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NUMERICAL BUCKLING ANALYSIS OF CYLINDRICAL HELICAL COIL SPRINGS IN A DYNAMIC MANNER

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Abstract

The free vibration equations of cylindrical isotropic helical springs loaded axially, developed by the author are solved numerically based on the transfer matrix method to perform buckling analysis in a dynamic manner. The axial and shear deformation effects together with the rotatory inertia effects are all considered based on the first order shear deformation theory. For the determination of the vertical tip deflection of helical springs with large pitch angles, closed-form equation obtained by the author based on Castigliano's first theorem is used to take into account for the whole effect of the stress resultants such as axial and shearing forces, bending and torsional moments on the tip deflection. A good agreement is observed with related benchmark studies.

Keywords: Linear analysis, helical spring, numeric, transfer matrix, buckling.

1. Introduction

The vibration of helical springs subjected to an axial load was paid less attention in the literature due to the complexity in the formulation of the problem. Haringx [1] studied analytically both buckling and free vibration behavior of cylindrical helical springs with circular sections and with small helix pitch angles. By using the straight beam elements, Unlüsoy [2] and Yalçın [3] studied the free vibration of helical springs subjected to axial static load based on the finite element method. Haktanır [4] handled the free vibration and buckling analyses of cylindrical helical springs under a static axial force based on the both transfer and stiffness matrix methods. Haktanır and Kıral [5] examined the free vibration of helical springs subjected to a static axial force with the help of the stiffness matrix formulation and straight beam elements. Xiong and Tabarrok [6] developed a spatially curved and twisted rod finite element based on the energy formulation to study the vibration analysis of spatially curved and twisted rods under various applied loads. Recently Becker et al [7] have broadened their previous works [8-10] for the free vibration analysis of a helical spring subjected to a static axial compressive load by incorporating inertial terms into previous buckling equations. For helices with clamped ends and circular cross-section, they examined the free vibration analysis numerically based on the transfer matrix method [7]. Buckling behavior of a such helical spring was studied in a static manner in their previous works [8-10].

In the present work first, the linearized disturbance equations derived by the author are presented. Then these equations are solved numerically with the help of the transfer matrix method by employing an effective numerical algorithm developed previously by the author [11]. To determine static axial tip deflections of helical springs with large pitch angles due to the static axial force, the author used analytical expression, which takes into account for the whole effect of the stress resultants such as axial and shearing forces, bending and torsional moments, given in reference [12]. Finally, the critical buckling loads are obtained for a cylindrical helical spring based on the dynamic approach and compared with the benchmark studies.

2. Governing Equations of a Spatial Bar Subjected to Axial Static Loads

Consider a curved spatial bar having the curvilinear position coordinate *s*, and let *t* be the time. Let $\boldsymbol{T}^*(s,t)=(T_t^*,T_n^*,T_b^*)$ be the internal force vector, let $\boldsymbol{M}^*(s,t)=(M_t^*,M_n^*,M_b^*)$ be the internal moment vector. Where T_t is the axial force; T_n and T_b are shearing forces; M_t is the torsional moment; M_n and M_b are the bending moments, respectively. Let $W^*(s,t)=(W_t^*,W_n^*,W_b^*)$ be the rotation vector, and $\boldsymbol{U}^*(s,t)=(U_t^*,U_n^*,U_b^*)$ be the displacement vector in Frenet coordinates ($\boldsymbol{t}$,$\boldsymbol{n}$,$\boldsymbol{b}$). Initial internal static force and moment vectors are denoted by $\boldsymbol{T}^o(s)$ and $\boldsymbol{M}^o(s)$, respectively. The external distributed force and moment vectors are illustrated by ***p(s,t)*** and ***m(s,t)***, respectively.

The author derived the following governing equations in a vector form for a spatial bar to study the static, buckling and vibration problems.

$$\frac{\partial \boldsymbol{U}^*}{\partial s} - Q_T \boldsymbol{T}^* + \boldsymbol{t}\ x\ W^* = 0 \qquad (1a)$$

$$\frac{\partial W^*}{\partial s} - Q_M \boldsymbol{M}^* = 0 \qquad (1b)$$

$$\frac{\partial \boldsymbol{T}^*}{\partial s} + (Q_M \boldsymbol{M}^*) x \boldsymbol{T}^o + \boldsymbol{p} + \boldsymbol{p}^{in} = 0 \qquad (1c)$$

$$\frac{\partial \boldsymbol{M}^*}{\partial s} + (Q_M \boldsymbol{M}^*) x \boldsymbol{M}^o + \boldsymbol{t}\ x\ \boldsymbol{T}^* + (Q_T \boldsymbol{T}^*) x\, \boldsymbol{T}^o + \boldsymbol{m} + \boldsymbol{m}^{in} = 0 \qquad (1d)$$

where

$$\boldsymbol{\Theta_T} = \begin{bmatrix} \frac{1}{EA} & 0 & 0 \\ 0 & \frac{K_n}{GA} & 0 \\ 0 & 0 & \frac{K_b}{GA} \end{bmatrix} \qquad (2a)$$

$$\boldsymbol{\Theta_M} = \begin{bmatrix} \frac{1}{GJ_b} & 0 & 0 \\ 0 & \frac{1}{EI_n} & 0 \\ 0 & 0 & \frac{1}{EI_b} \end{bmatrix} \qquad (2b)$$

In the above, the undeformed cross-sectional area is denoted by *A*, the moments of inertia with respect to the normal and binormal axes are denoted by I_n and I_b, respectively. J_b is the torsional moment of inertia, *G* is the shear modulus and *E* is the Young's modulus. K_n and K_b represent the Timoshenko's *k*-factors. For symmetrical cross sections, Timoshenko *k*-factors are equal to each other, $K_n = K_b$.

For the unit length, the inertia force and moments in (1*c*) and (1*d*) are given by

$$\boldsymbol{p}^{in}(t) = -m\left(\frac{\partial^2 U_t}{\partial t^2}\boldsymbol{t} + \frac{\partial^2 U_n}{\partial t^2}\boldsymbol{n} + \frac{\partial^2 U_b}{\partial t^2}\boldsymbol{b}\right) \tag{3a}$$

$$\boldsymbol{m}^{in}(t) = -m\left(k_t^2\frac{\partial^2 W_t}{\partial t^2}\boldsymbol{t} + k_n^2\frac{\partial^2 W_n}{\partial t^2}\boldsymbol{n} + k_b^2\frac{\partial^2 W_b}{\partial t^2}\boldsymbol{b}\right) \tag{3b}$$

Here, m is the mass per unit length of the rod, k_t, k_n, k_b are the radii of gyration with respect to the (***t, n, b)*** axes, respectively.

$$m = \frac{m}{L} = rA \tag{4a}$$

$$k_t^2 = \frac{J_b}{A}, \quad k_n^2 = \frac{I_n}{A}, \; k_b^2 = \frac{I_b}{A} \tag{4b}$$

where r is the density of the rod material.

3. Free Vibration Equations of a Cylindrical Helical Spring under Static Force

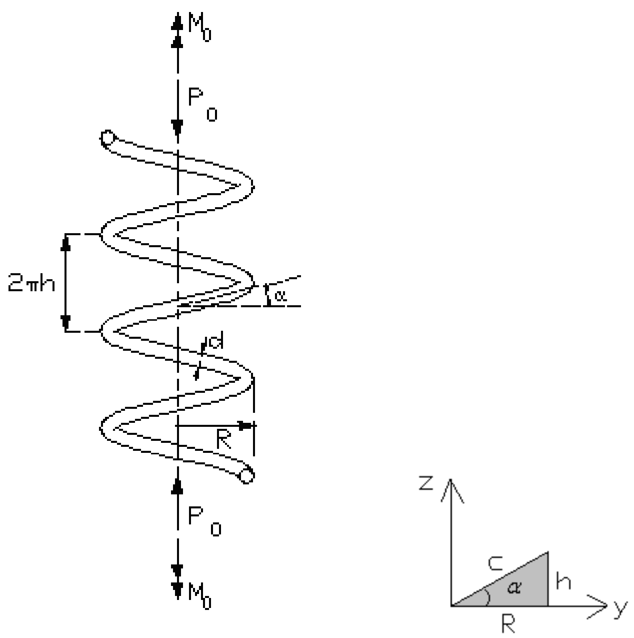

Fig.1. Geometry of a helix subjected to an axial compressive force and a torque

Let a be the helix pitch angle, q be the angular coordinate, and *R=(D/2)* be the radius of the cylinder (Figure 1). Frenet-Serret relations for cylindrical helical springs are given by

$$\frac{d\boldsymbol{t}}{ds} = c\ \boldsymbol{n} = \frac{R}{c^2}\boldsymbol{n} \tag{5a}$$

$$\frac{d\boldsymbol{n}}{ds} = t\ \boldsymbol{b} - c\ \boldsymbol{t} = \frac{h}{c^2}\boldsymbol{b} - \frac{R}{c^2}\boldsymbol{t} \tag{5b}$$

$$\frac{d\boldsymbol{b}}{ds} = -t\ \boldsymbol{n} = -\frac{h}{c^2}\boldsymbol{n} \tag{5c}$$

where the curvature and tortuosity are

$$c\ = \frac{R}{c^2} = \frac{cos\, a}{c} = \frac{cos^2\, a}{R} = \ constant \tag{6a}$$

$$t\ = \frac{h}{c^2} = \frac{\sin a}{c} = \frac{\sin a\ \cos a}{R}\ = constant \tag{6b}$$

and

$$\sin a\ = \frac{h}{c}\ ;\ \cos a\ = \frac{R}{c};\ \ ds = cdq \tag{7a}$$

$$c = (R^2 + h^2)^{1/2} = R(1+\tan^2 a\,)^{1/2} = \frac{R}{\cos a} = \frac{D}{2\cos a} \tag{7b}$$

The free axial length of the helix is defined by

$$\frac{L_0}{D} = np\ \tan a \tag{8}$$

In equation (8), the total number of active turns is denoted by ***n***. The Frenet components of the initial force and moment vectors will be in the following form of (Figure 1)

$$\boldsymbol{T}^o\ = (\,-P_o\ sin\, a\ ,0,-P_0\ cos\, a\ ) \tag{9a}$$

$$\boldsymbol{M}^o\ = (\,-P_o R\, cos\, a\ + M_0\ sin\, a\ ,0,P_0 R\, sin\, a\ + M_0\ cos\, a\ ) \tag{9b}$$

For the free vibration analysis, the following harmonic solution may be assumed for equations (1)

$$\begin{aligned}
\boldsymbol{U}(\,s,t\,) &= \boldsymbol{U}^*(\,s\,)sin\, wt \\
W(\,s,t\,) &= W^*(\,s\,)sin\, wt \\
\boldsymbol{T}(\,s,t\,) &= \boldsymbol{T}^*(\,s\,)sin\, wt \\
\boldsymbol{M}(\,s,t\,) &= \boldsymbol{M}^*(\,s\,)sin\, wt
\end{aligned} \tag{10}$$

without external loads, ***p(s,t)=0*** and ***m(s,t)=0***. Where w (rad/s) is the circular frequency. Substituting equation (10) into equation (1), employing Frenet-Serret relations given in equation (5), considering $ds = cdq$ for helical elements, and then using equation (9), a set of twelve linear differential scalar equations in Frenet trihedral, which govern the free vibration of the cylindrical helical springs subjected to an axial load, are written as follows:

$$\frac{dU_t}{dq} = \frac{R}{c}U_n + \frac{1}{EA}T_t \tag{11a}$$

$$\frac{dU_n}{dq} = -\frac{R}{c}U_t + \frac{h}{c}U_b + cW_b + \frac{cK_n}{GA}T_n \tag{11b}$$

$$\frac{dU_b}{dq} = -\frac{h}{c}U_n - cW_n + \frac{cK_b}{GA}T_b \tag{11c}$$

$$\frac{dW_t}{dq} = \frac{R}{c}W_n + \frac{c}{GJ_b}M_t \tag{11d}$$

$$\frac{dW_n}{dq} = -\frac{R}{c}W_t + \frac{h}{c}W_b + \frac{c}{EI_n}M_n \tag{11e}$$

$$\frac{dW_b}{dq} = -\frac{h}{c}W_n + \frac{c}{EI_b}M_b \tag{11f}$$

$$\frac{dT_t}{dq} = \frac{R}{c}T_n + \frac{cP_o \cos a}{EI_n}M_n - cmw^2U_t \tag{11g}$$

$$\frac{dT_n}{dq} = -\frac{R}{c}T_t + \frac{h}{c}T_b - \frac{cP_o \cos a}{GJ_b}M_t + \frac{cP_o \sin a}{EI_b}M_b - cmw^2U_n \tag{11h}$$

$$\frac{dT_b}{dq} = -\frac{h}{c}T_n - \frac{cP_o \sin a}{EI_n}M_n - cmw^2U_b \tag{11i}$$

$$\frac{dM_t}{dq} = \frac{K_n cP_o \cos a}{GA}T_n + \left(\frac{R}{c} - \frac{c(P_oR\sin a + M_0 \cos a)}{EI_n}\right)M_n - \frac{cmJ_b}{A}w^2W_t \tag{11j}$$

$$\begin{aligned}\frac{dM_n}{dq} = &-\frac{cP_o \cos a}{EA}T_t + \left(c + \frac{K_b cP_o \sin a}{GA}\right)T_b + \left(-\frac{R}{c} + \frac{c(P_oR\sin a + M_0\cos a)}{GJ_b}\right)M_t \\ &+ \left(\frac{h}{c} + \frac{c(P_oR\cos a - M_0 \sin a)}{EI_b}\right)M_b - \frac{cmI_n}{A}w^2W_n\end{aligned} \tag{11k}$$

$$\frac{dM_b}{dq} = -\left(\frac{K_n cP_o \sin a}{GA} + c\right)T_n - \left(\frac{h}{c} + \frac{c(P_oR\cos a - M_0\sin a)}{EI_n}\right)M_n - \frac{cmI_b}{A}w^2W_b \tag{11l}$$

These linear equations given above have the constant coefficients and valid for isotropic cylindrical helical springs with constant sections. The free vibration equations presented by Becker et al [7] coincide with the equations presented above in the absence of the first terms of equations (11*j*), (11*k*) and (11*l*). In other words, the first terms of equations (11*j*), (11*k*) and (11*l*) are the major contribution of the author to the literature.

Equations (11) may be expressed in a matrix notation as

$$\frac{d\boldsymbol{S}(q)}{dq} = \boldsymbol{DS}(q) \tag{12}$$

where the state vector is given by

$$\boldsymbol{S}(q) = \{U_t \quad U_n \quad U_b \quad W_t \quad W_n \quad W_b \quad T_t \quad T_n \quad T_b \quad M_t \quad M_n \quad M_b\}^T \tag{13}$$

and $\boldsymbol{D}(q, w, P_o, M_o)$ is the dynamic differential matrix.

For the determination of the vertical tip deflection of cylindrical helical compression springs subjected to an axial static force the following equation derived by the author [12] is used

$$d = \frac{2PnpR\cos a}{EI_b} + \frac{2PnpR\sin a\tan a}{EA} + \frac{2PnpR^3\sin a\tan a}{GI_b} + \frac{2PnpR^3\cos a}{GJ_b} \tag{14}$$

Instead of either

$$d = \frac{64nPR^3}{Gd^4} = \frac{8D^3nP}{Gd^4} \tag{15}$$

or

$$d = \frac{64PnR^3\cos a}{Gd^4} \tag{16}$$

While equation (14) considers the effect of shear force, axial force, bending moment and torsional moment, respectively, both equation (15) and (16) account for just the effect of the torsional moment on the tip deflection. Equation (15) is also valid for just helical springs with small pitch angles. A quick inspection shows that there is just a $\cos a$ factor between equations (15) and (16).

While equation (14), which is obtained based on the Castigliano's first theorem, is valid for helical springs having doubly symmetric cross-sections such as solid circle, square, rectangle, ellipse, and hollow circle etc., both equation (15) and (16) are valid for just circular cross section.

The computation truly of the tip deflection of the spring under static axial load is one of the crucial steps of the vibration analysis of such springs. It may be noted that those effects gain considerably importance for especially rectangular cross-sections

4. Solution with the Transfer Matrix Approach

In the transfer matrix method, the solution to Equation (12) is given by [13] as follows

$$\boldsymbol{S}(q) = \boldsymbol{F}(q, w, P_0)\boldsymbol{S}(0) \tag{17}$$

where ***S(0)*** is the state vector at section $q = 0$ and $\boldsymbol{F}(q)$ is the overall transfer matrix. The solution (17) is valid for both varying and constant cross-sections. The transfer matrix also satisfies the following differential equation

$$\frac{d\boldsymbol{F}(q)}{dq} = \boldsymbol{D}\boldsymbol{F}(q) \tag{18}$$

with the initial conditions

$$\boldsymbol{F}(0)=\boldsymbol{I} \tag{19}$$

*where **I*** is the unit matrix.

Numerical computation of the overall transfer matrix is a crucial step in the transfer matrix method. If it is obtained in an accurate manner, then the results will be acceptable. The standard solution of the overall transfer matrix in (18) for constant cross-sections is [13].

$$\boldsymbol{F}(q) = e^{\boldsymbol{D}q} = \boldsymbol{I} + q\boldsymbol{D} + \frac{q^2\boldsymbol{D}^2}{2!} + \frac{q^3\boldsymbol{D}^3}{3!} + ... \tag{20}$$

This solution is generally preferred in the literature due to its simplicity [7-10]. However, when applying this series it should be necessary to take some numerical precautions for especially large helix angles.

Based on the Cayley-Hamilton theorem, equation (20) may be put in the form of

$$\boldsymbol{F}(q) = \sum_{k=0}^{11} F_k(q)\boldsymbol{D}^k \tag{21}$$

where $\Phi_k(q)$ are functions of scalar infinite series in q. Utilization of each term of the series $\Phi_k(q)$ of Equation (21) corresponds to twelve terms in (20). The number of terms taken from the infinite series $\Phi_k(q)$ determines the accuracy of the solution. In the present study, 1000 terms are taken in each $\Phi_k(q)$ series of Equation (21) for each $q = p$ to calculate the overall transfer matrix. This number of terms corresponds to the 12000 terms in Equation (20). The number of terms can be increased without any trouble to increase the accuracy of solution in the numerical algorithm developed by the author [11].

For a certain axial load, after computation of ***F*** by attributing numerical values to the natural frequency, the frequency equation can be obtained from the boundary conditions given at both ends (*q =0 and q =2pn*) using Equation (17). For example, the eigen value equation for fixed-free ends (*W=0, **U**=0)* is reduced to the following

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{q=2np} = \begin{bmatrix} F_{7,7} & F_{7,8} & F_{7,9} & F_{7,10} & F_{7,11} & F_{7,12} \\ F_{8,7} & F_{8,8} & F_{8,9} & F_{8,10} & F_{8,11} & F_{8,12} \\ F_{9,7} & F_{9,8} & F_{9,9} & F_{9,10} & F_{9,11} & F_{9,12} \\ F_{10,7} & F_{10,8} & F_{10,9} & F_{10,10} & F_{10,11} & F_{10,12} \\ F_{11,7} & F_{11,8} & F_{11,9} & F_{11,10} & F_{11,11} & F_{11,12} \\ F_{12,7} & F_{12,8} & F_{12,9} & F_{12,10} & F_{12,11} & F_{12,12} \end{bmatrix} \begin{bmatrix} T_t \\ T_n \\ T_b \\ M_t \\ M_n \\ M_b \end{bmatrix}_{q=0} \qquad (22)$$

As can be seen from equation (22), the order of determinant is only six for the spring supported at both ends. As described, the transfer matrix method offers an exact solution with minimal computation memory requirement for dynamic problems. The values making the determinant zero are the natural frequencies of the helix

5. Numerical Examples

Two examples with two boundary conditions are studied in this section, namely, free vibration without static force and determination of the buckling force in a dynamic manner.

Example 1 (Free vibration without static force): In this example a very slender helical spring is considered to study the widely held formula for the fundamental frequency in axial mode, f_{axial} (in Hz), in the literature [1].

$$f_{axial} = \frac{1}{8pnCR}\sqrt{\frac{2G}{r}} = \frac{w_{axial}}{2p} \qquad (23)$$

The features of the spring are: L_o/D=40 d/L_o=0, n=30, $C=D/d$=10, d=1mm, $r = 7900 kg/m^3$, $u = 0.3$, $K_n=K_b=1.1$, $a_0 = 22.99701^o$, E=20.6 GPa, Fixed-Fixed ends.

Comparison of the natural frequencies in the literature with the present natural frequencies represented in both dimensional and dimensionless form is shown in Table 1. There is an excellent agreement between Becker et al's [7] results and the present results. However, for the large value of the helix angles, agreement with Haringx's results becomes poor. As stated Becker et al [7], surprisingly, the difference can be accounted for almost entirely by dividing either Becker et al's [7] estimate or the present estimate by $\cos a$. The author agree absolutely to the conclusion given by Becker et al [7] that "Significant deviations from the elementary theory occur at small number of turns, where the spring cannot be treated as an equivalent rod even for the lower modes, or at large helix angles, where the effective rigidities used by Haringx are inaccurate."

Table 1. Natural frequencies for Example 1 (TMM: Transfer Matrix Method)

Mode No	Becker et al [7] (TMM)	Haringx [1] (Rod model))	Present Study (TMM)	
	$\dfrac{f}{f_{axial}}$			w (rad/s)
1	0.08780	0.09444	0.0877957	10.4301
2	0.08781	0.09444	0.0878086	10.4317
3	0.2411	0.2595	0.241135	28.6469
4	0.2412	0.2595	0.241249	28.6604
5	0.4703	0.5063	0.470325	55.8747
6	0.4704	0.5063	0.470374	55.8805
7	0.7723	0.8321	0.772333	91.7534
8	0.7727	0.8321	0.772689	91.7957
9	0.9180	1.000	0.918055	109.065
10	1.047	1.140	1.04731	124.420
11	--	--	1.14445	135.961
12	--	--	1.14642	136.195
13	--	--	1.58179	187.917
14	--	--	1.58328	188.094
15	--	--	1.83676	218.208
16	--	--	2.08017	247.124

-- not given

Example 2 (Determination of the buckling force in a dynamic manner): As it is well known, the critical buckling loads may also be determined in a dynamic manner. In this method, for a given static axial load, from P_0 to P_{cr} at which the fundamental frequency of the spring becomes zero, the fundamental frequencies of the spring are searched (Figure 2).

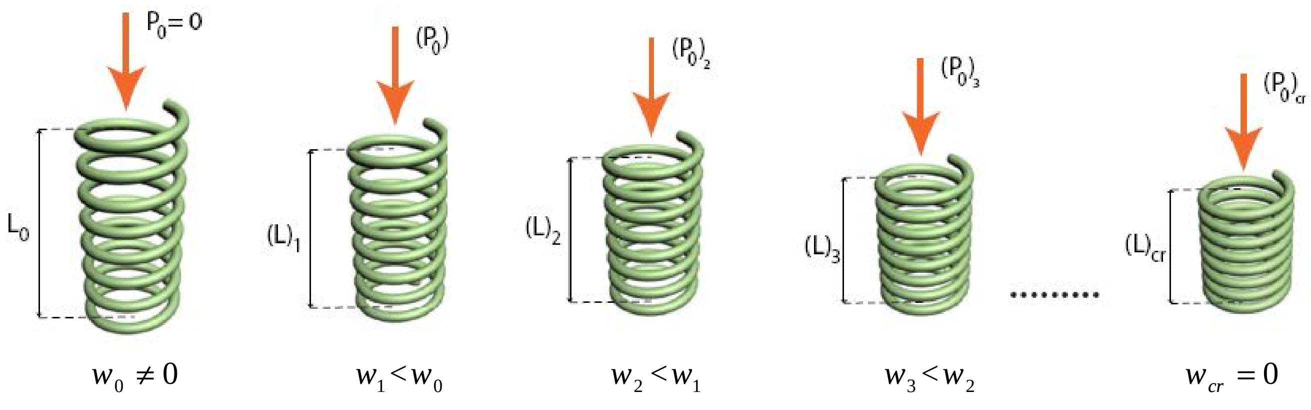

Fig. 2. Relation between frequencies and static axial load for the same spring

The properties included in this example are: L_o=36*mm, D*=10*mm, d*=1*mm, C*=10, *R*=5*mm, n*=7.6, *E*=20.684 *GPa*, $r = 7886.6 kg/m^3$, L_o/D = 3.6, a_0 = 8,5744°, K_n=K_b=*1.1,* Boundary Conditions: At $q = 0$ $W = 0$, ***U***=0 and at $q = 2pn$ $W = 0$, U_x=0, U_y=0, and T_z=0. It may be noted that the vertical tip deflection of the helix is calculated by employing equation (15) as references [6] and [1] did for the sake of comparison. However the additional terms that is the first terms of equations (11*j*), (11*k*) and (11*l*) are included in the analysis.

Table 2. Critical buckling loads for Example 2
(AA: Analytical Approach, DA: Dynamic Approach)

	P_{cr} *(N)*	
Present Study (DA)	Xiong and Tabarrok [6] (DA)	Haringx [1] (AA)
11.90	11.85	11.99

Critical buckling loads for this example are shown in Table 2. Xiong and Tabarrok [6] employed the finite element procedure.

Table 3 shows the variation of the present natural frequencies with the different value of static axial force applied on the same spring. In Table 3, it is seen that, the present results and Xiong and Tabarrok's [6] results show a good harmony at all modes. While Figure 3 illustrates variation of the fundamental natural frequency with the numerical value of static axial force, variation of the second, third and fourth natural frequencies with the numerical value of static axial force is illustrated by Figure 4.

Table 3. Variation of the present natural frequencies (in Hz) with the value of static axial force

	P_o *(N)*	**Modes**			
		1	**2**	**3**	**4**
Present Study	0	114.915	115.269	233.126	512.305
Xiong and Tabarrok [6]		114.8	115.2	232.8	512.1
Present Study	1	111.064	111.451	233.123	511.507
Present Study	2	106.902	107.326	233.120	510.661
Xiong and Tabarrok [6]		106.7	107.2	232.4	510.1
Present Study	3	102.383	102.847	233.118	509.775
Present Study	4	97.4449	97.9542	233.116	508.861
Xiong and Tabarrok [6]		97.21	97.73	231.9	508.0
Present Study	5	92.0084	92.5700	233.115	507.926
Present Study	6	85.9652	86.5890	233.114	506.981
Xiong and Tabarrok [6]		85.67	86.31	231.5	505.9
Present Study	7	79.1600	79.8609	233.114	506.034
Present Study	8	71.3562	72.1589	233.115	505.094
Xiong and Tabarrok [6]		71.00	71.81	231.1	503.7
Present Study	9	62.1561	63.1048	233.116	504.171
Present Study	10	50.7764	51.9683	233.118	503.271
Xiong and Tabarrok [6]		50.30	51.51	230.7	501.7
Present Study	11	35.1155	36.8683	233.120	502.405
Xiong and Tabarrok [6]		34.47	36.26	230.5	500.7
Present Study	11.8	10.5594	15.5168	233.123	501.741
Present Study	11.9	$\cong 0$	9.90088	233.123	501.660
Present Study	11.96		3.47311	233.124	501.611
Present Study	11.967		1.23284	nc	nc
Present Study	11.97		$\cong 0$	nc	nc

nc: not computed

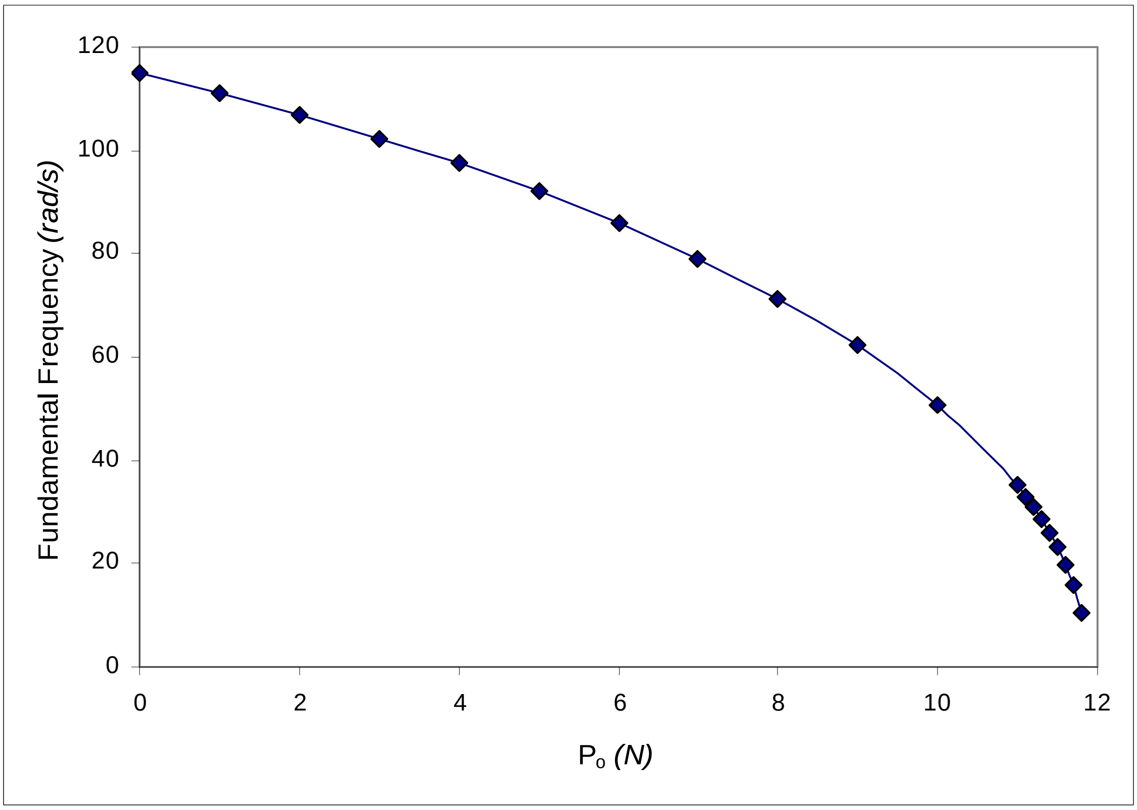

Fig. 3. Variation of the fundamental natural frequency with the numerical value of static axial force

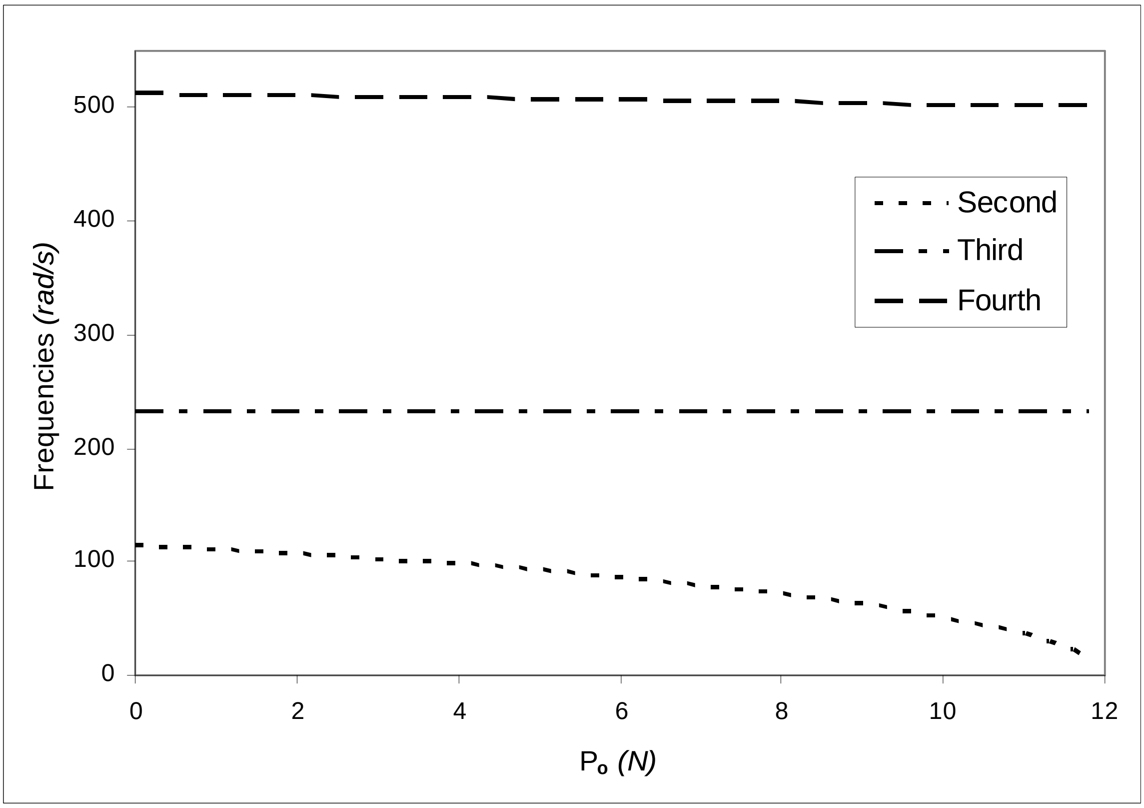

Fig. 4. Variation of the second, third and fourth natural frequencies with the numerical value of static axial force

As shown from Table 3, besides its time consuming operations, inspection of the critical buckling loads based on the dynamic approach has some risk. With a small increment of the value of the applied load may get the second buckling load instead the critical buckling load. This situation may be faced up frequently to helical springs having very close natural frequencies. Consequently, it may be better to use the static approach to determine the critical buckling loads without confronting this kind of ambiguity. Or, before vibration analysis, the buckling force should be computed.

6. Comparison of Equations (14), (15) and (16)

The effect of especially bending moment on the tip deflection increases with increasing helix pitch angle. To increase the helix pitch angle, the ratio L_o/D should be increased while the number of active turns decreases. To see the effect of bending moment on both the critical buckling load and the free vibration equations of a compressed spring, here a cylindrical helical spring having the same properties of the spring in the second example, except n=5 and L_o/D=10, is studied. Those properties are in contact with $a_0 = 32.4816366^o$ and $f_{axial} = 712.8014$ *Hz* ($w_{axial} = 4478.663$ rad/s).

For this example present critical buckling loads which are obtained by equations (14), (15) and (16) are given in Table 4. In Table 4, the critical buckling load which is obtained by equation (14) corresponds P_{cr}=21.283 *(N)* at which the fundamental frequency is almost near the zero, w_1 = 1.31086 (Hz), critical helix pitch angle is a_{cr}=29.287 *(^{o})*, and the relative compression is $\frac{d_{cr}}{L_0}$ = 0.11897. As stated above, equation (16), $d = \frac{64PnR^3 \cos a}{Gd^4}$, consider the deflection due to the just torsional moment for large helix angles. As it is well known, the common equation numbered by (15), $d = \frac{64nPR^3}{Gd^4}$, consider the deflection due to the just torsional moment for small helix angles. From Table 4, it may be concluded that critical buckling loads obtained by using equation (14) gives the higher values. This means that critical buckling loads found by equation (15) and (16) fell in a safe region in design for this example.

Table 4. Critical buckling loads and corresponding deformations in the spring

Equation used	$P_{cr}(N)$	$a_{cr}(^o)$	$\frac{d_{cr}}{L_0}$ (%)
14	21.283	29.287	11.9
16	20.4	30.17	8.7
15	20.9	29.66	10.6

7. Conclusions

Vibration analysis of the helical coil springs under axial static loads requires a precise theory and its application. So, the present work offers a complete and accurate study of the buckling behavior of the cylindrical helical springs in a dynamic manner. In the numerical examples, the validity of the present results is shown with the benchmark studies in the literature.

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