



Research Article

Five different distributions and metaheuristics to model wind speed distribution

Mohammed WADI^{1,*}

¹Electrical & Electronics Engineering Department, Istanbul S. Zaim University, Istanbul, Turkey

ARTICLE INFO

Article history

Received: 06 February 2021

Accepted: 17 April 2021

Keywords:

Wind Energy, Wind Speed, Statistical distributions, Probability distribution function (PDF), Cumulative distribution function (CDF), Inverse CDF (ICDF), Grey Wolf Optimization (GWO), Whale Optimization Algorithm (WOA)

ABSTRACT

This paper presents a comprehensive empirical study of five distribution functions to analyze wind energy potential: Rayleigh, Weibull, Gamma, Burr Type XII, and Generalized Extreme Value. In addition, two metaheuristics optimization methods, Grey Wolf optimization and Whale optimization algorithm, are utilized to determine the optimal parameter values of each distribution. Five error measures are investigated and compared to test the accuracy of the introduced distributions and optimization methods, such as mean absolute error, root mean square error, regression coefficient, correlation coefficient, and net fitness. The Catalca site in Istanbul, Turkey, was selected to be the case study to conduct this analysis. The obtained results confirm that all introduced distributions based on optimization methods efficiently model wind speed distribution in the selected site. Although Gamma distribution based on GWO and WOA outperformed other distributions for all datasets at all heights, it was the worst in terms of computation complexity. Rayleigh distribution occupied the latest rank, but it was the best in terms of computation complexity. MATLAB 2020b and Excel 365 were used to perform this study.

Cite this article as: Mohammed W. Five different distributions and metaheuristics to model wind speed distribution. J Ther Eng 2021;7,Supp14:1898–1920.

INTRODUCTION

Wind energy is one of the most rapidly growing renewable energies worldwide due to its many advantages. Wind speed distribution for a specific site shows the available wind potential. Once the distribution of wind patterns is established, the wind energy potential can be determined. Wind energy availability allows investors and developers to collect more accurate feasibility of the underlying site. The

feasibility study process consists of several steps: however, wind regime characterization via distribution functions is one of the most critical steps. During the last decades, many Probability Density Functions (PDFs) have been appeared in literature to represent wind speed distribution, such as Weibull [1], Rayleigh [2], Gamma [3], Normal [4], Lognormal [5], Logistical [6], Beta [7], Nakagami [8], Burr

*Corresponding author.

*E-mail address: mohammed.wadi@izu.edu.tr

This paper was recommended for publication in revised form by Regional Editor Ali Celen



[9] distribution functions, and others. However, Rayleigh, Gamma, and Weibull functions are the most widely used among these functions [10], [11], [12].

Wadi et al. [12] presented a statistical study based on the two-parameter Weibull distribution function for wind data for three years at Catalca in Istanbul, Turkey. Three estimation methods for Weibull parameters: approximation, graphical, and Energy Pattern Factor (EPF) methods, are studied and compared. Wind data for two years from the southern area in Pakistan at four heights were used to evaluate the wind potential based on Weibull distribution [13]. The Weibull parameters were modeled based on three different algorithms: Grey Wolf Optimization (GWO), Particle Swarm Optimization (PSO), and Cuckoo Search Optimization (CSO). Moreover, four numerical methods; EPF, Method of Moments (MOM), Empirical Method of Justus (EMJ), and Modified Maximum Likelihood (MML) estimation methods were used. Junk and Schindler [14] presented 24 distributions to assess their Goodness-Of-Fit (GOF) based on four years of wind speed data at different sites worldwide. Parameters of distributions were varied between one and five parameters. MOM, L-Moment (LM), Maximum Likelihood (ML), and least-squares (LS) methods are used to assess these parameters.

Two-parameter Weibull distribution function based on four estimation methods: graphical, empirical, EPF, and MML to estimate the capacity factor of wind turbines in Jaisalmer district of western Rajasthan in India was presented in [15]. MML method provided the best matching while the graphical method had the least matching. Two-parameter Weibull, three-parameter Weibull, two-parameter Gamma, and two-parameter Lognormal were presented to model wind speed at the airport site in Dolny Hricov [16]. ML method was applied to estimate the parameters of the distributions. The three-parameter Weibull and the two-parameter Weibull achieved the best first and second fitness, respectively.

Due to the inability of the Weibull distribution function to achieve the required matching in some wind regimes and the calculations complexity of mixture distribution functions, many researchers introduced different distribution functions like Birnbaum Saunders [17], [18]. Mohammadi et al. [19] presented the Birnbaum Saunders distribution to model the wind speed frequency distribution for ten sites distributed in the Ontario province of Canada. The GOF of the two-parameter Birnbaum Saunders distribution was compared with earlier nine one-component distributions, and the obtained results have confirmed the best fitness of the Birnbaum Saunders distribution. Several types of research based on Burr and inverse Burr distribution functions have appeared in the literature [9], [20], [21], [22]. A statistical analysis of wind speed data based on a four-parameter Burr distribution function in Antakya, Turkey, was proposed in [9]. GOF proved that the Burr distribution was more convenient than the Weibull and

generalized-Gamma distributions for describing the real distribution of wind speed data. Chiodo and Falco in [21] presented the two-parameter inverse Burr distribution function to represent the extreme values of wind speed. Three estimation methods: moment, ML, and quantile methods are used. The inverse Burr distribution was provided the best matching.

Wind speed and frequency are basically based on the site. Hence, some distribution functions that can successfully describe wind patterns at a specific site could fail at another. Therefore, this paper presents an extensive study to compare the performance of five distributions, namely, Rayleigh, Gamma, Weibull, Burr XII, and Generalized Extreme Value. Two metaheuristics optimization methods: GWO and Whale Optimization Algorithm (WOA) are utilized to evaluate the optimal parameter values of each distribution. As various GOF measures can lead to varied fit evaluation results, five measures such like Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Regression Coefficient (R^2), Correlation Coefficient (R), and net fitness are applied to assess the accuracy of introduced distributions. Wind data for three years from 2008 to 2010 at three different heights, i.e., 30, 60, and 80 m, from the Marmara area (Catalca) are utilized to carry out this study.

The rest of this paper is organized as follows: Section II is dedicated to exploring the statistical distribution functions, including the formulas of PDF, Cumulative Density Function (CDF), and Inverse CDF (ICDF) for each distribution. Furthermore, Section III introduces the methodology of selecting the optimal parameters of the distributions using GWO and WOA. Section IV elaborates the wind speed extrapolation procedure and wind energy potential. Section V explains the accuracy measures that are exploited to examine the performance of each distribution. Besides, Section VI discusses the obtained results. Finally, Section VII concludes the paper.

STATISTICAL DISTRIBUTION

As said, determination of the wind speed distribution is an essential step in assessing wind energy potential at a particular location. Once the wind speed distribution pattern is accurately determined, the technical and economic features belonging to the location can be appropriately specified. To describe the distributions of wind speed data, five different distributions have been studied. Hereinafter, a concise explanation of these distributions is given.

RAYLEIGH DISTRIBUTION

Rayleigh distribution (RD) was derived by British physicist Lord Rayleigh. The one-parameter Rayleigh distribution is a particular case of Weibull distribution in which the shape parameter (k_w) is assumed to equal 2 [23]. Due to its simplicity and the ability to accurately describe

wind regimes, many researches used Rayleigh distribution to evaluate the potential of wind at various locations worldwide [10], [24], [25]. One-parameter Rayleigh PDF is defined as follows [26].

$$f(v) = \frac{2v}{b_R^2} \exp\left(-\frac{v^2}{b_R^2}\right) \quad (1)$$

where $f(v)$ is the probability of observing wind speed v , $v = 0, 1, 2, \dots, N$, N is the size of wind speed vector, and $b_R > 0$. The Rayleigh CDF, $F(v)$ and ICDF, $G(p)$ are defined according to equation (2) and equation (3), respectively.

$$F(v) = 1 - \exp\left(-\frac{v^2}{2b_R^2}\right) \quad (2)$$

$$G(p) = b_R \sqrt{-2\ln(p-1)} \quad (3)$$

Where $0 \leq p \leq 1$.

WEIBULL DISTRIBUTION

Weibull distribution (WD) is one of the commonly used functions to represent wind speed measurements due to its high flexibility and accuracy [2], [15], [25]. Bi-parameter Weibull distribution depends on the shape (k_W) and the scale (c_W) parameters. The PDF of Weibull distribution is given by:

$$f(v) = \frac{k_W}{c_W} \left(\frac{v}{c_W}\right)^{k_W-1} \exp\left(-\left(\frac{v}{c_W}\right)^{k_W}\right) \quad (4)$$

where k_W and c_W are the shape and scale parameters. Weibull distribution has several vital characteristics that its parameters are evaluated at a particular height, and it is conceivable to extrapolate them for different heights [27]. The Weibull CDF, $F(v)$ and ICDF, $G(p)$ [28], [29] are defined as equations (5) and (6), respectively

$$F(v) = 1 - \exp\left(-\left(\frac{v}{c_W}\right)^{k_W}\right) \quad (5)$$

$$G(p) = -c_W [\ln(1-p)]^{\frac{1}{k_W}} \quad (6)$$

Gamma Distribution

Gamma distribution (GD) is also a widely used distribution; it is largely related to Exponential and Normal distributions. Besides, many distributions like Exponential, Chi-squared, and Erlang distributions are special cases of the Gamma distribution. The Gamma PDF is expressed as [10]:

$$f(v) = \frac{c_G^{k_G} v^{k_G-1} \exp(-c_G v)}{\Gamma(k_G)} \quad (7)$$

where k_G and $c_G > 0$, and Γ is the gamma function and can be computed by the following formula:

$$\Gamma(k_G) = \int_0^\infty t^{k_G-1} \exp(-t) dt \quad (8)$$

The Gamma distribution formulas for the CDF, $F(v)$ and ICDF, $G(p)$ [10] are according to equations (9) and (11), respectively.

$$F(v) = \frac{\Gamma_v(\gamma)}{\Gamma(\gamma)}, v \geq 0; \gamma > 0 \quad (9)$$

where $\Gamma_v(\gamma)$ can be computed using the following formula.

$$\Gamma_v(k_G) = \int_0^v t^{k_G-1} \exp(-t) dt \quad (10)$$

$$G(p) = \frac{1}{c_G^{k_G} \Gamma(k_G)} \int_0^v t^{k_G-1} \exp\left(-\frac{t}{c_G}\right) dt \quad (11)$$

Burr XII Distribution

Burr distribution (BD), also known as Burr Type XII distribution, is a continuous distribution which Irving W. Burr originally introduces. The PDF of the three-parameter Burr distribution is given by the following equation [30], [31], [32]:

$$f(v) = \frac{k_{B2} k_{B1} \left(\frac{v}{c_B}\right)^{k_{B1}-1}}{\left(1 + \left(\frac{v}{c_B}\right)^{k_{B1}}\right)^{k_{B2}+1}}, v > 0, c_B > 0, k_{B1} > 0, k_{B2} > 0 \quad (12)$$

The Burr distribution CDF, $F(v)$ [33] and ICDF, $G(p)$ [34] can be computed from equations (13) and (14), respectively.

$$F(v) = 1 - \frac{1}{\left(1 + \left(\frac{v}{c_B}\right)^{k_{B1}}\right)^{k_{B2}}}, v > 0, c_B > 0, k_{B1} > 0, k_{B2} > 0 \quad (13)$$

$$G(p) = \left((1-p)^{\frac{-1}{k_{B2}}} - 1 \right)^{\frac{1}{k_{B1}}} \quad (14)$$

Generalized Extreme Value Distribution

Generalized Extreme Value distribution (GEVD) is a continuous probability function developed by combining three simpler distributions: Extreme Value, Frechet,

Table 1. All used distributions and their parameters

Distributions	Number, (Name) of parameters	Parameters
Rayleigh	1, (Defining parameter)	$P1 = b_R$
Weibull	2, (Scale, Shape)	$P1 = c_W, P2 = k_W$
Gamma	2, (Shape, Scale)	$P1 = k_G, P2 = c_G$
Burr	3, (Scale, Shape1, Shape2)	$P1 = c_B, P2 = k_{B1}, P3 = k_{B2}$
Generalized Extreme Value	3, (Shape, Scale, Location)	$P1 = k_{GE}, P2 = c_{GE}, P3 = \lambda_{GE}$

and Weibull. Generalized Extreme Value distribution is often used as an approximation to model the maxima of long (finite) sequences of random variables. The following formula gives the PDF of the three-parameter Generalized Extreme Value distribution [35].

$$f(v) = \begin{cases} \frac{1}{c_{GE}} \exp\left(-\frac{v - \lambda_{GE}}{c_{GE}}\right) \exp\left(-\exp\left(-\frac{v - \lambda_{GE}}{c_{GE}}\right)\right), k_{GE} = 0 \\ \frac{1}{c_{GE}} \left(1 + k_{GE} \frac{(v - \lambda_{GE})}{c_{GE}}\right)^{-\frac{1}{k_{GE}}} \\ \exp\left(-\left(1 + k_{GE} \frac{v - \lambda_{GE}}{c_{GE}}\right)^{\frac{-1}{k_{GE}}}\right), k_{GE} \neq 0 \end{cases} \quad (15)$$

where k_{GE} , c_{GE} , and λ_{GE} are the shape, scale, and location parameters, respectively. Furthermore, Generalized Extreme Value distribution is pivotal in modeling the extreme wind events [36]. Generalized Extreme Value distribution formulas for the CDF, $F(v)$ and ICDF, $G(p)$ are as follows [35]:

$$F(v) = \begin{cases} \exp\left(-\exp\left(\frac{v - \lambda_{GE}}{c_{GE}}\right)\right), k_{GE} = 0 \\ \exp\left(-\left(1 + k_{GE} \frac{v - \lambda_{GE}}{c_{GE}}\right)^{\frac{-1}{k_{GE}}}\right), 1 + \\ k_{GE} \frac{v - \lambda_{GE}}{c_{GE}} > 0, k_{GE} \neq 0 \end{cases} \quad (16)$$

$$G(p) = \begin{cases} \lambda_{GE} - c_{GE} \log(-\log(p)), k_{GE} = 0 \\ \lambda_{GE} + \frac{c_{GE}}{k_{GE}} \left((- \log(p))^{-k_{GE}} - 1\right), k_{GE} \neq 0 \end{cases} \quad (17)$$

Table 1 summarizes all the used distributions along with the name and notation of their parameters.

MATERIALS AND METHODOLOGY

Evolutionary Algorithms (EAs) are prevalent in many research areas. This is because such algorithms are simple, flexible, and capable of avoiding local optima. On the other hand, EAs may suffer from various drawbacks such as long computation time, no guarantee to converge, and having several operating parameters to be adjusted before starting [37]. In order to overcome the limitations mentioned above, GWO and WOA are utilized. In this section, GWO and WOA are introduced briefly. Then, the methodology of exploiting the metaheuristics to select the optimal parameters of the introduced distributions is explained.

Grey Wolf Optimization

GWO algorithm imitates the leadership hierarchy and hunting mechanism of grey wolves in nature. Four types of grey wolves, ordered from the top of the pack hierarchy as alpha, beta, delta, and omega, are used for simulating this mechanism. Besides, three fundamental steps of hunting, namely, searching for prey, encircling prey, and attacking prey, are implemented to perform optimization [37].

The process of GWO can be summarized as follows. Firstly, all grey wolves (search agents) in the pack (population) are initialized randomly in [LB, UB], where LB and UB are the lower and upper bounds of the problem variables, respectively. Afterward, the fitness score for each search agent is evaluated using the accompanying objective function. The fittest solution so far is alpha, followed by beta and delta, respectively. Meanwhile, the rest of the wolves are grouped under the omega. The optimization in GWO is carried out by alpha, beta, and delta, whereas omega follows them. Then, search agents update their position based on prey position using the following equations:

$$\begin{aligned} D_\alpha &= |C_1 X_\alpha - X(t)|, \\ D_\beta &= |C_2 X_\beta - X(t)|, D_\delta = |C_3 X_\delta - X(t)| \end{aligned} \quad (18)$$

$$X_\alpha = X_\alpha - A_1 D_\alpha, X_\beta = X_\beta - A_2 D_\beta, X_\delta = X_\delta - A_3 D_\delta \quad (19)$$

$$X(t+1) = \frac{X_\alpha + X_\beta + X_\delta}{3} \quad (20)$$

where t , $X(t)$, $X(t+1)$, X_α , X_β , X_δ denote the current iteration, the current position of the prey, the position of the prey in the next iteration, the position of alpha, the position of beta, the position of the delta, respectively. The coefficient vectors A and C are computed according to the following equations.

$$A = 2.a.r - a \quad (21)$$

$$C = 2.r \quad (22)$$

where r_1 and r_2 are random vectors uniformly distributed in $[0, 1]$, and the control vector a is linearly decreased from 2 to 0 for better exploration and exploitation of candidate solutions. GWO continues in the same procedure until the maximum number of iterations is reached. Figure 1 (a) shows the flowchart of GWO.

Whale Optimization Algorithm

WOA is a new metaheuristic for optimization problems proposed in 2016 [38]. It has been inspired by the unique hunting behavior of humpback whales. In WOA, a population of whales (search agents) evolves to find the global optima after a specified number of iterations. WOA begins with the initialization of search agents randomly upon the interval of LB and UB of the problem variables. After that, WOA evaluates the fitness score for each search agent by using the fitness function. The best solution is saved for further processing later.

Then, WOA updates the position of each search agent depending on the following cases. If a random number (z) and the A vector are less than 1, then the particular search agent applies the Encircling method by updating its position for the next iteration using the following formulas:

$$X(t+1) = X^*(t) - A.D \quad (23)$$

$$D = |C.X^*(t) - X(t)| \quad (24)$$

$$A = 2.a.r - a \quad (25)$$

$$C = 2.r \quad (26)$$

where $X^*(t)$ is the position of the best solution so far. Furthermore, A and C are coefficient vectors. The a vector is linearly decreased from 1 to 0. The r vector is a random vector in $[0, 1]$. Else if a random number (z) is less than 0.5, but A vector is bigger than 1, then the particular search agent applies the Exploration method by updating its position for the next iteration using the following formulas:

$$D = |C.X_{rand} - X(t)| \quad (27)$$

$$X(t+1) = X_{rand} - A.D \quad (28)$$

where X_{rand} is the random whale in the current iteration, otherwise, the particular search agent applies the Spiral method by updating its position for the next iteration using the following equation:

$$X(t+1) = \begin{cases} X^*(t) - A.D, & \text{if } z < 0.5 \\ D'.\exp(bl).\cos(2\pi l) + X^*, & \text{if } z \geq 0.5 \end{cases} \quad (29)$$

where $D' = |X^* - X(t)|$ represents the distance between the search agent to the prey, b is a constant, and l is a random number $[-1, 1]$. Finally, WOA continues in the same process until the maximum number of iterations is reached. Figure 1 (b) depicts the flowchart of WOA.

Methodology

The parameters selection of a distribution may be defined as a non-linear optimization task that minimizes the mean absolute error between the collected and predicted wind speed vectors. Mathematically as shown in Equation (30).

$$\min\{MAE(V_m, V_d)\} \quad (30)$$

where V_m is the collected wind speed vector, and V_d is the predicted wind speed vector. Furthermore, V_d can be calculated based on the ICDF of the distribution, as expressed in Section II.

To solve the optimization as mentioned earlier problem, EAs can be used. Many algorithms were appeared in the literature to solve non-linear optimization problems and applied in many engineering fields [39], [40], [41], [42], [43]. In GWO and WOA, a group of search agents designates candidate solutions to the optimization problem. Concerning the parameter selection problem of the distribution, every search agent consists of integer values representing the values of the distribution parameters. The first group of search agents is generated randomly within specified boundaries of the parameters.

Then, within the initial iteration, the fitness score values per search agent are calculated via Equation (30). Next, the population increases by seeking the optimum solution applying the specified operation of the used EA. It continues in the same manner until the condition of the maximum number of iterations is satisfied. Table 2 gives the simulation parameters of GWO and WOA decided after performing a comprehensive exploration for all parameters over their recommended range. Table 3 displays the simulation time of GWO and WOA in seconds. It can be perceived that GWO is faster than WOA in convergence regarding all used distributions and datasets. Table 4 presents the resulting parameter values. Notably, GWO and WOA selected almost the same parameter values in most cases.

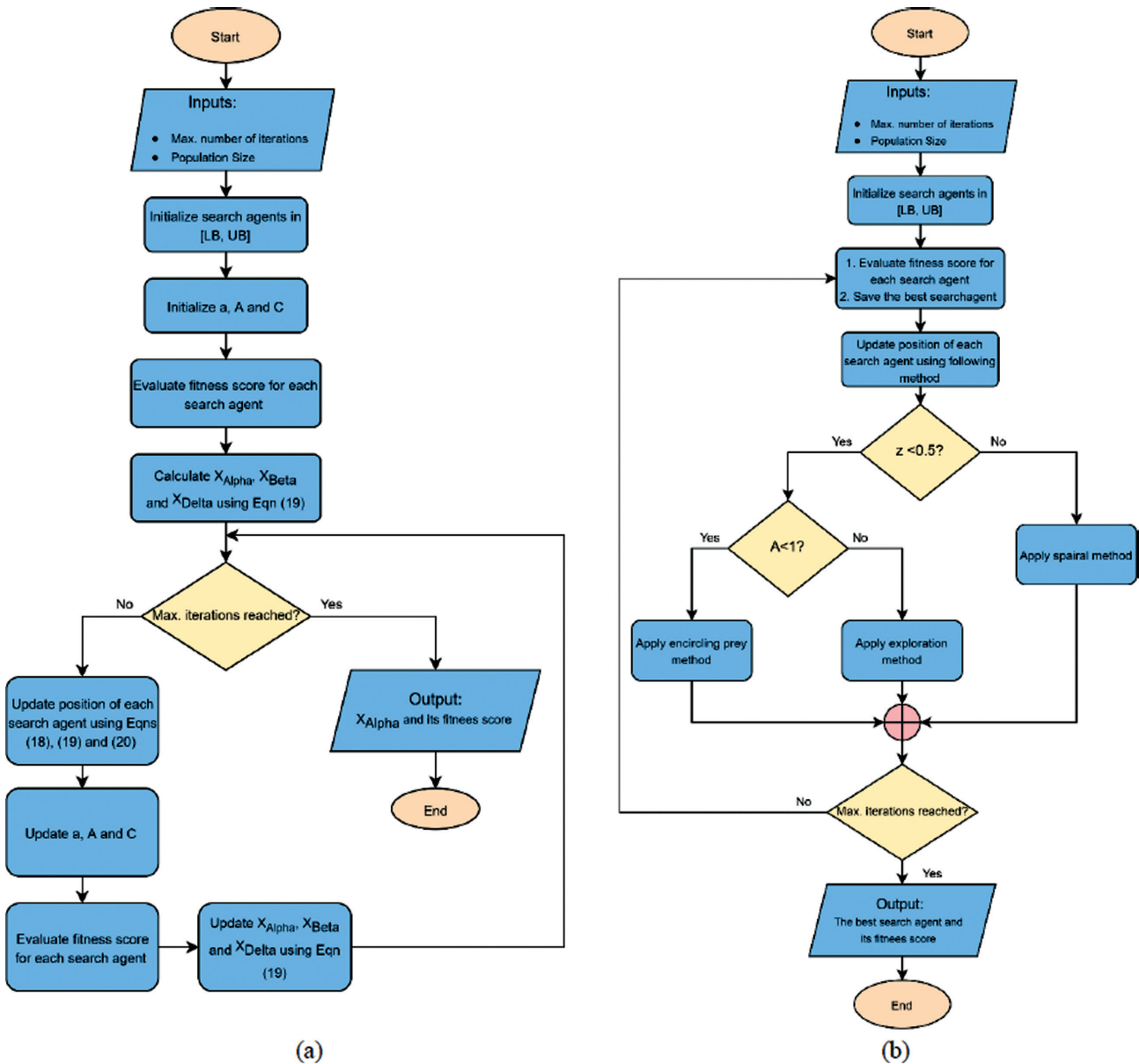


Figure 1. Flowchart of: (a) GWO (b) WOA.

Table 2. The main operating parameters of GWO and WOA

Parameter	Domain	Selected value	
		GWO	WOA
Population size	[5, 50]	50	50
Maximum number of iterations	[50, 300]	200	200
Stopping threshold	$[1 \times 10^{-4}, 1 \times 10^{-6}]$	1×10^{-6}	1×10^{-6}

WIND SPEED EXTRAPOLATION AND WIND ENERGY POTENTIAL

Wind data are generally collected at 10 m hub height; then, extrapolation can be done for any different height at the same site. The extrapolated wind speed at any height can be calculated as follows [44]:

$$\frac{V_2}{V_1} = \left(\frac{h_2}{h_1}\right)^\alpha \tag{31}$$

where V_1 and V_2 are the real and extrapolated wind speed values at the heights of h_1 and h_2 , respectively, the roughness

Table 3. Elapsed running time of distributions in seconds

Dataset	Distribution	Height (m)					
		30		60		80	
		GWO	WOA	GWO	WOA	GWO	WOA
2008	Rayleigh	3.170	2.963	2.529	3.501	2.446	3.756
	Weibull	8.744	9.345	8.550	10.897	8.555	8.744
	Gamma	134.579	139.608	135.060	145.694	134.748	146.300
	Burr Type XII	36.394	37.866	33.281	41.979	33.260	41.074
	Generalized Extreme Value	8.320	11.165	8.647	13.202	8.486	13.548
2009	Rayleigh	2.485	7.923	3.055	8.261	2.734	7.483
	Weibull	9.333	19.264	9.005	18.287	9.028	12.579
	Gamma	134.964	261.173	146.046	263.064	144.645	291.190
	Burr Type XII	33.359	77.508	36.215	79.466	37.084	80.539
	Generalized Extreme Value	8.507	23.445	9.365	21.627	9.332	22.555
2010	Rayleigh	2.779	3.690	2.515	3.714	2.989	4.017
	Weibull	10.766	11.150	8.592	11.082	8.692	11.113
	Gamma	137.239	144.505	140.942	144.865	139.263	150.100
	Burr Type XII	34.995	42.134	35.477	42.392	41.292	77.311
	Generalized Extreme Value	8.303	12.183	9.601	12.241	9.631	22.066

coefficient α varies due to many factors such as the nature of the site surface, steadiness of atmospheric, wind speed shape, and height interval [45].

Table 5 provides different values of α based on different surface characteristics [44]. Besides, the value of α can be calculated for any site when the wind speed values are available at any two different hub heights as follows [46], [47], [48]:

$$\alpha = \frac{\ln(v_2) - \ln(v_1)}{\ln(h_2) - \ln(h_1)} \quad (32)$$

where v_1 and v_2 are the wind speed values at the two different hub heights of h_1 and h_2 , respectively.

There is another way to calculate α if the two wind speed values are not available using the under-analyzed site's roughness length (z_0), as shown in Equation (33) [49].

$$\alpha = \left(\frac{z_0}{h_1} \right)^{0.2} [1 - 0.55 \log(v_1)] \quad (33)$$

In this study, z_0 is 0.0095 m [50] while h_1 equals 10 m. Moreover, α is computed by Equation (33) and found to be 0.143. Finally, α is checked at two different 10 m and 30 m heights using Equation (32).

Wind turbines transform kinetic wind energy into electrical energy. Wind turbines performance is fundamentally

correlated to the shape of wind distribution. Wind power expectation is mainly based on power curves matching with wind distribution. The wind energy stored in the actual wind data is computed as follows [41]:

$$E_m = 0.5 \rho \bar{v}^3 T \quad (34)$$

where \bar{v}^3 is the average of wind speed cubes, ρ is the air density ($\rho = 1.225 \text{ kg/m}^3$), and T is the hourly time equals 730 and 8760 hours for monthly and annual periods, respectively.

The annual average wind speed is a preliminary sign of the suitability of wind energy generation at the selected location. Wind locations with average wind speed greater than 7.9 m/s at 30 m height are classified as excellent locations for generating wind energy [51]. Table 6 presents the annual average wind speeds at the three different heights at the studied site. These values reflect a good to a very good implication of wind potential compared to corresponding values in Table 7 [44]. Table 8 shows the annual average power density classes.

Evaluation metrics

Many statistical metrics have appeared in the literature to determine the fittest method to represent the actual wind data. In this study, five error metrics are utilized. The list of the applied evaluation metrics is briefly explained here:

Table 4. Distributions parameter values generated by GWO and WOA

Dataset	Optimization method	Distribution	Height (m)								
			30			60			80		
			P1	P2	P3	P1	P2	P3	P1	P2	P3
2008	GWO	Rayleigh	5.368	–	–	5.940	–	–	6.190	–	–
		Weibull	7.662	1.914	–	8.471	1.917	–	8.826	1.917	–
		Gamma	3.522	1.948	–	3.542	2.141	–	3.542	2.232	–
		Burr Type XII	11.302	2.365	3.163	12.891	2.343	3.325	14.304	2.304	3.701
		Generalized Extreme Value	–0.002	2.873	5.165	–0.006	3.173	5.719	–0.001	3.295	5.957
	WOA	Rayleigh	5.368	–	–	5.940	–	–	6.190	–	–
		Weibull	7.661	1.913	–	8.467	1.919	–	8.824	1.919	–
		Gamma	3.689	1.851	–	3.412	2.221	–	4.773	1.639	–
		Burr Type XII	12.688	2.288	3.832	12.136	2.394	3.015	9.424	2.690	1.784
		Generalized Extreme Value	–0.022	3.009	5.138	0.031	3.092	5.720	–0.007	3.334	5.945
2009	GWO	Rayleigh	5.636	–	–	5.956	–	–	6.219	–	–
		Weibull	8.044	1.912	–	8.497	1.912	–	8.876	1.916	–
		Gamma	3.524	2.044	–	3.529	2.157	–	3.527	2.253	–
		Burr Type XII	11.938	2.359	3.189	12.414	2.372	3.112	13.239	2.355	3.215
		Generalized Extreme Value	0.012	2.985	5.427	–0.007	3.193	5.733	–0.001	3.325	5.983
	WOA	Rayleigh	5.636	–	–	5.956	–	–	6.218	–	–
		Weibull	8.044	1.913	–	8.498	1.918	–	8.874	1.912	–
		Gamma	3.006	2.372	–	3.549	2.146	–	3.455	2.301	–
		Burr Type XII	12.330	2.336	3.360	11.286	2.454	2.652	13.349	2.351	3.260
		Generalized Extreme Value	0.077	2.962	5.359	–0.042	3.445	5.669	0.002	3.326	5.982
2010	GWO	Rayleigh	6.119	–	–	6.789	–	–	7.088	–	–
		Weibull	8.725	1.917	–	9.682	1.920	–	10.000	1.939	–
		Gamma	3.540	2.207	–	3.549	2.442	–	3.548	2.552	–
		Burr Type XII	13.184	2.351	3.293	14.448	2.364	3.232	14.975	2.367	3.187
		Generalized Extreme Value	–0.004	3.268	5.887	–0.012	3.658	6.529	–0.009	3.807	6.812
	WOA	Rayleigh	6.119	–	–	6.789	–	–	7.088	–	–
		Weibull	8.719	1.920	–	9.676	1.919	–	10.000	1.939	–
		Gamma	3.872	1.998	–	3.583	2.419	–	3.455	2.620	–
		Burr Type XII	8.676	2.826	1.568	10.800	2.642	1.939	11.337	2.639	1.960
		Generalized Extreme Value	0.042	3.178	5.874	0.113	3.414	6.489	0.070	3.661	6.764

Table 5. Roughness values according to surface characteristics

Surface Characteristic	α
Icy land and sea	[0.09, 0.12]
Lands with distributed trees	[0.14, 0.16]
Densely forested areas	[0.21, 0.24]
Small metropolis	[0.26, 0.31]
Crowded cities with towering structures	[0.35, 0.41]

Table 6. Yearly average wind speed at the Catalca location in (m/s)

Wind Data	Height		
	30m	60m	80m
2008	6.91	7.25	7.88
2009	7.65	7.66	8.74
2010	7.97	8.00	9.12

- *Mean Absolute Error (MAE)* is the mathematical mean between the collected (x) and the predicted (y) wind speed vectors, which created using any of the distributions as in Equation (35) [53].

$$MAE = \frac{\sum_{i=1}^N |y_i - x_i|}{N} \quad (35)$$

where N is the vector length.

- *Root Mean Square Error (RMSE)* is the square root of the differences between the collected and predicted wind speeds [54]. It can be calculated as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (y_i - x_i)^2}{N}} \quad (36)$$

- *Regression Coefficient (R^2)* shows the degree of linearity between the collected and predicted wind speed data, as given in Equation (37). If R^2 equals one, the relationship between the collected and predicted data can be linearly drawn [55].

$$R^2 = \frac{\sum_{i=1}^N (x_i - \mu_i)^2 - \sum_{i=1}^N (x_i - y_i)^2}{\sum_{i=1}^N (x_i - \mu_i)^2} \quad (37)$$

where μ_i is the i^{th} mean of collected wind speed data.

- *Correlation Coefficient (R)* expresses the degree of correlation between two datasets. Its values are

Table 7. Site potentiality based on annual average wind speed (m/s)

Annual mean wind speed at height (m)				Indication
10 [44]	30	60	80	
<4.5	<5.3	<5.9	<6.15	Poor
4.5 to 5.4	5.3 to 6.4	5.9 to 7.1	6.15 to 7.4	Marginal
5.4 to 6.7	6.4 to 7.9	7.1 to 8.8	7.4 to 9.15	Good to Very good
>6.7	>7.9	>8.8	>9.15	Excellent

Table 8. Site class based on annual average power density [52]

Indication	Class	Power density (W/m ²)
Poor	1	50 to 199
Marginal	2	200 to 299
Moderate	3	300 to 399
Good	4	400 to 499
Excellent	5	500 to 599
Excellent	6	600 to 799
Excellent	7	<800

Table 9. Location information of the site

Location	Country	Latitude	Longitude	Altitude (m)	Years
Catalca	Turkey	41°8.5944'	28°27.6924'	88	2008–2010

between -1 (opposite correlation) and 1 (a perfect positive correlation). However, its zero value indicates that the two datasets are completely different (no correlation). The correlation coefficient is given by the following equation [56].

$$R = \frac{1}{N-1} \sum_{i=1}^N \frac{(x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y} \quad (38)$$

where (\bar{x}, \bar{y}) and (σ_x, σ_y) denote the mean and the standard deviation of the measured wind speed vector and the predicted wind speed vector, respectively.

- *Net Fitness* is an outstanding metric that is used to average a group of metrics in a fairly way. It is used to make a decision over a group of metrics and simplify performing the ranks for comparisons goals. In this paper, the four metrics mentioned above are utilized to calculate net fitness. To find the net fitness,

Table 10. Statistical analysis and average power of 2008 dataset for all distributions by GWO and WOA

Height (m)	Optimization method	Distribution	Mean	Standard Deviation	Variance	Min	Max	Skewness	Kurtosis	Average Power (W/m ²)
30	GWO	Real	6.906	3.650	13.322	0	25.400	0.993	1.202	400.308
		Rayleigh	6.781	3.513	12.338	0.437	23.038	0.632	0.252	361.445
		Weibull	6.853	3.693	13.640	0.388	24.438	0.691	0.368	390.149
		Gamma	6.912	3.657	13.376	0.858	29.178	1.068	1.695	404.131
		Burr Type XII	6.894	3.755	14.100	0.622	37.813	1.311	3.644	421.773
		Generalized Extreme Value	6.872	3.675	13.508	0.149	31.381	1.137	2.322	403.876
	WOA	Rayleigh	6.781	3.513	12.338	0.437	23.038	0.632	0.252	361.421
		Weibull	6.852	3.695	13.654	0.388	24.453	0.692	0.370	390.321
		Gamma	6.879	3.556	12.646	0.913	28.335	1.044	1.619	387.933
		Burr Type XII	6.885	3.724	13.869	0.582	34.805	1.168	2.716	412.296
		Generalized Extreme Value	6.867	3.751	14.070	-0.205	30.230	1.025	1.832	409.021
		60	GWO	Real	7.646	4.013	16.105	0	28.047	1.012
Rayleigh	7.504			3.887	15.110	0.090	25.496	0.631	0.253	489.895
Weibull	7.576			4.078	16.627	0.074	26.973	0.689	0.365	526.356
Gamma	7.643			4.033	16.263	0.348	32.167	1.065	1.686	544.554
Burr Type XII	7.626			4.147	17.197	0.160	40.911	1.270	3.368	568.102
Generalized Extreme Value	7.591			4.038	16.304	-1.326	34.139	1.111	2.216	540.140
WOA	Rayleigh		7.504	3.887	15.110	0.090	25.496	0.631	0.253	489.886
	Weibull		7.572	4.071	16.575	0.075	26.924	0.687	0.362	524.938
	Gamma		7.636	4.105	16.854	0.318	32.787	1.085	1.749	555.151
	Burr Type XII		7.629	4.151	17.231	0.173	42.600	1.347	3.905	572.418
	Generalized Extreme Value		7.663	4.144	17.171	-0.871	38.732	1.343	3.382	575.822
	80		GWO	Real	7.967	4.182	17.486	0	29.225	1.012
Rayleigh		7.820		4.051	16.408	0.094	26.568	0.631	0.253	554.347
Weibull		7.894		4.249	18.051	0.077	28.104	0.689	0.365	595.403
Gamma		7.965		4.203	17.662	0.362	33.521	1.065	1.686	616.288
Burr Type XII		7.935		4.290	18.401	0.158	40.567	1.187	2.843	631.634
Generalized Extreme Value		7.918		4.222	17.825	-1.320	36.168	1.141	2.351	615.920
WOA		Rayleigh	7.819	4.050	16.406	0.094	26.566	0.631	0.253	554.234
		Weibull	7.891	4.244	18.013	0.078	28.069	0.688	0.363	594.330
		Gamma	7.874	3.579	12.810	0.660	28.456	0.919	1.258	510.130
		Burr Type XII	8.076	4.664	21.753	0.260	64.091	2.189	11.487	781.374
		Generalized Extreme Value	7.908	4.236	17.945	-1.467	35.646	1.104	2.185	615.039

Table 11. Statistical analysis and average power of 2009 dataset for all distributions by GWO and WOA

Height (m)	Optimization method	Distribution	Mean	Standard Deviation	Variance	Min	Max	Skewness	Kurtosis	Average Power (W/m ²)
30	GWO	Real	7.251	3.832	14.688	0	26.670	0.993	1.202	463.407
		Rayleigh	7.120	3.688	13.602	0.459	24.189	0.632	0.252	418.364
		Weibull	7.195	3.881	15.061	0.407	25.684	0.692	0.371	451.977
		Gamma	7.258	3.839	14.738	0.902	30.628	1.068	1.694	467.693
		Burr Type XII	7.243	3.947	15.582	0.650	39.632	1.306	3.606	489.286
	Generalized Extreme Value	7.244	3.894	15.164	0.281	34.556	1.224	2.740	478.948	
	WOA	Rayleigh	7.120	3.688	13.602	0.459	24.190	0.632	0.252	418.390
		Weibull	7.195	3.880	15.052	0.407	25.675	0.692	0.370	451.841
		Gamma	7.186	4.116	16.945	0.694	33.060	1.155	1.981	500.387
		Burr Type XII	7.246	3.942	15.542	0.637	38.752	1.264	3.324	487.352
Generalized Extreme Value		7.371	4.260	18.147	0.531	45.110	1.685	5.550	570.878	
60	GWO	Real	7.663	4.050	16.401	0	28.183	0.993	1.202	546.837
		Rayleigh	7.524	3.897	15.190	0.485	25.563	0.632	0.252	493.744
		Weibull	7.600	4.101	16.814	0.429	27.139	0.692	0.372	532.922
		Gamma	7.670	4.055	16.440	0.955	32.346	1.067	1.692	551.646
		Burr Type XII	7.647	4.168	17.374	0.694	42.266	1.326	3.742	576.779
	Generalized Extreme Value	7.615	4.060	16.481	0.137	34.261	1.111	2.201	546.598	
	WOA	Rayleigh	7.524	3.897	15.189	0.485	25.562	0.632	0.252	493.716
		Weibull	7.600	4.089	16.716	0.433	27.041	0.688	0.362	531.122
		Gamma	7.674	4.045	16.361	0.964	32.263	1.064	1.682	550.619
		Burr Type XII	7.660	4.206	17.691	0.741	45.871	1.486	4.918	591.954
Generalized Extreme Value		7.584	4.192	17.571	-0.556	32.010	0.922	1.425	553.598	
80	GWO	Real	8.000	4.228	17.880	0	29.426	0.993	1.202	622.416
		Rayleigh	7.856	4.069	16.559	0.506	26.689	0.632	0.252	561.967
		Weibull	7.938	4.274	18.268	0.451	28.274	0.689	0.365	605.755
		Gamma	8.008	4.234	17.931	0.996	33.782	1.068	1.693	628.030
		Burr Type XII	7.991	4.354	18.961	0.714	43.571	1.300	3.560	656.643
	Generalized Extreme Value	7.963	4.261	18.155	0.186	36.512	1.145	2.359	629.143	
	WOA	Rayleigh	7.855	4.069	16.559	0.506	26.689	0.632	0.252	561.955
		Weibull	7.937	4.282	18.334	0.448	28.339	0.692	0.371	606.926
		Gamma	8.012	4.280	18.322	0.967	34.172	1.079	1.727	636.519
		Burr Type XII	7.993	4.350	18.922	0.712	43.266	1.287	3.476	655.476
Generalized Extreme Value		7.970	4.276	18.287	0.195	36.872	1.160	2.428	633.468	

Table 12. Statistical analysis and average power of 2010 dataset for all distributions by GWO and WOA

Height (m)	Optimization method	Distribution	Mean	Standard Deviation	Variance	Min	Max	Skewness	Kurtosis	Average Power (W/m ²)
30	GWO	Real	7.875	4.133	17.086	0	28.888	1.012	1.216	590.160
		Rayleigh	7.729	4.004	16.031	0.092	26.261	0.631	0.253	535.321
		Weibull	7.803	4.200	17.640	0.077	27.783	0.689	0.365	575.150
		Gamma	7.874	4.156	17.273	0.358	33.151	1.065	1.687	595.748
		Burr Type XII	7.851	4.262	18.166	0.167	42.169	1.274	3.398	618.780
	WOA	Generalized Extreme Value	7.822	4.171	17.400	-1.352	35.458	1.124	2.274	593.191
		Rayleigh	7.729	4.004	16.030	0.092	26.261	0.631	0.253	535.312
		Weibull	7.798	4.190	17.560	0.077	27.709	0.686	0.360	572.938
		Gamma	7.793	3.933	15.465	0.433	31.299	1.019	1.544	549.206
		Burr Type XII	8.001	4.670	21.805	0.298	69.319	2.497	15.108	790.000
60	GWO	Generalized Extreme Value	7.909	4.329	18.737	-0.816	41.689	1.419	3.820	645.795
		Rayleigh	8.738	4.586	21.035	0	32.053	1.012	1.216	806.183
		Rayleigh	8.577	4.443	19.738	0.103	29.139	0.631	0.253	731.353
		Weibull	8.659	4.654	21.657	0.086	30.774	0.687	0.361	784.499
		Gamma	8.733	4.604	21.193	0.399	36.718	1.064	1.682	811.490
	WOA	Burr Type XII	8.709	4.722	22.302	0.189	47.040	1.285	3.481	844.305
		Generalized Extreme Value	8.666	4.620	21.340	-1.647	38.425	1.078	2.066	803.426
		Rayleigh	8.576	4.443	19.736	0.103	29.138	0.631	0.253	731.257
		Weibull	8.654	4.653	21.646	0.085	30.768	0.687	0.361	783.486
		Gamma	8.732	4.581	20.987	0.408	36.529	1.059	1.667	806.822
80	GWO	Burr Type XII	8.818	5.002	25.022	0.271	64.963	1.983	9.336	977.390
		Generalized Extreme Value	8.955	5.224	27.286	-0.179	61.752	2.005	8.090	1063.756
		Real	9.124	4.789	22.931	0	33.467	1.012	1.216	917.605
		Rayleigh	8.955	4.638	21.515	0.107	30.423	0.631	0.253	832.356
		Weibull	8.940	4.763	22.687	0.093	31.432	0.673	0.334	854.900
	WOA	Gamma	9.122	4.809	23.130	0.416	38.360	1.064	1.683	925.036
		Burr Type XII	9.098	4.943	24.433	0.198	49.555	1.299	3.574	965.809
		Generalized Extreme Value	9.046	4.826	23.289	-1.670	40.447	1.094	2.138	915.837
		Rayleigh	8.955	4.638	21.515	0.107	30.423	0.631	0.253	832.324
		Weibull	8.940	4.763	22.686	0.093	31.431	0.673	0.333	854.878
	Gamma	9.122	4.874	23.751	0.391	38.904	1.078	1.727	939.424	
	Burr Type XII	9.200	5.200	27.039	0.282	66.994	1.954	9.059	1102.200	
	Generalized Extreme Value	9.221	5.207	27.115	-0.717	54.180	1.627	5.151	1080.278	

Table 13. Accuracy measures of GWO and WOA for 2008 dataset

Height (m)	Optimization method	Distribution	Accuracy measure				Net Fitness	Rank
			MAE	RMSE	R ²	R		
30	GWO	Rayleigh	0.2521	0.3828	0.9890	0.9956	0.1626	5
		Weibull	0.2409	0.3053	0.9930	0.9967	0.1391	4
		Gamma	0.0775	0.1426	0.9985	0.9992	0.0556	1
		Burr Type XII	0.1248	0.3569	0.9904	0.9958	0.1239	3
		Generalized Extreme Value	0.1020	0.2143	0.9966	0.9984	0.0803	2
	WOA	Rayleigh	0.2521	0.3829	0.9890	0.9956	0.1626	5
		Weibull	0.2409	0.3051	0.9930	0.9967	0.1391	4
		Gamma	0.0976	0.1667	0.9979	0.9993	0.0668	1
		Burr Type XII	0.1261	0.2771	0.9942	0.9974	0.1029	3
		Generalized Extreme Value	0.1229	0.2094	0.9967	0.9988	0.0842	2
60	GWO	Rayleigh	0.2860	0.4233	0.9889	0.9954	0.1812	5
		Weibull	0.2746	0.3491	0.9924	0.9966	0.1587	4
		Gamma	0.0816	0.1450	0.9987	0.9994	0.0571	1
		Burr Type XII	0.1448	0.3738	0.9913	0.9964	0.1327	3
		Generalized Extreme Value	0.1253	0.2388	0.9965	0.9984	0.0923	2
	WOA	Rayleigh	0.2860	0.4233	0.9889	0.9954	0.1812	5
		Weibull	0.2747	0.3498	0.9924	0.9965	0.1589	4
		Gamma	0.0975	0.1755	0.9981	0.9993	0.0689	1
		Burr Type XII	0.1445	0.4182	0.9891	0.9953	0.1445	3
		Generalized Extreme Value	0.1185	0.3722	0.9914	0.9964	0.1257	2
80	GWO	Rayleigh	0.2980	0.4408	0.9889	0.9954	0.1886	5
		Weibull	0.2862	0.3638	0.9924	0.9966	0.1652	4
		Gamma	0.0850	0.1512	0.9987	0.9994	0.0595	1
		Burr Type XII	0.1521	0.3350	0.9936	0.9972	0.1241	3
		Generalized Extreme Value	0.1273	0.2600	0.9961	0.9982	0.0982	2
	WOA	Rayleigh	0.2980	0.4411	0.9889	0.9954	0.1887	3
		Weibull	0.2862	0.3643	0.9924	0.9965	0.1654	2
		Gamma	0.4497	0.6337	0.9770	0.9990	0.2768	4
		Burr Type XII	0.2113	1.0285	0.9395	0.9791	0.3303	5
		Generalized Extreme Value	0.1317	0.2526	0.9963	0.9984	0.0974	1

all metrics must have the same importance in the numerator, and it can be computed as follows [57].

$$NetFitness = \frac{\sum_{i=1}^n |MAE_i| + \sum_{i=1}^n |RMSE_i| + \sum_{i=1}^n (1 - R_i^2) + \sum_{i=1}^n (1 - R_i)}{4n} \quad (39)$$

where n represents the total number of error metrics.

RESULTS AND DISCUSSION

The hourly data gathered from the Catalca site in the Marmara area for three years from 2008 to 2010 at different

heights have been utilized to test the performance of the introduced five distributions. Table 9 provides the location information of the studied site. The wind speed data at this site were gathered at 10 m height then extrapolated at three different heights, namely, 30, 60, and 80 m, respectively.

Two optimization algorithms, GWO and WOA, were exploited to model the parameters for each distribution. The statistical descriptors such as mean, standard deviation, variance, minimum, maximum, skewness, kurtosis, and average power describe the characteristics of wind data for all datasets are shown in Tables 10, 11, and 12. It is obvious that the mean wind speeds of the collected data are (6.906, 7.646, 7.967), (7.251, 7.663, 8.000), and (7.875, 8.738, 9.124)

Table 14. Accuracy measures of GWO and WOA for 2009 dataset

Height (m)	Optimization method	Distribution	Accuracy measure				Net Fitness	Rank
			MAE	RMSE	R ²	R		
30	GWO	Rayleigh	0.2648	0.4021	0.9890	0.9956	0.1705	5
		Weibull	0.2529	0.3202	0.9930	0.9967	0.1458	4
		Gamma	0.0814	0.1495	0.9985	0.9992	0.0583	1
		Burr Type XII	0.1311	0.3732	0.9905	0.9958	0.1295	3
		Generalized Extreme Value	0.1030	0.2708	0.9950	0.9977	0.0953	2
	WOA	Rayleigh	0.2648	0.4020	0.9890	0.9956	0.1705	4
		Weibull	0.2529	0.3203	0.9930	0.9967	0.1459	2
		Gamma	0.2267	0.3500	0.9917	0.9988	0.1466	3
		Burr Type XII	0.1311	0.3492	0.9917	0.9964	0.1231	1
		Generalized Extreme Value	0.1684	0.7119	0.9655	0.9905	0.2311	5
60	GWO	Rayleigh	0.2798	0.4247	0.9890	0.9956	0.1800	5
		Weibull	0.2673	0.3385	0.9930	0.9967	0.1540	4
		Gamma	0.0860	0.1577	0.9985	0.9992	0.0615	1
		Burr Type XII	0.1385	0.4047	0.9900	0.9956	0.1394	3
		Generalized Extreme Value	0.1158	0.2264	0.9969	0.9985	0.0867	2
	WOA	Rayleigh	0.2798	0.4248	0.9890	0.9956	0.1800	5
		Weibull	0.2673	0.3401	0.9929	0.9967	0.1544	3
		Gamma	0.0864	0.1569	0.9985	0.9993	0.0614	1
		Burr Type XII	0.1412	0.5080	0.9843	0.9931	0.1679	4
		Generalized Extreme Value	0.1695	0.2594	0.9959	0.9988	0.1085	2
80	GWO	Rayleigh	0.2921	0.4435	0.9890	0.9956	0.1877	5
		Weibull	0.2791	0.3543	0.9930	0.9967	0.1609	4
		Gamma	0.0898	0.1648	0.9985	0.9992	0.0642	1
		Burr Type XII	0.1446	0.4075	0.9907	0.9959	0.1414	3
		Generalized Extreme Value	0.1175	0.2521	0.9964	0.9983	0.0937	2
	WOA	Rayleigh	0.2921	0.4435	0.9890	0.9956	0.1878	5
		Weibull	0.2790	0.3533	0.9930	0.9967	0.1607	4
		Gamma	0.0933	0.1773	0.9982	0.9992	0.0683	1
		Burr Type XII	0.1446	0.3987	0.9911	0.9961	0.1390	3
		Generalized Extreme Value	0.1164	0.2611	0.9962	0.9982	0.0958	2

m/s at 30, 60, and 80 m hub heights, for 2008, 2009, and 2010 data sets, respectively.

The mean wind speed value is a vital indication of wind potential at a particular site. When comparing the annual mean wind speed values with the corresponding values in Table 7, it can be observed that the wind potential at the analyzed site occupies a good to very good rank. The standard deviation value slightly increases with an increase in the wind tower height. Variance, the square of standard deviation, is a measurement of the spread between wind speed values and their mean value. Also, variance slightly

increases with an increase in the wind tower height. The minimum real wind speed was zero, whereas the maximum was varied between 25 and 33.5 m/s.

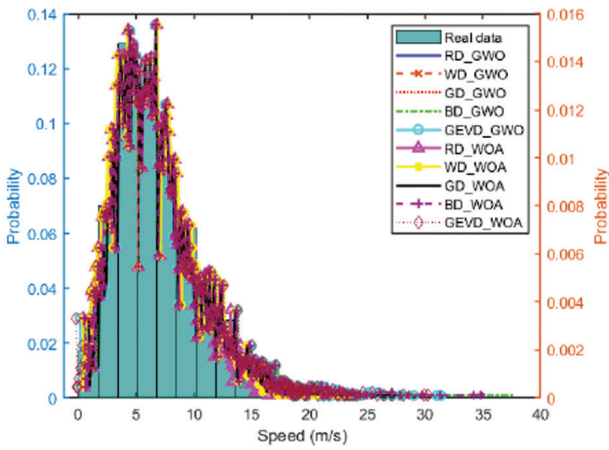
The skewness indicates the level of asymmetry from the average wind speed. The skewness values of all data sets demonstrate that the actual data conform to the positive-skewness shape. Kurtosis indicates the tops of a recurrence distribution. There are three types of kurtosis: zero, positive, and negative. Zero kurtosis distribution generally follows Normal distribution; positive kurtosis has heavier tails and a higher peak than Normal, whereas

Table 15. Accuracy measures of GWO and WOA for 2010 dataset

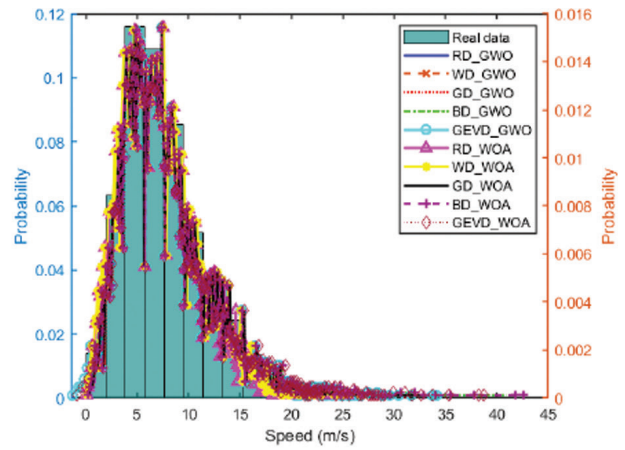
Height (m)	Optimization method	Distribution	Accuracy measure				Net Fitness	Rank
			MAE	RMSE	R ²	R		
30	GWO	Rayleigh	0.2945	0.4360	0.9889	0.9954	0.1866	5
		Weibull	0.2829	0.3595	0.9924	0.9966	0.1634	4
		Gamma	0.0841	0.1498	0.9987	0.9994	0.0589	1
		Burr Type XII	0.1491	0.3846	0.9913	0.9963	0.1365	3
		Generalized Extreme Value	0.1276	0.2514	0.9963	0.9983	0.0961	2
	WOA	Rayleigh	0.2945	0.4360	0.9889	0.9954	0.1866	4
		Weibull	0.2830	0.3608	0.9924	0.9965	0.1637	3
		Gamma	0.1545	0.2604	0.9960	0.9994	0.1049	1
		Burr Type XII	0.2674	1.1778	0.9188	0.9719	0.3886	5
		Generalized Extreme Value	0.1281	0.4538	0.9879	0.9953	0.1497	2
60	GWO	Rayleigh	0.3268	0.4836	0.9889	0.9954	0.2065	5
		Weibull	0.3140	0.3996	0.9924	0.9965	0.1812	4
		Gamma	0.0933	0.1649	0.9987	0.9994	0.0650	1
		Burr Type XII	0.1653	0.4323	0.9911	0.9961	0.1526	3
		Generalized Extreme Value	0.1480	0.2657	0.9966	0.9985	0.1046	2
	WOA	Rayleigh	0.3268	0.4838	0.9889	0.9954	0.2066	3
		Weibull	0.3139	0.3999	0.9924	0.9965	0.1812	2
		Gamma	0.0948	0.1630	0.9987	0.9994	0.0649	1
		Burr Type XII	0.2056	0.9628	0.9559	0.9837	0.3072	4
		Generalized Extreme Value	0.2760	1.1111	0.9413	0.9837	0.3655	5
80	GWO	Rayleigh	0.3412	0.5050	0.9889	0.9954	0.2155	5
		Weibull	0.3345	0.4523	0.9911	0.9963	0.1999	4
		Gamma	0.0974	0.1725	0.9987	0.9994	0.0680	1
		Burr Type XII	0.1725	0.4642	0.9906	0.9960	0.1625	3
		Generalized Extreme Value	0.1519	0.2831	0.9965	0.9984	0.1100	2
	WOA	Rayleigh	0.3412	0.5051	0.9889	0.9954	0.2155	3
		Weibull	0.3345	0.4524	0.9911	0.9963	0.1999	2
		Gamma	0.1058	0.1948	0.9983	0.9993	0.0757	1
		Burr Type XII	0.2122	0.9780	0.9583	0.9843	0.3119	5
		Generalized Extreme Value	0.1862	0.7680	0.9743	0.9919	0.2470	4

Table 16. Ranking of distributions using GWO and WOA

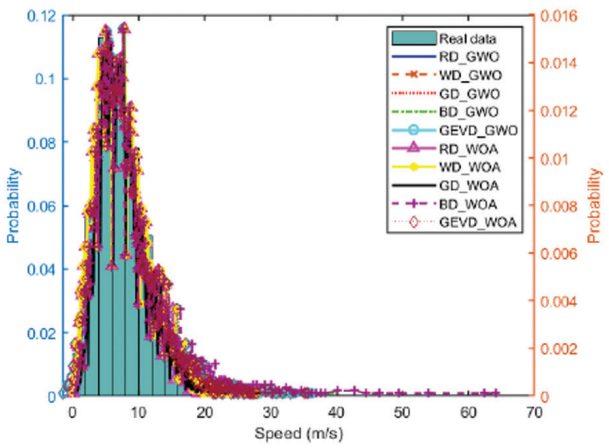
Distribution	GWO					WOA				
	1 st	2 nd	3 rd	4 th	5 th	1 st	2 nd	3 rd	4 th	5 th
Rayleigh	-	-	-	-	9	-	-	3	2	4
Weibull	-	-	-	9	-	-	4	2	3	-
Gamma	9	-	-	-	-	7	-	1	1	-
Burr Type XII	-	-	9	-	-	1	-	3	2	3
Generalized Extreme Value	-	9	-	-	-	1	5	-	1	2
Best	GD	GEVD	BD	WD	RD	GD	GEVD	BD	WD	RD



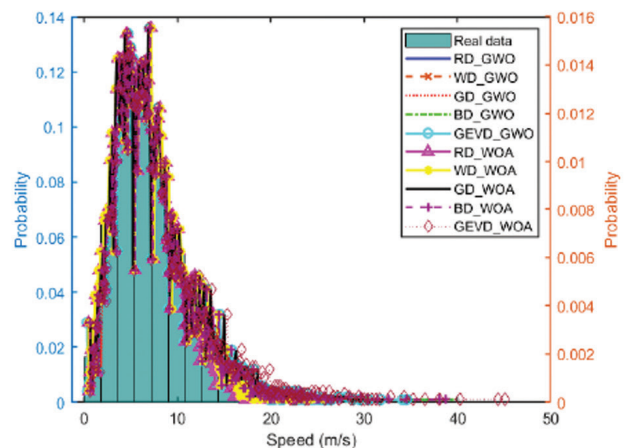
(a)



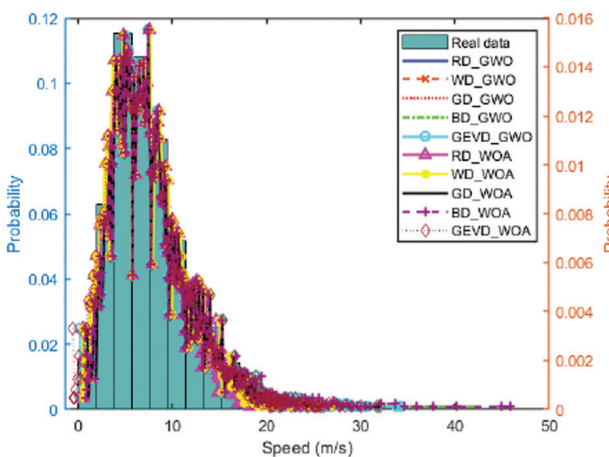
(b)



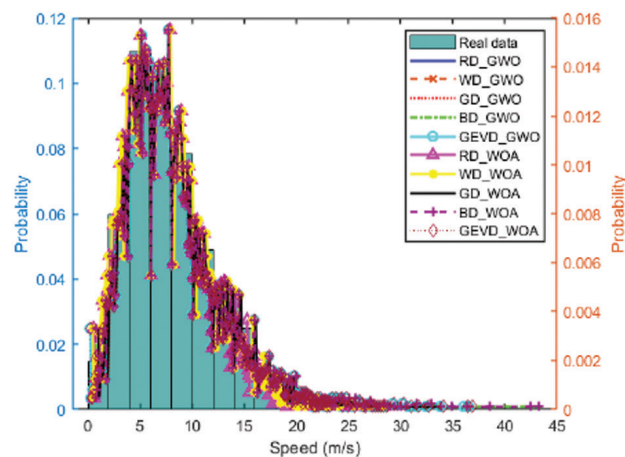
(c)



(d)



(e)



(f)

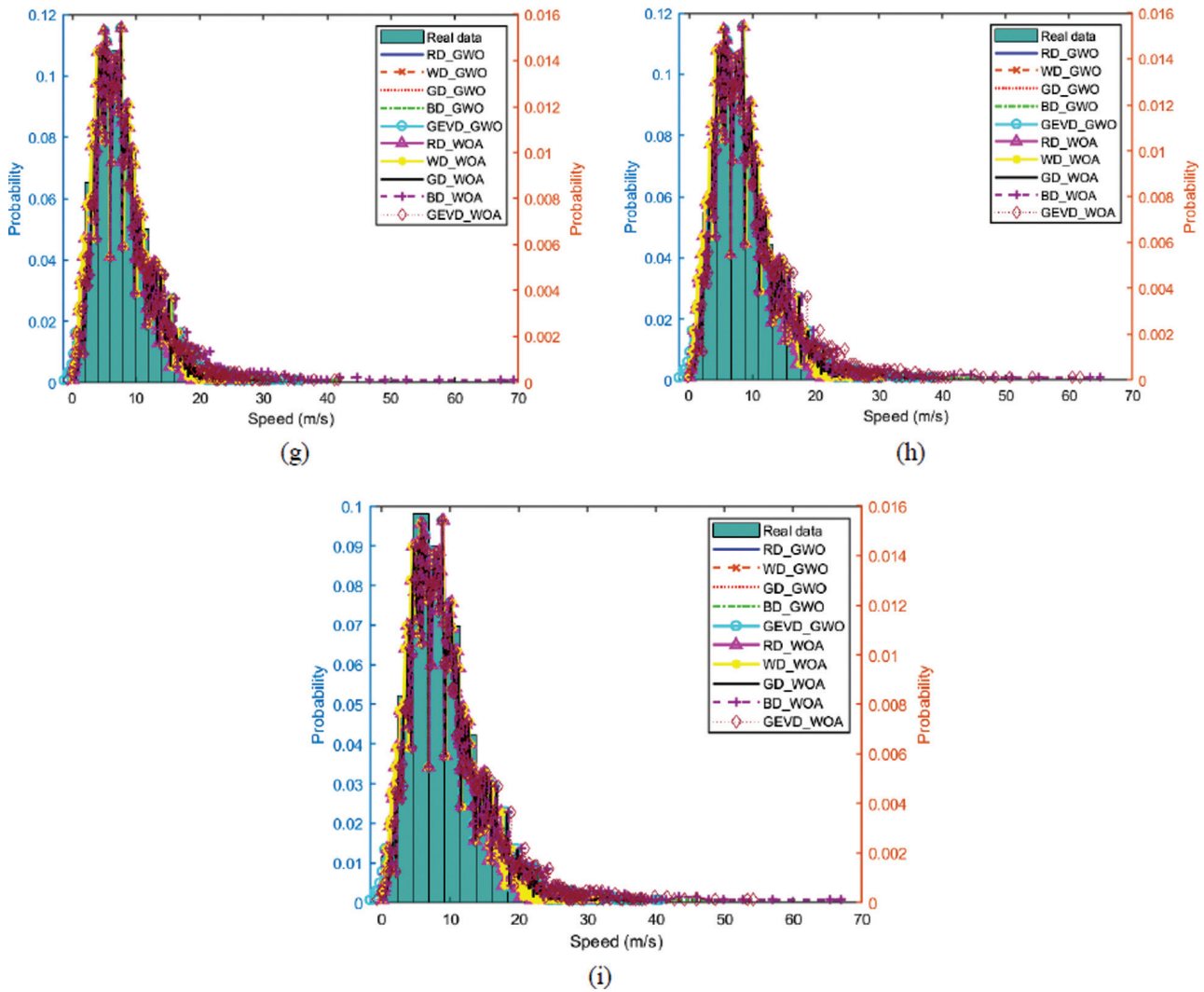


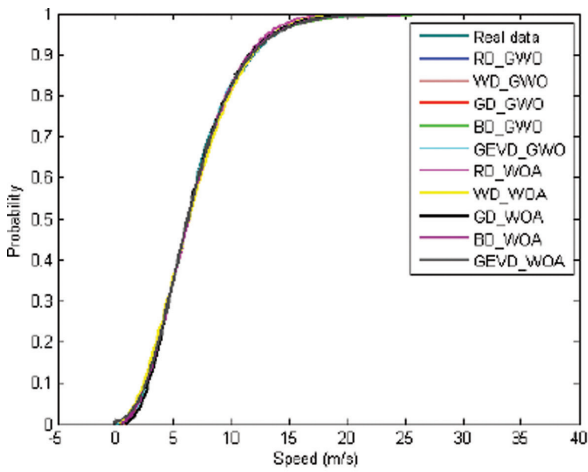
Figure 2. The PDF curves of introduced distributions when using GWO and WOA: (a) 2008-data set at 30m, (b) 2008-data set at 60m, (c) 2008-data set at 80m, (d) 2009-data set at 30m, (e) 2009-data set at 60m, (f) 2009-data set at 80m, (g) 2010-data set at 30m, (h) 2010-data set at 60m, and (i) 2010-data set at 80m.

the negative kurtosis has lighter tails and is flatter [58]. The real data of all datasets tend to the positive kurtosis (Leptokurtic).

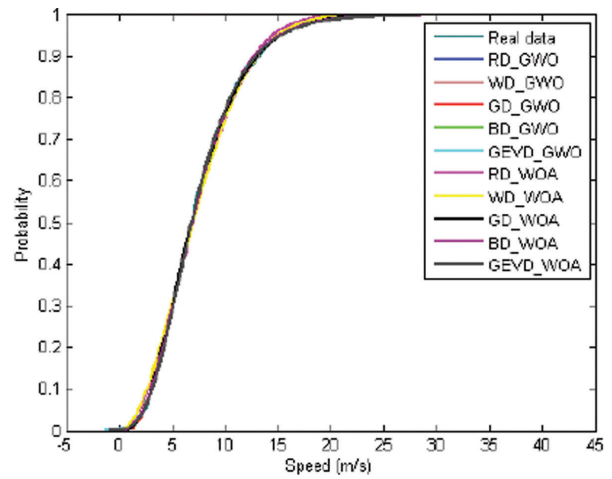
It is evident that the actual average power densities at the site are (400.308, 540.080, 611.022), (463.407, 546.837, 622.416), and (590.16, 806.183, 917.605) W/m² at 30, 60, and 80 m hub heights for 2008, 2009, and 2010 data sets, respectively. The average power density increases with an increase in the wind tower height. Besides, comparing the average power densities at the analyzed site with Table 8 [52], [59], [60], the underlying location can be classified in the fifth and sixth classes, which prove the location appropriateness for the installation of large-scale wind power turbines [61], [62].

Regarding the statistical descriptor values in Tables 10, 11, and 12, it can be noticed that Gamma distribution achieved the best matching, whereas Rayleigh distribution presented the worst matching.

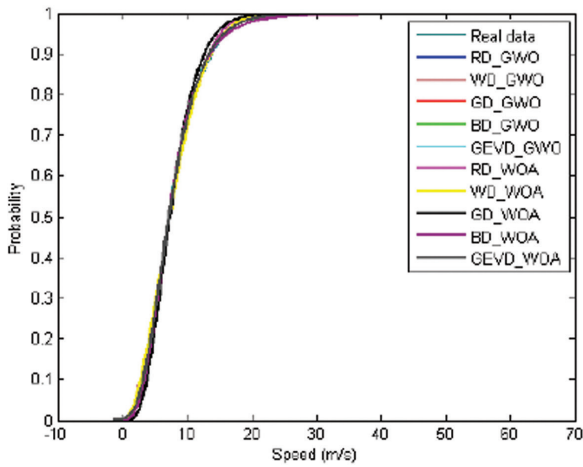
The distribution function reaches the optimal matching when the difference between the collected and the predicted wind speed values approaches zero [63], [64], [65]. Four evaluation metrics are used in this paper, namely, MAE, RMSE, R², and R. Tables 13, 14, and 15 summary the GOF of the introduced distribution functions based on GWO and WOA. The bold values in these tables point to the best at each height, whilst the underlined values point to the best between GWO and WOA.



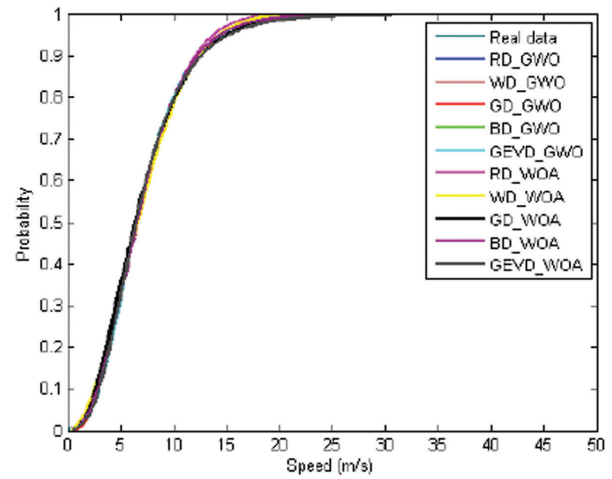
(a)



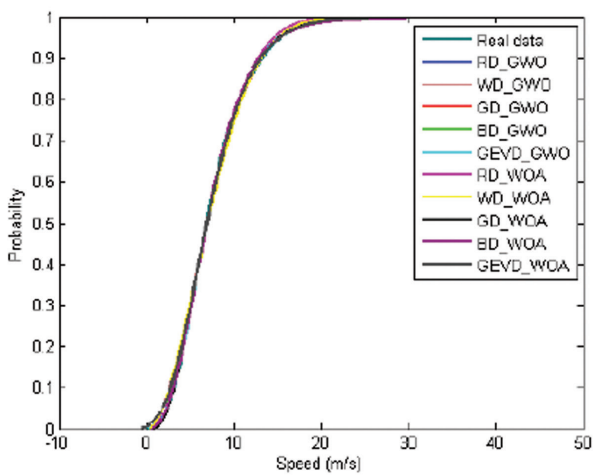
(b)



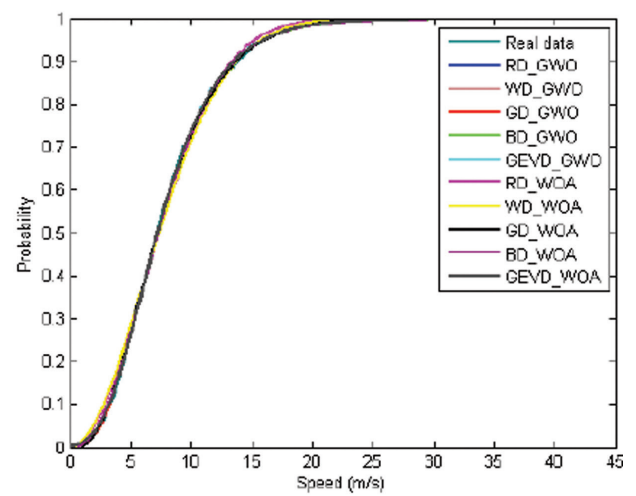
(c)



(d)



(e)



(f)

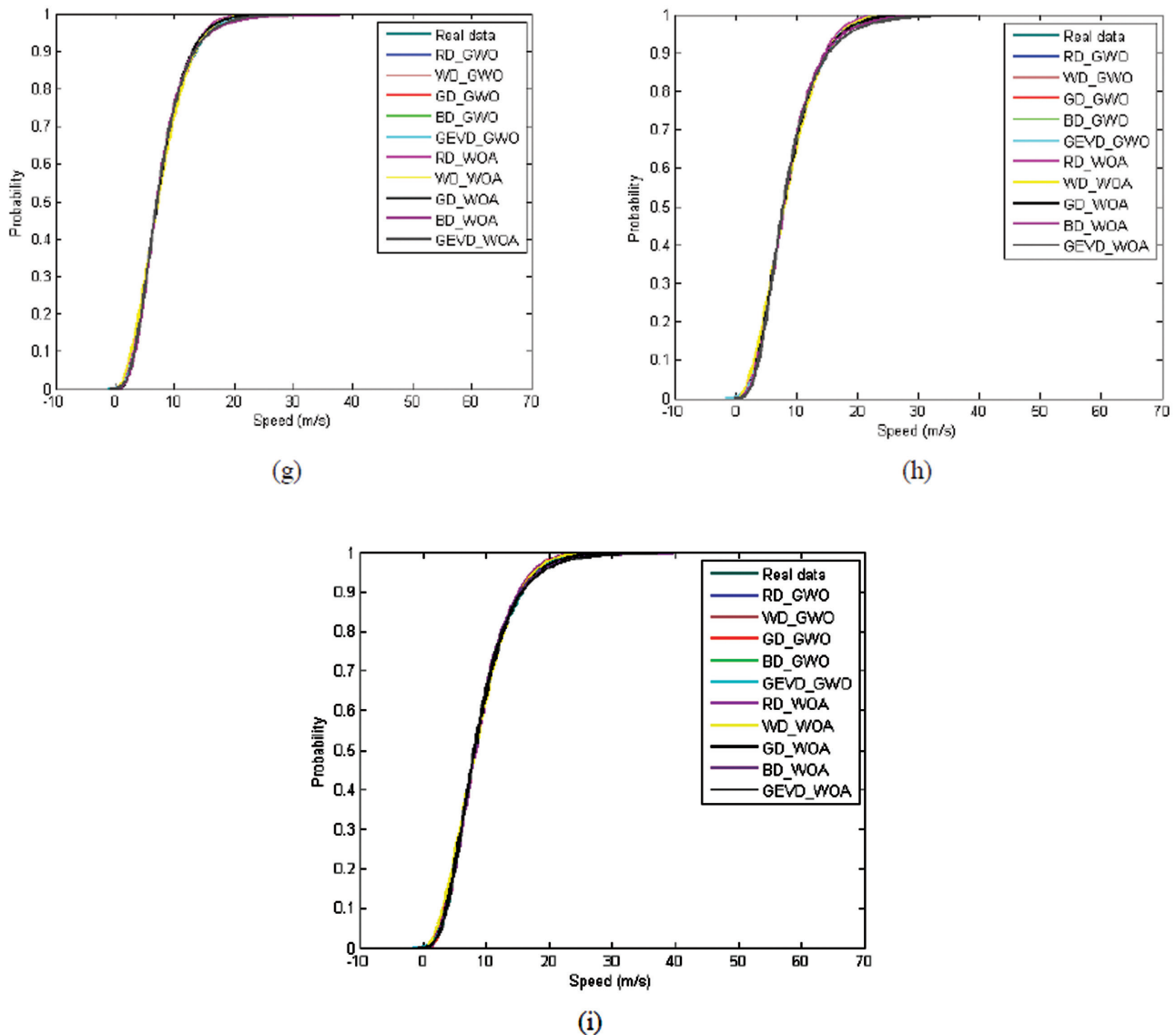


Figure 3. The CDF curves of introduced distributions when using GWO and WOA for: (a) 2008-data set at 30m, (b) 2008-data set at 60m, (c) 2008-data set at 80m, (d) 2009-data set at 30m, (e) 2009-data set at 60m, (f) 2009-data set at 80m, (g) 2010-data set at 30m, (h) 2010-data set at 60m, and (i) 2010-data set at 80m.

In most cases, Gamma distribution achieved the best GOF. On the other hand, Rayleigh was the worst. In some cases, the 2008 dataset at 80 m height, Generalized Extreme Value distribution based on WOA provided the best matching in MAE, RMSE, and R^2 measures.

The rank evaluation based on the net fitness metric is required to specify the accuracy of the best distribution. Distributions are ranked according to four GOF criteria. The rankings are performed by a maximum of R^2 and R , whereas a minimum of MAE and RMSE.

Based on net fitness, the top-down rank of the five distributions based on GWO and WOA is Gamma, Generalized Extreme Value, Burr Type XII, Weibull, and

Rayleigh. Table 16 shows the ranking of the five distributions. In addition, based on WOA, it can be observed that there is a slight difference between Weibull and Generalized Extreme Value to occupy the second rank. Hence, distributions based on different optimization methods provide different degrees of matching. Hence, it can be concluded that there is no unique, fully trusted, and best optimization method to assess the parameters of any distribution. Although Gamma distribution based on GWO and WOA outperformed the other distributions in terms of matching, it was the worst in computations. The computation time of Gamma was about sixteenth times of Generalized Extreme Value.

Figures 2 and 3 depict the fitted PDFs and CDFs, respectively, to explicate the obtained results visually for all datasets. For the PDF and CDF plots, the horizontal axis represents the wind speed data in m/s. Regarding PDF plots, two different-scale vertical axes are used; the left axis is for the histogram of the measured wind data, whereas the right axis is for the introduced distributions. These vertical axes represent the probability density which varies between the lowest and highest possible values. The vertical axis for CDF plots represents the cumulative density. It is observed from Figures 2 and 3 that all the introduced distributions achieved good matching. Rayleigh distribution occupied the last rank in terms of matching, whereas it was the best in computation complexity.

Many valuable inferences can be deduced from this study as follows:

- One of the most paramount deductions is the distribution pattern of wind regime. This study confirms that wind distribution patterns vary from one location to another; therefore, different distribution functions should be applied to describe its pattern accurately.
- The second important inference is that wind regime varies from one location to another and from height to another at the same location. Therefore, both utilized distributions and estimation methods need to be tested at different heights.
- The third important deduction is the selection of the optimization method. The success of the optimization method mainly depends on the characteristics of wind speed data. For instance, both GWO and WOA with the Gamma distribution achieved the best matching; however, Rayleigh distribution did not. Therefore, various optimization methods need to be tested and compared.
- The fourth deduction is the applied error metric. For instance, a specific error metric may demonstrate a particular distribution function as the best, but for another the worst. Hence, it is crucial to utilize many error metrics. Next, to decide the rank of the estimation technique accurately, the net fitness calculation is indispensable.

CONCLUSION

Selecting convenient distribution functions for representing wind speed frequency distribution is a critical for feasibility studies, wind turbine design, and long-term investment decisions. Moreover, selecting the suitable estimation method is also crucial since one method can achieve best GOF with specific distribution but may fail with others. This paper presents five different distributions to estimate the wind energy potential. The optimal parameter values for each distribution were selected based on GWO and WOA methods. The statistical characteristics of the analyzed site and the GOF of each distribution function

were studied and compared via many statistical descriptors and different error measures. In most cases, Gamma distribution based on both GWO and WOA outperformed the other distributions for all datasets. On the contrary, it was the worst in respect of computation complexity. Conversely, Rayleigh distribution was the worst in matching, but the best in computation time. In most cases, GWO was more robust and faster than WOA. The skewness and kurtosis statistical descriptors are also crucial to describe the wind regime since they can display the whole wind distribution pattern. In this study, skewness and kurtosis values are positive, therefore, the wind pattern takes the shape of positively skewed and leptokurtic. Accordingly, the selection of the convenient distribution can be perceived. Besides, it can be observed that particular distribution with particular error measures can achieve the best matching, for example, in terms of RMSE and R2, Weibull distribution based on WOA at 30 m height for 2009 dataset achieved the best fitness, and in terms of MAE, Burr distribution was the best. Therefore, to accurately specify the best GOF, net fitness computation is required. Ultimately, the used distribution function, optimization method and error measure are essential factors to make an accurate decision of which the best GOF for wind pattern at any site. This study can be extended to include other distribution functions, optimization algorithms, and error measures.

NOMENCLATURE

PDF	Probability Distribution Function, $f(v)$
CDF	Cumulative Distribution Function, $F(v)$
ICDF	Inverse Cumulative Distribution Function, $G(p)$
GWO	Grey Wolf Optimization
WOA	Whale Optimization Algorithm
EPF	Energy Pattern Factor
PSO	Particle Swarm Optimization
CSO	Cuckoo Search Optimization
MOM	Method of Moments
EMJ	Empirical Method of Justus
MML	Modified Maximum Likelihood
ML	Maximum Likelihood
LM	L-Moment
LS	Least-Squares
GOF	Goodness-Of-Fit
MAE	Mean Absolute Error
RMSE	Root Mean Square Error
R ²	Regression Coefficient
R	Correlation Coefficient
LB	Lower Bound
UB	Upper Bound
RD	Rayleigh Distribution
WD	Weibull Distribution
GD	Gamma Distribution

BD Burr Type XII Distribution
 GEVD Generalized Extreme Value Distribution

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

REFERENCES

- [1] Chang TP. Estimation of wind energy potential using different probability density functions. *Appl Energy* 2011;88:1848–1856. [\[CrossRef\]](#)
- [2] Pishgar-Komleh SH, Keyhani A, Sefeedpari P. Wind speed and power density analysis based on Weibull and Rayleigh distributions (a case study: Firouzkoh county of Iran). *Renew Sust Energ Rev* 2015;42:313–322. [\[CrossRef\]](#)
- [3] Morgan EC, Lackner M, Vogel RM, Baise LG. Probability distributions for offshore wind speeds. *Energy Convers Manag* 2011;52:15–26. [\[CrossRef\]](#)
- [4] Harolad L., Crutcher; Ledoleh B. Computations from elliptical wind distribution statistics. *J Appl Meteorol* 1962;1:522–530. [\[CrossRef\]](#)
- [5] Garcia A, Torres JL, Prieto E, de Francisco A. Fitting wind speed distributions: a case study. *Sol Energy* 1998;62:139–144. [\[CrossRef\]](#)
- [6] Scerri E, Farrugia R. Wind data evaluation in the Maltese Islands. *Renew Energy* 1996;7:109–114. [\[CrossRef\]](#)
- [7] Çelik HE, Yilmaz V. A statistical approach to estimate the wind speed distribution: the case of Gelibolu region. *Doğuş Üniversitesi Dergisi* 2008;9:122–132. [\[CrossRef\]](#)
- [8] Alavi O, Mohammadi K, Mostafaeipour A. Evaluating the suitability of wind speed probability distribution models: A case of study of east and southeast parts of Iran. *Energy Convers Manag* 2016;119:101–108. [\[CrossRef\]](#)
- [9] Mert İ, Karakuş C. A statistical analysis of wind speed data using Burr, generalized gamma, and Weibull distributions in Antakya, Turkey. *Turk J Electr Eng Comput Sci* 2015;23:1571–1586. [\[CrossRef\]](#)
- [10] Wadi M, Elmasry W. Statistical analysis of wind energy potential using different estimation methods for Weibull parameters: a case study. *Electr Eng* 2021;103:2573–2594. [\[CrossRef\]](#)
- [11] Sohoni V, Gupta S, Nema R. A comparative analysis of wind speed probability distributions for wind power assessment of four sites. *Turk J Electr Eng Comput Sci* 2016;24:4724–4735. [\[CrossRef\]](#)
- [12] Wadi M, Kekezoglu B, Baysal M, Tur MR, Shobole A. Feasibility study of wind energy potential in Turkey: case study of Catalca District in Istanbul. 2nd International Conference on Smart Grid and Renewable Energy, SGRE 2019 - Proceedings 2019:1–6. [\[CrossRef\]](#)
- [13] Gul M, Tai N, Huang W, Nadeem MH, Yu M. Evaluation of wind energy potential using an optimum approach based on maximum distance metric. *Sustainability (Switzerland)* 2020;12:1999. [\[CrossRef\]](#)
- [14] Jung C, Schindler D. Global comparison of the goodness-of-fit of wind speed distributions. *Energy Convers Manag* 2017;133:216–234. [\[CrossRef\]](#)
- [15] Saxena BK, Rao KVS. Comparison of Weibull parameters computation methods and analytical estimation of wind turbine capacity factor using polynomial power curve model: case study of a wind farm. *Renewables: Wind, Water, and Solar* 2015;2:3. [\[CrossRef\]](#)
- [16] Pobočiková I, Sedliačková Z, Michalková M. Application of four probability distributions for wind speed modeling. *Procedia Engineering* 2017;192:713–718. [\[CrossRef\]](#)
- [17] Leiva V, Barros M, Paula GA, Galea M. Influence diagnostics in log-Birnbaum-Saunders regression models with censored data. *Comput Stat Data Anal* 2007;51:5694–5707. [\[CrossRef\]](#)
- [18] Abu-Hamdeh NH, Almitani KH. Construction and numerical analysis of a collapsible vertical axis wind turbine. *Energy Convers Manag* 2017;151:400–413. [\[CrossRef\]](#)
- [19] Mohammadi K, Alavi O, McGowan JG. U Use of Birnbaum-Saunders distribution for estimating wind speed and wind power probability distributions: A review. *Energy Convers Manag* 2017;143:109–122. [\[CrossRef\]](#)
- [20] Chiodo E. The burr XII model and its bayes estimation for wind power production assessment. *Int Rev Electr Eng* 2013;8:737–751.
- [21] Chiodo E, de Falco P. Inverse Burr distribution for extreme wind speed prediction: Genesis, identification and estimation. *Electr Power Syst Res* 2016;141:549–561. [\[CrossRef\]](#)
- [22] Kose B, Duz M, Guneser MT, Recebli Z. Estimating wind energy potential with predicting burr LSM parameters: a different approach. *Sigma J Eng Nat Sci* 2018;36:389–404.
- [23] Hoxha B, Selimaj R, Osmanaj S. An experimental study of Weibull and Rayleigh distribution

- functions of wind speeds in Kosovo. *Telkomnika* 2018;16:2451–2457. [\[CrossRef\]](#)
- [24] Arikan Y, Arslan ÖP, Çam E. The analysis of wind data with rayleigh distribution and optimum turbine and cost analysis in Elmadag, Turkey. *Istanbul Univ - J Electr Electron Eng* 2015;15:1907–1912.
- [25] Bidaoui H, el Abbassi I, el Bouardi A, Darcherif A. Wind speed data analysis using weibull and rayleigh distribution functions, case study: five cities northern morocco. *Procedia Manuf* 2019;32:786–793. [\[CrossRef\]](#)
- [26] Maleki Jebely F, Zare K, Deiri E. Efficient estimation of the PDF and the CDF of the inverse Rayleigh distribution. *J Stat Comput Simul* 2018;88:75–88. [\[CrossRef\]](#)
- [27] Hemanth Kumar MB, Balasubramaniyan S, Padmanaban S, Holm-Nielsen JB. Wind energy potential assessment by weibull parameter estimation using multiverse optimization method: a case study of tirumala region in India. *Energies* 2019;12:2158. [\[CrossRef\]](#)
- [28] Justus CG, Hargraves WR, Mikhail A, Graber D. Methods for estimating wind speed frequency distributions. *J Appl Meteorol Climatol* 1978;17:350–353. [\[CrossRef\]](#)
- [29] Khan MS, Pasha GR, Pasha AH. Theoretical analysis of inverse weibull distribution. *WSEAS Trans Math* 2008;7:30–38.
- [30] Tsogt K, Zandraabal T, Lin C. Diameter and height distributions of natural even-aged pine forests (*pinus sylvestris*) in western khentey, mongolia. *Taiwan Lin Ye Ke Xue* 2013;28:29–41.
- [31] Ismail NHB, Khalid ZBM. EM algorithm in estimating the 2- and 3-parameter Burr Type III distributions. *AIP Conference Proceedings, American Institute of Physics Inc.*; 2014;1605:881–887. [\[CrossRef\]](#)
- [32] Kumar D. The burr type Xii distribution with some statistical properties. *J Data Sci* 2017;15:509–534. [\[CrossRef\]](#)
- [33] Kim C, Kim W. Estimation of the parameters of burr type iii distribution based on dual generalized order statistics. *Sci World J* 2014;2014:512039. [\[CrossRef\]](#)
- [34] Pant MD, Headrick TC. A method for simulating burr type iii and type xii distributions through L -moments and L -correlations. *ISRN Appl Math* 2013;2013:1–14. [\[CrossRef\]](#)
- [35] Markose S, Alentorn A. The generalized extreme value distribution, implied tail index, and option pricing. *J Deriv* 2011;18:35–60. [\[CrossRef\]](#)
- [36] Molina-Aguilar JP, Gutierrez-Lopez A, Raynal-Villaseñor JA, Garcia-Valenzuela LG. Optimization of parameters in the generalized extreme-value distribution type 1 for three populations using harmonic search. *Atmosphere* 2019;10:1–19. [\[CrossRef\]](#)
- [37] Mirjalili S, Mirjalili SM, Lewis A. Grey wolf optimizer. *Adv Eng Softw* 2014;69:46–61. [\[CrossRef\]](#)
- [38] Mirjalili S, Lewis A. The whale optimization algorithm. *Adv Eng Softw* 2016;95:51–67. [\[CrossRef\]](#)
- [39] Elmasry W, Akbulut A, Zaim AH. Deep learning approaches for predictive masquerade detection. *Secur Commun Netw* 2018;2018:9327215. [\[CrossRef\]](#)
- [40] Elmasry W, Akbulut A, Zaim AH. Evolving deep learning architectures for network intrusion detection using a double PSO metaheuristic. *Comput Netw* 2020;168:107042. [\[CrossRef\]](#)
- [41] Wadi M, Elmasry W. Modeling of wind energy potential in marmara region using different statistical distributions and genetic algorithms. 2021 International Conference on Electric Power Engineering - Palestine, ICEPE-P 2021, Institute of Electrical and Electronics Engineers Inc; 2021. [\[CrossRef\]](#)
- [42] Wadi M, Elmasry W. An Anomaly-based Technique for Fault Detection in Power System Networks. 2021 International Conference on Electric Power Engineering - Palestine, ICEPE-P 2021, Institute of Electrical and Electronics Engineers Inc.; 2021. [\[CrossRef\]](#)
- [43] Wadi M. Fault detection in power grids based on improved supervised machine learning binary classification. *J Electr Eng* 2021;72:315–22. [\[CrossRef\]](#)
- [44] Mostafaeipour A. Feasibility study of harnessing wind energy for turbine installation in province of Yazd in Iran. *Renew Sust Energy Rev* 2010;14:93–111. [\[CrossRef\]](#)
- [45] Okorie ME, Inambao F, Chiguvare Z. Evaluation of wind shear coefficients, surface roughness and energy yields over inland locations in namibia. *Procedia Manuf* 2017;7:630–638. [\[CrossRef\]](#)
- [46] Gualtieri G, Secci S. Wind shear coefficients, roughness length and energy yield over coastal locations in Southern Italy. *Renew Energy* 2011;36:1081–1094. [\[CrossRef\]](#)
- [47] Lackner MA, Rogers AL, Manwell JF, McGowan JG. A new method for improved hub height mean wind speed estimates using short-term hub height data. *Renew Energy* 2010;35:2340–2347. [\[CrossRef\]](#)
- [48] Laban ON, Maghanga CM, Joash K. Determination of the surface roughness parameter and wind shear exponent of kisii region from the on-site measurement of wind profiles. *J Energy* 2019;2019:1–12. [\[CrossRef\]](#)
- [49] Gualtieri G, Secci S. Comparing methods to calculate atmospheric stability-dependent wind speed profiles: A case study on coastal location. *Renew Energy* 2011;36:2189–2204. [\[CrossRef\]](#)
- [50] Ağçay M. Türkiye'nin Elektrik Enerjisi Arz Talep Dengesinin Tespiti, Üretim Projeksiyonuna Yönelik

- Rüzgar Elektrik Santrali Tasarımı RES'in Kurulum Maliyetlerinin ve Üretim Parametrelerinin Analizinin Matlab&Simulink İle Yazılan Programda Yapılması. Bitirme Tezi. Yıldız Teknik Üniversitesi Elektrik Elektronik Fakültesi Elektrik Mühendisliği Bölümü, İstanbul, 2017.
- [51] Irwanto M, Gomesh N, Mamat MR, Yusoff YM. Assessment of wind power generation potential in Perlis, Malaysia. *Renew Sust Energy Rev* 2014;38:296–308. [\[CrossRef\]](#)
- [52] Gul M, Tai N, Huang W, Nadeem MH, Yu M. Assessment of wind power potential and economic analysis at hyderabad in Pakistan: powering to local communities using wind power. *Sustainability (Switzerland)* 2019;11:1391. [\[CrossRef\]](#)
- [53] De Myttenaere A, Golden B, Le Grand B, Rossi F. Mean absolute percentage error for regression models. *Neurocomputing* 2016;192:38–48. [\[CrossRef\]](#)
- [54] Hyndman RJ, Koehler AB. Another look at measures of forecast accuracy. *Int J Forecast* 2006;22:679–688. [\[CrossRef\]](#)
- [55] Akgun O, Cetin Akinci T. Wind speed analysis using signal processing technique. In Al Qubeissi, M, El-kharouf A, Soyhan HS, editors. *Renewable Energy - Resources, Challenges and Applications*. London: IntechOpen; 2020.
- [56] Korun M, Vodenik B, Zorko B. Calculation of the correlation coefficients between the numbers of counts (peak areas and backgrounds) obtained from gamma-ray spectra. *Appl Radiat Isot* 2016;118:1–6. [\[CrossRef\]](#)
- [57] Papoulis A, Pillai SU. *Probability, random variables, and stochastic processes*. 4th ed. McGraw-Hill; 2002.
- [58] DeCarlo LT. On the meaning and use of kurtosis. *Psychol Methods* 1997;2:292–307. [\[CrossRef\]](#)
- [59] Saeidi D, Mirhosseini M, Sedaghat A, Mostafaeipour A. Feasibility study of wind energy potential in two provinces of Iran: North and South Khorasan. *Renew Sust Energy Rev* 2011;15:3558–3569. [\[CrossRef\]](#)
- [60] Nogay HS, Akinci TC, Eidukeviciute M. Application of artificial neural networks for short term wind speed forecasting in Mardin, Turkey. *J Energy South Africa* 2012;23:2–7. [\[CrossRef\]](#)
- [61] Kavak Akpınar E. Statistical analysis of wind speed distribution with Sinop-Turkey Application. *J Therm Eng* 2019;5:277–292. [\[CrossRef\]](#)
- [62] Sharifi F, Hashemi N. An analysis of current and future wind energy gain potential for central Iowa. *J Therm Eng* 2015;1:245–250. [\[CrossRef\]](#)
- [63] Wadi M, Baysal M. Reliability Assessment of radial networks via modified RBD analytical technique 2017;35:717–726.
- [64] Elaydi H, Jaddu H, Wadi M. An iterative technique for solving nonlinear optimal control problems using legendre scaling function. *Int J Emerg Technol* 2021;2:7–11.
- [65] Cetin B, Alpkaya MD. Parametric analysis of electrical energy production by wind energy for Bozcaada. *Yildiz Technical University Press* 2019;5:271–276. [\[CrossRef\]](#)