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Research Article

Empirical modelling for work piece temperature during end milling of inconel 625 using a green's function approach based on dirac delta function

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ABSTRACT

Milling is a very versatile process for the manufacturing of dies and aerospace components. Especially in the manufacturing of thin walled components, the dimensional accuracy is greatly affected due to heat generation and deflection in the wall. Therefore, minimization of heat generation during milling by optimizing controllable input process parameters, leads to improved accuracy in thin walls. In the present study, an empirical model for work piece temperature by solving the non-homogeneous partial differential equation using Green's function has been simplified with Dirac delta approach during the end milling of Inconel625 work-piece with the assumption of single-pass cutting and one-point observation. This technique is normally used for solving the complex higher-order partial differential equation, but it is rarely applied in the heat dissipation problem during manufacturing in the recent past. The empirical approach is more effective and accurate as compared to experimental approaches used earlier in the recent past. To verify the adequacy of the empirical model of work piece temperature, 9 conformational experiments have been performed at 3 different cutting speeds with a constant depth of cut 5 mm and feed per tooth 0.05 mm. A good compromise has been observed among the responses obtained among the results obtained from an empirical approach and experimental observations at different cutting speeds. However, by little modification and adding a small algorithm, this single pass problem can be implemented on the multi-pass problems and even can also be applied to complex shapes.

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INTRODUCTION

Most of the components are being manufactured with the help of different types of dies and moulds and these dies and moulds having thin walls are mostly being machined by computer numerical controlled milling operations. (1). Nowadays, thin walled components have become more important in the aerospace and automobile industry due to the metallurgical advancement and development in high strength alloys. Many times, during the milling operation of the complex shaped dies and moulds, the work piece temperature increases due to shearing of the material and friction. This causes the length of large size thin walled components to increase, especially if the material has a high thermal coefficient. However, metals with low thermal coefficients are found to be less sensitive to temperature increases during milling. In the present work, Inconel625 material has been considered for milling due to a unique combination of properties such as high rupture strength, better thermal & fatigue strength, high corrosion strength and excellent weldability & brazeability with a high density and high melting point (2). This material has greater importance in the aviation, aerospace and automobile industry. Its high strength enables it to be used in thinner walled components, thus improving heat transfer and saving weight. It has been observed that the component of inconel625 has a thermal expansion coefficient of 1.28 × 10-5 1/K (3) and a length of 200 mm has increased by 205 μm on rise in work piece temperature by 110^{0}C approximately. Most dies and moulds require high accuracy with less than 50 μm by a rise in work piece temperature of more than 100^{0}C, and their accuracy may be compromised. Many times, components with large sized thin walled components with bounded sides may buckle if the temperature rises above a certain threshold. Therefore, it is very essential to study the temperature of the work piece to improve the accuracy of the components.

In the recent past, a lot of techniques and methods have been adopted by researchers for reducing the temperature of the work piece, such as improvement in cutting strategy, geometry of the tools, optimization of machining parameters etc. For optimizing the work piece's temperature, it is necessary to build a mathematical model to predict the temperature with the input parameter. These models developed by the researchers can be classified into 3 categories, such as statistical, artificial intelligence (AI) and analytical techniques.

Many researchers have opted for statistical modelling techniques for estimation of work piece temperature and other responses during the milling process. On the basis of design of experiment approaches such as response surface methodology (4) (5)(6)(7)(8) and Taguchi(9)(10), a large number of experiments have been performed and on the basis of these experiments regression models have been developed for establishment the relation among the dependent & independent variables (11) (12). Kaushik et al.(13) conducted 32 experiments based on the central composite design of RSM, and regression models were developed on the basis of these experiments to establish the relationship between temperature rise and cutting speed, feed rate, and axial depth of cut during dry milling. These statistical techniques provide good estimation and are easy to implement in manufacturing processes. However, realistic mathematical models provide better optimization of the responses.

Therefore, in the recent past, most researchers have preferred artificial intelligence techniques such as Artificial Neural Network (10) (14) (15), Fuzzy Logic Methodology(5) (10) (14), Support Vector Machine(16) (17) etc. These techniques are based upon the learning algorithm and are found more realistic as compared to the statistical methods. Gupta et al. 2020 (18) applied artificial intelligence technique based on a learning algorithm for obtaining the optimum combination of input parameters and compared with the Taguchi statistical method. A 58.28% reduction in pressure die casting defects has been observed using GA as compared with the Taguchi methodology. The AI techniques also provide better results with a large number of experiments. Sometimes, the small size of an experimental database provides poor estimation of the responses.

Therefore, optimum numbers of experiments are preferred for such types of AI mathematical models. These techniques are frequently applied by researchers and even in real life production applications. In the concept of Industry 4.0(18), AI techniques play a critical role in real-time optimization of manufacturing processes during mass production. However, in small size or batch production, it is uneconomical to perform large number of experiments and hence, the statistical and AI techniques are not viable for such situations. In such cases, analytical mathematical modelling techniques are more feasible as these techniques are based on empirical relations and mathematical derivations.

In recent past, many researchers have applied analytical modelling techniques to estimate the various responses such as thermal stresses (19) (20), heat transfer (21), tool and chip temperature (22) (23), work piece temperature (4) (12) (24), cutting force (6) (25) and face milling temperature(26). In these techniques, very few experiments are required to be performed just for validation of the technique. Xiong et al. 2018(27) proposed an analytical model for work piece temperature using the moving heat source method and presented good reliability and accuracy between analytical and experimental results with less than 18% error after conducting 5 numbers of experiments. Therefore, analytical modelling is found more suitable for estimation of responses in the manufacturing of dies and mould manufacturing as these components are manufactured in small size production. In the analytical modelling technique, complex differential equations are required to be solved with the help of various approaches like Green's function (20) (23) (24)(28) (29), moving heat source method(27) Finite difference method(30), Kommanduri and Huang-Liang(31), finite element method(32) and theories like Oxley's theory(33) and inverse heat conduction theory(30) provided by researchers in the recent past. The Separation of variable method has also been used by most researchers for solving non Fourier heat transfer problem (34) (35). Many researchers have also been applied Green's function approach for the heat and temperature measurement (36) (37) (38) (39) (40). Although these analytical techniques are much more complex to solve and require deep knowledge of the manufacturing process mechanism, these techniques provide good estimation and are more economical due to the lower number of required experiments. A solution of a large number of differential equations is required to solve heat and temperature problems with the help of an analytical approach.

Green function is an empirical method used by a lot of researchers in the recent past to solve such types of complex differential equations (41) (42) (43). Liu et al. 2014 (28) presented a Green's function approach for solving two types of heat sources that is described using the Dirac delta function in helical milling. According to Ribeiro et al. 2018 (24), a Green's function approach is a better technique for the estimation of the heat and work piece temperature without use of optimization technique. We only observe nearby values in simulation work such as FEM analysis, but a mathematical technique such as Green's function gives the exact value at the specific point of moving heat source. This approach is simple without iterative processes and therefore extremely fast. According to Kitetu et al 2013 (36), Green's function method is a flexible and powerful method and it can be used for any type of source. For complex geometrics, this method reduces the possibility of error and saves time.

Therefore, in the present work, an empirical model for work piece temperature by solving the non-homogeneous partial differential equation using Green's function has been simplified with Dirac delta approach during the end milling of Inconel625 work-piece with the assumption of single-pass cutting and one-point observation. To minimize the complexity, a 100 mm length of span with a thin wall of 3mm thickness has been machined in unidirectional with a depth of 5mm in a single pass. However, by little modification and adding small algorithms, this single pass problem can be implemented on the multi-pass problems and even can also be applied to complex shapes. An empirical relation has been obtained by applying the theory of moving heat source and direct delta function for solving the non-homogeneous partial differential equation for single pass cutting with one-point observation. Further, the analytical model for work-piece temperature has been validated with the help of experiments. A 100 mm length of span has been considered, and 60 observations of work-piece temperature at various cutting speeds have been taken using a thermocouple installed in the middle of the cutting span as a single point observation. However, the other parameters such as feed per tooth, DOC, and Stopover are considered to be constant for the present problem.

MATERIALS AND METHOD

For estimating the work piece temperature during the single pass end milling, a Green's function approach has been proposed. The empirical equations used that are bounded with some assumptions are explained in the latter section. To validate the estimated work piece temperature using green's function approach, the experimental work has been performed on a CNC vertical milling machine (make AMS mcv 450 of Ace Micromatic group) with a cutting speed of 9000 rpm and power of 15 KW as shown in figure 1. A work piece of Inconel625: a nickel based alloy material with 100×50 cross-section, 40 mm depth and initial temperature of 22°C has been considered in the present work. Since the study is at the single point only, a K-type thermocouple of diameter 1 mm is fixed at the centre of the work piece and the probe of the thermocouple just touches the edge for accurate temperature measurement as shown in figure 2. A tungsten carbide tool having two flutes with a diameter of 20 mm has been considered for milling operation. To verify the adequacy of the empirical model of work piece temperature, 9 conformational experiments have been performed at 3 different cutting speeds with a constant depth of cut 5 mm and feed per tooth 0.05 mm as shown in table 2.

For defining and solving the empirical method, some assumptions/conditions have been considered.

- The reference temperature was considered 22^0C at the beginning of experimentation.
- The coefficient of friction is constant in tool-work piece interaction.
- The convective heat transfer coefficient is assumed to be 10.0 W/m2 K (45).
- Study of work piece temperature at a single point only.
- Moving heat source is unidirectional.
- Uniform heat flux distribution
- Material properties are isotropic & homogeneous.
- The thickness of the thin-walled work piece is 3 mm.

Table 1 Parameter considered for experimentation.

Parameters	Units	value
Cutting speed	rpm	6000, 7000, 8000
Depth of cut	mm	5
Feed per tooth	mm	0.05

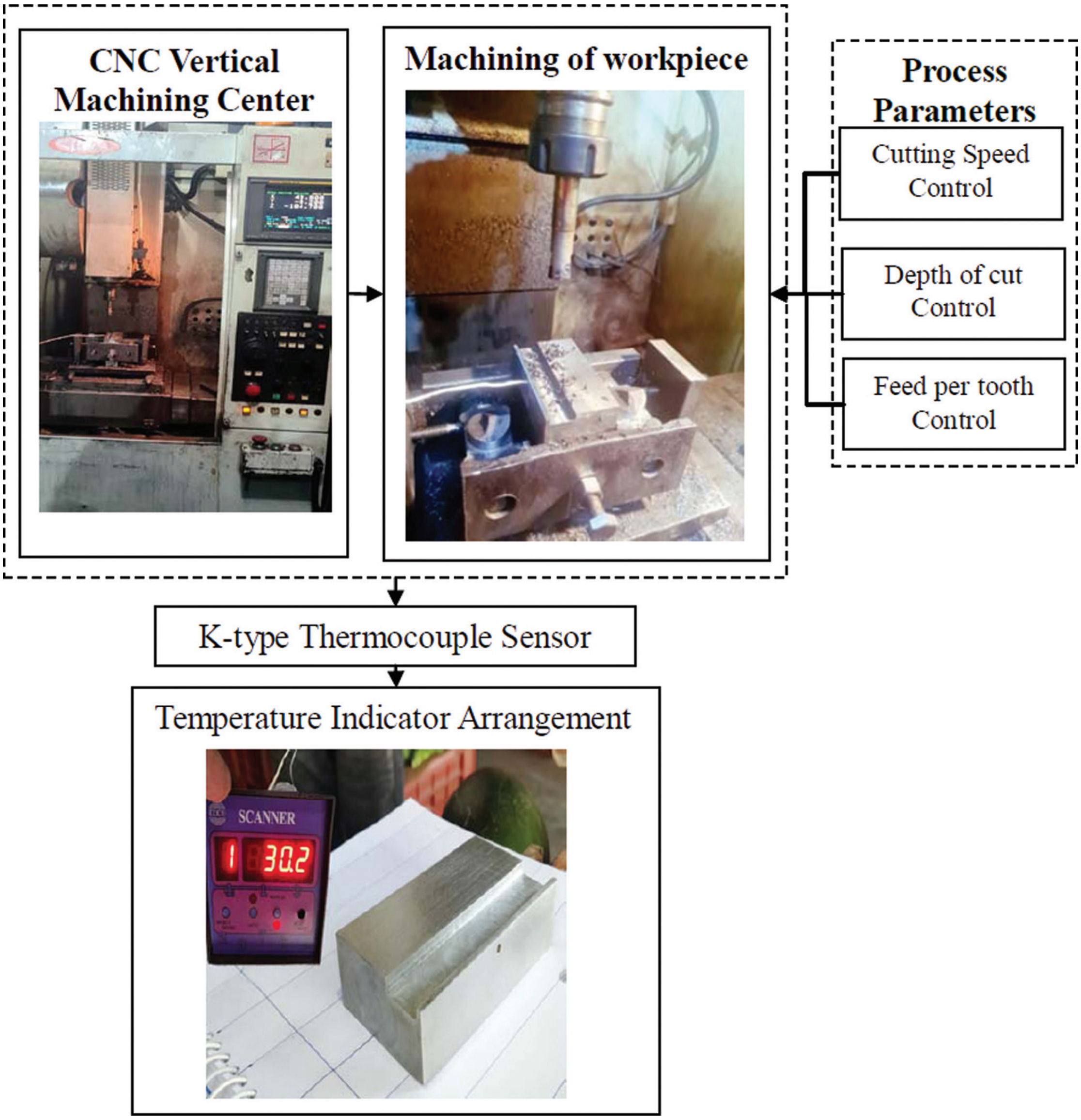

Figure 1. CNC end milling experimental set up.

Table 2. Properties of tool and work-piece(3)(44)

	Inconel625	Tungsten carbide
Density (g/cm^3)	8.44	15.8
Thermal conductivity (W/m · K)	9.8	84.02
Specific heat (J/(Kg·°C)	410	500
Thermal Expansion coefficient μm/m · K	12.8	
Poisson's ratio	0.3	0.3
Hardness(HB)	320	444

MATHEMATICAL MODELLING USING MOVING HEAT SOURCE AND GREEN'S FUNCTION

During milling operation, the cutting tool is moving while work piece remains stationary. The moving tool performed cutting and the sources of heat during cutting due to shearing, rubbing and friction in the tool is being transformed in the chip, tool and work piece and if we consider this tool as a moving heat source then heat can be estimated easily. Xiong et al. 2018(27) also considered the tool as a moving heat source and proposed an analytical model for work piece temperature using moving heat source method. Daniel Rosenthal (1935)(46) applied moving heat source in the theory of heat flow to arc welding first time. In the present work, moving heat source method has been considered for calculating the work piece temperature during a single pass cutting. A one dimensional heat conduction model with different boundary condition has been shown in figure 2. Initially the temperature is at room temperature. When cutting performed heat is generated due to the friction among tool and work piece. Some amount of heat is escaped out due to convection at the boundary but the heat generated within the work piece by the heat source is called volume heat source. This hypothesis has been used in many engineering cutting application.(47) In this work, the volume heat-source has been taken a point heat-source of invariable strength g_p^c (S) and moving with a speed *v* in the positive *x* direction by continuously releasing its energy as shown in figure 3.

The 3-dimensional heat conduction equation for the figure 3(a) presuming constant properties, is taken as

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{1}{k}g(x,y,z,t)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (1)$$

Where T ≡ $T(x,y,z,t)$ in equation (1) and boundary conditions as per our problem is given by

$$\begin{aligned}T(0,y,z,t)&=T(L_1,y,z,t)=T_0\\T(x,0,z,t)&=T(x,L_2,z,t)=T_0\\T(x,y,0,t)&=T(x,y,L_3,t)=T_0\end{aligned} \quad (2)$$

Let the heat source be a point heat source of invariable strength g_p^c (S) in watt, located at y = 0, z = 0 and releasing its energy continually as move about along the x axis in the positive x direction with a constant velocity *v*. Such a point heat source is correlated to the volumetric heat source $g(x,y,z,t)$ by the delta function notation. From equation 1, we have

$$g(x,y,z,t)=g_p^c\delta(x-vt)\delta(y-0)\delta(z-0) \quad (3)$$

Where δ(–) denotes the Dirac delta function and ξ = (x – *v*t) moving the x coordinate with a new coordinate (ξ). The point source g_p^c therefore represents the total quantity of energy released by the point source and has the dimensions in watts. Subscript and superscript of *g* function g_p^c represents the point (p) and continuous (c) heat source.

Figure 3(b) shows the new coordinate system ξ, y, z moving with the source. The heat conduction equation (1) has been transformed by the application of the coordinate system given by

$$\frac{\partial^2 S}{\partial \xi^2}+\frac{\partial^2 S}{\partial y^2}+\frac{\partial^2 S}{\partial z^2}+g_p^c\delta(\xi)\delta(y)\delta(z)=\frac{1}{\alpha}\left(\frac{\partial S}{\partial t}-v\frac{\partial S}{\partial \xi}\right) \quad (4)$$

The solid is moving with a velocity *v* in the negative ξ direction with respect to an viewer positioned at the source and this is the cause for the negative sign in front of the velocity term in equation (3). A new solution has been generated in moving solids by introducing a new variable *S* with the help of transformations for removing the last term in equation (4) (47). From equation (1) and (4)

$$T(x,y,z,t)=S(x,y,z,t)exp\left(\frac{v_x}{2\alpha}-\frac{v^2 t}{4\alpha}\right) \quad (5)$$

The value of $S(x,y,z,t)$ in equation (5) has been calculated with the help of Green' function [15] by considering equation (4) and Subjected to first kind boundary conditions

$$\begin{aligned}S(0,y,z,t)&=S(L_1,y,z,t)=0\\S(\xi,0,z,t)&=S(\xi,L_2,z,t)=0\\S(\xi,y,0,t)&=S(\xi,y,L_3,t)=0\end{aligned} \quad (7)$$

and according to Dirichlet and convective boundary conditions (47)

$$k\left.\frac{\partial S}{\partial x}\right|_{x=L_1}+S(L_1,t)+\left(h+\frac{kv}{2\alpha}\right)=hS_\infty e^{-\frac{L_1}{2\alpha}-\frac{v^2 t}{4\alpha}} \quad (8)$$

Where

$$h+\frac{kv}{2\alpha}=h_c \quad (9)$$

The term h_c in (9) is the convective heat transfer coefficient,

The desired solution of $S(x,y,z,t)$ is given by with the help of Green's function (48) by replacing $t = (t - \tau)$.

$$S(x,y,z,t)=\int_{\tau=0}^{t}\int_{\xi'=0}^{L_1}\int_{y'=0}^{L_2}\int_{z'=0}^{L_3}G^*(t-\tau)q_g(\tau)dz'dy'd\xi'd\tau \quad (10)$$

Where

$$G^*(\xi,\text{y,z,t}\,|\,\xi',y',z',t-\tau)=G(t-\tau)\delta(\xi')\delta(y')\delta(z')\times e^{\frac{vp_\xi}{2\alpha}-\frac{v^2\tau}{4\alpha}} \quad (11)$$

The solution of G*(ξ, y, z, t | ξ′, y′, z′, t –τ) with the help of Green's function (48)

$$G\left(\xi, y, z, t \mid \xi', y', z', t-\tau\right)=\frac{8}{L_1 L_2 L_3}$$
$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty} e^{-\beta_m^2 v} e^{-\beta_n^2 v} e^{-\beta_P^2 v} \times sin\left(\beta_m x\right) sin\left(\beta_m x'\right) \quad (12)$$
$$sin\left(\beta_n y\right) \times sin\left(\breve{a}_n y'\right) sin\left(\beta_P z\right) sin\left(\beta_P z'\right) \frac{\beta_P^2 + B^2}{\beta_P^2 + B^2 + B'}$$

Where

$$\beta_m = \frac{m\pi}{L_1}, \beta_n = \frac{n\pi}{L_2}, \beta_P cot\left(\beta_P\right) = -B, B = \frac{hL_3}{k}, \; and \; v = \alpha(t-\tau)$$

Now direct solution of work piece temperature can be found easily with the help of equation (4) and (12) and (13) i.e.

$$T\left(x, y, z, t\right) = S\left(x, y, z, t\right) exp\left(\frac{v_x}{2\alpha} - \frac{v^2 t}{4\alpha}\right) \qquad (13)$$

If the value of the energy generation, $q_g(t)$, is known(49), the temperature equation (13) gives the direct solution of the problem. Using equation (10), (11) and (12), above equation can be written as

$$T\left(x, y, z, t\right) = \int_{\tau=0}^{t}\int_{\xi'=0}^{L_1}\int_{y'=0}^{L_2}\int_{z'=0}^{L_3} \frac{8}{L_1 L_2 L_3} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}$$
$$\sum_{p=1}^{\infty} e^{-\beta_m^2 v} e^{-\beta_n^2 v} e^{-\beta_P^2 v} \times sin\left(\beta_m x\right) sin\left(\beta_m x'\right)$$
$$sin\left(\beta_n y\right) \times sin\left(\beta_n y'\right) sin\left(\beta_P z\right) sin\left(\beta_P z'\right) \qquad (14)$$
$$\frac{\beta_P^2 + B^2}{\beta_P^2 + B^2 + B} \times q_g\left(\tau\right) dz' dy' d\xi' d\tau \times$$
$$exp\left(\frac{v_x}{2\alpha} - \frac{v^2 t}{4\alpha}\right)$$

The above Eq. (14) has been solved convolution of time since it has two different expressions $G^*(t-\tau)$ and q_g which both are function of time. Based on the empirical model equation, the average temperature of work piece and heat generation into the work piece during cutting span has been estimated using the above equation (14), the results being shown in Figure 4 and 5.These results are comparable with reference(1).

RESULTS AND DISCUSSION

An empirical model for work piece temperature by solving the non-homogeneous partial differential equation using Green's function has been simplified with Dirac delta approach during the end milling of Inconel625 work-piece with the assumption of single-pass cutting and one-point observation. This modal has also been compared with the experimental observations. For authenticate the modal a series of experiments has been conducted according to the various cutting speeds. Nine random experiments have been carried out and each experiment has been stopped after 100 mm cutting length. A cutting pass has been conducted with cutting speed, feed and depth of cut shown in table 1. During the cutting of 100 mm length of span, 60 observations have been taken whereas cutting speed varying from 6000 to 8000rpm. However, the feed per tooth is constant 0.05mm/tooth. Therefore, feed rate varying from 600 to 800mm/min. In the present work, as we have taken 60 observations during cutting of span of 100mm. Therefore, the frequency of observation of thermocouple installed in the mid of the length of the span varying from 6 Hz to 8 Hz during the cutting.

The results obtained by the experiments and empirical model have been compared in figure 6. The results shown

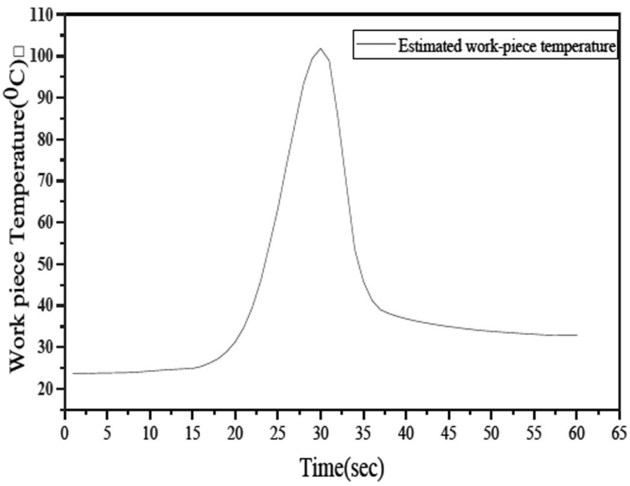

Figure 4. Variation of workpiece temperature with time.

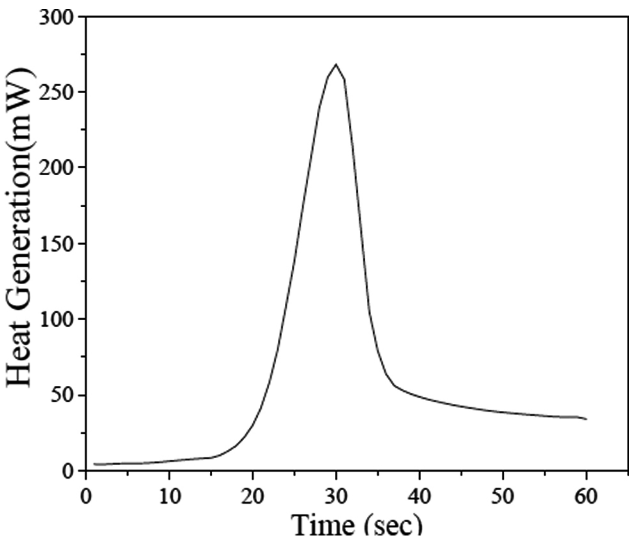

Figure 5. Estimation of heat generation.

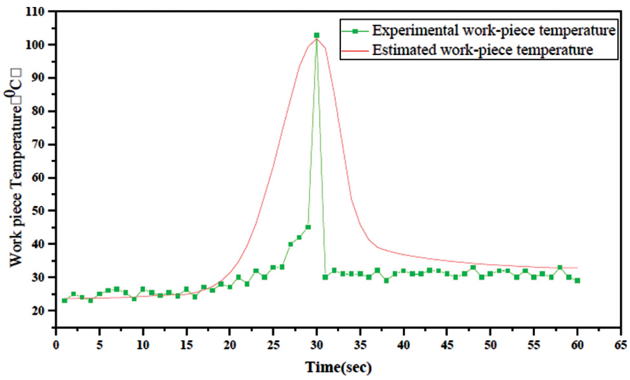

Figure 6. Comparison of work piece temperature obtained from empirical model and from practical experiments.

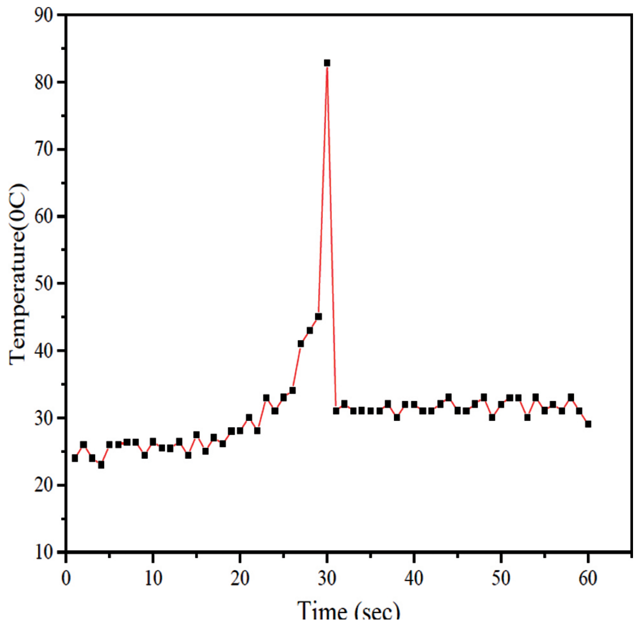

Figure 7. Variation of work piece temperature with time at 6000rpm.

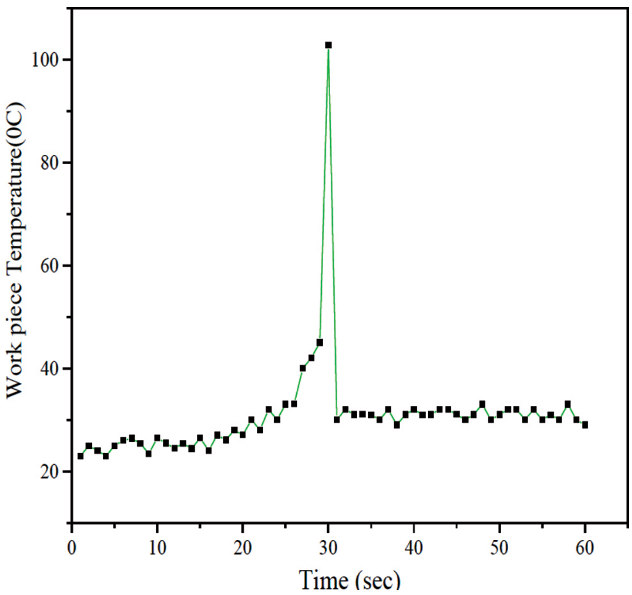

Figure 8. Variation of work piece temperature with time at 7000rpm.

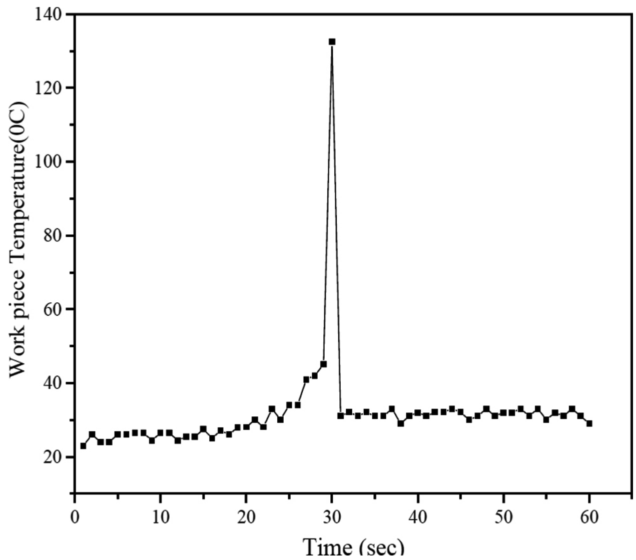

Figure 9. Variation of work piece temperature with time at 8000rpm.

in the figure consist of the observations with single thermocouple installed in the mid of the length of the cutting span. When the cutting tool moves from zero to 100 mm, initially the work piece temperature recorded by thermocouple is low. However, it rises slowly initially and then increase exponentially as reaches nearer to the thermocouple. Similar, comparable results have been obtained from empirical modal for work-piece temperature observation. The maximum work piece temperature 132°C has been observed in the middle of the work piece at 50 mm of cutting span, whereas empirical model has estimate the peak temperature of 129°C. Figure 6 shows the overlapped observation obtained from empirical model of work-piece temperature and 60 experimental observations during the single pass milling at the cutting speed of 7000 rpm. The comparative analysis of three average peak values obtained from experiments and empirical model has been tabulated in table 3. It is clearly illustrated in the overlapped trends that both the observations are come under the close compromise. Therefore, it can be concluded that empirical model provides more realistic results for single pass cutting with one middle-point observation during end milling.

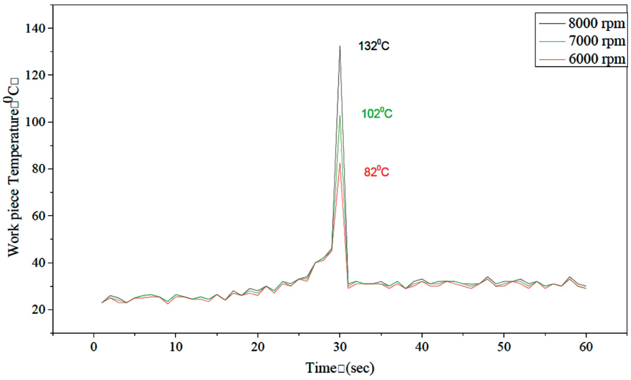

Figure 10. Comparison of work piece temperature at different speed.

Table 3 Average peak temperatures of the three trials at different cutting speed

S. no	Cutting speed (rpm)	Predicted peak Temperature (°C)	Experimental trials [average peak temperature (°C)]	%error
1	6000	79	82	3.66
2	7000	97	102	4.90
3	8000	129	132	2.27

The average peak temperature obtained from experimental observations at three different cutting speeds has been plotted in figure 7, 8 and 9 respectively. Figure 10 shows the comparison of 3 average peak temperatures at different cutting speeds. It has been observed that the peak work piece temperature observed at the mid of the span increases with the cutting speed. At high cutting speed, there is not enough time to escape heat and hence, the residual heat causes the rise in work-piece temperature with the rise in cutting speed(4). Table 3 shows a good agreement between predicted and experimental work piece temperature with the rise in cutting speed. Similar results have also been published in the earlier research work(22) (26) (28) (28)(1), et al had predicted the work piece temperature 140°C at the similar feed/tooth of 0.04mm/tooth. Similarly, the maximum temperature of work piece has been increased up to 1500C at the similar feed 0.05 mm/tooth.

CONCLUSION

In the present research work, an empirical model for work piece temperature by solving the non-homogeneous partial differential equation using Green's function has been simplified with Dirac delta approach during the end milling of Inconel625 work-piece with the assumption of single-pass cutting and one-point observation. The solution of the proposed Green's function model has been validated experimentally. The results obtained from an empirical approach have been judged against the experimental data and good trend has been obtained. The comparison of estimated and experimental results has showed the success of the proposed Green's function model.

Following conclusions has been obtained.

1. It has been observed that the peak work piece temperature is observed at the middle of the span with the help of the Green's Function based empirical model. As there is only one point of observation, initially the temperature increases slowly and then increase exponentially as the tool comes nearer to the middle observation point and then decreases at the same pace and then decreases slowly afterward.
2. It has been observed that the peak work piece temperature observed at the mid of the span increases with the cutting speed. At high cutting speed, there is not enough time to escape heat and hence, the residual heat causes the rise in work-piece temperature with the rise in cutting speed.
3. To verify the adequacy of empirical model of work piece temperature 9 conformational experiments have been performed at 3 different cutting speeds with constant depth of cut 5 mm and feed per tooth 0.05. A good agreement with 3 to 5% error has been observed between both theoretical and experimental results of work piece temperature

However, by little modification and adding small algorithm this single pass problem can be implemented on the multi-pass problems and even can also be applied for complex shapes. This approach can also be applied for other material work piece by altering just the material property parameter in the final equation.

FUTURE SCOPE OF THE WORK

This approach can also be applied for other material work piece by altering just the material property parameter in the final equation for the thin-wall parts used in the aeronautics and astronautics industries as well as in the die and mould industries. The proposed empirical model will further be used for automation. However, by little modification and adding small algorithm this single pass problem can be implemented on the multi-pass problems and even can also be applied for complex shapes.

CONFLICT OF INTEREST

The author(s) declared that we have no potential conflicts of interest with respect to the research, authorship and publication of this article.

FUNDING

There is no funding agency for this research work.

NOMENCLATURE

$g(x,y,z,t)$	Heat generation
g_p^c	Continuous point source
$\beta_m, \beta_n, \beta_p$	Sets of Eigen value
v_x:	Feed rate or Heat source velocity
(x,y,z):	Position of heat source at time $t = 0$
(ξ', y', z'):	Position of heat source at instantaneous time
(τ):	Instantaneous time
$G(\xi, y, z, t \mid \xi', y', z', \tau)$:	Green's function at position and time
$G(\xi, y, z, t \mid \xi', y', z', t - \tau)$:	Green's function for instantaneous time releases energy continuously at $t = \tau$

Greek symbols

α	Heat diffusivity
β	Sets of Eigen function

Subscripts

x	Refers to direction
m,n,p	Refers to 1,2,3…
p	Refers to point source

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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