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UTILIZATING FUZZY SETS AND FUZZY RELATIONS IN COMMUNICATION

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ABSTRACT

In interpersonal communication, misunderstandings may occur due to personal or environmental factors. Some mathematical models based on fuzzy logic have been developed to minimize the occurance of these misunderstandings. In this paper, fuzzy sets and fuzzy relations originating from fuzzy logic have been introduced as simple as possible, and an application of a mathematical model (Yager, 1980), that utilizes fuzzy sets and relations for the recovery of communication errors, has been presented

Key words : Fuzzy Set, Fuzzy Relation, Communication.

Bulanık Kümeler ve Bulanık Bağıntıların İletişimde Kullanımı

ÖZET

Kişisel iletişimlerde kişisel ya da çevresel etkenlerden kaynaklanan çeşitli yanlış anlaşılmalar ortaya çıkabilir. Bu yanlışlıkların en aza indirilmesini amaçlayan, bulanık mantığın kullanıldığı bazı matematiksel modeller geliştirilmiştir. Bu makalede bulanık mantıktan hareketle ortaya çıkmış olan bulanık kümeler ve bulanık bağıntılar olabildiğince basitleştirilerek tanıtılmış ve bunların iletişimdeki yanlışlıkları düzeltmeye yönelik kullanımlarını içeren bir matematiksel modelin (Yager, 1980) bir uygulaması verilmiştir.

Anahtar kelimeler: Bulanık Küme, Bulanık Bağıntı, Haberleşme.

1. Introduction

"There are differences between, what we think, what we want to say, what we think we say, what we say, what they want to hear, what they hear, what they want to understand, what they think they understand, and what they understand. That's why there are at least nine reasons for people to misunderstand each other" (Yager, 1980).

Fuzzy set theory which has a wide abstract disperse in mathematics, has numerous applications in social sciences, engineering, pharmacy, medicine, traffic control, cryptology, criminology, military, management and communications. Application of the theory on communication is studied in this paper.

In interpersonal communication, distortions can occur in message transactions due to environmental factors and dissimilar properties of parties such as language, voice, style, tone, clothing, and behavior. In this situation, transmitted or received signals establish a fuzzy set in the universe of possible signals. In order to determine the most appropriate responses to these fuzzy signals, mathematical models have been developed based on fuzzy set theory. Here, a model which is proposed in the paper "On Modeling Interpersonal Communication (1980)" by Yager,R.R. is introduced through examples by

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avoiding the mathematical details. The required fundamental concepts about fuzzy sets and fuzzy relations are also presented as simple as possible.

2. The Notion of Fuzzy Sets and Fuzzy Relations

The concept of fuzzy sets is first introduced in the paper titled "Fuzzy Sets", which is published in 1965 by an Azerbaijani Turkish mathematician, Lotfi Aliaskerzade (L.A. Zadeh) Since his publication, the concept of fuzzy sets has been studied and applied in almost all areas of mathematics (Ruan, 1997). Fuzzy sets are a generalization of crisp sets. In an universal set X, a crisp set A is composed of elements which makes an open proposition p(x) true. Formally it is written as A={x | p(x)}. This statement means "Set A is composed of elements x of the universe such that p(x) open proposition is true for the elements x". Set A is also characterized by the function φ_A which is called as the characteristic function of set A and is designated by the open proposition p(x). It is formulated as follows:

$$\boldsymbol{\varphi_A: X \rightarrow \{0, 1\}, \quad \varphi_A(x) = \text{truth value of } p(x) = \begin{cases} 0, & x \notin A \\ 1, & x \in A \end{cases}}.$$

A fuzzy set A in universe X is defined as a fuzzy open proposition p(x). A fuzzy open proposition p(x) becomes a fuzzy proposition when any element of the universe replaces x and in that case it does not need to be certainly true or certainly false, and it can have a truth value between 0 and 1. Fuzzy set A, which is defined by a fuzzy open proposition p(x), does not need to absolutely include or absolutely exclude an element x of the universe X. But the element x has a degree of belonging to set A. This degree is called the element degree of x and is a reel number between 0 and 1, and corresponds to the truth value of p(x) that defines A. A fuzzy set A is represented by the following function which is again denoted by $\varphi_{A,}$ and determined by open fuzzy proposition p(x) and is called the characteristic or membership function of fuzzy set A.

$$\boldsymbol{\varphi_A: X \rightarrow [\,0, 1\,], \quad \varphi_A(x) = \text{truth value of } p(x) \in [0,1] = I}$$

As a result, fuzzy sets in universe X can be regarded as functions from X to closed interval I; therefore, the family of all fuzzy sets in universe X is denoted by the symbol I^X. As an example from the case of a person-to-person communication, let the universe X be the set of the possible signals generated by one party (say John) against a proposal of the other:

$$\boldsymbol{X = \{\ \text{yes}(x_1),\ \text{no}(x_2),\ \text{let me think}(x_3),\ \text{nonsense}(x_4)\ \}}$$

Assume that the signals are not received clearly by the other party (say David) and let the truth values of the fuzzy propositions for the values that x can have in the open proposition p(x) = "John generated the signal x" be as $p(x_1)$ = p(yes) = "John gave the answer *yes*" = 0.6, $p(x_2)$ = p(no) = 0.4, $p(x_3)$ = p(let me think) = 0.9, $p(x_4)$ = p(nonsense) = 0.1. Then, the element function of fuzzy set A defined by the open proposition p(x) becomes

$$\boldsymbol{\varphi_A = \{ (x_1, 0.6), (x_2, 0.4), (x_3, 0.9), (x_4, 0.1) \}}$$

The element degree of element "nonsense" in A is 0.1; in other words the proposition "Signal '*nonsense*' is in set A" can only have a truth degree of 0.1, and this is near to zero (though not definitely wrong). Contrary to this, the signal *letMeThink* not definitely an element of set A has an element degree of 0.9, which is quite high.

A crisp relation from X to Y, where X and Y are any sets, is a subset of Cartesian product set X×Y; in other words it is a set in universe X×Y. Similarly, a fuzzy relation form X to Y is a subset in universe X×Y and therefore is represented by the characteristic function

$$\boldsymbol{\varphi_\beta : X\times Y \to I, \quad \varphi_\beta = \{ ((x,y), \varphi_\beta(x,y)) \mid (x,y) \in X\times Y \}}$$

The real number $\varphi_\beta(x,y)$ between 0 and 1, which is an image of an element (x,y) of universe X×Y under the element function $\varphi_{\beta,}$ is the element degree of the ordered pair (x,y) in the fuzzy relation β. This real number can be interpreted as the degree of dependency of an element x in set X to an element y in set Y according to the relation β. As the element degree of ordered pair (x,y) in β goes to zero, dependency of x to y as compared to β decreases; as it goes to 1, the dependency increases. As an example, let X be the set of "John's possible signals" as defined above; and Y be the set of "David's possible responses",

Y = { pleased(y_1), offended(y_2), surprised(y_3), shocked(y_4), think again(y_5) }.

Let us consider the fuzzy relation β from Y to X, defined as the open proposition "the best appropriate answer to signal x is y". In case the sets X and Y are finite, it is practical to denote two sided fuzzy relations in matrix form. If

$$\boldsymbol{\varphi_\beta = \{ ((y_i, x_j), \varphi_\beta(y_i,x_j)) \mid (y_i, x_j) \in Y\times X \}}$$

is a characteristic function of a fuzzy relation from Y to X, where

$$\mathbf{X = \{x_i \mid i = 1,\ldots n\} \text{ and } Y = \{y_j \mid j = 1,\ldots m\}}$$

are two finite sets, then the matrix form of β is in the form

$$\boldsymbol{\beta = [\,\varphi_\beta(y_i, x_j)\,]_{m\times n}}$$

Element degrees (assume that they are specified by an expert) of the elements of universe Y×X (in our example) in β, the matrix form of β is as follows:

$$\boldsymbol{\beta \;=\; [\,\varphi_\beta(y_i, x_j)\,]_{5\times 4}}$$

$$= \begin{pmatrix} \varphi_\beta(y_1,x_1) & \varphi_\beta(y_1,x_2) & \varphi_\beta(y_1,x_3) & \varphi_\beta(y_1,x_4) \\ \varphi_\beta(y_2,x_1) & \varphi_\beta(y_2,x_2) & \varphi_\beta(y_2,x_3) & \varphi_\beta(y_2,x_4) \\ \varphi_\beta(y_3,x_1) & \varphi_\beta(y_3,x_2) & \varphi_\beta(y_3,x_3) & \varphi_\beta(y_3,x_4) \\ \varphi_\beta(y_4,x_1) & \varphi_\beta(y_4,x_2) & \varphi_\beta(y_4,x_3) & \varphi_\beta(y_4,x_4) \\ \varphi_\beta(y_5,x_1) & \varphi_\beta(y_5,x_2) & \varphi_\beta(y_5,x_3) & \varphi_\beta(y_5,x_4) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 0.6 & 0.6 & 0.9 \\ 0.3 & 0.7 & 0.8 & 0.6 \\ 0.8 & 0.7 & 0.2 & 0.9 \\ 0 & 0.9 & 0.1 & 0.4 \end{pmatrix}$$

In this notation, for example, the real number $\varphi_\beta(y_2,x_3) = 0.2$ in second row third column (for i=4 and j=3), which is the element degree of pair (y_4,x_3) of Y×X in fuzzy relation β, shows the truth value of the fuzzy proposition “The most appropriate answer to signal *let me think*(x_3) is *shocked*(y_4)” and describes that the proposition has a truth value close to false; in other words the message *shocked* is far from being an appropriate answer to the message *let met think* .

3. Application of Fuzzy Sets to Communication

For a communication case, let X be the set of possible messages of the sender. Let us assume that messages of the sender cannot be transmitted to the receiver clearly due to environmental or personal reasons and that the transmitted messages constitute a fuzzy set A in universe X. The maximum value of the set

$$\boldsymbol{\varphi_A[\,X\,] = \{\,\varphi_A(x) \mid x \in X\,\}}$$

which is the set of images under the element function φ_A: X→ [0,1] is called the strength of the transmission. As an example, strength of the transmission represented by the fuzzy set A above is

$$\mathbf{\max\ \varphi_A[\,X\,] = \max\{\,0.6,\ 0.4,\ 0.9,\ 0.1\,\} = 0.9\ .}$$

If the set φ_A[X] takes the maximum value in points x more than once, than this message is called *ambiguous.* The clarity of the message is measured by the difference between the maximum element degree of the fuzzy set, which represents the message, and the next element degree. The clarity of the message in our example is 0.9 - 0.6 = 0.3. As the strength and clarity of the message increases, the quality of communication increases.

The transmitted signals have possibilities according to the expectations of the receiver. Let us assume that these possibilities has a distribution b, and let $b(x) \in I$ be the degree of receiver's belief on the reception of signal x. The total possibility (not probability!) of fuzzy message A is defined as

$$\mathbf{b(A) = max\ \{ min\{ \varphi_A(x),\ b(x)\}\ |\ x \in X \}.}$$

In our example, if the distribution of the David's expectations is b={ (x_1, 0.8), (x_2, 0.3), (x_3, 0.2), (x_4, 0) } than the total possibility of A is

$$\mathbf{b(A) = max\{ min\{ 0.6, 0.8 \}, min\{ 0.9, 0.3 \}, min\{ 0.4, 0.2 \}, min\{ 0.1, 0 \} \} = 0.6.}$$

Due to the inconsistencies between the receiver's expectations and the message, there can be deviations in the perception of message A by the receiver. If the message perceived by the receiver after these deviations is fuzzy set B, then, for each $x \in X$, the relation between messages A and B is given by the formula

$$\boldsymbol{\varphi_B (x) = [\ \varphi_A(x)\ ]^{b(A)}.}$$

According to this, the perception B of David in our example is the fuzzy set

$$\boldsymbol{\varphi_B = \{ (\ x_1, (\ 0.6\ )^{0.6}\ ), (\ x_2, (\ 0.4\ )^{0.6}\ ), (\ x_3, (\ 0.9\ )^{0.6}\ ), (\ x_4, (\ 0.1\ )^{0.6}\ ) \}}$$

Due to this perception of the receiver, there may be variations on the distribution of possibilities of his expectations. For each $x \in X$, the new distribution of possibilities arising due to these variations is defined by the formula

$$\boldsymbol{c(x) = min\ \{ (\ b(x)\ )^{1-b(A)},\ \varphi_B (x) \}}$$

According to the data in our example, the new possibility distribution c, the total possibility c(B), and the characteristic function φ_C of the new fuzzy set C, that results from this total possibility is calculated as

$$c = \{ (x_1, (0.6)^{0.6}), (x_2, (0.4)^{0.6}), (x_3, (0.2)^{0.4}), (x_4, 0) \},$$

$$c(B) = \max\{ (0.6)^{0.6}, (0.4)^{0.6}, (0.2)^{0.4}, 0 \}$$

$$\approx \max\{ 0.73, 0.57, 0.52, 0 \} = 0.73.$$

$$\varphi_C = \{ (x, [\varphi_B(x)]^{c(B)}) \mid x \in X \}$$

$$\approx \{ (x_1, 0.79), (x_2, 0.66), (x_3, 0.95), (x_4, 0.36) \}$$

This final form of the received message is stronger and clearer. Once the final form of the message is calculated as a fuzzy set C, the most appropriate response to the message can now be created. The most appropriate response D to message C is defined in the form (max-min composition)

$$D = \beta \circ C, \quad \varphi_D(y) = \max\{ \min\{ \varphi_\beta(y, x), \varphi_{C(X)} \} \mid x \in X \}$$

where X is the set of possible transmitted messages, Y is the set of possible response messages to the transmitted message, $\beta = [\varphi_\beta(y_i, x_j)]_{m\times n}$ is the membership matrix of the fuzzy set "the most appropriate response to message x is y" (i.e. the fuzzy relation from Y to X) in universe Y×X [1] (Hence the response of David is

$$D = [\varphi_D(y_i)]_{5\times 1} = \beta \circ C = [\varphi_\beta(y_i,x_j)]_{5\times 4} \circ [\varphi_C(x_j)]_{4\times 1}$$

$$= [\max\{ \min\{ \varphi_\beta(y_i,x_j), \varphi_{C(Xj)} \} \mid x_i \in X \}]_{5\times 1} \circ [\varphi_C(x_j)]_{4\times 1}$$

$$= \begin{pmatrix} 1 & 0 & 0.5 & 0 \\ 0 & 0.6 & 0.6 & 0.9 \\ 0.3 & 0.7 & 0.8 & 0.6 \\ 0.8 & 0.7 & 0.2 & 0.9 \\ 0 & 0.9 & 0.1 & 0.4 \end{pmatrix} \circ \begin{pmatrix} 0.79 \\ 0.66 \\ 0.95 \\ 0.36 \end{pmatrix} =$$

$$\begin{pmatrix} \min\{1, 0.79\}, \ \min\{0, 0.66\}, \ \min\{0.5, 0.95\}, \ \min\{0, 0.36\}\} \\ \max\{ \min\{0, 0.79\}, \min\{0.6, 0.66\}, \min\{0.6, 0.95\}, \min\{0.9, 0.36\}\} \\ \max\{\min\{0.3, 0.79\}, \min\{0.7, 0.66\}, \min\{0.8, 0.95\}, \min\{0.6, 0.36\}\} \\ \max\{ \min\{0.8, 0.79\}, \min\{0.7, 0.66\}, \min\{0.2, 0.95\}, \min\{0.9, 0.36\}\} \\ \max\{ \min\{0, 0.79\}, \min\{0.9, 0.66\}, \min\{0.1, 0.95\}, \min\{0.4, 0.36\}\} \end{pmatrix} = \begin{pmatrix} 0.79 \\ 0.60 \\ 0.80 \\ 0.79 \\ 0.66 \end{pmatrix},$$

$$\varphi_Y = \{ (y_1, \mathbf{0.79}), (y_2, \mathbf{0.60}), (y_3, \mathbf{0.80}), (y_4, \mathbf{0.79}), (y_5, \mathbf{0.66}) \}$$

This response message has a strength value of 0.8, and is definite since it has a maximum at only one point. Despite the high strength value, the clarity of the message is as low as 0.8 – 0.79 = 0.01. An exact response could not be created for the received fuzzy message. With a truth-value of 0.8, David tells that he is surprised by John's message. Other response messages have also levels of appropriateness as calculated. The proposition that "*pleased* is the most appropriate response for the message David received" has only a truth-value of 0.6. The appropriateness or usefulness of the resulting values depends on the accuracy of numerical values used in message A and proportion β, and using accurate values requires expertise on communication. What mathematics can do is to deduce logical results from assumptions and definitions. The usefulness of results (note that this is not the subject of pure mathematics) depends on accurate initial conditions.

4. Conclusion and Discussion

Generally, mathematicians are not interested in whether their studies on pure mathematics will find an application area. Furthermore, "there is no field, where there is a perfect correspondence between empirical variables of the physical world and a mathematical model" (Stevens, 1968); "a mathematical proposition is not as certain as it concerns with the real world, and it does not concern with the real world as much as it is certain the efficacy of mathematics on science of nature is so mystical that there is no rational explanation (Stevens, 1968)". Contrary to this, mathematics is constantly used in all applied sciences hoping to acquire more certain results. However, certainty of mathematics comes from the fact, that acquired results are inevitable logical deductions of some assumptions. "Acquired results in mathematics are nothing else than different forms of their predecessors" (İnönü, 2003). Therefore, successful use of mathematics on an application area depends on the correct selection of initial assumptions. To derive useful results from the communication model we have worked on here, assumptions must be appropriate. Especially values attributed to the element functions of fuzzy sets and fuzzy relations are controversial. One approach to determine the appropriate element functions is to make surveys on experts of the field, and evaluate their statistical results.

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