Design and Implementation of MRAC & MRAC-PID Feedback for Turntable

Saibal MANNA*, Deepak Kumar SINGH†, Ashok Kumar AKELLA‡

Department of Electrical Engineering, National Institute of Technology Jamshedpur, Jharkhand-831014, India

Highlights

• An adaptive algorithm i.e., MRAC-PID is proposed for the Turntable.
• The performance of MRAC-PID is compared to PID and traditional MRAC.
• The robustness of the proposed controller is validated by introducing uncertainties in two aspects.

Abstract

The paper explains a control method for turntable by feedback a PID controller with the traditional model reference adaptive controller (MRAC) based on the MIT rule. The traditional MRAC is designed for a first-order system with the adjustment of a two-variable parameter. However, the majority of plants, including turntables, are second-order systems. Traditional MRAC tracking performance for second-order systems is unsatisfactory. The control law for the second order system along with extension from the first to the second order of MRAC is derived. The modified MRAC i.e., MRAC-PID controller is designed for the application of turntable. It enhanced the system’s dynamic performance. To assess the performance of the proposed controller, MATLAB/Simulink software was used. The article incorporates a detailed analysis and comparison of PID, MRAC as well as MRAC-PID controllers based on the MIT rule for the turntable system. The robustness of the proposed controller is validated by introducing uncertainties in two aspects i.e., mistuning of the controller gains and turntable system dynamics change. The probabilistic design assessment for the mistuning is carried out through levels of uncertainty in controller gain. It is observed that PID and MRAC would track the reference model but modified MRAC has better performance in terms of tracking accuracy, adaptability, and rapidity. Several performance indexes such as integral absolute error (IAE), integral time absolute error (ITAE), and integral square error (ISE) were employed to justify the proposed controller superiority.

Received: 03 Jan 2022
Accepted: 04 Aug 2022

Keywords

MRAC
MRAC-PID
Turntable
MIT rule
PID

1. INTRODUCTION

A country’s scientific and technological quality is reflected in its level of advancement in aerospace and aviation innovation. In aerospace and its fields, the turntable has long been a crucial piece of hardware in the loop [1]. It is a vital link in aerospace and its field. There has been a significant change in the performance of the turntable since the beginning, as the accuracy and reliability of guidance as well as inertial navigation systems continue to enhance. Initially, modelling of the turntable was not very accurate as there were interference and noise in the system. The challenging task, in the context of the turntable, for the researchers is to improve the tracking accuracy [2].

Feedback controllers have been deployed for decades and will continue to be used in the future. The PID controller is among the most frequently applied feedback controllers in industrial workplaces. To calculate the optimum gain of the PID controller is not an easy task. Furthermore, PID tuning necessitates the employment of a time-consuming and painstaking gain-phase margin method depending on the frequency response approach [3]. There are several tuning methods available such as Tyreus-Luyben, Z-N, Cohen-Coon, and automatic tuning methods. However, they are all trial and error with time-consuming as well. The system employing the PID controller is completely reliant on the controller's efficiency, which is directly proportional to tuning accuracy [4].
Insufficient tuning of the PID controller would degrade the performance of the system causing oscillations, lag, undershoot as well as overshoot in the process. The PID controller gains must be determined by computing the plant's transfer function, which necessitates linearization of the plant dynamics and this is the reason to ignore the non-linear behavior of the plant. In reality, not always the performance of the PID controller is the desired one because they are affected by system dynamic behavior, disturbances, modeling uncertainties, and time-varying parameters.

The MRAC was initially developed for flight control and other aerospace applications. The MRAC was first conceived at the Massachusetts Institute of Technology in the United States of America, with an emphasis on flight control i.e., known as the MIT rule [5]. MRAC is simply a control approach that does not depend on a plant model and has excellent anti-interference properties. It comes under adaptive control. The main concept is to select a reference model that has the required performance, and then use the control law to make the plant's output compatible with the reference model, resulting in the desired system outcome. In MRAC, the Lyapunov method and the MIT rule are the two methods for formulating control laws. Additional benefits of MRAC include its capacity to operate a system that is subject to ambient or parameter changes [6]. MRAC has been successfully used in a number of applications, including temperature [7, 8], pH [9], speed control of synchronous motor drive [10], speed control of ultrasonic motor [11], aortic pressure regulation [12], control of rotorcraft [13], nuclear reactor power control [14], etc.

The goal of this study is to develop a control technique for a turntable by employing traditional MRAC with a PID controller based on the MIT rule. The modified controller i.e., MRAC-PID has supremacy over both MRAC and PID. In comparison to a traditional MRAC, the modified controller has better dynamic performance. The following section is the outline of this paper. The controllers (MRAC and modified MRAC) are described in section 2. Afterward, section 3 describes several performance indices. Derivation of control law for modified MRAC and MRAC is explained in section 4. Section 5 incorporates the simulation results and discussion of all control schemes and section 6 concludes the paper.

2. TRADITIONAL AND MODIFIED MRAC

2.1. Traditional MRAC

MRAC comes under the adaptive control technique. It is typically used to establish a closed loop control with modified parameters in order to adjust the system's behavior. In MRAC, the system’s output is compared to the reference model and finds the error. Based on error values, its control parameters are updated. The aim is to find the desired response. Hence, plant response follows the reference model output [6]. Figure 1 depicts the MRAC general block diagram.

MRAC has the following features:

- It is less reliant on the plant model.
- It has a high level of anti-interference capability.
• It quickly adapts to the desired response.
• It has robust feedback control.

The MIT theory is employed as the control law to develop the proposed controller. It is a technique for adjusting local parameters [6]. Employing the gradient descent concept, the difference between the reference \(p_r\) and plant output \(p\) is defined as the error (Equation (1)). The error \(e_{rr}\)-related performance function is developed in a standard way (Equation (2)). The turntable variable is also modified along the performance function’s negative gradient direction to bring the error closer to zero

\[
e_{rr} = p - p_r \quad (1)
\]

\[
C_f(\phi) = \frac{e_{rr}^2}{2} \quad (2)
\]

To design the controller, performance function \(C_f(\phi)\) is devised i.e., related to the error in Equation 2. The MIT rule states that the control variable \(\phi\) must be the negative gradient direction of \(C_f(\phi)\) i.e., given in Equation 3 and \(C_f(\phi)\) tends to zero in order to have the plant output follow the reference model output [15]. The MRAC must have three variables as explained in Equation 4 so that the second-order system can follow the reference model

\[
\frac{d\phi}{dt} = -\lambda \frac{\partial C_f}{\partial \phi} = -\lambda e_{rr} \frac{\partial e_{rr}}{\partial \phi} \quad (3)
\]

\[
m(t) = \phi_1 m_c(t) - \phi_2 p(t) - \phi_3 p(t) \quad (4)
\]

where \(\lambda\) denotes the adaptation gain. \(m\) & \(m_c\) are the control and command signals respectively.

### 2.2. Modified MRAC

Using MRAC, the plant output can be tracked by the reference model output. By determining the reference model, desired results will definitely be achieved. However, employing only traditional MRAC to improve system performance is not sufficient. So, modified MRAC i.e., MRAC-PID controller is devised [16]. Figure 2 illustrates the block diagram of a modified MRAC. Here, traditional MRAC is feedback by the PID controller. After combining the MRAC-PID controller, the system’s performance is improved as compared to traditional MRAC. The MRAC-PID control law is given in Equation 5, where \(K_P\), \(K_I\) and \(K_D\) are gains of PID.

The proposed controller, the control law is stated as:

\[
m(t) = \phi_1 m_c(t) - \phi_2 p(t) - \phi_3 p(t) - \left( K_P e_{rr} + K_I \int e_{rr} dt + K_D \frac{de_{rr}}{dt} \right) \quad (5)
\]
3. PERFORMANCE INDICES

A widely employed performance index of an adaptive system is the error functional integral, that is utilized to define the dynamic features of the system. Functional error integration is a method of integral evaluation that leverages the system’s instantaneous error $e_r(t)$ as a functional form. The performance indices include IAE, ISE, and ITAE. Among these, the ITAE index is more selective and practical. The following is the mathematical expression for the aforesaid performance indices:

$$IAE = \int_0^\infty |e_r(t)| \, dt$$  \hspace{1cm} (6)

$$ISE = \int_0^\infty |e_r(t)|^2 \, dt$$  \hspace{1cm} (7)

$$ITAE = \int_0^\infty t|e_r(t)| \, dt.$$  \hspace{1cm} (8)

4. CONTROLLER DESIGN

In this section, the designing of plant, traditional, and modified MRAC for turntable are discussed.

4.1. Turntable Model

The turntable is a servo-electric system. It consists of a permanent magnet DC motor; turntable parameter values are presented in Table 1. The turntable transfer function is given in Equation (9).

$$\mathcal{G}_p(s) = \frac{a_t}{(M + v_0^2)(R_0 + L_0)} + a_t a_b.$$  \hspace{1cm} (9)

<table>
<thead>
<tr>
<th>Table 1. Parameters of turntables [12]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Resistance, $R_{01}$</td>
</tr>
<tr>
<td>Inductance, $L_{01}$</td>
</tr>
<tr>
<td>Moment of inertia, $M$</td>
</tr>
<tr>
<td>Back emf constant, $a_b$</td>
</tr>
<tr>
<td>Coefficient of viscous friction, $v_0$</td>
</tr>
<tr>
<td>Motor torque constant, $a_t$</td>
</tr>
</tbody>
</table>
4.2. MRAC for Turntable

According to the MIT theory, control variables must be equal to the negative gradient direction of $C_f(\phi)$, as discussed in Equation (3), for error tends to zero.

Let the plant transfer function be:

$$\frac{d^2 p(t)}{dt^2} = -a_{p1} \frac{dp(t)}{dt} - a_{p0} p(t) + b_p m(t).$$

(10)

The reference model is as follows:

$$\frac{d^2 p_r(t)}{dt^2} = -a_{r1} \frac{dp_r(t)}{dt} - a_{r0} p_r(t) + b_r m_c(t)$$

(11)

where $a_{p1}$, $a_{p0}$, and $b_p$ are the plant parameters. $a_{r0}$, $a_{r1}$ and $b_r$ are the reference model parameters.

Equations (4), (10), and (11) are used to develop a model that connects the reference input and the plant output

$$\frac{p}{m_c} = \frac{b_p \phi_1}{s^2 + (a_{p1} + b_p \phi_3)s + (a_{p0} + b_p \phi_2)}.$$  

(12)

Here $\phi_i$, $\phi_2$, and $\phi_3$ are control parameters.

Substituting the $m(t)$ value in Equations (4) and (10), we get

$$\frac{d^2 p(t)}{dt^2} = -a_{p1} \frac{dp(t)}{dt} - a_{p0} p(t) + b_p \left(\phi_1 m(t) - \phi_2 p(t) - \phi_3 \frac{dp(t)}{dt}\right).$$  

(13)

Comparing the Equations (11) and Equation (13) coefficients, we get

$$b_r = \phi_1 b_p$$  

(14)

$$a_{r0} = a_{p0} + \phi_2 b_p$$  

(15)

$$a_{r1} = a_{p1} + \phi_3 b_p.$$  

(16)

It converged as:

$$\phi_1 \approx \frac{b_r}{b_p}$$  

(17)

$$\phi_2 \approx \frac{a_{r0} - a_{p0}}{b_p}$$  

(18)

$$\phi_3 \approx \frac{a_{r1} - a_{p1}}{b_p}.$$  

(19)

Using the MIT rule, $\phi_1$, $\phi_2$, and $\phi_3$ can be determined as

$$\frac{d\phi_1}{dt} = -\lambda_1 \frac{dC_f}{d\phi_1} = -\lambda_1 \times \frac{dC_f}{de_{rr}} \times \frac{de_{rr}}{dp} \times \frac{dp}{d\phi_1}$$  

(20)

$$\frac{d\phi_2}{dt} = -\lambda_2 \frac{dC_f}{d\phi_2} = -\lambda_2 \times \frac{dC_f}{de_{rr}} \times \frac{de_{rr}}{dp} \times \frac{dp}{d\phi_2}$$  

(21)
\[
\frac{d\phi_3}{dt} = -\lambda_3 \frac{\partial C_f}{\partial \phi_3} = -\lambda_3 \left( \frac{\partial C_f}{\partial e_{rr}} \times \frac{\partial e_{rr}}{\partial p} \times \frac{\partial p}{\partial \phi_3} \right)
\]

where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are adaptation gains.

By solving Equations (1), (2), (12), (20), (21) and (22), the updated \( \phi_1, \phi_2, \) and \( \phi_3 \) can be calculated as

\[
\frac{d\phi_1}{dt} = -\lambda_1 e_{rr} \cdot \frac{b_p m_c}{s^2 + (a_{p1} + b_p \phi_3)s + (a_{p0} + b_p \phi_2)}
\]

\[
\frac{d\phi_2}{dt} = \lambda_2 e_{rr} \cdot \frac{b_p p}{s^2 + (a_{p1} + b_p \phi_3)s + (a_{p0} + b_p \phi_2)}
\]

\[
\frac{d\phi_3}{dt} = \lambda_3 \dot{e}_{rr} \cdot \frac{b_p \phi_3}{s^2 + (a_{p1} + b_p \phi_3)s + (a_{p0} + b_p \phi_2)}
\]

Equations (23), (24), and (25) demonstrate the change of \( \phi_1, \phi_2, \) and \( \phi_3 \) parameters with time. Now, the turntable system design is completed.

### 4.3. Modified MRAC for Turntable

The control law of the MRAC-PID controller is expressed in Equation (26)

\[
m(t) = \phi_1 m_c(t) - \phi_2 p(t) - \phi_3 \dot{p}(t) - \left( K_P e_{rr} + K_I \int e_{rr} dt + K_D \frac{de_{rr}}{dt} \right).
\]

The control law is used to make the plant output compatible with the reference model. The reference model is represented by Equation (27)

\[
G_m(s) = \frac{50}{s^2 + 15s + 50}.
\]

### 5. SIMULATION RESULTS AND DISCUSSION

MATLAB/Simulink is used to simulate the turntable model. The description of the plant is expressed by Equation (9) and Table 1 incorporates the turntable parameters.

#### 5.1. Simulation with Different Adaptation Gain

The adaptation gain \( \lambda \) is one of the parameters of the adaptive control law, which is user defined [6]. Three values of \( \lambda \) are defined i.e., \( \lambda = 0.08, \lambda = 0.6, \) and \( \lambda = 1.6 \) for MRAC and MRAC-PID controllers. After optimum tuning of the PID controller, the system minimizes deviation from the set point, and responds to disturbances or set point changes quickly but with minimal overshoot. Also, in this case, controller gains are obtained as \( K_P = 8.54, K_I = 8.40, K_D = 2.15 \) after optimum tuning for the turntable. Figures 3, 4 and 5 illustrate the comparison between the PID, traditional, and modified MRAC controller responses for \( \lambda = 0.08, 0.6, \) and 1.6, respectively.
Figure 3. PID, traditional, and modified MRAC response for adaptation gain 0.08

Figure 4. PID, traditional, and modified MRAC response for adaptation gain 0.6

Figure 5. PID, traditional, and modified MRAC response for adaptation gain 1.6
Table 2 shows the detailed comparison of set point tracking of PID, traditional, and modified MRAC controllers for three different adaptation gains. For PID controller the set point tracking i.e., settling time, overshoot, and rise time are 12.57 sec., 13.068%, and 1.031 sec. respectively. In MRAC, for \( \lambda = 0.08 \), the settling time, overshoot, and rise time are 17.68 sec., 0.504%, and 9.272 sec. respectively. In MRAC-PID, for \( \lambda = 0.08 \), the set point tracking i.e., settling time, overshoot, and rise time are 4.7 sec., 2.577%, and 937.218 msec. respectively. After analysing all cases, it is observed that for \( \lambda = 0.08 \) the overall performance of the MRAC-PID controller for the turntable is enhanced.

The detailed performance indices of the PID, traditional, and modified MRAC with three different adaptation gains are listed in Table 3. For PID controller, the performance indices i.e., IAE, ISE, and ITAE are \( 8.207 \times 10^{-6} \), \( 9.059 \times 10^{-7} \), and 0.000272 respectively. In MRAC, for \( \lambda = 0.08 \), IAE, ISE, and ITAE are \( 3.214 \times 10^{-5} \), \( 1.033 \times 10^{-19} \), and 0.001607 respectively. In MRAC-PID, for \( \lambda = 0.08 \), the performance indices i.e., IAE, ISE, and ITAE are \( 2.018 \times 10^{-10} \), \( 4.074 \times 10^{-20} \), and 1.009 \times 10^{-8} \) respectively. After analysing all scenarios, it is concluded that for \( \lambda = 0.08 \), the modified MRAC controller enhances the dynamic performance of the turntable.

Table 2. Set point tracking for PID, traditional and modified MRAC

<table>
<thead>
<tr>
<th>Controller</th>
<th>Adaptation Gain (( \lambda ))</th>
<th>Set point tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Settling Time</td>
</tr>
<tr>
<td>PID</td>
<td>-</td>
<td>12.57 sec.</td>
</tr>
<tr>
<td>MRAC</td>
<td>0.08</td>
<td>17.68 sec.</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>14.29 sec.</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>18.15 sec.</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>0.08</td>
<td>4.70 sec.</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>6.07 sec.</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>7.80 sec.</td>
</tr>
</tbody>
</table>

Table 3. Performance evaluation for different control techniques

<table>
<thead>
<tr>
<th>Controller</th>
<th>Adaptation Gain (( \lambda ))</th>
<th>Performance Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IAE</td>
</tr>
<tr>
<td>PID</td>
<td>-</td>
<td>( 8.207 \times 10^{-6} )</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>( 3.214 \times 10^{-5} )</td>
</tr>
<tr>
<td>MRAC</td>
<td>0.6</td>
<td>( 7.704 \times 10^{-10} )</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>( 3.255 \times 10^{-8} )</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>0.08</td>
<td>( 2.018 \times 10^{-10} )</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>( 1.408 \times 10^{-5} )</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>( 7.16 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

The change of adaptive parameters (\( \phi_1 \), \( \phi_2 \), and \( \phi_3 \)) for MRAC and modified MRAC at \( \lambda = 0.08, 0.6, 1.6 \) are displayed in Figures 6, 7, and 8 respectively. It is clearly seen that the reliance of the modified controller on the traditional controller is lessened. The system can stay up with the reference model with a minor change in the adaptive parameter. The adaptive parameters’ final values are summarised in Table 4. Again for \( \lambda = 0.08 \), the performance of the MRAC-PID controller is quite impressive.
Figure 6. Adaptation parameter ($\phi_1$)

Figure 7. Adaptation parameter ($\phi_2$)

Figure 8. Adaptation parameter ($\phi_3$)
Table 4. Comparison of adaptive parameters

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>Adaptation Gain ($\lambda$)</th>
<th>Adaptive Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.08</td>
<td>8.8</td>
</tr>
<tr>
<td>MRAC</td>
<td>0.6</td>
<td>15.5</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>40</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>0.08</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>4.65</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>11</td>
</tr>
</tbody>
</table>

5.2. Robustness Test of the Proposed Controller

The turntable servo systems are affected by the surrounding environment and working status changing in actual work, thus bringing some uncertain factors, which may cause difficulties in accurate modeling, tracking accuracy, and rapidity of the system. The uncertain factors can be introduced in two aspects i.e., mistuning of the controller gains and turntable system dynamics change.

5.2.1. Simulation with mistuned controller gains

The most prevalent difficulty in control theory is a mistuned controller gain. When tuning the controller, it is clear that some aspects of the plant's dynamics are being overlooked. So, it is important to analyse the response of the system under the mistuning of the controller gain.

To ensure robustness, the probabilistic design assessment is carried out through levels of uncertainty in controller gain i.e., decreased and increased controller gain. Under decreased controller gain, the value of $K_P = 3.50$, $K_I = 2.0$, $K_D = 1.50$ are considered while under increased controller gain, the value of $K_P = 17.33$, $K_I = 13.11$, $K_D = 10.30$ are chosen. The probabilistic controller gain variation is listed in Table 5 and the adaptation gain is fixed at $\lambda = 0.08$ in all cases.

Table 5. Probabilistic controller gain variation

<table>
<thead>
<tr>
<th>Controller gain</th>
<th>$K_P$</th>
<th>$K_I$</th>
<th>$K_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best tuned</td>
<td>8.74</td>
<td>8.40</td>
<td>2.15</td>
</tr>
<tr>
<td>Decreased gain</td>
<td>3.50</td>
<td>2.0</td>
<td>1.50</td>
</tr>
<tr>
<td>Increased gain</td>
<td>17.33</td>
<td>13.11</td>
<td>10.30</td>
</tr>
</tbody>
</table>

The responses of the classical PID, MRAC, and modified MRAC controller for decreased gain are depicted in Figure 9. The modified controller takes 7.71 sec. to follow the reference model whereas the traditional MRAC and PID controller take more than 23.96 sec. and 28.57 sec. respectively. Also, the error of the proposed controller is less compared to MRAC and PID controllers. Table 6 shows the detailed comparison of three controllers in terms of settling time and error rates.
Figure 9. PID, traditional and modified MRAC response for decreased controller gain

Table 6. Comparison of three controllers under decreased controller gain

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>Settling Time (Sec.)</th>
<th>Performance Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IAE</td>
</tr>
<tr>
<td>PID</td>
<td>28.57</td>
<td>0.0183</td>
</tr>
<tr>
<td>MRAC</td>
<td>23.96</td>
<td>0.004729</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>7.71</td>
<td>2.86×10^-9</td>
</tr>
</tbody>
</table>

Now the controller gain is increased from the best-tuned value as given in Table 5. The comparison between the PID, traditional, and modified MRAC response for a turntable is shown in Figure 10. The classical PID controller takes more time i.e., 36.52 sec. to track the reference model. The MRAC takes 23.02 sec. and the proposed controller takes only 3.63 sec. to follow the desired response. The error is the lowest in the case of modified controllers compared to PID and MRAC. Table 7 shows that the modified controller is superior to PID and MRAC controllers in terms of performance indices and settling time.

Figure 10. PID, traditional and modified MRAC response for increased controller gain
Table 7. Comparison of three controllers under increased controller gain

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>Settling Time (Sec.)</th>
<th>Performance Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>IAE</td>
</tr>
<tr>
<td>PID</td>
<td>36.52</td>
<td>0.02017</td>
</tr>
<tr>
<td>MRAC</td>
<td>23.03</td>
<td>0.0003606</td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>3.63</td>
<td>2.154×10⁻¹¹</td>
</tr>
</tbody>
</table>

Table 8 shows the comparison of settling time under varying controller gains. After probabilistic design assessment, it is concluded that PID controller performance is degraded when there is a change in the controller gain. Whereas the proposed controller persistently gives better performance. The proposed controller can also guarantee a fast convergence time for the turntable regardless of mistune controller gains. Figure 11 illustrates the graphical presentation of settling time under best and mistuned controller gain. Therefore, the proposed MRAC-PID approach is more adaptive and suitable for the turntable than PID and MRAC controller.

Table 8. Comparison of settling time under varying controller gain

<table>
<thead>
<tr>
<th>Type of Controller</th>
<th>Settling Time (Sec.)</th>
<th>Best Tuned</th>
<th>Decreased Gain</th>
<th>Increased Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>12.57</td>
<td>28.57</td>
<td>36.52</td>
<td></td>
</tr>
<tr>
<td>MRAC</td>
<td>17.68</td>
<td>23.96</td>
<td>23.03</td>
<td></td>
</tr>
<tr>
<td>MRAC-PID</td>
<td>4.7</td>
<td>7.71</td>
<td>3.63</td>
<td></td>
</tr>
</tbody>
</table>

Figure 11. Comparative analysis of settling time

5.2.2. Simulation with changing turntable system dynamics

In reality, it is not always possible to achieve desired turntable performance due to dynamic behavior, modeling uncertainty, disturbances, and time-varying parameter. Simulation with turntable parameters (system-I) is performed in section 5.1 and its values are listed in Table 1.

Again the robustness of the proposed controller is verified after changing the turntable system dynamics i.e., system-II and its parameters are listed in Table 9. Here, adaptation gain is fixed at 0.08, and simulation is performed under best-tuned controller gains for step input. Figure 12 illustrates the comparison between the PID, traditional, and modified MRAC response for the system-II turntable.

The modified controller takes 4.93 sec. to follow the reference model whereas the traditional MRAC and PID controller takes 10.86 sec. and 18.51 sec. respectively. Table 10 incorporates the detailed analysis of the turntable under different system dynamics (system-I and system-II). After changing the system...
dynamics (system-II), the performance of the PID is degraded such that it takes 18.51 sec. to reach the desired response with a significant amount of error while the proposed controller adapts to the new system dynamics in 4.93 sec. with negligible error. Therefore, the proposed MRAC-PID approach is more adaptive and appropriate for the turntable application.

Table 9. Turntable parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$R_{o1}$ (Ω)</th>
<th>$L_{o1}$ (H)</th>
<th>$M$ (kg.m$^2$)</th>
<th>$a_0$ (V.sec/rad)</th>
<th>$v_0$ (Nm/rad/sec)</th>
<th>$a_0$ (Nm/A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>System-I</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>System-II</td>
<td>1</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 12. PID, traditional and modified MRAC response under system-II parameters

Table 10. Comparison of controllers with two different system dynamics (system-I & system-II)

<table>
<thead>
<tr>
<th>System with Transfer Function</th>
<th>Type of Controller</th>
<th>Settling Time (Sec.)</th>
<th>Performance Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{p1} = \frac{0.5}{s^2 + 2s + 1}$</td>
<td>PID</td>
<td>12.57</td>
<td>IAE: 8.207×10$^{-6}$, ISE: 9.059×10$^{-7}$, ITAE: 0.000272</td>
</tr>
<tr>
<td></td>
<td>MRAC</td>
<td>17.68</td>
<td>IAE: 3.214×10$^{-5}$, ISE: 1.033×10$^{-19}$, ITAE: 0.001607</td>
</tr>
<tr>
<td></td>
<td>MRAC-PID</td>
<td>4.70</td>
<td>IAE: 2.018×10$^{-10}$, ISE: 4.074×10$^{-20}$, ITAE: 1.009×10$^{-8}$</td>
</tr>
<tr>
<td>$G_{p2} = \frac{1}{s^2 + s + 1}$</td>
<td>PID</td>
<td>18.51</td>
<td>IAE: 0.0008786, ISE: 7.719×10$^{-7}$, ITAE: 0.02636</td>
</tr>
<tr>
<td></td>
<td>MRAC</td>
<td>10.86</td>
<td>IAE: 2.44×10$^{-7}$, ISE: 5.953×10$^{-14}$, ITAE: 7.32×10$^{-6}$</td>
</tr>
<tr>
<td></td>
<td>MRAC-PID</td>
<td>4.93</td>
<td>negligible</td>
</tr>
</tbody>
</table>

6. CONCLUSION

In the article, the extension of MRAC from first to second order is carried out and modified MRAC i.e., MRAC-PID is devised. The second-order traditional and modified MRAC systems are designed and analysed. The simulation findings show that the second-order MRAC controller would allow the system to follow the reference model. Desired output is obtained by selecting a reference model that meets the system requirements. Several advantages of modified MRAC as compared to MRAC and PID such as better performance, fast tracking speed and has robust nature. The modified MRAC also has a lower error function (IAE, ISE, and ITAE) than traditional MRAC and PID. Further, The robustness of the proposed controller is validated by introducing uncertainties in two aspects i.e., mistuning of the controller gains and turntable system dynamics change. Under mistuning of the controller gains, probabilistic design assessment is carried
out and it is found that the performance of the proposed MRAC-PID controller is superior in all considered aspects. Under turntable system dynamics change, the proposed controller adapts to the new system dynamics quickly with negligible error. The MRAC-PID minimises reliance on adaptive variables and it is implemented with minor changes only. The second-order MRAC system was designed successfully, and the controller has improved performance.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES


