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ON NEW SYMPLECTIC APPROACH FOR EXACT FREE VIBRATION SOLUTIONS OF MODERATELY THICK RECTANGULAR PLATES WITH TWO OPPOSITE EDGES SIMPLY SUPPORTED

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Abstract

The purpose of this paper is to report the effective application of a new symplectic approach for exact free vibration solutions of moderately thick rectangular plates. By way of a simple but rigorous derivation, the governing differential equations for free vibration of the plates are transferred into Hamilton canonical equations. The whole state variables are then separated. Using the method of eigenfunction expansion in the symplectic geometry, the free vibration analysis of moderately thick rectangular plates with two opposite edges simply supported is performed and exact vibration solutions are obtained. The method eliminates the need to pre-determine any trial functions hence more reasonable than other available methods. Comprehensive numerical results are presented to validate the approach proposed here by comparison with those established in the open literature.

Keywords: Moderately thick plate; free vibration; exact solution; symplectic approach

1. Introduction

The moderately thick rectangular plates are commonly used components encountered in various engineering structures and research on their vibration is significant for its practical importance. However, most previous investigations were focused on the vibration of thin plates, e.g. [1,2], while research on the problem of moderately thick rectangular plates was fewer on account of its complexity. A comprehensive survey of previous studies on the vibration of thick plates has been made by Liew et al. [3], which is helpful in locating relevant existing literature quickly. The conventional exact solutions for free vibration of moderately thick rectangular plates are based on the semi-inverse method with trial mode shape functions [4], which, however, do not always exist except in some special cases of support conditions such as fully simply supported plates. For the plates with other combinations of boundary conditions, some numerical and approximate methods have been widely employed by scientists and researchers in their valuable work.

Representative approximate methods include the finite element method [5-8], finite difference method [9-11], Rayleigh-Ritz method [12-14], Galerkin method [15,16], finite strip method [17-20], spline strip method [21], collocation method [22], differential quadrature (DQ) method [23] and discrete singular convolution (DSC) method [24].

As mentioned above, the applications of various approximate methods for free vibration of moderately thick plates are due to the unavailability of exact analytical solutions up to the present.

A new symplectic methodology developed for the theory of elasticity has shown considerable promise [25-27]. This systematic approach has been successfully applied to derive exact solutions to the problem of thin or thick plate bending [28-33]. The most powerful advantage of the approach lies in its rational and exactness in solution without any pre-selection of trial functions, which, however, can scarcely be avoided in the traditional semi-inverse approaches.

The solution of the dynamical problem for plates has been explored in the symplectic space. Bao and Deng [34] made an investigation of free vibration for rectangular thin plates in Hamilton systems, which presents an interesting endeavor in the problem. Recently, Lim et al. [35] developed the symplectic elasticity approach based on the conservative energy principle and analyzed the examples for Lévy-type thin plates. Xing and Liu [36] employed the same solution method to solve Hamiltonian dual form of eigenvalue problem for transverse free vibrations of thin plates and performed the formulation of the natural mode in closed form for selected cases. As for vibration analysis of moderately thick plates, there was also an attempt made by Zou [37] for Reissner plates simply supported at one pair of parallel edges, but the trial mode functions were still predetermined which differs from the rational symplectic approach.

In the present paper, a new symplectic approach is proposed to derive the exact free vibration solutions of moderately thick rectangular plates based on Reissner plate theory [38-40] in which we neglect the transverse contraction e_z for convenience [41]. Exact free vibration analysis of the plates with two opposite edges simply supported and the others arbitrarily supported is analytically performed by first transferring the basic vibration equations into Hamilton canonical equations in a simple but efficient manner before separation of variables. By eigenfunction expansion in the symplectic geometry, the exact solutions of free vibration are obtained accordingly.

The solution procedure presented here goes beyond the usual limitations of the classical semi-inverse method and extends the scope of the analytical solutions for plate vibration problems. Numerical results are presented for an easy comparison with those from available literature to verify the accuracy and validity of the formulations derived.

2. Hamilton canonical equations for free vibration of moderately thick rectangular plate

The coordinate system of a moderately thick rectangular plate under consideration is illustrated in Fig. 1, where $0 \le x \le a$ and $0 \le y \le b$. The directions of the vectors, taken as positive, are indicated.

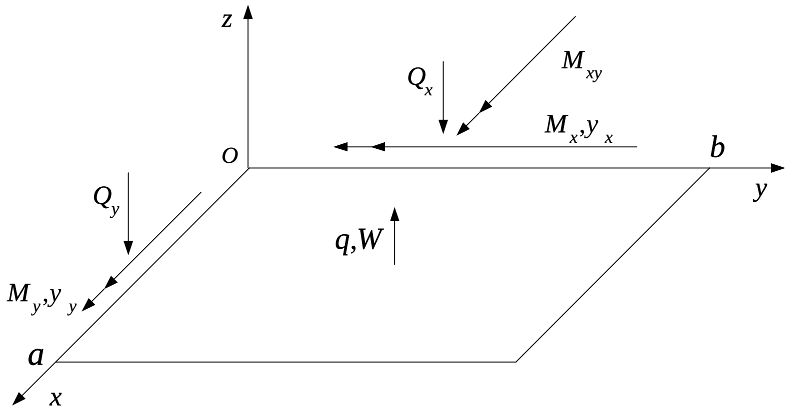

Fig. 1. Coordinates, displacements, load and stress resultants of a moderately thick rectangular plate

Based on the classical Reissner plate theory [38-40], the equations of motion for free vibration of a moderately thick rectangular plate expressed in the frequency domain are

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\rho\omega^2 W=0;\ \frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0;\ \frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}-Q_y=0 \tag{1a-c}$$

where ρ is the area density, ω is the natural frequency, W is the transverse displacement, M_x, M_y, M_{xy}, Q_x and Q_y are the bending moments, torsional moment and shear forces, respectively.

The internal forces of the plate can be presented as

$$M_x=-D\left(\frac{\partial \psi_x}{\partial x}+\nu\frac{\partial \psi_y}{\partial y}\right);\ M_y=-D\left(\frac{\partial \psi_y}{\partial y}+\nu\frac{\partial \psi_x}{\partial x}\right);\ M_{xy}=-\frac{D(1-\nu)}{2}\left(\frac{\partial \psi_x}{\partial y}+\frac{\partial \psi_y}{\partial x}\right) \tag{2a-c}$$

$$Q_x=C\left(\frac{\partial W}{\partial x}-\psi_x\right);\ Q_y=C\left(\frac{\partial W}{\partial y}-\psi_y\right) \tag{3a,b}$$

where D is the flexural rigidity, C is the shear stiffness, ψ_x and ψ_y are the angles of rotation, ν is the Poisson's ratio. There exist some mathematical relations: $C=\frac{5Eh}{12(1+\nu)}$, $D=\frac{Eh^3}{12(1-\nu^2)}$, where E is the modulus of elasticity and h is the thickness of the plate. From Eq. (1b) and Eqs. (2a-c), we have

$$Q_x=\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}=-D\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi_x}{\partial x}+\nu\frac{\partial \psi_y}{\partial y}\right)+\frac{1-\nu}{2}\frac{\partial}{\partial y}\left(\frac{\partial \psi_x}{\partial y}+\frac{\partial \psi_y}{\partial x}\right)\right] \tag{4}$$

$$Q_y=\frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x}=-D\left[\frac{\partial}{\partial y}\left(\frac{\partial \psi_y}{\partial y}+\nu\frac{\partial \psi_x}{\partial x}\right)+\frac{1-\nu}{2}\frac{\partial}{\partial x}\left(\frac{\partial \psi_x}{\partial y}+\frac{\partial \psi_y}{\partial x}\right)\right] \tag{5}$$

Summation of Eqs. (2a) and (2b) gives

$$M_x + M_y = -D\left(\frac{\partial y_x}{\partial x} + n\frac{\partial y_x}{\partial x} + \frac{\partial y_y}{\partial y} + n\frac{\partial y_y}{\partial y}\right) = -D(1+n)\left(\frac{\partial y_x}{\partial x} + \frac{\partial y_y}{\partial y}\right) \tag{6}$$

Let

$$-\frac{M_x + M_y}{D(1+n)} = \frac{\partial y_x}{\partial x} + \frac{\partial y_y}{\partial y} = M \tag{7}$$

From Eq. (4) we get

$$\begin{aligned} Q_x &= -D\left[\frac{\partial}{\partial x}\left(\frac{\partial y_x}{\partial x} + \frac{\partial y_y}{\partial y} - \frac{\partial y_y}{\partial y} + n\frac{\partial y_y}{\partial y}\right) + \frac{1-n}{2}\frac{\partial}{\partial y}\left(\frac{\partial y_x}{\partial y} + \frac{\partial y_y}{\partial x}\right)\right] \\ &= -D\left[\frac{\partial}{\partial x}\left(\frac{\partial y_x}{\partial x} + \frac{\partial y_y}{\partial y}\right) - (1-n)\frac{\partial^2 y_y}{\partial x \partial y} + \frac{1-n}{2}\frac{\partial^2 y_x}{\partial y^2} + \frac{(1-n)}{2}\frac{\partial^2 y_y}{\partial x \partial y})\right] \\ &= -D\left[\frac{\partial}{\partial x}\left(\frac{\partial y_x}{\partial x} + \frac{\partial y_y}{\partial y}\right) - \frac{1-n}{2}\frac{\partial}{\partial y}\left(\frac{\partial y_y}{\partial x} - \frac{\partial y_x}{\partial y}\right)\right] \end{aligned} \tag{8}$$

Let

$$\frac{\partial y_y}{\partial x} - \frac{\partial y_x}{\partial y} = \Psi \tag{9}$$

From Eqs. (7), (8) and (9), we have

$$Q_x = -D\left[\frac{\partial M}{\partial x} - \frac{1-n}{2}\frac{\partial \Psi}{\partial y}\right] \tag{10}$$

Similarly, using Eq. (5), we obtain

$$Q_y = -D\left[\frac{\partial M}{\partial y} + \frac{1-n}{2}\frac{\partial \Psi}{\partial x}\right] \tag{11}$$

Substituting Eqs. (10) and (11) into Eq. (1a) yields

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + rw^2 W = -D\left(\frac{\partial^2 M}{\partial x^2} + \frac{\partial^2 M}{\partial y^2}\right) + rw^2 W = -D\nabla^2 M + rw^2 W = 0 \tag{12}$$

or

$$\nabla^2 M = \frac{rw^2}{D} W \tag{13}$$

From Eqs. (3a,b), (10) and (11),

$$C\left(\frac{\partial W}{\partial x} - y_x\right) = -D\left[\frac{\partial M}{\partial x} - \frac{1-n}{2}\frac{\partial \Psi}{\partial y}\right] \tag{14}$$

$$C\left(\frac{\partial W}{\partial y} - y_y\right) = -D\left[\frac{\partial M}{\partial y} + \frac{1-n}{2}\frac{\partial \Psi}{\partial x}\right] \tag{15}$$

After differentiations of both sides of Eq. (14) with respect to x, Eq. (15) with respect to y, we get

$$C\left(\frac{\partial^2 W}{\partial x^2}-\frac{\partial y_x}{\partial x}\right)=-D\left[\frac{\partial^2 M}{\partial x^2}-\frac{1-n}{2}\frac{\partial^2 \Psi}{\partial x \partial y}\right] \tag{16}$$

$$C\left(\frac{\partial^2 W}{\partial y^2}-\frac{\partial y_y}{\partial y}\right)=-D\left[\frac{\partial^2 M}{\partial y^2}+\frac{1-n}{2}\frac{\partial^2 \Psi}{\partial x \partial y}\right] \tag{17}$$

Summation of Eqs. (16) and (17), using Eq. (7), yields

$$C\left(\nabla^2 W - M\right) = -D\nabla^2 M \tag{18}$$

Using Eq. (13), we arrive at the expression

$$C\left(\nabla^2 W - M\right) = -rw^2 W \tag{19}$$

After differentiations of both sides of Eq. (14) with respect to y, Eq. (15) with respect to x, we get

$$C\left(\frac{\partial^2 W}{\partial x \partial y}-\frac{\partial y_x}{\partial y}\right)=-D\left[\frac{\partial^2 M}{\partial x \partial y}-\frac{1-n}{2}\frac{\partial^2 \Psi}{\partial y^2}\right] \tag{20}$$

$$C\left(\frac{\partial^2 W}{\partial x \partial y}-\frac{\partial y_y}{\partial x}\right)=-D\left[\frac{\partial^2 M}{\partial x \partial y}+\frac{1-n}{2}\frac{\partial^2 \Psi}{\partial x^2}\right] \tag{21}$$

Subtraction of Eq. (20) and Eq. (21), using Eq. (9), yields

$$\nabla^2 \Psi = \frac{2C}{D(1-n)}\Psi \tag{22}$$

By virtue of the above derivation, all physical quantities of moderately thick plates can be represented by three functions: M, W and Ψ.

For the angles of rotation, Eqs. (14) and (15) give

$$y_x = \frac{\partial W}{\partial x}+\frac{D}{C}\left(\frac{\partial M}{\partial x}-\frac{1-n}{2}\frac{\partial \Psi}{\partial y}\right) \tag{23}$$

$$y_y = \frac{\partial W}{\partial y}+\frac{D}{C}\left(\frac{\partial M}{\partial y}+\frac{1-n}{2}\frac{\partial \Psi}{\partial x}\right) \tag{24}$$

For the bending moments and torsional moment, from Eqs. (2a-c), (23) and (24), we obtain

$$\begin{aligned} M_x &= -D\left(\frac{\partial y_x}{\partial x}+n\frac{\partial y_y}{\partial y}\right) \\ &= -D\left\{M-(1-n)\frac{\partial}{\partial y}\left[\frac{\partial W}{\partial y}+\frac{D(1-n)}{2C}\frac{\partial \Psi}{\partial x}+\frac{D}{C}\frac{\partial M}{\partial y}\right]\right\} \end{aligned} \tag{25}$$

$$\begin{aligned} M_y &= -D\left(\frac{\partial y_y}{\partial y}+n\frac{\partial y_x}{\partial x}\right) \\ &= -D\left\{M-(1-n)\frac{\partial}{\partial x}\left[\frac{\partial W}{\partial x}-\frac{D(1-n)}{2C}\frac{\partial \Psi}{\partial y}+\frac{D}{C}\frac{\partial M}{\partial x}\right]\right\} \end{aligned} \tag{26}$$

$$
\begin{aligned}
M_{xy} &= -\frac{D(1-n)}{2}\left(\frac{\partial y_x}{\partial y}+\frac{\partial y_y}{\partial x}\right) \\
&= -\frac{D(1-n)}{2}\left[2\frac{\partial^2 W}{\partial x\partial y}+\frac{2D}{C}\frac{\partial^2 M}{\partial x\partial y}+\frac{D(1-n)}{2C}\left(\frac{\partial^2\Psi}{\partial x^2}-\frac{\partial^2\Psi}{\partial y^2}\right)\right]
\end{aligned}
\tag{27}
$$

It can be seen that the problem of bending of moderately thick rectangular plates reduces to solving the following equations:

$$
\begin{aligned}
&\nabla^2 M = \frac{rw^2}{D}W \\
&C\left(\nabla^2 W - M\right) = -rw^2 W \\
&\nabla^2\Psi = \frac{2C}{D(1-n)}\Psi
\end{aligned}
\tag{28a,b,c}
$$

from which the problem can be led to Hamiltonian system.

Let

$$\frac{\partial\Psi}{\partial y} = q \tag{29}$$

From Eq. (28c), observing that $\frac{2C}{D(1-n)}=\frac{10}{h^2}$ where h is the thickness of the plate, we obtain

$$\frac{\partial q}{\partial y} = \frac{10}{h^2}\Psi - \frac{\partial^2\Psi}{\partial x^2} \tag{30}$$

Let

$$\frac{\partial W}{\partial y} = a \tag{31}$$

$$\frac{\partial M}{\partial y} = \frac{rw^2}{D}b \tag{32}$$

Eqs. (28a,b) can be represent as

$$\frac{\partial a}{\partial y} = M - \frac{\partial^2 W}{\partial x^2} - \frac{rw^2}{C}W \tag{33}$$

$$\frac{\partial b}{\partial y} = -\frac{D}{rw^2}\frac{\partial^2 M}{\partial x^2} + W \tag{34}$$

Eqs. (29), (30), (31), (32), (33) and (34) can be expressed in the matrix form as

$$\frac{\partial \mathbf{Z}}{\partial y} = \mathbf{HZ} \tag{35}$$

Where

$$\mathbf{H}=\begin{bmatrix}\mathbf{0} & \mathbf{F}\\ \mathbf{G} & \mathbf{0}\end{bmatrix},\ \mathbf{F}=\begin{bmatrix}rw^2/D & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix},\ \mathbf{G}=\begin{bmatrix}-\frac{D}{rw^2}\frac{\partial^2}{\partial x^2} & 1 & 0\\ 1 & -\frac{\partial^2}{\partial x^2}-\frac{rw^2}{C} & 0\\ 0 & 0 & \frac{10}{h^2}-\frac{\partial^2}{\partial x^2}\end{bmatrix},\ \mathbf{Z}=\left[M,W,\Psi,b,a,q\right]^{\mathrm{T}}.$$

It is obvious that $\mathbf{H}^{\mathrm{T}}=\mathbf{JHJ}$, in which $\mathbf{J}=\begin{bmatrix}\mathbf{0} & \mathbf{I}_3\\ -\mathbf{I}_3 & \mathbf{0}\end{bmatrix}$ is the symplectic metric matrix and $\mathbf{I}_3$ is 3×3 unit matrix. Therefore $\mathbf{H}$ is a Hamiltonian operator matrix and Eq. (35) is the Hamiltonian dual equation for moderately thick rectangular plates.

3. Symplectic solution methodology for free vibration of plates with two opposite edges simply supported

Applying the method of separation of variables to $\mathbf{Z}$ yields

$$\mathbf{Z}=\mathbf{X}(x)Y(y) \tag{36}$$

where $\mathbf{X}(x)=\left[M(x),W(x),\Psi(x),b(x),a(x),q(x)\right]^{\mathrm{T}}$. Substituting Eq. (36) into Eq. (35) gives

$$\frac{dY(y)}{dy}=mY(y),\ \ \mathbf{HX}(x)=m\mathbf{X}(x) \tag{37a,b}$$

where m is the eigenvalue and $X(x)$ is the corresponding eigenvector. Eq. (37b) gives an eigenvalue problem. Its characteristic equation is

$$\begin{vmatrix}-m & 0 & 0 & k & 0 & 0\\ 0 & -m & 0 & 0 & 1 & 0\\ 0 & 0 & -m & 0 & 0 & 1\\ -\frac{l^2}{k} & 1 & 0 & -m & 0 & 0\\ 1 & -l^2-kd & 0 & 0 & -m & 0\\ 0 & 0 & \frac{10}{h^2}-l^2 & 0 & 0 & -m\end{vmatrix}=0 \tag{38}$$

where $k=\frac{rw^2}{D}$, $d=\frac{D}{C}$.

Determinantal expansion yields the eigenvalue equation

$$\left\{\left(l^2+m^2\right)^2+k\left[d\left(l^2+m^2\right)-1\right]\right\}\left(l^2+m^2-\frac{10}{h^2}\right)=0 \tag{39}$$

Accordingly, the eigenvalues of the characteristic equation can be obtained.

$\left(l^2+m^2\right)^2+k\left[d\left(l^2+m^2\right)-1\right]=0$ gives

$$l_{1,2}=\pm a_1\mathrm{i},\quad l_{3,4}=\pm a_2 \tag{40}$$

Where

$$a_1=\sqrt{\frac{1}{2}\sqrt{k\left(4+kd^2\right)}+\frac{kd}{2}+m^2},\ a_2=\sqrt{\frac{1}{2}\sqrt{k\left(4+kd^2\right)}-\frac{kd}{2}-m^2} \tag{41}$$

When $m^2=\frac{10}{h^2}$, $l^2+m^2-\frac{10}{h^2}=0$ gives $l=0$ (double roots); but in this way we get inconsistent equations via Eq. (37b) thus indicating invalidity of the case. When $m^2\neq\frac{10}{h^2}$, $l^2+m^2-\frac{10}{h^2}=0$ gives

$$l_{3,4}=\pm a_3\ , \tag{42}$$

where

$$a_3=\sqrt{\frac{10}{h^2}-m^2} \tag{43}$$

Thus the general solutions are represented in the following form:

$$\begin{aligned}
M &= A_1\cos(a_1x)+B_1\sin(a_1x)+C_1\,\mathrm{ch}(a_2x)+D_1\,\mathrm{sh}(a_2x)+E_1\,\mathrm{ch}(a_3x)+F_1\,\mathrm{sh}(a_3x)\\
W &= A_2\cos(a_1x)+B_2\sin(a_1x)+C_2\,\mathrm{ch}(a_2x)+D_2\,\mathrm{sh}(a_2x)+E_2\,\mathrm{ch}(a_3x)+F_2\,\mathrm{sh}(a_3x)\\
\Psi &= A_3\sin(a_1x)+B_3\cos(a_1x)+C_3\,\mathrm{sh}(a_2x)+D_3\,\mathrm{ch}(a_2x)+E_3\,\mathrm{sh}(a_3x)+F_3\,\mathrm{ch}(a_3x)\\
b &= A_4\cos(a_1x)+B_4\sin(a_1x)+C_4\,\mathrm{ch}(a_2x)+D_4\,\mathrm{sh}(a_2x)+E_4\,\mathrm{ch}(a_3x)+F_4\,\mathrm{sh}(a_3x)\\
a &= A_5\cos(a_1x)+B_5\sin(a_1x)+C_5\,\mathrm{ch}(a_2x)+D_5\,\mathrm{sh}(a_2x)+E_5\,\mathrm{ch}(a_3x)+F_5\,\mathrm{sh}(a_3x)\\
q &= A_6\sin(a_1x)+B_6\cos(a_1x)+C_6\,\mathrm{sh}(a_2x)+D_6\,\mathrm{ch}(a_2x)+E_6\,\mathrm{sh}(a_3x)+F_6\,\mathrm{ch}(a_3x)
\end{aligned} \tag{44}$$

in which the constants are not all independent. Substituting Eq. (44) back into Eq. (37b) yields the relations between these constants as

$$\begin{aligned}
&A_1=kRA_2;\ A_4=RmA_2;\ A_5=mA_2;\ A_3=A_6=0\\
&B_1=kRB_2;\ B_4=RmB_2;\ B_5=mB_2;\ B_3=B_6=0\\
&C_1=kSC_2;\ C_4=SmC_2;\ C_5=mC_2;\ C_3=C_6=0\\
&D_1=kSD_2;\ D_4=SmD_2;\ D_5=mD_2;\ D_3=D_6=0\\
&E_6=mE_3;\ E_1=E_2=E_4=E_5=0\\
&F_6=mF_3;\ F_1=F_2=F_4=F_5=0
\end{aligned} \tag{45}$$

where $R=\frac{1}{2}\left(d-\sqrt{\frac{4}{k}+d^2}\right)$, $S=\frac{1}{2}\left(d+\sqrt{\frac{4}{k}+d^2}\right)$.

The boundary conditions of a plate with two opposite edges simply supported at $x=0$ and $x=a$ are

$$W=0, y_y=0, M_x=0 \quad \text{for} \quad x=0 \quad \text{and} \quad x=a \tag{46}$$

Substituting Eqs. (44) and (45) into Eq. (46) and then equating the determinant of the coefficient matrix to zero yields the transcendental equation of eigenvalues for free vibration of a plate simply supported on opposite edges at $x=0$ and $x=a$ as

$$\sin(a_1a)\,\mathrm{sh}(a_2a)\,\mathrm{sh}(a_3a)=0 \tag{47}$$

which gives the roots

$$a_1=\pm\frac{mp}{a}\text{, or }\ a_2=\pm\frac{mp}{a}\mathrm{i}\text{, or }\ a_3=\pm\frac{mp}{a}\mathrm{i}\quad\text{for}\quad m=1,2,3,\mathbf{L} \tag{48}$$

Substituting Eq. (48) into Eqs. (41) and (43) leads to

$$m_{\pm m}^{(1)} = \pm\sqrt{\left(\frac{mp}{a}\right)^2 - kS} \quad \text{or} \quad m_{\pm m}^{(2)} = \pm\sqrt{\left(\frac{mp}{a}\right)^2 - kR} \quad \text{or} \quad m_{\pm m}^{(3)} = \pm\sqrt{\left(\frac{mp}{a}\right)^2 + \frac{10}{h^2}} \tag{49}$$

The corresponding eigenvector of $m_m^{(1)}$ is

$$\mathbf{X}_m^{(1)}(x) = \left[k\sin(a_m x), \sin(a_m x)/R, 0, m_m^{(1)}\sin(a_m x), m_m^{(1)}\sin(a_m x)/R, 0\right]^{\mathrm{T}} \tag{50}$$

where $a_m = \dfrac{mp}{a}$ $(m = 1, 2, 3, \mathbf{L})$. The corresponding eigenvector of $m_m^{(2)}$ is

$$\mathbf{X}_m^{(2)}(x) = \left[k\sin(a_m x), \sin(a_m x)/S, 0, m_m^{(2)}\sin(a_m x), m_m^{(2)}\sin(a_m x)/S, 0\right]^{\mathrm{T}} \tag{51}$$

The corresponding eigenvector of $m_m^{(3)}$ is

$$\mathbf{X}_m^{(3)}(x) = \left[0, 0, \cos(a_m x), 0, 0, m_m^{(3)}\cos(a_m x)\right]^{\mathrm{T}} \tag{52}$$

The eigenvectors of $m_{-m}^{(1)}$, $m_{-m}^{(2)}$ and $m_{-m}^{(3)}$, i.e. $\mathbf{X}_{-m}^{(1)}(x)$, $\mathbf{X}_{-m}^{(2)}(x)$ and $\mathbf{X}_{-m}^{(3)}(x)$, can be readily obtained by replacing $m_m^{(1)}$, $m_m^{(2)}$ and $m_m^{(3)}$ in $\mathbf{X}_m^{(1)}(x)$, $\mathbf{X}_m^{(2)}(x)$ and $\mathbf{X}_m^{(3)}(x)$ with $-m_m^{(1)}$, $-m_m^{(2)}$ and $-m_m^{(3)}$ respectively. According to Yao and Zhong [29], the state vectors $\mathbf{Z}_1(x)$ and $\mathbf{Z}_2(x)$ are symplectic adjoint orthogonal when they satisfy $\int_0^a \mathbf{Z}_1(x)^{\mathrm{T}} \mathbf{J}\mathbf{Z}_2(x)\mathrm{d}x = 0$. Any two eigenvectors of a Hamiltonian matrix satisfy the symplectic adjoint orthogonality property, i.e. $\int_0^a \mathbf{X}_m^{(i)}(x)^{\mathrm{T}} \mathbf{J}\mathbf{X}_{-m}^{(i)}(x)\mathrm{d}x \neq 0\ (m = 1, 2, 3, \mathbf{L}\,;\ i = 1, 2, 3)$ while any other two eigenvectors are symplectic orthogonal to each other. From the above property and expansion of eigenvectors, the state vector $\mathbf{Z}$ can be expanded as

$$\mathbf{Z} = \sum_{m=1}^{\infty}\left(f_m^{(1)}\mathrm{e}^{m_m^{(1)}y}\mathbf{X}_m^{(1)} + f_{-m}^{(1)}\mathrm{e}^{m_{-m}^{(1)}y}\mathbf{X}_{-m}^{(1)} + f_m^{(2)}\mathrm{e}^{m_m^{(2)}y}\mathbf{X}_m^{(2)} + f_{-m}^{(2)}\mathrm{e}^{-m_m^{(2)}y}\mathbf{X}_{-m}^{(2)} + f_m^{(3)}\mathrm{e}^{m_m^{(3)}y}\mathbf{X}_m^{(3)} + f_{-m}^{(3)}\mathrm{e}^{m_{-m}^{(3)}y}\mathbf{X}_{-m}^{(3)} \right) \tag{53}$$

where $f_m^{(k)}, f_{-m}^{(k)}$ $(k = 1, 2, 3)$ are constants to be determined by imposing the remaining boundary conditions at $y = 0$ and $y = b$ which will result in the frequency equation.

4. Exact frequency equations

Assume that the edges $x = 0$ and $x = a$ of the rectangular plate, shown in Fig. 2, are simply supported and that the other two edges are S-S, S-C, S-F, C-C, C-F or F-F (S denotes simply supported, C clamped and F free). For a fully simply supported (SSSS) plate, in addition to the boundary conditions expressed in Eq. (46), the boundary conditions at the remaining two edges are

$$W = 0, y_x = 0, M_y = 0 \quad \text{for} \quad y = 0 \quad \text{and} \quad y = b \tag{54}$$

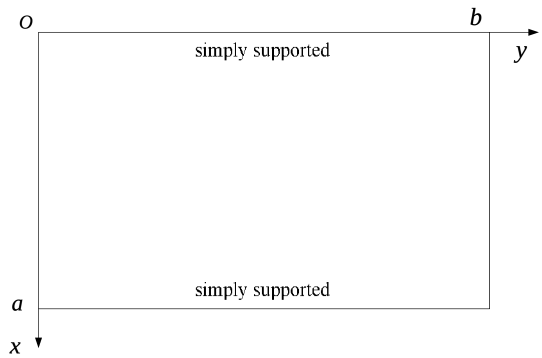

Fig. 2. A moderately thick rectangular plate simply supported at $x=0,a$

Combination of Eqs. (53) and (54) leads to six linear simultaneous equations with respect to unknown constants. Observing the condition for existence of nontrivial solutions, the determinant of the coefficient matrix of the above equations must be zero, which yields the transcendental frequency equation

$$\begin{vmatrix} 1 & 1 & p & p & 0 & 0 \\ 1 & 1 & 1 & 1 & m_m^{(3)}/a_m & -m_m^{(3)}/a_m \\ a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(2)2} & a_m^2 n - m_m^{(2)2} & a_m m_m^{(3)}(n-1) & a_m m_m^{(3)}(1-n) \\ e^{bm_m^{(1)}} & e^{-bm_m^{(1)}} & e^{bm_m^{(2)}} p & e^{-bm_m^{(2)}} p & 0 & 0 \\ e^{bm_m^{(1)}} & e^{-bm_m^{(1)}} & e^{bm_m^{(2)}} & e^{-bm_m^{(2)}} & e^{bm_m^{(3)}} m_m^{(3)}/a_m & -e^{-bm_m^{(3)}} m_m^{(3)}/a_m \\ e^{bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & e^{-bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & e^{bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & e^{-bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & e^{bm_m^{(3)}} a_m m_m^{(3)}(n-1) & e^{-bm_m^{(3)}} a_m m_m^{(3)}(1-n) \end{vmatrix} = 0 \quad (55)$$

where $p = \frac{2+kd^2 - d\sqrt{k\left(4+kd^2\right)}}{2+kd^2 + d\sqrt{k\left(4+kd^2\right)}}$, $q = \frac{\sqrt{k}d - \sqrt{4+kd^2}}{\sqrt{k}d + \sqrt{4+kd^2}}$, $r = \frac{1}{kd^2 + d\sqrt{k\left(4+kd^2\right)}}$. Accordingly the exact natural frequency parameter of vibration for SSSS plate is obtained via Eq. (55), after simplification, as

$$\bar{k} = \frac{\left(m^2 b^2 + n^2\right)^2}{1 + d_b\left(m^2 b^2 + n^2\right)} \quad (56)$$

where $\bar{k} = \frac{rw^2}{D}\frac{b^4}{p^4}$, $d_b = \frac{p^2 D}{b^2 C}$, $b = \frac{b}{a}$, $m,n = 1,2,3,\mathbf{L}$.

The exact frequency solutions for other cases can be obtained similarly. The boundary conditions at the remaining two edges ($y=0$ and $y=b$) and frequency equations for these cases are presented as follows:

SSSC:

$$W=0, y_x=0, M_y=0 \quad \text{for} \quad y=0; \quad W=0, y_x=0, y_y=0 \quad \text{for} \quad y=b \quad (57)$$

$$\begin{vmatrix} 1 & 1 & p & p & 0 & 0 \\ 1 & 1 & 1 & 1 & m_m^{(3)}/a_m & -m_m^{(3)}/a_m \\ a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(2)2} & a_m^2 n - m_m^{(2)2} & a_m m_m^{(3)}(n-1) & a_m m_m^{(3)}(1-n) \\ e^{bm_m^{(1)}} & e^{-bm_m^{(1)}} & e^{bm_m^{(2)}} p & e^{-bm_m^{(2)}} p & 0 & 0 \\ e^{bm_m^{(1)}} & e^{-bm_m^{(1)}} & e^{bm_m^{(2)}} & e^{-bm_m^{(2)}} & e^{bm_m^{(3)}} m_m^{(3)}/a_m & -e^{-bm_m^{(3)}} m_m^{(3)}/a_m \\ e^{bm_m^{(1)}} m_m^{(1)} & -e^{-bm_m^{(1)}} m_m^{(1)} & e^{bm_m^{(2)}} m_m^{(2)} & -e^{-bm_m^{(2)}} m_m^{(2)} & e^{bm_m^{(3)}} a_m & e^{-bm_m^{(3)}} a_m \end{vmatrix} = 0 \quad (58)$$

SSSF:

$$W=0,y_{x}=0,M_{y}=0 \quad \text{for} \quad y=0\,;\quad M_{y}=0,Q_{y}=0,M_{xy}=0 \quad \text{for} \quad y=b \tag{59}$$

$$\begin{vmatrix} 1 & 1 & p & p & 0 & 0 \\ 1 & 1 & 1 & 1 & m_m^{(3)}/a_m & -m_m^{(3)}/a_m \\ a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(2)2} & a_m^2 n - m_m^{(2)2} & a_m m_m^{(3)}(n-1) & a_m m_m^{(3)}(1-n) \\ 2\mathrm{e}^{bm_m^{(1)}} a_m m_m^{(1)} & -2\mathrm{e}^{-bm_m^{(1)}} a_m m_m^{(1)} & 2\mathrm{e}^{bm_m^{(2)}} a_m m_m^{(2)} & -2\mathrm{e}^{-bm_m^{(2)}} a_m m_m^{(2)} & \mathrm{e}^{bm_m^{(3)}}\left(a_m^2+m_m^{(3)2}\right) & \mathrm{e}^{-bm_m^{(3)}}\left(a_m^2+m_m^{(3)2}\right) \\ \mathrm{e}^{bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & \mathrm{e}^{-bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & \mathrm{e}^{bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & \mathrm{e}^{-bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & \mathrm{e}^{bm_m^{(3)}} a_m m_m^{(3)}(n-1) & \mathrm{e}^{-bm_m^{(3)}} a_m m_m^{(3)}(1-n) \\ -\mathrm{e}^{bm_m^{(1)}} m_m^{(1)} & \mathrm{e}^{-bm_m^{(1)}} m_m^{(1)} & -\mathrm{e}^{bm_m^{(2)}} m_m^{(2)} q & \mathrm{e}^{-bm_m^{(2)}} m_m^{(2)} q & 2\mathrm{e}^{bm_m^{(3)}} a_m r & 2\mathrm{e}^{-bm_m^{(3)}} a_m r \end{vmatrix} = 0 \tag{60}$$

SCSC:

$$W=0,y_{x}=0,y_{y}=0 \quad \text{for} \quad y=0 \quad \text{and} \quad y=b \tag{61}$$

$$\begin{vmatrix} 1 & 1 & p & p & 0 & 0 \\ 1 & 1 & 1 & 1 & m_m^{(3)}/a_m & -m_m^{(3)}/a_m \\ m_m^{(1)} & -m_m^{(1)} & m_m^{(2)} & -m_m^{(2)} & a_m & a_m \\ \mathrm{e}^{bm_m^{(1)}} & \mathrm{e}^{-bm_m^{(1)}} & \mathrm{e}^{bm_m^{(2)}} p & \mathrm{e}^{-bm_m^{(2)}} p & 0 & 0 \\ \mathrm{e}^{bm_m^{(1)}} & \mathrm{e}^{-bm_m^{(1)}} & \mathrm{e}^{bm_m^{(2)}} & \mathrm{e}^{-bm_m^{(2)}} & \mathrm{e}^{bm_m^{(3)}} m_m^{(3)}/a_m & -\mathrm{e}^{-bm_m^{(3)}} m_m^{(3)}/a_m \\ \mathrm{e}^{bm_m^{(1)}} m_m^{(1)} & -\mathrm{e}^{-bm_m^{(1)}} m_m^{(1)} & \mathrm{e}^{bm_m^{(2)}} m_m^{(2)} & -\mathrm{e}^{-bm_m^{(2)}} m_m^{(2)} & \mathrm{e}^{bm_m^{(3)}} a_m & \mathrm{e}^{-bm_m^{(3)}} a_m \end{vmatrix} = 0 \tag{62}$$

SCSF:

$$W=0,y_{x}=0,y_{y}=0 \quad \text{for} \quad y=0\,;\quad M_{y}=0,Q_{y}=0,M_{xy}=0 \quad \text{for} \quad y=b \tag{63}$$

$$\begin{vmatrix} 1 & 1 & p & p & 0 & 0 \\ 1 & 1 & 1 & 1 & m_m^{(3)}/a_m & -m_m^{(3)}/a_m \\ m_m^{(1)} & -m_m^{(1)} & m_m^{(2)} & -m_m^{(2)} & a_m & a_m \\ 2\mathrm{e}^{bm_m^{(1)}} a_m m_m^{(1)} & -2\mathrm{e}^{-bm_m^{(1)}} a_m m_m^{(1)} & 2\mathrm{e}^{bm_m^{(2)}} a_m m_m^{(2)} & -2\mathrm{e}^{-bm_m^{(2)}} a_m m_m^{(2)} & \mathrm{e}^{bm_m^{(3)}}\left(a_m^2+m_m^{(3)2}\right) & \mathrm{e}^{-bm_m^{(3)}}\left(a_m^2+m_m^{(3)2}\right) \\ \mathrm{e}^{bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & \mathrm{e}^{-bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & \mathrm{e}^{bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & \mathrm{e}^{-bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & \mathrm{e}^{bm_m^{(3)}} a_m m_m^{(3)}(n-1) & \mathrm{e}^{-bm_m^{(3)}} a_m m_m^{(3)}(1-n) \\ -\mathrm{e}^{bm_m^{(1)}} m_m^{(1)} & \mathrm{e}^{-bm_m^{(1)}} m_m^{(1)} & -\mathrm{e}^{bm_m^{(2)}} m_m^{(2)} q & \mathrm{e}^{-bm_m^{(2)}} m_m^{(2)} q & 2\mathrm{e}^{bm_m^{(3)}} a_m r & 2\mathrm{e}^{-bm_m^{(3)}} a_m r \end{vmatrix} = 0 \tag{64}$$

SFSF:

$$M_{y}=0,Q_{y}=0,M_{xy}=0 \quad \text{for} \quad y=0 \quad \text{and} \quad y=b \tag{65}$$

$$\begin{vmatrix} 2a_m m_m^{(1)} & -2a_m m_m^{(1)} & 2a_m m_m^{(2)} & -2a_m m_m^{(2)} & a_m^2+m_m^{(3)2} & a_m^2+m_m^{(3)2} \\ a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(1)2} & a_m^2 n - m_m^{(2)2} & a_m^2 n - m_m^{(2)2} & a_m m_m^{(3)}(n-1) & a_m m_m^{(3)}(1-n) \\ -m_m^{(1)} & m_m^{(1)} & -m_m^{(2)} q & m_m^{(2)} q & 2a_m r & 2a_m r \\ 2\mathrm{e}^{bm_m^{(1)}} a_m m_m^{(1)} & -2\mathrm{e}^{-bm_m^{(1)}} a_m m_m^{(1)} & 2\mathrm{e}^{bm_m^{(2)}} a_m m_m^{(2)} & -2\mathrm{e}^{-bm_m^{(2)}} a_m m_m^{(2)} & \mathrm{e}^{bm_m^{(3)}}\left(a_m^2+m_m^{(3)2}\right) & \mathrm{e}^{-bm_m^{(3)}}\left(a_m^2+m_m^{(3)2}\right) \\ \mathrm{e}^{bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & \mathrm{e}^{-bm_m^{(1)}}\left(a_m^2 n - m_m^{(1)2}\right) & \mathrm{e}^{bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & \mathrm{e}^{-bm_m^{(2)}}\left(a_m^2 n - m_m^{(2)2}\right) & \mathrm{e}^{bm_m^{(3)}} a_m m_m^{(3)}(n-1) & \mathrm{e}^{-bm_m^{(3)}} a_m m_m^{(3)}(1-n) \\ -\mathrm{e}^{bm_m^{(1)}} m_m^{(1)} & \mathrm{e}^{-bm_m^{(1)}} m_m^{(1)} & -\mathrm{e}^{bm_m^{(2)}} m_m^{(2)} q & \mathrm{e}^{-bm_m^{(2)}} m_m^{(2)} q & 2\mathrm{e}^{bm_m^{(3)}} a_m r & 2\mathrm{e}^{-bm_m^{(3)}} a_m r \end{vmatrix} = 0 \tag{66}$$

It should be pointed out that the exact frequency transcendental equations for non SSSS plates are so complicated in form that they are presented in the form of different determinants as given above.

5. Numerical examples

In order to verify the validity of the results obtained in present study, the natural frequencies of all cases in the present work are solved for comprehensive numerical comparison. The Poisson's ratio n is taken to be 0.3 throughout. For rectangular SSSS and SCSC plates, comparisons with the results from Institute of Mechanics (IMECH), Chinese Academy of Science [4], obtained by the semi-inverse method, are tabulated in Tables 1-2 respectively, in which perfect agreement is observed. Tables 3-6 present the lowest natural frequency parameters for SSSC, SSSF, SCSF and SFSF plates with different aspect ratios a/b and thickness ratios h/b. The results used for comparison are obtained by Liew et al. [14] using the pb-2 Rayleigh-Ritz method. Satisfactory accordance is observed. From all comparison studies presented in this paper, the applicability and validity of the symplectic approach proposed and expressions derived are demonstrated.

Table 1. The lowest three natural frequency parameters $\bar{k}$ of moderately thick SSSS plates ($\bar{k}=\frac{rw^2}{D}\frac{b^4}{p^4}$, $d_b=\frac{p^2D}{b^2C}$, $b=\frac{b}{a}$)

d_b	b	0.2		0.6		1		2	
	Mode	IMECH (1977)	Present	IMECH (1977)	Present	IMECH (1977)	Present	IMECH (1977)	Present
0	1	1.082	1.08160	1.850	1.84960	4.000	4.00000	25.00	25.0000
	2	1.346	1.34560	5.954	5.95360	25.00	25.0000	64.00	64.0000
	3	1.850	1.84960	17.98	17.9776	64.00	64.0000	169.0	169.000
0.05	1	1.028	1.02814	1.732	1.73184	3.636	3.63636	20.00	20.0000
	2	1.272	1.27183	5.306	5.30624	20.00	20.0000	45.71	45.7143
	3	1.732	1.73184	14.83	14.8330	45.71	45.7143	102.4	102.424
0.10	1	0.980	0.979710	1.628	1.62817	3.333	3.33333	16.67	16.6667
	2	1.206	1.20573	4.786	4.78585	16.67	16.6667	35.56	35.5556
	3	1.628	1.62817	12.63	12.6247	35.56	35.5556	73.48	73.4783
0.20	1	0.895	0.895364	1.454	1.45409	2.857	2.85714	12.50	12.5000
	2	1.092	1.09221	4.001	4.00108	12.50	12.5000	24.62	24.6154
	3	1.454	1.45409	9.728	9.72814	24.62	24.6154	46.94	46.9444
0.40	1	0.764	0.763842	1.198	1.19793	2.222	2.22222	8.333	8.33333
	2	0.919	0.919126	3.013	3.01296	8.333	8.33333	15.24	15.2381
	3	1.198	1.19793	6.668	6.66825	15.24	15.2381	27.26	27.2581
0.60	1	0.666	0.666010	1.019	1.01850	1.818	1.81818	6.250	6.25000
	2	0.793	0.793396	2.416	2.41623	6.250	6.25000	11.03	11.0345
	3	1.019	1.01850	5.073	5.07269	11.03	11.0345	19.21	19.2045
0.80	1	0.590	0.590393	0.886	0.885824	1.538	1.53846	5.000	5.00000
	2	0.698	0.697925	2.017	2.01680	5.000	5.00000	8.649	8.64865
	3	0.886	0.885824	4.093	4.09326	8.649	8.64865	14.83	14.8246
1.00	1	0.530	0.530196	0.784	0.783729	1.333	1.33333	4.167	4.16667
	2	0.623	0.622963	1.731	1.73070	4.167	4.16667	7.111	7.11111
	3	0.784	0.783729	3.431	3.43084	7.111	7.11111	12.07	12.0714

Table 2. The lowest natural frequency parameters $\bar{k}$ of moderately thick SCSC plates

d_b	b	0.2	0.4	0.6	0.8	1.0	1.5	2.0
0	IMECH(1977)	5.240	5.562	6.162	7.132	8.604	15.19	30.76
	Present	5.24005	5.56251	6.16210	7.13233	8.60445	15.6861	30.7651
0.01	IMECH(1977)	4.987	5.287	5.846	6.753	8.131	14.74	28.62
	Present	4.98720	5.28707	5.84618	6.75305	8.13065	14.7379	28.6244
0.05	IMECH(1977)	4.179	4.415	4.861	5.593	6.710	12.02	22.70
	Present	4.17854	4.41512	4.86143	5.59286	6.70985	12.0169	22.7048
0.10	IMECH(1977)	3.472	3.662	4.025	4.626	5.549	9.879	18.23
	Present	3.47220	3.66195	4.02492	4.62644	5.54927	9.87950	18.2344
0.20	IMECH(1977)	2.593	2.773	3.007	3.466	4.172	7.390	13.21
	Present	2.59302	2.73318	3.00685	3.46635	4.17199	7.38997	13.2088
0.40	IMECH(1977)	1.721	1.819	2.013	2.341	2.839	4.991	8.585
	Present	1.72060	1.81852	2.01326	2.34143	2.83890	4.99132	8.58540
0.60	IMECH(1977)	1.288	1.366	1.522	1.783	2.173	3.792	6.376
	Present	1.28769	1.36595	1.52212	1.78325	2.17255	3.79179	6.37605
0.80	IMECH(1977)	1.029	1.096	1.228	1.446	1.767	3.064	5.075
	Present	1.02913	1.09551	1.22762	1.44619	1.76706	3.06362	5.07494
1.00	IMECH(1977)	0.857	0.915	1.031	1.219	1.492	2.572	4.216
	Present	0.857241	0.91544	1.03064	1.21919	1.49238	2.57246	4.21612

Table 3. The lowest natural frequency parameters $\sqrt{\frac{rw^2}{D}}\frac{b^2}{p^2}$ of moderately thick SSSC plates

a/b	h/b					
	0.001		0.1		0.2	
	Liew et al. (1993)	Present	Liew et al. (1993)	Present	Liew et al. (1993)	Present
0.4	7.4408	7.4408	6.5903	6.7248	5.2319	5.4250
0.6	4.0543	4.0544	3.7546	3.8021	3.1829	3.2778
0.8	2.9074	2.9074	2.7340	2.7592	2.3783	2.4357
1.0	2.3958	2.3959	2.2684	2.2854	1.9964	2.0374
1.25	2.0805	2.0805	1.9785	1.9910	1.7538	1.7851
1.5	1.9150	1.9151	1.8256	1.8360	1.6247	1.6510
1.75	1.8177	1.8179	1.7356	1.7448	1.5485	1.5719
2.0	1.7561	1.7561	1.6782	1.6867	1.4998	1.5214
2.25	1.7143	1.7143	1.6395	1.6475	1.4669	1.4874
2.5	1.6847	1.6847	1.6121	1.6197	1.4436	1.4632

Table 4. The lowest natural frequency parameters $\sqrt{\frac{rw^2}{D}}\frac{b^2}{p^2}$ of moderately thick SSSF plates

a/b	h/b					
	0.001		0.1		0.2	
	Liew et al. (1993)	Present	Liew et al. (1993)	Present	Liew et al. (1993)	Present
0.4	6.4121	6.4119	5.7705	5.7628	4.6886	4.7865
0.6	2.9585	2.9580	2.8006	2.8312	2.4760	2.5160
0.8	1.7480	1.7472	1.6860	1.6978	1.5521	1.5866
1.0	1.1847	1.1838	1.1523	1.1582	1.0840	1.1026
1.25	0.8204	0.8193	0.8017	0.8047	0.7653	0.7755
1.5	0.6191	0.6174	0.6056	0.6075	0.5870	0.5894
1.75	0.4945	0.4924	0.4837	0.4849	0.4679	0.4723
2.0	0.4112	0.4087	0.4017	0.4026	0.3898	0.3931
2.25	0.3515	0.3491	0.3432	0.3439	0.3338	0.3364
2.5	0.3082	0.3048	0.2997	0.3003	0.2919	0.2941

Table 5. The lowest natural frequency parameters $\sqrt{\frac{rw^2}{D}\frac{b^2}{p^2}}$ of moderately thick SCSF plates

a/b	h/b					
	0.001		0.1		0.2	
	Liew et al. (1993)	Present	Liew et al. (1993)	Present	Liew et al. (1993)	Present
0.4	6.4520	6.4517	5.7941	5.7628	4.6987	4.7865
0.6	3.0203	3.0202	2.8474	2.8789	2.5032	2.5786
0.8	1.8302	1.8301	1.7548	1.7675	1.5994	0.6354
1.0	1.2853	1.2854	1.2411	1.2477	1.1512	1.1717
1.25	0.9416	0.9415	0.9126	0.9163	0.8559	0.8680
1.5	0.7577	0.7575	0.7358	0.7383	0.6944	0.7028
1.75	0.6480	0.6480	0.6305	0.6324	0.5977	0.6041
2.0	0.5779	0.5779	0.5632	0.5647	0.5356	0.5409
2.25	0.5302	0.5302	0.5175	0.5189	0.4937	0.4983
2.5	0.4965	0.4965	0.4853	0.4865	0.4641	0.4682

Table 6. The lowest natural frequency parameters $\sqrt{\frac{rw^2}{D}\frac{b^2}{p^2}}$ of moderately thick SFSF plates

a/b	h/b					
	0.001		0.1		0.2	
	Liew et al. (1993)	Present	Liew et al. (1993)	Present	Liew et al. (1993)	Present
0.4	6.1807	6.1803	5.5840	5.6902	4.5595	4.7362
0.6	2.7342	2.7337	2.6011	2.6272	2.3178	2.3823
0.8	1.5318	1.5309	1.4860	1.4949	1.3817	1.4083
1.0	0.9768	0.9758	0.9565	0.9603	0.9102	0.9226
1.25	0.6235	0.6219	0.6136	0.6152	0.5934	0.5989
1.5	0.4330	0.4304	0.4262	0.4269	0.4162	0.4189
1.75	0.3185	0.3154	0.3130	0.3135	0.3075	0.3090
2.0	0.2437	0.2410	0.2395	0.2398	0.2362	0.2372
2.25	0.1945	0.1901	0.1892	0.1893	0.1871	0.1877
2.5	0.1596	0.1600	0.1532	0.1533	0.1518	0.1521

6. Conclusions

In this paper the new symplectic method is developed for free vibration analysis of moderately thick rectangular plates. Exact frequency equations of the plates with two opposite edges simply supported are derived analytically. Unlike the traditional semi-inverse approaches in classical vibration analysis, where trial functions are pre-selected inevitably, the present symplectic procedure is completely rational and rigorous without any trial functions. The distinct advantage provides the approach an excellent applicability to the vibration problems of moderately thick rectangular plates, as described in the present work, thus it exhibits a breakthrough in solving the problems exactly. The present analysis has provided a significant extension of the symplectic approach while more studies interest the researchers will be explored in future such as free and forced vibrations for plates based on various higher-order theories with any other combinations of supported conditions.

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