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Mathematical Modeling of European Countries' Telecommunication Investments

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Abstract: This study investigates the amounts of countries' telecommunication investments and seeks a decent method to model the data mathematically. By using fractional calculus, two methods are proposed, called model 1 and model 2 in the study. A comparison is performed between the conventional polynomial model and both models 1 and 2 using the yearly data of telecommunication investments from France, Germany, Italy, Spain, Turkey, and the OECD total. The proposed methods outperform the conventional polynomial model.

Keywords: Fractional calculus, MAPE, telecommunications investments

Avrupa Ülkelerinin Haberleşme Sektöründeki Yatırımlarının Matematiksel Modellenmesi

Öz: Bu çalışma, ülkelerin telekomünikasyon yatırımlarının miktarlarını araştırmakta ve verileri matematiksel olarak modellemek için uygun bir yöntem aramaktadır. Çalışmada kesirli kalkülüs kullanılarak model 1 ve model 2 olarak adlandırılan iki yöntem önerilmiştir. Fransa, Almanya, İtalya, İspanya, Türkiye ve OECD toplamından elde edilen telekomünikasyon yatırımlarının yıllık verileri kullanılarak geleneksel polinom modeli ile 1 ve 2 modelleri arasında bir karşılaştırma yapılmıştır. Önerilen yöntemler, geleneksel polinom modelinden daha iyi performans gösterdiği gözlenmiştir.

Anahtar Kelimeler: haberleşme sektöründeki yatırımlar, kesirli türev, MAPE

1. Introduction

While the internet is becoming more common worldwide, the impacts of telecommunication technologies scaled up [1-3]. The sector strengthened its place as one of the economy's flagships. There are plenty of hypotheses to promote investments in telecommunications which contain various amounts of competition and regulation [2-5]. Telecommunication investments may lead to positive progress in economic growth and productiveness in the country, reducing the costs of market entry facilitating business relationships to build [1]. One can also say that the problem of inadequate investment has detrimental effects such as lack of competition an increase in market abdication, leading to a loss in the other sectors [6-8].

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The study pushes forward a mathematical approach that analyses and examines the factors continuously related to telecommunications investments with the assistance of the theory of fractional calculus [9,10]. The concept of fractional calculus can be mentioned as the former, but it is still a current topic. Over 300 years, fractional calculus found a solid place in the literature. In the near past, mathematicians, physicists, chemists, engineers, economists, and social scientists have shared an unignorable interest in this topic [11-15]. With the help of fractional calculus, more preferable curves which reduce the errors are prepared. Evidence from 6 different OECD countries throughout the 1997-2018 time period is used for the investigation [6,14].

France, Germany, Italy, Spain, Turkey, and the OECD are taken to consider in the present research because these five countries and OECD total hold some critical similarities in terms of economic development, geographical structure, competition for foreign direct investment, and population density, yet acquire particular cultural and origination atmospheres and pursue one of a kind regulatory policies and economic approaches that have impacts on telecommunication investments [12-17].

The structure of the study is as follows. In Section 2, the problem statement and the mathematical background are presented. Later, in Section 3, the numerical results are given according to the proposed approach. Finally, the conclusion is drawn.

2. Problem Statement

The study applies two different modeling techniques using fractional calculus to model the discrete data that we have as a continuous curve representing the discrete data with minimum error. Section below includes both mathematical background of model 1 and model 2. The following section compares the proposed models and the polynomial model [8].

3. Mathematical Background

In this section, two models are presented. These models employ the fractional-order differential equations and the least-squares method. Both models are originated from similar approaches. The derivations and detailed explanations are given for each model below.

3.1. Fractional Model-1

First, a continuous and differentiable function $g(x)$ is expanded as the Taylor Series.

$$g(x) = \sum_{n=1}^{\infty} a_n (x)^n \tag{1}$$

Then, the first derivative of equation (1) becomes as:

$$g'(x) = \sum_{n=0}^{\infty} a_n n (x)^{n-1} = \sum_{n=1}^{\infty} a_n n (x)^{n-1}. \tag{2}$$

Here, $g'(x)$ stands for the first derivative of $g(x)$ with respect to x and from equation (2), our purpose is to generalize the derivative of the function, and we propose that α'th derivative of another function $f(x)$ is equal to the right-hand side of equation (2). By generalizing the derivative order, the model gains another parameter for optimum modeling [18,19].

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}} = \sum_{n=1}^{\infty} a_n n x^{n-1} \tag{3}$$

where $f(x)$ is the function that we want to find and $\frac{d^{\alpha}f(x)}{dx^{\alpha}}$ means that α^{th} order derivative of $f(x)$ function.

Here, $f(x)$ corresponds to Telecommunication Investments in a specific time interval. Laplace Transform, which is denoted as $\mathcal{L}$ can be used for solving the equation (3),

$$\mathcal{L}\left\{\frac{d^{\alpha}f(x)}{dx^{\alpha}}\right\} = \mathcal{L}\left\{\sum_{n=1}^{\infty} a_n n x^{n-1}\right\} \tag{4}$$

where $0 < \alpha < 1$ is assumed. According to the Laplace Transform Table [9,18,19],

$$\mathcal{L}\left\{\frac{d^{\alpha}f(x)}{dx^{\alpha}}\right\} = s^{\alpha}F(s) - s^{\alpha-1}f(0) \tag{5}$$

$$\mathcal{L}\{x^{n-1}\} = \frac{\Gamma(n)}{s^n} \tag{6}$$

where, $F(s) = \mathcal{L}\{f(x)\}$ and Γ is the Gamma function. The goal here is to reduce the differential equations to an algebraic equation. Laplace Transform of equation (3) is,

$$s^{\alpha}F(s) - s^{\alpha-1}f(0) = \sum_{n=1}^{\infty} a_n n \frac{\Gamma(n)}{s^n} \tag{7}$$

Equation (7) can be rewritten as,

$$F(s) = \frac{f(0)}{s} + \sum_{n=1}^{\infty} a_n \frac{\Gamma(n+1)}{s^{n+\alpha}} \tag{8}$$

If we take the inverse Laplace Transform, we find $f(x)$:

$$\mathcal{L}^{-1}\{F(s)\} = f(x) \tag{9}$$

$$\mathcal{L}^{-1}\left\{\frac{f(0)}{s}\right\} = f(0) \tag{10}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^{n+\alpha}}\right\} = \frac{x^{n+\alpha-1}}{\Gamma(n+\alpha)} \tag{11}$$

which yields,

$$f(x) = f(0) + \sum_{n=1}^{\infty} a_n \frac{\Gamma(n+1)x^{\alpha+n-1}}{\Gamma(n+\alpha)} \tag{12}$$

After finding $f(x)$ in the form of equation (12), the only unknowns are $f(0)$, a_n constant coefficients. To find these unknowns, we use the Least Squares Method. For the computation, summation in equation (12) is truncated to N as denoted below,

$$f(x) \cong f(0) + \sum_{n=1}^{N} a_n \frac{\Gamma(n+1)x^{\alpha+n-1}}{\Gamma(n+\alpha)} \tag{13}$$

After obtaining the general expression for $f(x)$, it is time to model our given data. Note that the number of unknowns is *N+1*. To employ regression and curve-fitting on the dataset, first, the following notations are presented in equation (14),

$$P_K = [P_0 P_1 \dots P_K]\,, \quad x_K = [x_0 x_1 \dots x_K] \tag{14}$$

Here, x_i represents the time, and P_i corresponds to the investment amounts. The error between the value P_i and $f(x_i)$ is shown as ε_i,

$$(\varepsilon_i)^2 = (p_i - f(x_i))^2 \tag{15}$$

Square of the errors' summation,

$$\varepsilon_T^2 = \sum_{i=0}^{K}\left[p_i - \left\{f(0) + \sum_{n=1}^{N} a_n \frac{\Gamma(n+1)x_i^{\alpha+n-1}}{\Gamma(n+\alpha)}\right\}\right]^2 \tag{16}$$

The least-squares method is employed to obtain a minimum error. Therefore, the set of equations given in equation (17) needs to be satisfied. By equation (17), there exist *N*+1 equations regarding *N*+1 numbers of unknowns. Then, this system can be solved [4, 18].

$$\frac{\partial \varepsilon_T^2}{\partial f(0)} = 0, \frac{\partial \varepsilon_T^2}{\partial a_1} = 0, \frac{\partial \varepsilon_T^2}{\partial a_2} = 0 \dots, \frac{\partial \varepsilon_T^2}{\partial a_N} = 0 \tag{17}$$

Equations (16) and (17) must be satisfied to reduce the total error. After that, a system of linear algebraic equations (SLAE) is found,

$$[\mathrm{A}]_{\mathrm{N+1\times N+1}}\,[\Omega]_{\mathrm{N+1\times 1}} = [\mathrm{B}]_{\mathrm{N+1\times 1}} \tag{18}$$

where,

$$[\mathrm{A}] = \begin{bmatrix} k+1 & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{\alpha} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{\alpha+1} & \cdot\cdot & \frac{N!}{\Gamma(\alpha+N)}\sum_{i=0}^{K} x_i^{\alpha+N-1} \\ \sum_{i=0}^{K} x_i^{\alpha} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{2\alpha} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{2\alpha+1} & \cdot\cdot & \frac{N!}{\Gamma(\alpha+N)}\sum_{i=0}^{K} x_i^{2\alpha+N-1} \\ \sum_{i=0}^{K} x_i^{\alpha+1} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{2\alpha+1} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{2\alpha+2} & \cdot\cdot & \frac{N!}{\Gamma(\alpha+N)}\sum_{i=0}^{K} x_i^{2\alpha+N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum_{i=0}^{K} x_i^{\alpha+N-1} & \frac{1}{\Gamma(\alpha+1)}\sum_{i=0}^{K} x_i^{2\alpha+N-1} & \frac{2.1!}{\Gamma(\alpha+2)}\sum_{i=0}^{K} x_i^{2\alpha+N} & \cdot\cdot & \frac{N!}{\Gamma(\alpha+N)}\sum_{i=0}^{K} x_i^{2(\alpha+N-1)} \end{bmatrix}$$

$$[\Omega] = [f(0)\ a_1\ a_2 \dots a_N]^T$$

$$[\mathrm{B}] = \left[\sum_{i=0}^{K} P_i \sum_{i=0}^{K} P_i x_i^{\alpha} \sum_{i=0}^{K} P_i x_i^{\alpha+1} \dots \sum_{i=0}^{K} P_i\, x_i^{\alpha+N-1}\right]^T \tag{19}$$

Here, the transpose of the matrix is denoted as T. The coefficients that are unknown in the Ω vector can be found by,

$$[\Omega]_{\mathrm{N+1\times 1}} = [\mathrm{A}]^{-1}_{\mathrm{N+1\times N+1}}[\mathrm{B}]_{\mathrm{N+1\times 1}} \tag{20}$$

where $[A]^{-1}$ is the inverse of [A] matrix.

3.2. Fractional Model-2

A continuous function $g(x)$ can be represented as a Taylor Series [16-19],

$$g(x) = \sum_{n=0}^{\infty} a_n (x)^{n+\alpha} \tag{21}$$

$$g'(x) = \sum_{n=0}^{\infty} a_n (n+\alpha) x^{n+\alpha-1} \tag{22}$$

From $g(x)$ and $g'(x)$ functions, the fractional derivative of $f(x)$ function can be inspired as,

$$\frac{d^{\alpha} f(x)}{dx^{\alpha}} = \sum_{n=0}^{\infty} a_n (n+\alpha) x^{n+\alpha-1} \tag{23}$$

In Equation (23), the right-hand side of equation (22) is assumed to equal the α^{th} derivative of $f(x)$. Here, $f(x)$ can be any dataset. In the present work, it is again, Telecommunication Investments. We choose a range of 0< α<1 for α. To convert the differential equation into the algebraic equation, the Laplace Transform is employed.

$$\mathcal{L}\{x^{\alpha}\} = \frac{\Gamma(\alpha+1)}{s^{\alpha+1}} \tag{24}$$

where Gamma function is denoted as Γ. The Laplace Transform of the equation (23) is,

$$\mathcal{L}\left\{\frac{d^{\alpha} f(x)}{dx^{\alpha}}\right\} = s^{\alpha} F(s) - s^{\alpha-1} f(0) = \sum_{n=0}^{\infty} a_n \frac{\Gamma(n+\alpha+1)}{s^{n+2\alpha}} \tag{25}$$

Then, we take the inverse Laplace Transform [18,19]:

$$\mathcal{L}^{-1}\{F(s)\} = f(x) \cong f(0) + \sum_{n=0}^{N-1} a_n \frac{\Gamma(n+\alpha+1)}{\Gamma(n+2\alpha)} x^{n+2\alpha-1} \tag{26}$$

We can truncate equation (23) to $N-1$ as given in equation (26). This yields the same number of unknowns as the previous method (Model 1). Now, the only unknowns are $f(0)$ and a_n. To find them, Least Squares Method can be used. Our dataset,

$$P_K = [P_1 P_2 \ldots P_K], \quad x_K = [x_1 x_2 \ldots x_K] \tag{27}$$

The error between the value P_i and $f(x_i)$ is shown as ε_i

$$(\varepsilon_i)^2 = (\mathrm{p}_i - f(x_i))^2 \tag{28}$$

Square of the errors' summation [18],

$$\varepsilon_{\mathrm{T}}^2 = \sum_{i=1}^{K} (\varepsilon_i)^2 \tag{29}$$

$$(\varepsilon_k)^2 = \left[p_k - \left\{f(0) + \sum_{n=0}^{N-1} a_n \frac{\Gamma(n+\alpha+1)x^{2\alpha+n-1}}{\Gamma(n+2\alpha)}\right\}\right]^2 \quad (30)$$

To minimize the error, we take derivatives according to the unknowns and assume they equal 0 [19].

$$\frac{\partial \varepsilon_T^2}{\partial f(0)} = 0, \quad \frac{\partial \varepsilon_T^2}{\partial a_0} = 0, \qquad \ldots\ldots, \frac{\partial \varepsilon_T^2}{\partial a_{N-1}} = 0 \quad (31)$$

Then, one can obtain the Matrix of,

$$[\mathrm{A}]_{\mathrm{N+1\times N+1}}\,[\Omega]_{\mathrm{N+1\times 1}} = [\mathrm{B}]_{\mathrm{N+1\times 1}} \quad (32)$$

$$[\Omega] = [\mathrm{A}]^{-1}[\mathrm{B}] \quad (33)$$

$$[\mathrm{A}] = \begin{bmatrix} K+1 & \sum_{i=1}^{k} c_0(x_i) & \sum_{i=1}^{k} c_1(x_i) & .. & \sum_{i=0}^{K} c_{N-1}(x_i) \\ \sum c_0(x_i) & \sum c_0(x_i)c_0(x_i) & \sum c_0(x_i)c_1(x_i) & .. & \sum c_0(x_i)c_{N-1}(x_i) \\ \sum c_1(x_i) & \sum c_1(x_i)c_0(x_i) & \sum c_1(x_i)c_1(x_i) & .. & \sum c_1(x_i)c_{N-1}(x_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \sum c_{N-1}(x_i) & \sum c_{N-1}(x_i)c_0(x_i) & \sum c_{N-1}(x_i)c_1(x_i) & .. & \sum_{i=1}^{K} c_{N-1}(x_i)c_{N-1}(x_i) \end{bmatrix}$$

$$[\Omega] = [f(0)\ a_0\ a_1\ a_2 \ldots a_{N-1}]^T$$

$$[\mathrm{B}] = \left[\sum_{i=0}^{K} P_i \sum_{i=0}^{K} P_i c_0(x_i) \sum_{i=0}^{K} P_i c_1(x_i) \ldots \sum_{i=0}^{K} P_i c_{N-1}(x_i)\right]^T$$

where,

$$c_m(x,\alpha) = \frac{\Gamma(m+\alpha+1)}{\Gamma(m+2\alpha)}\mathrm{x}^{2\alpha+m-1} \quad (34)$$

Here $m = 0, 1, \ldots N-1$.

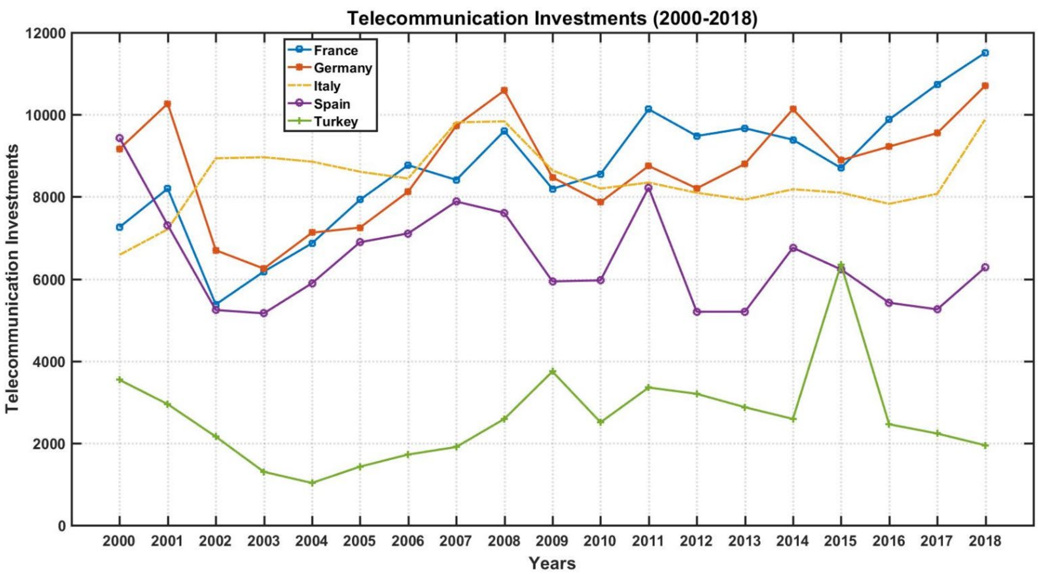

Figure 1. Telecommunication Investments of the countries

4. Experimental Data

From the dataset, countries' telecommunication investment values are used to model. See (Figure 1) and (Table A.1) in [6]. 6 European countries are selected among 35 different countries from all over the world to be modeled.

5. Results and Discussion

Fractional Model and Polynomial Model are used and compared for ideally modeling the dataset with different exponent numbers N =5 and N =10 from equation (13). The comparison between models is made by the Mean Absolute Percentage Error Method with equation (35). Mean Absolute Percentage Error (MAPE) is formulated as follows,

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{P(i) - f(x_i)}{P(i)}\right| \times 100 \tag{35}$$

where $P(i)$ is the real value from the dataset and $f(x_i)$ is the predicted value. To achieve the best comparison between models, MAPE values were calculated for each model. In (Table 1), the MAPE and corresponding optimized α values can be found for each polynomial model [8] and fractional models [10] for two different N values 5 and 10.

From these results, it can be obtained that the fractional models are better for modeling the data than the polynomial model. However, the fractional models' results are so close to each other one of them shines out more. (Table 1) makes it easy to realize that the increase of the exponent number N affects the majority of the results positively. Germany's MAPE values are 10.3822, 10.3804, and 10.5401 for models 1, 2, and polynomial models, respectively. For the case of Germany, the models performed well with the higher N value, yet in Italy's case, it has not performed the same. Italy has the MAPE of 4.1554 for Model 1 and 2 and 4.1859 for polynomial.

Tablo 1. Modeling results of telecommunication investments (current USD millions).

	MODELS	**Results**	**France**	**Germany**	**Italy**	**Spain**	**Turkey**	**UK**
N=5	**Fractional Model-1**	MAPE	9.9032	10.3822	4.1554	14.4371	12.2359	17.5792
		α value	1	0.01	0.613	0.228	0.01	0.01
	Fractional Model-2	MAPE	9.9032	10.3804	4.1554	14.4371	12.1639	17.5774
		α value	1	0.501	0.806	0.614	0.501	0.501
	Polynomial	MAPE	9.9032	10.5401	4.1859	16.3108	32.5308	19.5389
N=10	**Fractional Model-1**	MAPE	5.5037	5.0291	4.2305	8.6983	6.1121	8.7691
		α value	0.01	0.01	1	1	0.01	1
	Fractional Model-2	MAPE	5.4964	5.0250	4.2305	8.6983	6.1015	8.7691
		α value	0.501	0.501	1	1	0.501	1
	Polynomial	MAPE	6.2900	5.4441	4.2305	8.6983	8.2000	8.7688

As seen in (Table 1), Turkey has a 12.2359 Mape value obtained by Model 1 12.1639 Mape value for Model 2 whereas, the Polynomial model gives MAPE as 32.5308 with N=5. Meanwhile, the α values of Fractional Model 1 and 2 are 0.01 and 0.501, respectively. According to the results from (Table 1) and (Figure 1), it can be observed that in most cases, Model 2 slightly steps forward.

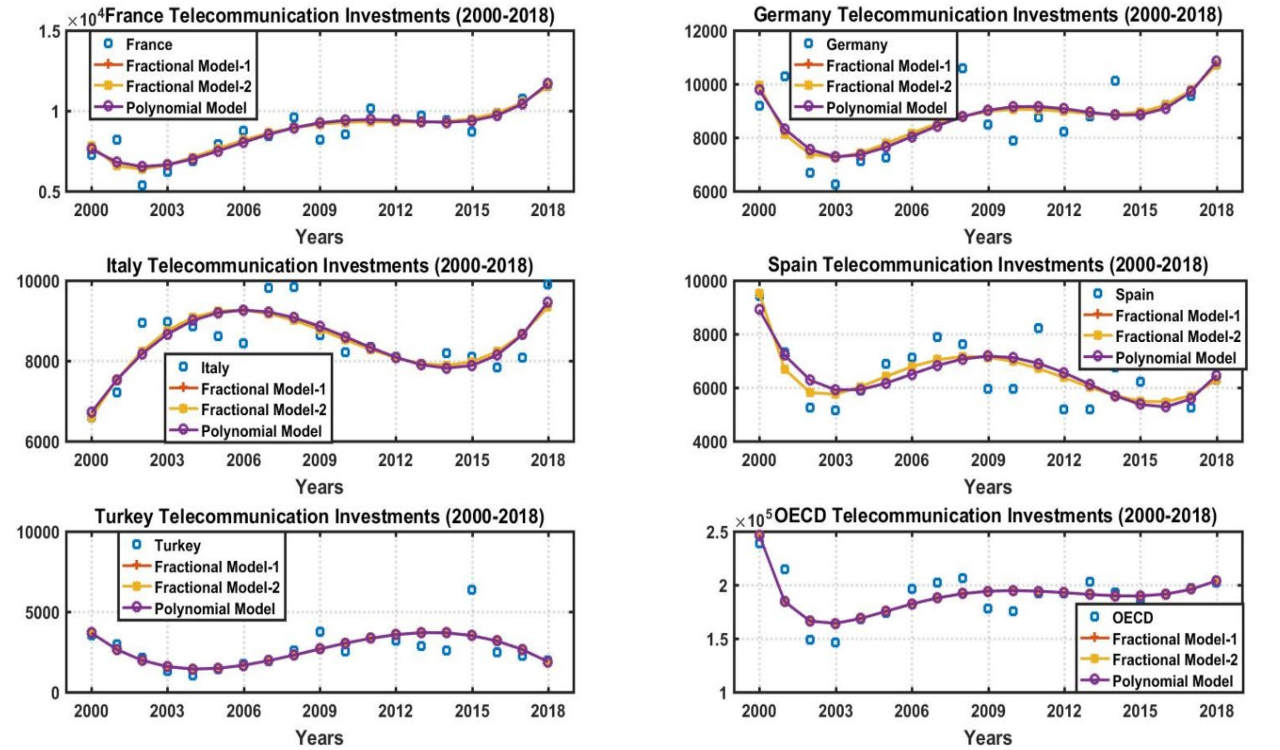

Figure 2. Modeling of the countries for N=5

The graphs in (Figure 1) and (Figure 2) show that some countries are more suitable to be modeled according to their telecommunication investments, among others. There is a noticeable difference between the results in various aspects: the number of exponents, model type, and countries themselves. The best results have occurred with the N=10 exponent value as given in (Figure 3). As expected, when the fractional order equals 1, all models become the same and give the same error. Including the fractional calculus leads to more flexible modeling than the conventional polynomials method, which utilizes only first derivatives in modeling.

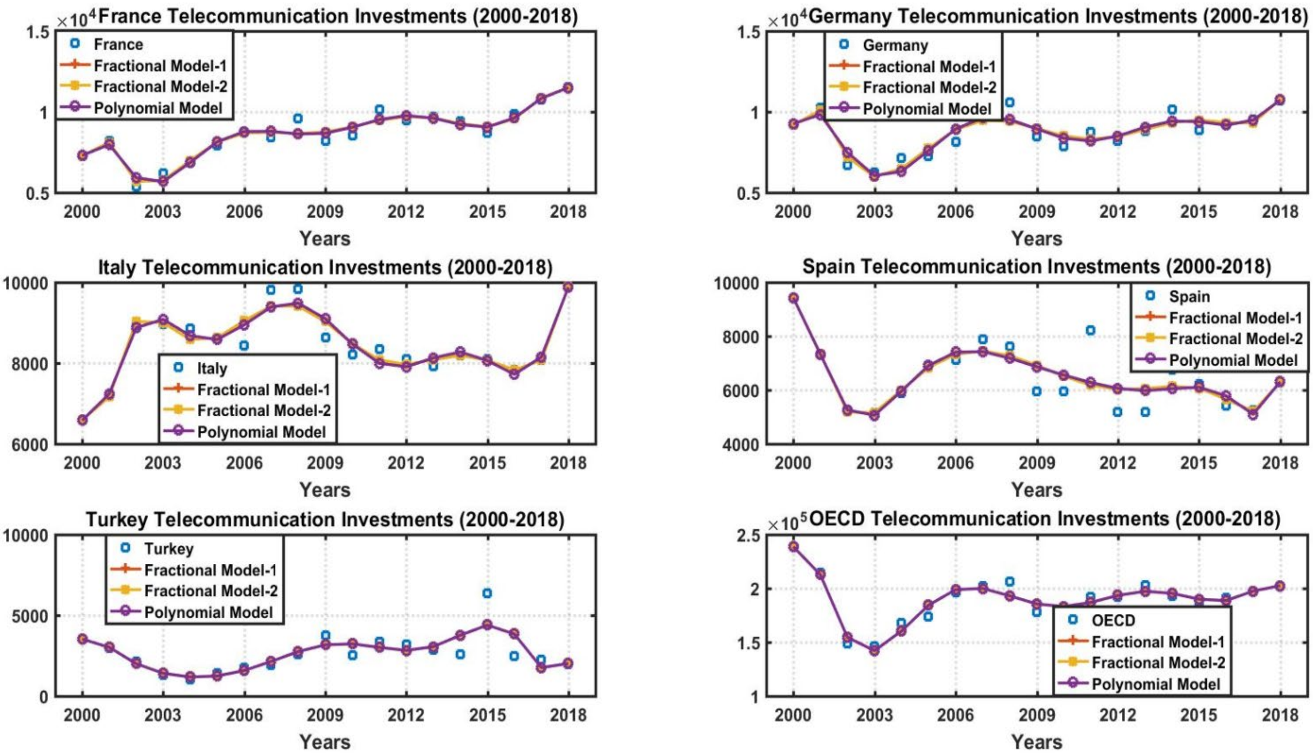

Figure 3. Modeling of the countries for N=10.

The fractional models are more accurate than polynomial according to (Figure 1) and (Figure 2) as mentioned before. Finally, some countries yield preferable predictions. Turkey's modeling can be used as an example because proposed fractional models outperformed and caused better results.

6. Conclusion

The paper focuses on mathematically modeling the investment amounts of telecommunication sector of countries. The values include only year-based numbers, which form a discrete function. Although the data may seem detailed and adequate, they lack much certainty in yearly intervals. These gaps create challenges for scientists, mathematicians, economists, and analysts.

In the case of telecommunications, it is pretty vital to acquire qualified and satisfactory data for governments to arrange the regulations, for economists to analyze and make predictions, and for companies to plan how, when, and where to enter or pull out of the market or start their investments, employment, and dismissals.

To eliminate the uncertainty from the discrete data, two mathematical models are proposed and compared in the paper. After the modeling process, continuous curve predictions are obtained instead of discrete and assumably deficient data. Throughout the study, the computing environment MATLAB is used for calculation realizations and generating graphs.

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Authors' Contributions

KK took part in the mathematical part of the study, in the examination of the manuscript. SAS was responsible for writing the article and finding the data. EE contributed to the literature review. NÖT took part in editing the article, numerical calculations and software development. BG played a role in writing the article and finding data. EK made important contributions to theoretical studies and determination of the subject. The all authors have read and approved the final version of the article.

Conflict of Interests

The authors reported no conflict of interest between the authors and their respective institutions.

References

[1]. Aytun C., Akın C. S., and Okyay U., Relationship between telecommunication investments and foreign direct investments in developing and developed countries, Ege Acad. Rev., 2015, 15(2), 207–216.
[2]. Beil R. O., Ford G. S., and Jackson J. D., On the relationship between telecommunications investment and economic growth in the United States, Int. Econ. J., 2005, 19(1), 3–9.
[3]. Kotakorpi K., Access price regulation, investment, and entry in telecommunications, Int. J. Ind. Organ., 2006, 24(5), 1013–1020.
[4]. Önal N. Ö., Karaçuha K., Erdinç G. H., Karaçuha B. B., and Karaçuha E., A mathematical approach with fractional calculus for the modeling of children's physical development, Comput. Math. Methods Med., 2019, 2019.
[5]. Machado J. T., Kiryakova V., and Mainardi F., Recent history of fractional calculus, Commun. nonlinear Sci. Numer. Simul., 2011, 16(3), 1140–1153.

[6]. Key ict indicators, OECD. [Online]. Available: https://www.oecd.org/digital/broadband/oecdkeyictindicators.htm. [Accessed: 15-Sep-2021].
[7]. Doh J. P. and Teegen H. J., Private telecommunications investment in emerging economies: Comparing the Latin American and Asian experience, Manag. Res. J. Iberoam. Acad. Manag., 2003.
[8]. Royston P. and Altman D. G., Regression using fractional polynomials of continuous covariates: parsimonious parametric modelling, J. R. Stat. Soc. Ser. C Applied Stat., 1994, 43(3), 429–453.
[9]. Podlubny I., Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications, Elsevier, 1998.
[10]. Loverro A., Fractional calculus: history, definitions and applications for the engineer, Rapp. Tech. Univeristy Notre Dame Dep. Aerosp. Mech. Eng., 2004, 1–28,.
[11]. Chang H., Koski H., and Majumdar S. K., Regulation and investment behaviour in the telecommunications sector: Policies and patterns in US and Europe, Telecomm. Policy, 2003, 27(10–11), 677–699.
[12]. Henisz W. J. and Zelner B. A., The institutional environment for telecommunications investment, J. Econ. Manag. Strateg., 2001, 10(1), 123–147.
[13]. Madden G. and Savage S. J., CEE telecommunications investment and economic growth, Inf. Econ. Policy, 1998, 10(2), 173–195.
[14]. Roller L. H. and Waverman L., Telecommunications infrastructure and economic development: A simultaneous approach, Am. Econ. Rev., 2001, 91(4), 909–923.
[15]. Karaçuha K., Tabatadze V., and Veliev E. I., Plane wave diffraction by strip with an integral boundary condition, Turkish J. Electr. Eng. Comput. Sci., 2020, 28(3), 1776–1790.
[16]. Karaçuha E. et al., Modeling and Prediction of the Covid-19 Cases With Deep Assessment Methodology and Fractional Calculus, IEEE Access, 2020, 8, 164012–164034.
[17]. E. Karaçuha, V. Tabatadze, K. Karaçuha, N. Ö. Önal, and E. Ergün, Deep Assessment Methodology Using Fractional Calculus on Mathematical Modeling and Prediction of Gross Domestic Product per Capita of Countries, Mathematics, 2020, 8(4), 633.
[18]. Önal, N. Ö., Karacuha, K., and Karacuha, E., A Comparison of Fractional and Polynomial Models: Modelling on Number of Subscribers in the Turkish Mobile Telecommunications Market, Int. J. Appl. Phys. Math., 2019, 10.
[19]. Önal, N. Ö., Karacuha, K., & Karacuha, E., Modelling on Economic Growth and Telecommunication Sector of Turkey Using a Fractional Approach Including Error Minimizing, in 3rd Asia-Pacific Conference on Applied Mathematics and Statistics (AMS 2020), Feb. 2020.