $(f^{-1}(V))^{-}$, that is, $x \in (f^{-1}(V))^{-\circ}$. Therefore, $(f^{-1}(V))^{-\circ}$ is a neighbourhood of x.

Definition 1.1.[1] Let f be a mapping of a Hausdorff topological space E into another Hausdorff topological space F. F is said to be almost continuous at $x \in X$, if for each neigbourhood V of $f(x) \in F$, $(f^{-1}(V))^{-}$ is a neighbourhood of x.

[1] Selcuk University, Department of Mathemathics, [42031] Campus/Konya/TURKEY

Remark 1.1. [1] A continuous a mapping is almost continuous. But the converse is not true.

Example 1.1.The real function defined by f(x) =1 or according as x is a rational or irrational number, is a discontinuous function. But it is almost continuous, as easy to verify.

Definition 1.2.[1] A mapping a topological space E_U onto another topological space F_V is said to be completely closed if for each closed subset A of E. f(A) is V*-closed, where V* is defined on F ($\ni V^* \subset V$).

Lemma 1.1. [1] Let f be a mapping of a topological space E_U onto another topological space F_V such that the graph of f is closed in ExF. Then F_{V^*} ($V^* \subset V$) is a T_1 - space.

Theorem 1.1.[1] Let f be an almost continuous competely closed mapping of a topological space E_U onto a Hausdorff compact topological space F_V such that the graph of f is closed in ExF. Then f is continuous function.

Definition 1.3. [2] Let X be a topological space. For each $x \in X$ if x has closed compact neigbourhood, then X is called strong localy compact space.

Remark 1.2. Every localy compact space is strong localy compact space. But the converse of this statement is not true as seen by "Lynn A.Steen and J.Arthur See bach Jr. Counterexamples in Topology, New York, 1970" Ex.73.

Theorem 1.2. Let $f : X \to Y$ is almost continuous with the closed graph where Y is strong localy compact space. Then f is continuous function.

Proof. Let $x \in X$ be a point. Since Y is strong localy compact space, then an closed compact neigbourhood system of f(x) form a basis for an neigbourhood system of f(x). Thus, for every $V \in \vartheta(f(x))$ is open neigbourhood which contains f(x), there exists V_1 is closed compact of neigbourhood basis [3] such that

$V_1 \subset V$ (1)

Also, since the graph of f is closed (See Theorem 3.6.[4]), $f^{-1}(V_1) \subset X$ is closed set. Hence

$$(f^{-1}(V_1))^- = f^{-1}(V_1) \qquad (2)$$

In (1), if we take the inverse image under mapping of f of both side, we get

$$x \in f^{-1}(V_1) \subset f^{-1}(V) \qquad (3)$$

By (2) and (3) statements,

$$x \in (f^{-1}(V_1))^- \subset f^{-1}(V) \qquad (4)$$

Since f is almost continuous function, $(f^{-1}(V_1))^-$ is a neigbourhood of x. Therefore, $f^{-1}(V_1)$ is a neigbourhood of x. Consequently, f is continuous function.

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