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LARGE DEFLECTION STATIC ANALYSIS OF A CANTILEVER BEAM SUBJECTED TO A POINT LOAD

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Abstract

This work presents geometrically non-linear static analysis of a cantilever beam subjected to a non-follower transversal point load at the free end of the beam. The material of the beam is assumed as isotropic and hyperelastic. In this study, finite element model of the beam is constructed by using total Lagrangian finite element model of two dimensional continua for a twelve-node quadratic element. The considered highly non-linear problem is solved by using incremental displacement-based finite element method in connection with Newton-Raphson iteration method. In the study, the effect of the large deflections and rotations on the displacements and the normal stress and the shear stress distributions through the thickness of the beam is investigated in detail. With the variation of the ratio of Lenght/height, the results of the total Lagrangian finite element model of two dimensional continua for a twelve-node quadratic element are compared with the results of SAP2000 packet program. Also, a few of the obtained results are compared with the previously published results. Numerical results indicate that with decrease the of ratio of lenght/height, using the total Lagrangian finite element model of two dimensional continua plays very important role in the static responses of the beam in geometrically non-linear static analysis.

Keywords: Cantilever Beam, Non-linear Finite Element Analysis, Large Displacements, Large Rotations.

1. Introduction

In recent years, with the development of technology, increasing demands for optimum or minimum-weight designed structural components makes it necessary to use non-linear theory of beams. Especially, developments in aerospace engineering, robotics and manufacturing make it inevitable to excessively use non-linear models that must be solved numerically. Because, a closed-form solution is not possible and hence more general numerical processes play an important role. Some of these studies concerning closed-form solutions are given in the following paragraphs: Chucheepsakul et al. [1, 2] studied the large deflections of beams under moment gradients whose deformed arc lengths are not fixed by using the elliptic integral method, the shooting-optimization method and the finite element method. Wang et al. [3] considered the large deflection problem of variable deformed arc-length beams considering one end of the beam being hinged and the beam being allowed to slide freely on a frictionless support located at a specified distance away from this hinged end under a point load. A similar problem was solved by He et al. [4] in which only the frictionless support in the previous study was assumed as a friction support. Some of the numerical studies are given in the following paragraphs: Kapania and Li [5] formulated and implemented exact curved beam elements incorporating finite strains and finite rotations. Pulngern et al. [6] investigated large static deflection due to uniformly distributed self weight and the critical or maximum applied uniform loading that a simply supported beam with variable-arc-length can resist by using both finite-element method discretization of the span length based on variational formulation and shooting method based on an elastic theory formulation. Al Sadder and Al Rawi [7] developed a quasi-linearization finite differences scheme for large deflection analysis of prismatic and non-prismatic slender cantilever beams subjected to various types of continuous and discontinuous external variable distributed and concentrated loads in horizontal and vertical global directions. Li and Zhou [8] investigated the post-buckling behavior of a hinged-fixed beam under uniformly distributed follower forces by deriving an exact mathematical model and using the shooting method for numerical results. Al Sadder et al. [9] developed an improved finite element formulation with a scheme of solution for the large deflection analysis of inextensible prismatic and nonprismatic slender beams. The considered problem was investigated by Reddy [10] by using an eight-node quadratic element. Banerjee et. al [11] proposed non-linear shooting and Adomian decomposition methods in order to investigate the large deflection of a cantilever beam under arbitrary loading. Shvartsman [12] studied the large deflection problem of non-uniform cantilever beams under a tip-concentrated and intermediate follower forces by reducing the governing non-linear boundary-value problem to an initial-value problem by change of variables. Brojan at al. [13] investigated the large deflections of nonlinearly elastic cantilever beams made from materials obeying the generalized Ludwick constitutive law. Nallathambi at all [14] studied a method to analyze for the large deflections of curved prismatic cantilever beams with uniform curvature subjected to a follower load at the tip.The quasi-static response and the stored and dissipated energies due to large deflections of a slender inextensible beam made of a linear viscoelastic material and subjected to a time-dependent inclined concentrated load at the free end are investigated by Vaz and Caire [15]. Akbaş [16] studied the geometrically non-linear static analysis of a cantilever beam under a non-follower uniformly distributed load using finite element model of the beam constructed by using total Lagrangian finite element model of two dimensional continuum for a twelve-node quadratic element. Large deflection static analysis of simple beams was investigated by Akbaş and Kocatürk [17]. In a recent study, geometrically non-linear static analysis of a simply supported beam made of hyperelastic material subjected to a non-follower transversal uniformly distributed load is analyzed by Kocatürk and Akbaş [18] using finite element model of the beam constructed by using total Lagrangian finite element model of two dimensional continuum for a twelve-node quadratic element.

The aim of this paper is to compute the displacements of the considered cantilever beam made of hyperelastic material. The development of the formulations of general solution procedure of nonlinear problems follows the general outline of the derivation given by Zienkiewichz [19]. The geometrically non-linear responses of a cantilevered beam subjected to a point load at the free end of the beam are obtained by using total Lagrangian finite element model of a two-dimensional solid continua. The TL finite element equations of two dimensional continua for a twelve-node quadratic element are used. These TL twelve-node quadratic element formulations were given Kocatürk and Akbaş [18].

2. Theory and Formulations

A cantilever beam made of isotropic, hyperelastic material, with material or Lagrangian coordinate system $\left({}^{0}x_1,\ {}^{0}x_2,\ {}^{0}x_3\right)$ and with spatial or Euler coordinate system $\left({}^{2}x_1,\ {}^{2}x_2,\ {}^{2}x_3\right)$ having the origin *O is* shown in Fig. 1. One of the supports of the beam is assumed to be fixed and the other free. The beam is subjected to a non-follower transversal point load in the transverse direction as seen from Fig. 1.

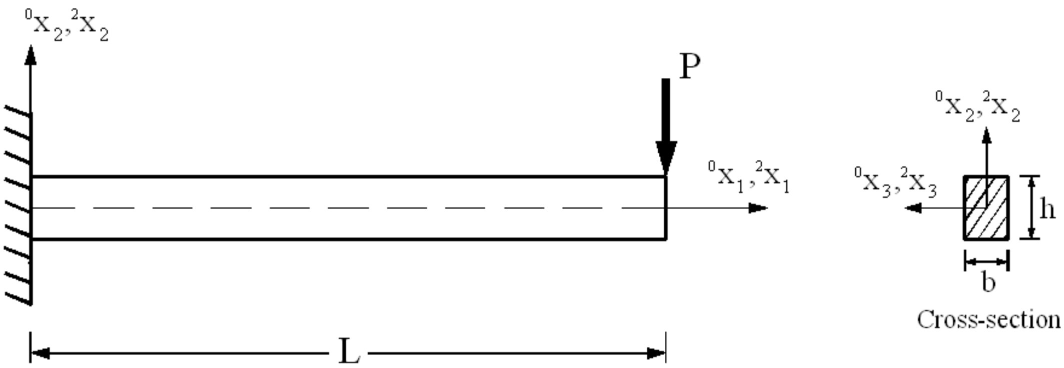


Figure 1. Cantilever beam subjected to a point load.

While the derivation of the governing equations for most problems is not unduly difficult, their solution by exact methods of analysis is a formidable task. In such cases, numerical methods of analysis provide an alternative means of finding solutions. Numerical methods typically transform differential equations to algebraic equations that are to be solved using computers. The considered problem is a nonlinear one. Even linear problems may not admit exact solutions due to geometric and material complexities, but it is relatively easy to obtain approximate solutions using numerical methods (Reddy [10]). There are some solutions for the special cases of boundary and loading conditions for large displacements of beams in the framework of Euler-Bernoulli beam theory. However, as far as the authors know, exact solution of a nonlinear problem in the framework of two or three-dimensional continua approach is not possible. For the analysis of the cantilever beam, the beam problem is considered as a two-dimensional continua problem: The total Lagrangian Finite element model of two dimensional continua based on the total Lagrangian formulation for a twelve-node quadratic element is used in the study. For the solution of the total Lagrangian formulations of TL two dimensional continua problem, small-step incremental approaches from known solutions are used. As it is known, it is possible to obtain solutions in a single increment of the external force only in the case of mild nonlinearity (and no path dependence).

In this study, small-step incremental approaches from known solutions with Newton-Raphson iteration method are used in which the solution for $n+1$th load increment and i th iteration is obtained in the following form:

$$d\,\mathbf{u}_n^i = \left(\mathbf{K}_T^i\right)^{\mathbf{-1}} \mathbf{R}_{n+1}^i \tag{1}$$

Where $\mathbf{K}_T^i$ is the stiffness matrix corresponding to a tangent direction at the i th iteration, $d\mathbf{u}_n^i$ is the solution increment vector at the i th iteration and $n+1$ th load increment, $\mathbf{R}_{n+1}^i$ is the residual vector at the i th iteration and $n+1$ th load increment. This iteration procedure is continued until the difference between two successive solution vectors is less than a selected tolerance criterion in Euclidean norm given by

$$\sqrt{\frac{\left[\left(d\,\mathbf{u}_n^{i+1}-d\,\mathbf{u}_n^{i}\right)^T\left(d\,\mathbf{u}_n^{i+1}-d\,\mathbf{u}_n^{i}\right)\right]^2}{\left[\left(d\,\mathbf{u}_n^{i+1}\right)^T\left(d\,\mathbf{u}_n^{i+1}\right)\right]^2}} \leq \zeta_{tol} \tag{2}$$

A series of successive approximations gives

$$\mathbf{u}_{n+1}^{i+1} = \mathbf{u}_{n+1}^{i} + d\mathbf{u}_{n+1}^{i} = \mathbf{u}_n + \Delta\mathbf{u}_n^{i} \tag{3}$$

where

$$\Delta\mathbf{u}_n^{i} = \sum_{k=1}^{i} d\,\mathbf{u}_n^{k} \tag{4}$$

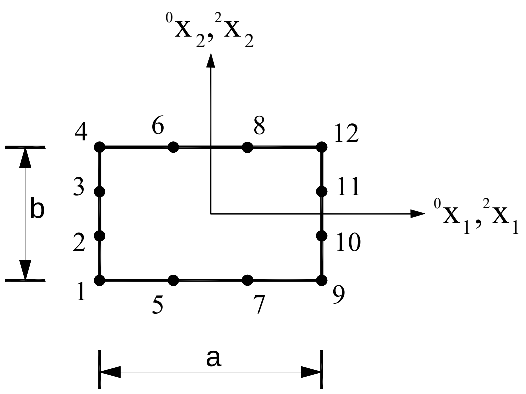


Fig. 2. A twelve-node quadratic plane element.

Total displacement fields and incremental displacement fields are expressed in terms of nodal displacements as follows:

$$\mathrm{u}=\begin{Bmatrix} u \\ v \end{Bmatrix}=\begin{Bmatrix} \sum_{j=1}^{12} u_j \psi_j\left({}^o x_1, {}^o x_2\right) \\ \sum_{j=1}^{12} v_j \psi_j\left({}^o x_1, {}^o x_2\right) \end{Bmatrix} \tag{5}$$

$$\widehat{\mathrm{u}}=\begin{Bmatrix} \widehat{u} \\ \widehat{v} \end{Bmatrix}=\begin{Bmatrix} \sum_{j=1}^{12} \bar{u}_j \psi_j\left({}^o x_1, {}^o x_2\right) \\ \sum_{j=1}^{12} \bar{v}_j \psi_j\left({}^o x_1, {}^o x_2\right) \end{Bmatrix} \tag{6}$$

where $\psi_j(x)$ are interpolation functions for a twelve-node quadratic element and can be found in Kocatürk and Akbaş [18], $\bar{u}_j$ and $\bar{v}_j$ are the components of vectors of nodal displacements in the ${}^0x_1$ and ${}^0x_2$ directions respectively. The tangent stiffness matrix $\mathbf{K}_T^i$ and the residual vector $\mathbf{R}_{n+1}^i$ which are to be used in Eq. (1) at the ith iteration for the total Lagrangian finite element model of two dimensional continuum for an twelve-node quadratic element are given below:

$$\mathbf{K}_T^i \, d\,\mathbf{u}_n^i = \mathbf{R}_{n+1}^i \rightarrow \begin{bmatrix} \mathbf{K}^{\mathbf{11L}} + \mathbf{K}^{\mathbf{11NL}} & \mathbf{K}^{\mathbf{12L}} \\ \mathbf{K}^{\mathbf{21L}} & \mathbf{K}^{\mathbf{22L}} + \mathbf{K}^{\mathbf{22NL}} \end{bmatrix}^i \begin{Bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{v}} \end{Bmatrix}^i = \begin{Bmatrix} {}_{\mathbf{0}}^{\mathbf{2}}\mathbf{F}^{\mathbf{1}} - {}_{\mathbf{0}}^{\mathbf{1}}\mathbf{F}^{\mathbf{1}} \\ {}_{\mathbf{0}}^{\mathbf{2}}\mathbf{F}^{\mathbf{2}} - {}_{\mathbf{0}}^{\mathbf{1}}\mathbf{F}^{\mathbf{2}} \end{Bmatrix}^i \tag{7}$$

The explicit expressions of $\mathbf{K}^{\mathbf{11L}}$, $\mathbf{K}^{\mathbf{11NL}}$, $\mathbf{K}^{\mathbf{12L}}$, $\mathbf{K}^{\mathbf{21L}}$, $\mathbf{K}^{\mathbf{22L}}$, $\mathbf{K}^{\mathbf{22NL}}$, ${}_{\mathbf{0}}^{\mathbf{1}}\mathbf{F}^{\mathbf{1}}$ and ${}_{\mathbf{0}}^{\mathbf{1}}\mathbf{F}^{\mathbf{2}}$ are given in Reddy [10] and Kocatürk and Akbaş [18].

$${}_0^2F_i^1 = h_e \int_{\Omega^e} {}_0^2f_{{}^0x_1}\, \psi_i \, d^0x_1 \, d^0x_1 + h_e \int_{\Gamma^e} {}_0^2t_{{}^0x_1}\, \psi_i \, d^0x_1 \tag{8a}$$

$$
{}^2_0F_i^2 = h_e\int_{\Omega^e} {}^2_0f_{{}^0x_2}\,\psi_i\, d^0x_1\, d^0x_2 + h_e\int_{\Gamma^e} {}^2_0t_{{}^0x_2}\,\psi_i\, d^0x_1
$$

(8b)

where ${}^2_0f_{{}^0x_1}$, ${}^2_0f_{{}^0x_1}$ are the body forces, ${}^2_0t_{{}^0x_1}$, ${}^2_0t_{{}^0x_2}$ are the surface forces in the ${}^0x_1$ and ${}^0x_2$ directions respectively.

The considered material is hyperelastic. In this case, the constitutive relation between the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor can be assumed as follows:

$$
{}^{\mathbf{1}}_{\mathbf{0}}\mathbf{S} = \begin{Bmatrix} {}^1_0S_{11} \\ {}^1_0S_{22} \\ {}^1_0S_{12} \end{Bmatrix} = \begin{bmatrix} {}_0C_{11} & {}_0C_{12} & 0 \\ {}_0C_{12} & {}_0C_{22} & 0 \\ 0 & 0 & {}_0C_{66} \end{bmatrix} \begin{Bmatrix} {}^1_0E_{11} \\ {}^1_0E_{22} \\ 2\,{}^1_0E_{12} \end{Bmatrix}
$$

(9)

where ${}^1_0S_{11}$, ${}^1_0S_{22}$, ${}^1_0S_{12}$ are the components of the second Piola-Kirchhoff stress tensor components in the C_1 configuration of the body, ${}_0C_{ij}$ are the components of the reduced constitutive tensor in the C_0 configuration of the body, α_1 and α_2 are coefficients of thermal expansion in the ${}^0x_1$ and ${}^0x_2$ directions respectively.. The components of the reduced constitutive tensor can be written in terms of Young modulus E and Poisson's ratio ν as follows:

$$ {}_0C_{11} = \frac{E}{1-v^2} \;,\; {}_0C_{12} = {}_0C_{21} = \frac{vE}{1-v^2} \;,\; {}_0C_{22} = \frac{E}{1-v^2} \;,\; {}_0C_{66} = \frac{E}{2(1+v)} \tag{10} $$

The Green-Lagrange strain tensor's expression in terms of displacements in the case of two-dimensional solid continuum is given by Reddy [10] . Numerical calculations of the integrals in the rigidity matrices will be calculated by using five-point Gauss rule. The strains are assumed as small.

The true stress, namely stress in the deformed configuration is defined to be the current force per unit deformed area. The relation between the Cauchy stress tensor components ${}^2\sigma_{ij}$ and the second Piola-Kirchhoff stress tensor components ${}^2_0S_{ij}$ can be written as follows;

$$ {}^2\sigma_{ij} = \frac{{}^2\rho}{{}^0\rho} \frac{\partial\, {}^2x_i}{\partial\, {}^0x_p} \frac{\partial\, {}^2x_j}{\partial\, {}^0x_q}\, {}^2_0S_{pq} \tag{11} $$

Where ${}^0\rho$ and ${}^2\rho$ represent the mass densities of the material in configurations C_0 and C_2 respectively. The relation between the ${}^0\rho$ and ${}^2\rho$ is as follows;

$$ {}^0\rho = {}^2\rho\, {}^2_0J \tag{12} $$

Where ${}^2_0J$ is the determinant of the deformation gradient tensor ${}^2_0\mathbf{F}$ (or the Jacobian of the transformation) and defined as follows:

$$ {}^2_0J = \det({}^2_0\mathbf{F}) = \left| \frac{\partial^2 x_i}{\partial^0 x_j} \right| = \begin{vmatrix} \frac{\partial^2 x_1}{\partial^0 x_1} & \frac{\partial^2 x_1}{\partial^0 x_2} & \frac{\partial^2 x_1}{\partial^0 x_3} \\ \frac{\partial^2 x_2}{\partial^0 x_1} & \frac{\partial^2 x_2}{\partial^0 x_2} & \frac{\partial^2 x_2}{\partial^0 x_3} \\ \frac{\partial^2 x_3}{\partial^0 x_1} & \frac{\partial^2 x_3}{\partial^0 x_2} & \frac{\partial^2 x_3}{\partial^0 x_3} \end{vmatrix} \tag{13} $$

The following transformation rule hold between the components of the elasticity tensors in different configurations:

$$ {}_2C_{ijkl} = \frac{{}^2\rho}{{}^0\rho} \frac{\partial^2 x_i}{\partial^0 x_p} \frac{\partial^2 x_j}{\partial^0 x_q} \frac{\partial^2 x_k}{\partial^0 x_r} \frac{\partial^2 x_l}{\partial^0 x_s}\, {}_0C_{pqrs} \tag{14} $$

It is assumed in the study that the components of the reduced constitutive tensor remain constant during the deformation. Namely, it is assumed that ${}_0\rho \approx {}_2\rho$ and therefore ${}_0C_{ij} = {}_2C_{ij}$. The error introduced by this assumption can be negligible if the strains are relatively small but the difference can be significant in large deformation problems.

The total displacements of a particle in the two configurations C_0 and C_2 can be written as

$$ {}^2_0u_i = {}^2x_i - {}^0x_i \tag{15} $$

From Eq. (32), the relation between ${}^{2}x_i$ and ${}^{0}x_i$ can be written as follows:

$$ {}^{2}x_i = {}^{0}x_i + {}_{0}^{2}u_i \tag{16} $$

A material line $d\mathbf{L}$ before deformation deforms to the line $d\,\mathbf{l}$ (consisting of the same material as $d\mathbf{L}$) after deformation as follows

$$ d\,\mathbf{l} = {}_{0}^{2}\mathbf{F}\, d\,\mathbf{L} \tag{17} $$

where

$$ {}_{0}^{2}\mathbf{F} = F_{ij}\,{}^{0}\mathbf{e}_i\,{}^{2}\mathbf{e}_j,\ F_{ij} = \frac{\partial\,{}^{2}x_i}{\partial\,{}^{0}x_j} \tag{18} $$

or more explicitly

$$ [F] = \begin{bmatrix} \dfrac{\partial\,{}^{2}x_1}{\partial\,{}^{0}x_1} & \dfrac{\partial\,{}^{2}x_1}{\partial\,{}^{0}x_2} & \dfrac{\partial\,{}^{2}x_1}{\partial\,{}^{0}x_3} \\ \dfrac{\partial\,{}^{2}x_2}{\partial\,{}^{0}x_1} & \dfrac{\partial\,{}^{2}x_2}{\partial\,{}^{0}x_2} & \dfrac{\partial\,{}^{2}x_2}{\partial\,{}^{0}x_3} \\ \dfrac{\partial\,{}^{2}x_3}{\partial\,{}^{0}x_1} & \dfrac{\partial\,{}^{2}x_3}{\partial\,{}^{0}x_2} & \dfrac{\partial\,{}^{2}x_3}{\partial\,{}^{0}x_3} \end{bmatrix} \tag{19} $$

The formulations given by Equations (5) to (19) are adopted from Reddy [10].

3. Solution of the system of equilibrium equations and numerical results

By use of usual assembly process, the system tangent stiffness matrix given in Eq. (1) is obtained by using the element stiffness matrixes given above for the total Lagrangian Finite element model of two dimensional continuum based on the total Lagrangian formulation for a twelve-node quadratic element. In the numerical integrations, five-point Gauss integration rule is used. The material of the beam is linear elastic and isotropic. An eight-node quadratic element was used by Reddy [10] to solve a similar problem: It was noted by Reddy [10] that the stresses could not be obtained truly apart from the Gauss points. Also, it is found by us that the boundary conditions at the free surfaces of the beam can not be satisfied in the eight-node quadratic element. Therefore, a twelve-node quadratic element is used in this study instead of eight-node quadratic element and the boundary conditions at the free surfaces are satisfied perfectly. Convergence analysis is performed for point load for various numbers of finite elements in ${}^{0}x_1$ and ${}^{0}x_2$ directions. When the number of finite elements in ${}^{0}x_1$ direction is $m=35$ and when the number of elements in ${}^{0}x_2$ direction is $n=8$ for the total Lagrangian finite element model of two dimensional continuum for an twelve-node quadratic element, the considered stresses and displacements converge perfectly.

In order to establish the accuracy of the present formulation and the computer program developed by the authors, results obtained from the present study are compared with the available results in the literature. For this purpose, the non-linear static deflections of an isotropic cantilever beam ($L=1000$ in. (25.40 m), $EI=180\times10^3$ kip-in^2 (516541 Nm^2)) under a non-follower point load P at the free end of the beam are compared with data presented in Fertis [21]. The horizontal deflections of the free end is found $u=-4{,}68$ m and Fertis [21] found the horizontal deflections $u=-181.67$ inch (4,61 m) and comparison of the results shows that the present results are comparatively near-by the results of Fertis [21] .

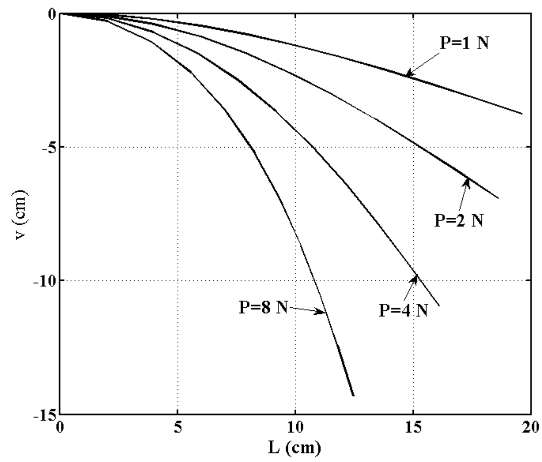


Figure 3. The displaced configuration of the axis of beam for some given point loads.

Fig. 3 shows the deflected shape of the axis of the beam for some given point loads and for L=20 cm, b=1 cm, h=1cm, $E=8300\ N/cm^2$.

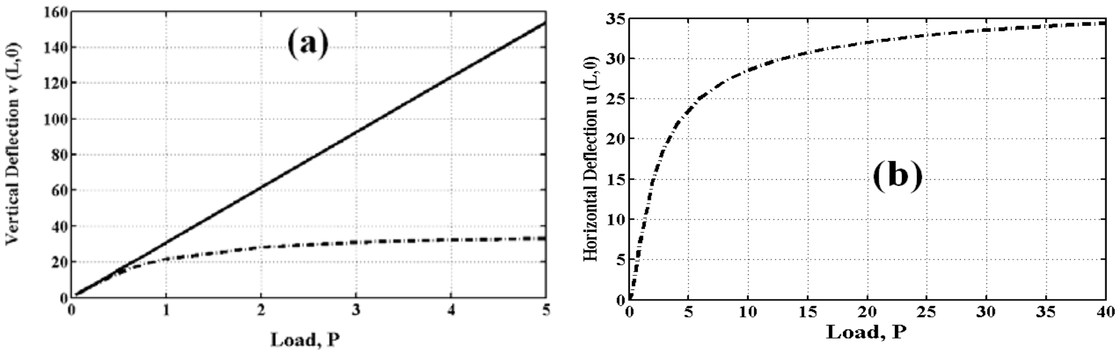
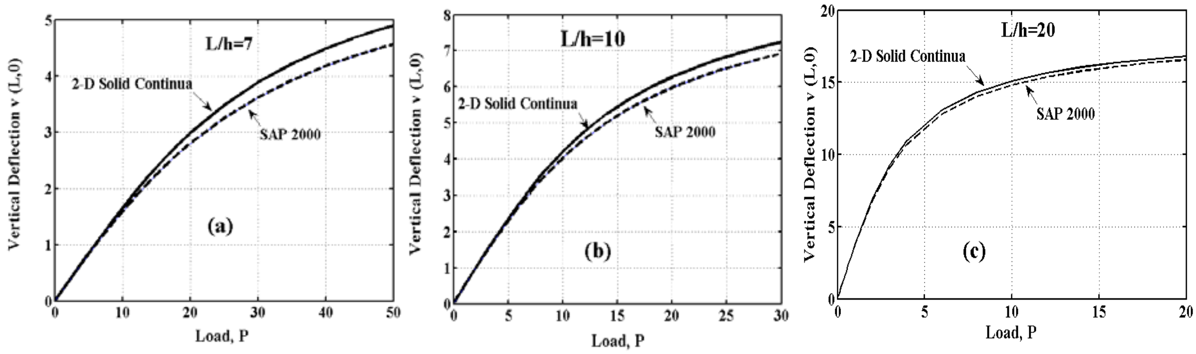


Figure 4. a) Load−vertical displacement ($\mathrm{v}(L,0)$) b) Load− horizontal displacement ($\mathrm{u}(L,0)$) curves for geometrically linear and geometrically nonlinear cases, Lineer (——) ; Nonlineer (−·−·−) .

Fig. 4 shows that increase in load causes increase in difference between the vertical and horizontal displacement values of the linear and the nonlinear solutions for L=20 cm, b=1 cm, h=1 cm and $E=8300\ N/cm^2$. Increase in load is more effective in the vertical and horizontal displacements of the linear solution compared to the geometrically nonlinear case. This situation may be explained as follows: In the linear case, arm of the external forces or arm of the external resultant force do not change with the magnitude of the external forces, and therefore the displacements depend on the external forces linearly. However, in the case of nonlinear analysis, the arm of the external forces change with the magnitude of the external force and, as the magnitude of the force increases the arm of these external forces decrease. However, as the forces increase the configuration of the beam become close to vertical direction and therefore the increase in the load does not cause a significant increase in the displacements after certain load level in which the configuration of the beam is close to the vertical direction. This situation is seen in Fig. 3 which shows the displaced configuration of the beam. After this load, it is expected that axial rigidity of the beam gains more importance than its flexural rigidity.



a) b) c)

Figure 5. Load – vertical displacement ($\mathrm{v}(L,0)$) curves for some given point loads and various ratios of lenght / height (L/h) for beam in the geometrically nonlinear case, a) $L/h=7$, b) $L/h=10$, c) $L/h=20$, 2-D Solid Continua (——); Sap 2000 ($-\cdot-\cdot-$) .

In Fig. 5, very great values of loads are used for obtaining vertical displacements at the free end of the beam in the geometrically nonlinear case. It is seen from Fig. 5 that the difference between the results of two dimensional solid continuum and SAP2000 packet program which uses Timoshenko beam theory increases with decrease in the ratio of length/beam height in the geometrically nonlinear case. It can be said that with decrease in the ratio of L/h, finite element model of two dimensional solid continuum must be used instead of Timoshenko beam theory in the geometrically nonlinear cases.

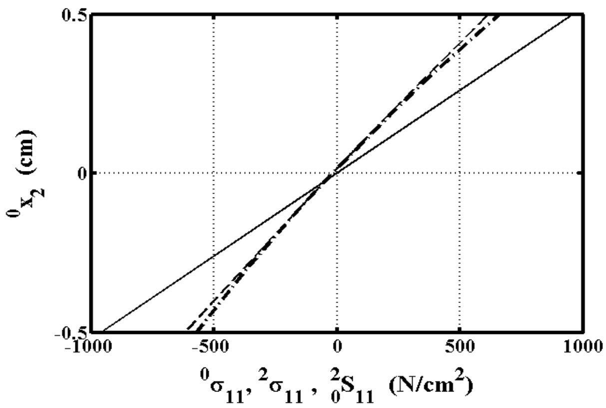


Fig. 6. Normal stresses of the cross section at the fixed end of the beam P=8 N. $^0\sigma_{11}$(——); $^2\sigma_{11}(-\cdot-\cdot-)$; $^2_0S_{11}(---)$.

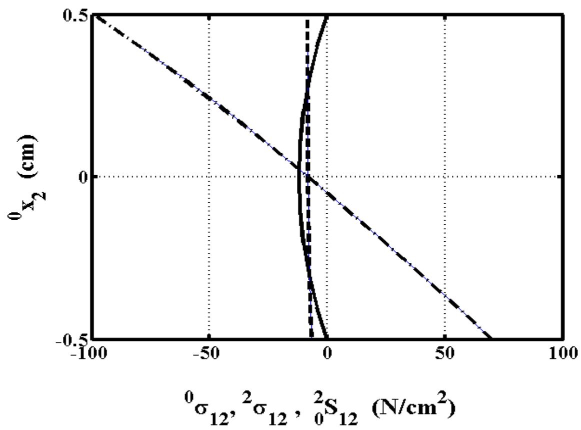


Fig. 7. Shear stresses of the cross section at the fixed end of the beam for P=8 N. $^0\sigma_{12}(\text{——})$; $^2\sigma_{12}(-\cdot-\cdot-)$; $^2_0S_{12}(---)$.

The stress diagrams at $^0x_1 = 0$ along the $^0x_2$ axis (namely for the cross section at the fixed support of the beam) are given in Figs. 6-7 for geometrically nonlinear and linear cases for L=20 cm, b=1 cm, h=1 cm, $E = 8300\ N/cm^2$ and P=8 N. In Figs. 6, 7, the stress distributions for geometrically linear case is denoted by $^0\sigma_{11}$, $^0\sigma_{12}$, the Cauchy stress distributions is denoted by $^2\sigma_{11}$, $^2\sigma_{12}$ and the second Piola-Kirchhoff stress tensor is denoted by $^2_0S_{11}$, $^2_0S_{12}$.

4. Conclusions

The geometrically non-linear static responses of a cantilevered beam subjected to a non-follower point load at the free end of the beam has been studied. In the study, the finite element model of the beam is constructed by using total Lagrangian finite element model of two dimensional solid continua for a twelve-node quadratic element. The considered highly non-linear problem is solved by using incremental displacement-based finite element method in connection with Newton-Raphson iteration method. There is no restriction on the displacements. The effects of the geometric non-linearity on the displacements and on the stresses are investigated. The comparison studies are performed.

It is observed from the investigations that geometrical non-linearity plays very important role on the responses of the beam as the displacements increase. In fact, as it is known, after some values of displacements which can be determined according to the parameters of the problem, it is inevitable to analyze the problem as geometrically non-linear. Also, with decrease in the ratio of beam length/beam height in the geometrically nonlinear case, the difference between the results of two dimensional solid continuum and SAP2000 packet program which uses Timoshenko beam theory increases. Therefore, for small ratios of beam length/beam height, finite element model of two dimensional solid continuum must be used instead of SAP2000 which uses Timoshenko beam theory.

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