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The Investigation of the Perceptions of Students with Disabilities on the Virtual Concrete-Representational-Abstract Model to Teach One Digit Multiplication Word Problem[1]

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ABSTRACT

In today's world, the knowledge of mathematics provides students opportunity to understand the world and helps them adopt themselves to the world easily. In this respect, students need to comprehend basic mathematical and word problem skills in elementary grades and combine new and old knowledge to be successful in future academic lives. Unfortunately, students with disabilities struggle in mathematics, and it affects their future academic achievement negatively. Many studies support evidence about concrete-representational-abstract (CRA) sequence to teach mathematics to students with disabilities. In addition to this, with development of technology, virtual manipulative began to be used in teaching mathematics. However, studies used virtual manipulative were limited. Therefore, in this study, the researcher created an iPad application that follows the CRA enriched with virtual manipulative. The iPad application has three phases and is designed to teach how to solve multiplication word problems conceptually and procedurally. The purpose of this study is to understand the perceptions of students with disabilities on teaching to solve multiplication word problems by using the CRA enriched with virtual manipulative. Systematic cluster sampling method is used since students are selected based on some criteria. In this study, the constant comparative method is used. The findings of this study revealed the iPad tool was effective for students with disabilities to teach one-digit multiplication word problems.

Keywords: Students with Disabilities, iPad application, Virtual Manipulatives, Mathematics, Word Problem.

[1] This article was produced from a doctoral thesis.

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Tek Basamaklı Sözel Kelime Problemlerinin Sanal Soyut-Yarı Somut-Somut Model ile Öğretimi Hakkında Yetersizliği Olan Öğrencilerin Görüşlerinin İncelenmesi[1]

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ÖZET

Günümüzde, matematik bilgisi öğrencilere dünyayı anlamaları ve kendilerini kolayca adapte etmeleri için olanak sunar. Bu yüzden, gelecek akademik hayatlarında başarılı olabilmeleri için öğrenciler temel matematik ve sözel problemleri çözme becerilerini ilkokul yıllarında kavramalı ve eski ile yeni bilgileri bir araya getirebilmelidir. Malesef, yetersizliği olan öğrenciler matematikte zorlanmaktadır ve gelecekteki akademik yaşantıları olumsuz etkilenmektedir. Literatürde, somut-yarı somut-soyut (SYS) öğrenme modelinin yetersizliği olan öğrencilere matematik öğretmek için uygun bir yöntem olduğunu kanıtlayan birçok araştırma vardır. Bununla beraber teknolojinin gelişmesi ile beraber sanal manipülatifler matematik öğretiminde kullanılmaya başlanmıştır. Fakat bu tür manipülatifleri kullanan araştırmalar sınırlı sayıdadır. Bu sebepten dolayı, bu araştırmada, araştırmacı sanal manipülatiflerin SYS öğrenme modeli içine yerleştirdiği bir iPad uygulaması geliştirmiştir. iPad uygulaması üç aşamadan oluşturulmuş olup, soruları çözerken öğrencilerin kavramsal ve yöntemsel anlayışlarını geliştirmek için tasarlanmıştır. Bu araştırmanın amacı yetersizliği olan öğrencilerin iPad uygulamasını kullanarak çarpma işlemli sözel problemlemleri çözmeyi öğrenirken sahip oldukları görüşleri incelemektir. Öğrenciler araştırma için bazı şartlara göre seçileceğinden dolayı sistematik zümrelere göre örnekleme yöntemi kullanılacaktır. Araştırmada sabit karşılaştırmalı yöntem kullanılanılmıştır. Bu araştırmanın bulguları İpad uygulamasının tek basamaklı sözel çarpma işlemlerinin öğretiminde yetersizliği olan öğrenciler için etkili olduğunu ortaya koymuştur.

Anahtar Kelimeler: Yetersizliği Olan Öğrenciler, iPad Uygulaması, Sanal Manipülatifler, Matematik, Sözel Problemler.

1. Introduction

Nowadays, mathematics achievement is a critical subject for students, especially students with disabilities (SWD) to be successful in the academic and professional lives. The mathematics achievement of students with disabilities is low and tends not to improve over time. And, this could be a potential barrier for their success (Ok & Kim, 2017). In this respect, students with disabilities need to comprehend necessary mathematical skills and knowledge in elementary grades.

Using the evidence-based strategies is an important aspect to enhance the mathematic achievement of SWD. Several evidence-based strategies have been used for SWD to teach mathematical concepts. The National Mathematics Advisory Panel technical report mentions five specific components, and one of them is sequencing or providing a range of examples (Gersten et al., 2008). The Concrete-Representational-Abstract (CRA) instructional sequence provides systematic pathway; starts with using concrete manipulatives, continues with representations of these manipulatives and finishes with an abstract equation of the problem (Flores, 2010). The CRA has been well documented as an effective approach to teach mathematics to SWD. The CRA was used to teach different mathematics subjects such as computation and word problems (Morin & Miller, 1998), place value (Peterson, Mercer, & O'Shea, 1988), addition (Mercer & Miller, 1992), subtraction (Flores, 2009, 2010), fraction (Butler, Miller, Crehan, Babbitt, & Pierce 2003), and multiplication (Flores, Hinton, & Schweck 2014; Harris, Miller, & Mercer, 1995).

With the advancement of technology, the new trend in mathematics is virtual manipulatives (Bouck, Working, & Bone, 2017). This type of manipulatives is interactive, web-based representations of objects that allow users to manipulate to construct mathematical skills and knowledge (Moyer, Bolyard, & Spikell, 2002). Research indicates positive effects of using virtual manipulatives on mathematics performance when teaching mathematics to SWD (Gersten et al., 2008; Maccini, Strickland, Gagnon, & Malmgren, 2008; Witzel, Riccomini, & Schneider, 2008). Virtual manipulatives were used to teach a variety of

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mathematical concepts and skills such as subtraction (Bouck, et al.), division, fractions (Reimer & Moyer,2005; Suh & Moyer, 2007), and word problems (Root, Browder, Saunders, & Lo, 2017).

The literature clearly demonstrated the effectiveness of CRA instruction and virtual manipulatives. However, the combination of them was not investigated in the study. In this regard, the researcher created an iPad application enhanced with virtual manipulatives that follows the CRA sequence to teach one-digit multiplication word problems. In this study, the researcher investigate how students with disabilities feel about CRA instruction, virtual manipulatives, and iPad application. The following research question is investigated: how do students with disabilities think about CRA instruction, virtual manipulatives, and iPad application?

2. Methodology

2.1. Participants

The participants collected from the pilot and dissertation studies of the author (Özdemir, 2017). The participants were chosen for these studies based on the following criteria, students must be: (a) performing below grade level benchmarks in mathematics, (b) able to use the iPad, virtual manipulatives and stylus, (c) performing below standards set for acceptable performance for curriculum based measures for mathematics. The researchers used the teachers' academic reports in participant selection. Eight students were selected as participants. The detailed information about students were provided in the table 1 below.

Table 1
Participants' Demographic Information and Academic Achievement

Participant	Age	Gender	Race	Grade	SES (free lunch)	Disability Classification	IQ
Kristine	10	Female	Black	3rd	Y	SLI	81[b]
Jane	9	Female	Hispanic	3rd	Y	OHI (504 Plan)	N/A
Susan	10	Female	Hispanic	3rd	Y	LD	76[a]
Amy	10	Female	Black	3rd	Y	Evaluation Progress	N/A
John	10	Male	Hispanic	3rd	Y	SLI	110[a]
Liam	9	Male	Black	3rd	Y	OHI	79[c]
John	8	Male	Black	3rd	Y	ED	96[d]
Eric	9	Male	Black	2nd	Y	ED	76[d]

Note. a Kaufman Brief Intelligence Test II; b Kaufman Assessment Battery for Children II; c DAS-II School-Age Battery; d Stanford-Binet Intelligence Scale V; SLI=Speech and Language Impairment; LD=Learning Disability; OHI=Other Health Impairment; ED=Emotional Disturbance

2.2. The iPad Application

The iPad application is created as an intervention based on the CRA model. The iPad application has three phases: (a) virtual manipulatives, (b) representation, and (c) abstract (See Figure 1). The iPad application has a question part, pencil and eraser icons, voice icon to listen the question, and done button to get a print screen of the product for all phases. In addition, there is a non-visible timer - starts with touching the voice icon and stops with touching the done button. Moreover, all three phases have two parts: (a) demonstrating the question, and (b) writing the equation. In the first phase, there are different types of virtual manipulatives. In the third phase, there is a "group of" section that students use them to demonstrate the questions

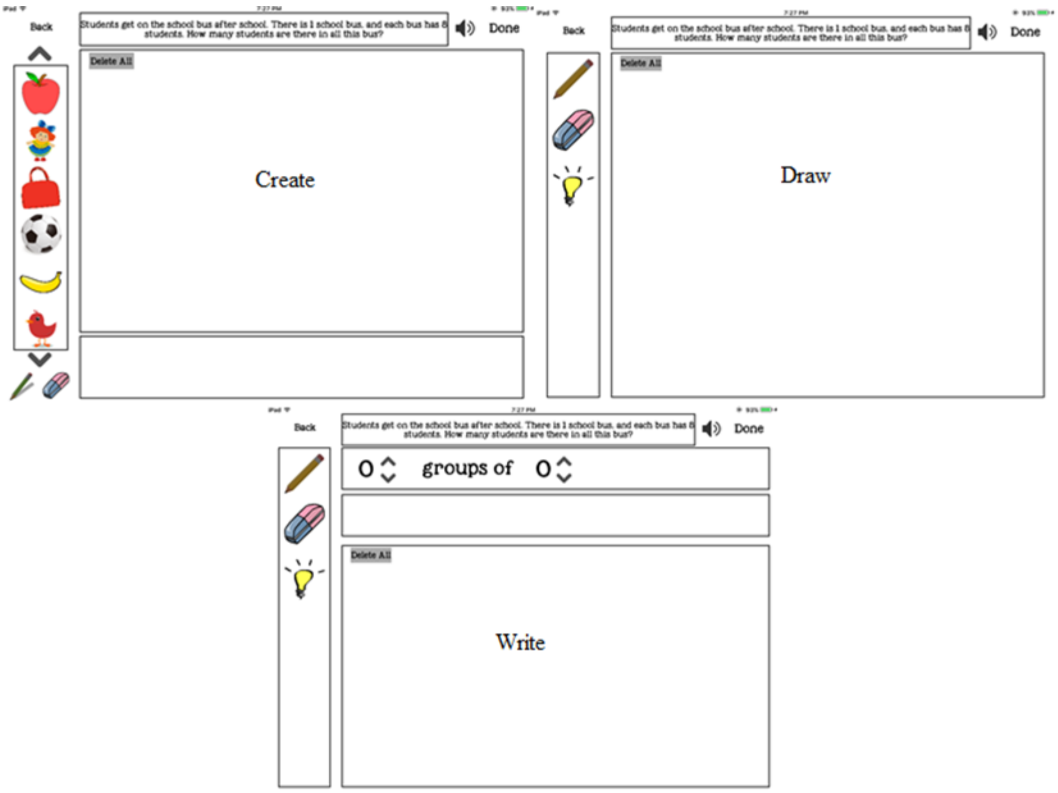

Figure 1. The Phases of the iPad Tool

2.3. Research Design

This is an expanded social validity part of a single subject research design study (Özdemir, 2017). A qualitative study used to understand what students think about CRA instruction, iPad application, and virtual manipulatives. Data were collected from interviews with students and artifacts from the students. The researcher conducted pre and post interviews with students. The pre semi-structured interviews were implemented before the intervention phase and post interviews were conducted after the intervention phase. All sessions were video recorded. The researcher also took field notes in addition to the resources.

The researcher used the constant comparative method to analyze codes, patterns, and trends from participants' data (Strauss & Corbin, 1998). In this method, the data analysis identifies the similar and different patterns by comparing one section of data with another section. Strauss and Corbin mention flexible guidelines for coding data that include open coding, axial coding, and selective coding. Additionally, Glaser (1965) mentions four steps to analyze data: (a) comparing incidents applicable to each category, (b) integrating categories and their properties, (c) delimiting the theory, and (d) writing the theory. Based on the procedure and coding system, the researcher analyzed video recordings of the participants, artifacts, and field notes when they solved multiplication word problems and answered interview questions.

The procedure of the qualitative study begins with transcription in this study. The video recordings were transcribed into a written text format. The researcher watched and listened to all video recordings and used a word document to type the text. This process was completed several times to ensure all details were recorded accurately. After transcribing the data, field notes and artifacts were provided to clarify the written text. The next step was open coding. At first, the researcher read and looked at the general event. In the second reading, the researcher focused on the general concepts. Then, the researcher investigated the specific sentences and thoughts for codes in the third reading. Later, the analysis of the constant comparative method was used to search for patterns and trends in the data. During the data analysis, similar concepts emerged and were grouped together through coding. These categories and subcategories were noted and trends and patterns were found.

In this study, the data were triangulated. Triangulation is a process to validate the study (Schwandt, 2007). In this process, the researcher used multiple views to examine the data. The idea of triangulation assumes data from different resources prove or disprove the outcomes from other resources. In this study, the interviews, artifacts, and field notes were triangulated to investigate the students' perceptions of the iPad application, CRA instruction, and virtual manipulatives.

3. Results

Analysis of data revealed four main themes regarding the perceptions of students: (a) conceptual understanding of multiplication, (b) strategy deficiency, (c) technology integration, and (d) the iPad application.

3.1. Conceptual understanding of multiplication

The researcher asked questions about how students felt solving multiplication word problems and what they could do to solve these problems. Students reported mixed feelings about solving multiplication word problems during the pre-interview. Kristine summarized her feeling stating: "I feel good because it is not that hard" - this also demonstrated her confidence. John summarized his confidence, as: "I feel good because I liked doing math". Additionally, Liam added: "I liked doing multiplication. It is easy, no problem". Eric moved his head as: "yes". They exhibited positive attitudes toward the multiplication word problems. On the other hand, other students showed some concerns about multiplication word problems. Jane said: "the questions are hard because math problems make me figure out what I am supposed to do". Susan also said "it feels like it is a kind of hard, and I am trying to not to give up". Finally, John mentioned: "I had a little confident about solving word problems, because some of them I do not know". Additionally, participants demonstrated a variety of weaknesses about solving word problems. Eric moved his head as: "no" when I asked "do you know how to find the answer?". John added: "add big numbers and little numbers". And Liam mentioned: "count what you have, then stop". This demonstrated students had limited conceptual understanding of multiplication. They confused the process with the addition process. The researcher supported this finding with the baseline probe sheets because students used addition to answer multiplication word problems.

At the post interview, all students verbalized good attitudes about solving word problems. John mentioned: "I am happy because I learned how to solve (multiplication word problems). It was difficult before". Additionally, Eric moved his head for: "yes" when I asked "do you feel great?". And, Liam noted: "I learned a lot". Jane said: "it helped my math problems. The problems are easy now". Additionally, Susan said: "I feel good. It is hard to draw problems in the classroom, but it is easy with iPad". John said "I feel comfortable. It seems easier now". As seen in these examples, participants enhanced their self-confidence about solving word problems. In addition to the comments, students solved the questions correctly by following the steps to solve multiplication word problems.

3.2. Strategy deficiency

The researcher asked students to explain what they do, when they solve word problems. When the students explained the steps, they displayed a lack of knowledge on how to solve word problems as a whole. All students emphasized some part of solving process. For example, Susan said "I use my finger to find answer and drawing the pictures". Amy mentioned: "I use pencil and paper, draw groups. When I am done with that, I figure out the problem, I count and decide groups. Then, try to figure it the answer". John said: "read the paragraph, you look at the numbers in the paragraph". These sentences clearly indicated that participants had strategy deficiencies on solving word problems. According to the baseline probes, students did not demonstrate conceptual understanding and did not write the equation properly. These data clearly indicated the participants had strategy deficiencies regarding how to solve multiplication word problems. Students, however, followed the steps the researcher taught during intervention and maintenance phases. They drew circles and dots to demonstrate the questions and write the equations. In sum, the intervention helped students to minimize their deficiencies.

3.3. Technology integration

In the pre-interview, the researcher asked what technological devices they used to solve questions and how these devices helped them during the process. Eric moved his head as "no" for any technological devices. Liam noted he used his phone to calculate numbers. Additionally, he mentioned they had a computer in their classroom and the teacher gave him permission to use it when he behaved well during the instruction. John emphasized the use of computer in his classroom. He said he used it for games. These answers indicated that technology was not a part of the mathematics instruction. Teachers used the computer as a reward and kept it apart from the instruction and technology in the classroom.

At the post interview, all students emphasized that they liked the iPad application. They mentioned they wanted to use it in their classrooms. All participants strongly recommended the iPad application as a way to improve word problem solving skills. Jane mentioned: "it helped me to improve my skill. Yes". Susan said: "I would just say to my teacher how I worked on this iPad tool. Oh yeah, I improved my solving skills". Amy also noted: "Yes, I want to use iPad in the classroom because it is much easier". John stated: "we do not have iPad in the classroom, but they can teach us with this iPad. They do and track it". As a result, it was evident the iPad tool or another similar technological device could be used to provide instruction in mathematics.

3.4. The iPad application

The students' views changed after experiencing the iPad tool as an intervention. Participants reported positive attitudes toward the iPad application and its phases. John said: "he preferred iPad instead of paper". John specifically liked the write part. He said: "it is simple and show what you need to do. Then, it just makes it easier for you". Kristine said draw part was helpful for her. Additionally, Eric selected draw part. When I asked the reason, he did not provide a response. On the other hand, Liam liked the write part most and he said: "it is better than others". Eric said: "yes" for the iPad. Additionally, Eric pointed to the virtual parts, when I asked why, he touched the virtual manipulatives. Other students demonstrated enthusiasm for create phase. Amy mentioned: "create part I liked a lot. But, it was new for me because you look at the pictures". Susan noted: "create. It was easy with pictures and write them [equations]". Jane specifically mentioned that "I liked this part because pictures look real". In sum, participants had positive perceptions about the phases of iPad tool, especially, the virtual manipulative phase. Students thought the create phase prepared them for the drawing part and made writing the equation easier. Additionally, the create phase was based on pictures, and they related pictures with the real objects. That was an important point of this iPad tool.

However, Liam mentioned "boring and waste of time". for create phase. When the researcher looked at the draw part. Liam mentioned the same comment. He said: "I am bored". Based on my anecdotal notes, Liam was frustrated when working on the create phase. He tried to complete this phase fast. He touched the delete all button a couple of times, and the iPad tool became frozen. Therefore, he did not like the create phase.

4. Discussions

According to the National Council of Teachers of Mathematics (NCTM, 2000), there are five components for math proficiency: (a) conceptual understanding, (b) procedural fluency, (c) adaptive reasoning, (d) strategic competence, and (e) productive disposition. The Council of Chief State School Officers (2010) emphasized these components are interrelated, but conceptual and procedural understandings are the first component to develop. Conceptual understanding is the ability to recognize and understand the rationale behind the subject such as relationships and reasons to solve math problems where procedural understanding is the knowledge of rules, symbols and steps (Burns, Walick, Simonson, Dominguez, Harelstad, Kincaid, & Nelson, 2015).

Alter, Brown, and Pyle (2011) examined the conceptual understanding, problem solving skills, and time on task behavior of students with EBD. These researchers mention students' conceptual understanding helped them to improve their problem solving skills and time on task behaviors. Jitendra, George, Stood, and Price (2010) used schema-based instruction (SBI) for students with EBD to make sense of and solve word problems. They note the SBI supported these students by promoting understanding and teaching them how to use problem-solving skills successfully. Mulcahy and Krezmien

(2009) used a contextualized instructional package that consists of conceptual and procedural understandings for teaching the area and perimeter skills. They found students improved their accuracy for computing area and perimeter. Based on previous studies, the current study supported the literature as the researcher taught both conceptual and procedural understandings and found both of them could be used to teach a mathematical skill for students with disabilities. This finding indicate the iPad tool was appropriate to teach conceptual and procedural understandings.

Strategy competence focuses on the process to solve mathematical problems (NCTM, 2008). In this study, the comments of students during the pre-interviews demonstrated the students did not know the steps to solve multiplication word problems. This indicated students lacked strategies to solve these mathematics problems. After the implementation of the intervention, students followed the steps explained during the instruction, and minimized their strategy deficiencies on solving multiplication word problems. Results indicated following the teaching protocol explicitly enhanced the conceptual and procedural understanding and minimized the strategy deficiencies of students with disabilities. Researchers have conducted studies that focus on teaching strategies to enhance the mathematics achievement of students such as: a strategy-based intervention (Alter et al., 2011), schema-based strategy instruction (Jitendra et al., 2010; Peltier & Vannest, 2016a), and STAR strategy (Peltier & Vannest, 2016b). The current study contributes to the literature as it found the iPad tool that follows CRA instructional sequence was effective in teaching one-digit multiplication word problems to students with disabilities.

This study used virtual manipulative in the CRA sequence and thus contributed to the word problem solving literature in a unique way. The researcher created an iPad tool that followed the CRA sequence. The concrete phase of CRA instructional sequence was introduced as a form of virtual manipulative that provides interactive and dynamic visual representation of concrete materials (Bouck et al. 2014; Martin, 2007; Martin & Lukong, 2005). This allowed students to manipulate the pictures to visually represent multiplication word problems. Additionally, virtual manipulatives have been used to help students construct mathematical knowledge and conceptual understanding to solve multiplication word problems (Kelly, 2006).

In the literature, Satsangi, Bouck, Taber-Doughty, Bofferding, and Roberts (2016) compared the concrete and virtual manipulatives to teach solving algebraic equations. They worked with three secondary students with learning disabilities. They found all three students accurately solved algebraic questions with concrete and virtual manipulatives. In addition to accuracy scores, the researchers concluded equations could be solved in less time than concrete manipulatives. Root, Browder, Saunders, and Lo (2017) compared the concrete and virtual by using a modified schema-based instruction to compare types of mathematical word problems skills. There were three elementary students with autism spectrum disorder. They concluded two of three students performed more steps in the virtual condition, and third student had the same performance in both virtual and concrete conditions. Additionally, students preferred to work with the virtual condition and maintained their knowledge when given an option during the maintenance phase. Bouck et al. (2014) compared the concrete and virtual manipulative by the use of a system of least prompts and suggested using virtual manipulatives with different instructional methods for a better glimpse of how students successfully solve mathematics problems.

The findings of this study support previous studies on virtual manipulatives. The students supported this idea when they explained that the virtual manipulative phase helped them to visualize the word problems in the next phases. Some students appreciated and enjoyed the manipulation of pictures to represent word problems. In this phase, virtual manipulatives helped students to demonstrate groups and units for the multiplication word problems. Students understood the question and decided what they needed to use for group and unit, and represented the question by manipulation of the pictures. In this way, students used virtual manipulatives to organize their conceptual understanding. In this study, the virtual manipulative phase was the learning phase for students with disabilities. During the baseline phase, students did not have conceptual and procedural understandings for multiplication word problems. After completing the virtual manipulative phase, students with disabilities learned the how to solve these problems conceptually and procedurally In summary, the use of virtual manipulatives and its integration into classroom instruction has potential to enhance the learning of students with disabilities.

The positive feedback for the iPad tool intervention provided strong support for the social validity of the virtual CRA model to solve multiplication word problems. All students liked the iPad application for learning multiplication word problems. This indicated the structure and sequence of iPad application was helpful for students with disabilities. Additionally, all students strongly suggested the iPad tool as a way to improve word problem solving skills and wanted to use this iPad tool in their classrooms. This indicated the potential for teachers to effectively use the iPad tool in their classroom and track students' progress. In addition to classroom use, this tool might be helpful for families as it provided clear directions with embedded virtual manipulatives, as well as, drawing and writing tools for parents to support their child's ability to solve multiplication word problems.

5. Conclusion

The purpose of this study was to examine the perceptions of students on the virtual CRA instructional approach to solve one digit multiplication word problems. The CRA instruction approach was selected because it has been shown to be effective on students with disabilities on a variety of mathematics subjects. Technology is integrated into the CRA approach and virtual manipulatives are embedded into the iPad application to investigate the effectiveness of this type of manipulative. The results of this study revealed the iPad tool was effective for students with disabilities to teach one-digit multiplication word problems. The findings support replication of this study across different disabilities types, grade levels, mathematical content generalize effects to the education field to further investigate its potential to be recognized as a research based intervention. This study contributes to the mathematics literature and the use of a CRA technology based intervention with students with disabilities (Özdemir, 2017).

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