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INVESTIGATION OF EFFECT THERMAL CONDUCTIVITY ON STRAIGHT FIN PERFORMANCE WITH DTM

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Abstract

In this study, considered new type of temperature dependent thermal conductivity and problem is solved analytically with differential transformation method (DTM). In this paper suppose thermal conductivity varies with temperature exponentially, and investigated effect of thermal conductivity on fin performance, and results compared with case that thermal conductivity varies with temperature linearly. The obtained differential transformation approximate analytic solution is in the form of an infinite power series, so that with obtained explicit form of temperature profile, the fin tip temperature, fin base heat transfer rate, and fin efficiency can be calculated directly from temperature profile easily. Temperature profile obtained for several assigned value of exponential parameter, that shown effect of exponential parameter on fin performance. In this paper for comparison of results, problem solved numerically with fourth-order Range-Kutta method that results of numerical have good agreement with results of DTM, as well as DTM results indicate that series converge rapidly with high accuracy.

Keyword: differential transformation method, exponentially, fin, numerical solution, heat transfer

1. Introduction

One of the most problems in the engineering is heat transfer, and fins play important role in heat transfer problems. Major of physical phenomena in the real world are described by nonlinear differential equation, and nonlinear phenomena play important role in mathematics. Large class of equations does not have an analytical solution, so that numerical methods have largely been used to handle these equations. There are some analytic method for solve of differential equation, such as Adomian decomposition method (ADM), HAM, sinh-cosh method, homotopy perturbation method (HPM), DTM and variational iteration method (VIM). The concept of differential transformation method was first introduced by Zho [1] in 1986 and it was used to solve both linear and nonlinear initial value problems in electric circuit analysis. The main advantage of this method is that it can be applied directly for linear and nonlinear differential equation without requiring linearization, discretization, or perturbation. In this study, suppose thermal conductivity varies with temperature exponentially, and investigated effect of exponential parameter on fin performance, results shown for efficiency, fin base heat transfer rate and temperature profile. In the previous studies, thermal conductivity varies with temperature linearly. Numerous studies have devoted to the analysis of fin performance of this type of problems due to its important application in engineering.Chang [2] solved a decomposition solution for temperature dependent surface heat flux, Joneidi, Ganji and Babaelahi [3] solved differential transformation method to determined fin efficiency of convective straight fins with temperature dependent thermal conductivity, that in this study used of results [3] for comparing with present results than investigate effect of thermal conductivity on straight fin performance. Liaw and Yeh [4] used the same model and further studies all possible type of heat transfer including the cases of film and transition boiling with and without heat transfer at fin tip, Abbasbandy and shivanian [5] made exact analytical solution of a nonlinear equation arising in heat transfer. Khani, ahmadzade Raji, and Hamidi Nejad [6] used HAM to evaluate the analytical approximate solution and efficiency of the nonlinear fin problem with temperature dependent thermal conductivity and heat transfer coefficient. Rashidi and Erfani [7] used DTM for solved fin efficiency of convective straight fins with temperature dependent thermal conductivity and comparison results with HAM, Hsiang Chang and Ling Chang [8,9] use of new algorithm for calculation one and two-dimensional differential transform of nonlinear functions, Keshin [10] using of reduced differential transformation method for solving gas dynamic problem, Chen and Ju [11] used of differential transformation to transient advective-dispersive transport equation. Jang [12] solving linear and nonlinear initial value problems by the projected differential transform method, that this method can be easily applied to the initial value problem by less computational work. A. H. Hassan [13] used of DTM for solving eigenvalue problems, that vibration problems is one example of eigenvalue problems in engineering. Arslanturk [14] and Rajabi [15] used the ADM and HPM to evaluate the efficiency of straight fins with temperature dependent thermal conductivity and to determine the temperature distribution within the fin. Franco [16] present analytical method for the optimum thermal design of convective longitudinal fin arrays. Lee and Lin [17] investigated boiling on a straight fin with variable thermal conductivity that thermal conductivity varies with temperature linearly. Recently, differential transformation method has been used to solve a wide range of physical problem. This method provides a direct scheme for solving linear and nonlinear deterministic and stochastic equation without the need for linearization and yield rapidly convergent series solution. In this study is to apply differential transformation method to investigate a straight fin governed by temperature dependent thermal conductivity that conductivity varies with temperature exponentially and results compared with [3] for investigation of effect conductivity on straight fin performance. Their results showed that the differential transformation method has many merits including fast convergence and high accuracy. Base of DTM, temperature on the fin surface can be expressed explicitly as a function of position along the fin, the effect of exponential function of conductivity and fin parameter on temperature profile as well as fin tip temperature can also be obtained quickly. In addition to, heat transfer rate and fin efficiency are presented in detail.

2. Fundamentals of differential transformation method

We suppose $y(t)$ to be analytic function in a domain *D* and $t=t_i$ represented any point in *D*. The function $y(t)$ is the represented by one power series whose center is located at t_i. The Taylor series expansion function of $y(t)$ is of the form [3]

$$y(t)=\sum_{j=0}^{\infty}\frac{(t-t_i)^j}{j!}\left[\frac{d^j y(t)}{dt^j}\right]_{t=t_i} \qquad \forall t\in D \tag{1}$$

The particular case of Eq. (1) when $t_i=0$ is referred to as the Maclurin series of $y(t)$ and is expressed as:

$$y(t)=\sum_{j=0}^{\infty}\frac{t^j}{j!}\left[\frac{d^j y(t)}{dt^j}\right]_{t=0} \qquad \forall t\in D \tag{2}$$

As explained in [16] that differential transformation method of the function $y(t)$ is defined as follow:

$$Y(j)=\sum_{j=0}^{\infty}\frac{H^j}{j!}\left[\frac{d^j y(t)}{dt^j}\right]_{t=0,} \tag{3}$$

Where $y(t)$ is the original function and $Y(j)$ is the transformed function. The differential spectrum of the $Y(j)$ is confined within the interval $t\in[0,H]$, where H is a constant. The differential inverse transform of $Y(j)$ is defined as follow:

$$y(t)=\sum_{j=0}^{\infty}\left(\frac{t}{H}\right)^j Y(j), \tag{4}$$

Some of the original function and transformed function is shown in Table 1. It is clear that the concept of differential transformation is Taylor series expansion. For assigned solution with high accuracy may be calculated more number of series in Eq. (4).

Table 1. The fundamental operations of differential transform method.

Original function	Transformed function
$f(x)=\alpha h(x)\pm\beta g(x)$	$F(j)=\alpha H(j)\pm\beta G(j)$
$f(x)=\frac{d\theta}{dx}$	$F(j)=(j+1)\Theta(j+1)$
$f(x)=\frac{d^2\theta}{dx^2}$	$F(j)=(j+1)(j+2)\Theta(j+2)$
$f(x)=h(x)g(x)$	$F(j)=\sum_{i=0}^{j}H(i)G(j-i)$
$f(x)=exp\,(\gamma x)$	$F(j)=\frac{\gamma^j}{j!}$

$f(x) = (1+x)^m$	$F(j) = \frac{j(j-1)\cdots(j-m-1)}{j!}$
$f(x) = x^n$	$F(j) = \delta(j-n) = \begin{Bmatrix} 1 & j = n \\ 0 & j \neq n \end{Bmatrix}$
$f(x) = \cos(\omega x + \beta)$	$F(j) = \frac{\omega^j}{j!} \cos(\pi \frac{j}{2!} + \beta)$
$f(x) = \sin(\omega x + \beta)$	$F(j) = \frac{\omega^j}{j!} \sin(\pi \frac{j}{2!} + \beta)$
$f(x) = \int_{x_0}^{x} U(t)\,dt$	$F(j) = \frac{U(j-1)}{j}, \quad j \geq 1$

3. Description of the problem

Consider a straight fin of length L with a uniform cross-section area A. The fin surface is exposed to a convective environment at temperature T_∞ and the local heat transfer coefficient h along the fin surface is constant. Thermal conductivity varies with temperature exponentially. The one dimensional energy equation can be expressed as

$$A\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - ph(T - T_\infty) = 0, \tag{5}$$

In the above equation, p is periphery of the fin, T_∞ is ambient temperature, and $k(T)$ defined follow as

$$k(T) = k_b e^{\beta(\frac{T - T_\infty}{T - T_b})} \tag{6}$$

In the above equation k_b is the thermal conductivity at the ambient fluid temperature of the fin, β is a constant of exponential function, T_b is the fin base temperature, with employing the following dimensionless parameters:

$$\theta = \frac{T - T_\infty}{T - T_b}, \qquad X = \frac{x}{L}, \qquad N = \left(\frac{hpL^2}{K_b A}\right)^{\frac{1}{2}} \tag{7}$$

The energy equation reduced to:

$$e^{\beta\theta}\frac{d^2\theta}{dX^2} + \beta e^{\beta\theta}\left(\frac{d\theta}{dX}\right)^2 - N^2\theta = 0, \tag{8}$$

Temperature in the fin base is T_b, and tip of the fin is insulated then with use of DTM, boundary condition defined as follow:

$$X = 0, \qquad \frac{d\theta}{dX} = 0, \tag{9}$$

$$X = 1, \qquad \theta = 1, \tag{10}$$

For fin with temperature dependent thermal conductivity that thermal conductivity defined as follow [3]:

$$k(T) = k_a[1 + \lambda(T - T_\infty)] \tag{11}$$

Energy equation with define dimensionless parameter [3] can be expressed follow as:

$$\frac{d^2\theta}{dX^2} + \gamma\theta\frac{d^2\theta}{dX^2} + \gamma(\frac{d\theta}{dX})^2 - N^2\theta = 0 \tag{12}$$

In the above equation, $\gamma = \lambda(T_b - T_\infty)$, $N = \left(\frac{hpL^2}{k_a A}\right)^{\frac{1}{2}}$, that λ is a constant.

4. Solution with differential transformation method

For solution of Eq. (8) by DTM for transformed function used of Table 1, but for transformed function of $e^{\beta\theta}$ can be referred to Appendix A. with use of differential transformation method for Eq. (8) gives:

$$\sum_{i=0}^{j} F(i)(j-i+1)(j-i+2)\Theta(j-i+2) + \beta\sum_{i=0}^{j}\sum_{l=0}^{j-i} F(l)(i+1)(j-i-l+1)\times \\ \Theta(i+1)\Theta(j-i-l+1) - N^2\Theta(j) = 0, \tag{13}$$

In the above equation $F(i), F(l)$ transformed function of $e^{\beta\theta}$ that described in the Appendix A.

Transformed boundary condition as follow:

$$\Theta(1) = 0, \tag{14}$$

$$\sum_{i=0}^{\infty}\Theta(\mathrm{i}) = 1, \tag{15}$$

By suppose $\Theta(0) = \alpha$ and use of Eq. (13) and Eq. (14) another value of $\Theta(i)$ can be calculated, the value of α can be calculated with used of Eq. (15), we will have:

$$
\begin{aligned}
&\Theta(2)=\frac{1}{2}e^{-\alpha\beta}N\alpha\\
&\Theta(3)=0\\
&\Theta(4)=\frac{1}{12}e^{-\alpha\beta}(\frac{1}{2}e^{-\alpha\beta}N^4\alpha-\frac{3}{2}e^{-\alpha\beta}N^4\alpha^2\beta)\\
&\Theta(5)=0\\
&\Theta(6)=\frac{1}{720}e^{-3\alpha\beta}N^6\alpha(1-18\alpha\beta+30\alpha^2\beta^2)\\
&\Theta(7)=0\\
&\vdots
\end{aligned}
\tag{16}
$$

The above process continuous, substituting Eq. (16) in the main equation base on DTM, it can be obtained that the closed form of the solution is:

$$
\begin{aligned}
&\theta(X)=\alpha+\frac{1}{2}e^{-\alpha\beta}N\alpha X^2+\frac{1}{12}e^{-\alpha\beta}\left(\frac{1}{2}e^{-\alpha\beta}N^4\alpha-\frac{3}{2}e^{-\alpha\beta}N^4\alpha^2\beta\right)X^4+\\
&\frac{1}{720}e^{-3\alpha\beta}N^6\alpha\left(1-18\alpha\beta+30\alpha^2\beta^2\right)X^6+\cdots
\end{aligned}
\tag{17}
$$

For obtained the value of α used of Eq. (15) and we will have:

$$
\begin{aligned}
&\theta(1)=\alpha+\frac{1}{2}e^{-\alpha\beta}N\alpha+\frac{1}{12}e^{-\alpha\beta}\left(\frac{1}{2}e^{-\alpha\beta}N^4\alpha-\frac{3}{2}e^{-\alpha\beta}N^4\alpha^2\beta\right)+\\
&\frac{1}{720}e^{-3\alpha\beta}N^6\alpha\left(1-18\alpha\beta+30\alpha^2\beta^2\right)+\cdots=1,
\end{aligned}
\tag{18}
$$

Solving Eq. (18), gives the value of α, that for this work used of one of the mathematics software.

Transformed form of Eq. (12) can be expressed as [3]:

$$
\begin{aligned}
&(j+1)(j+2)\Theta(j+2)+\gamma\sum_{i=0}^{j}\Theta(i)(j-i+1)(j-i+2)\Theta(j-i+2)+\\
&\gamma\sum_{i=0}^{j}(i+1)\Theta(i+1)(j-i+1)\Theta(j-i+1)-N^2\Theta(j)=
\end{aligned}
\tag{19}
$$

Above equation mentioned in this paper for comparison of results and investigate effect thermal conductivity on fin performance.

5. Results and discussion

First temperature profile assigned for several value of β at $N=1$ for Eq. (17), that results compared with numerical solution that showed in Fig. 1, in the DTM used of 40 terms of Eq. (17) and for numerical solution used of fourth-order Runge-Kutta method, it can be deduced that results have an excellent agreement with together, fin tip temperature $\theta(0)=\alpha$, increased when the value of N increased and with decrease of N, the value of fin tip temperature decreased, the variation between $\theta(0)$ and N for several assigned value of β is shown in Fig. 2. And for larger value of N the value of $\theta(0)$ decrease because increase of N cause to increase of convective heat transfer rate. For investigate of effect thermal conductivity on straight fin performance present results compared with [3] for $N=1$, in this study thermal conductivity varies with temperature exponentially whereas in the [3] thermal conductivity varies with temperature linearly.

Results showed that for the same values of β and γ that are nearer zero effect of thermal conductivity on temperature profile of fin is very low and with away from zero effect of thermal conductivity on distributed temperature is sensible, from results can be deduced effect of thermal conductivity on fin parameter at larger value of β and γ, is more sensible. Results of comparison showed in Fig. 3.

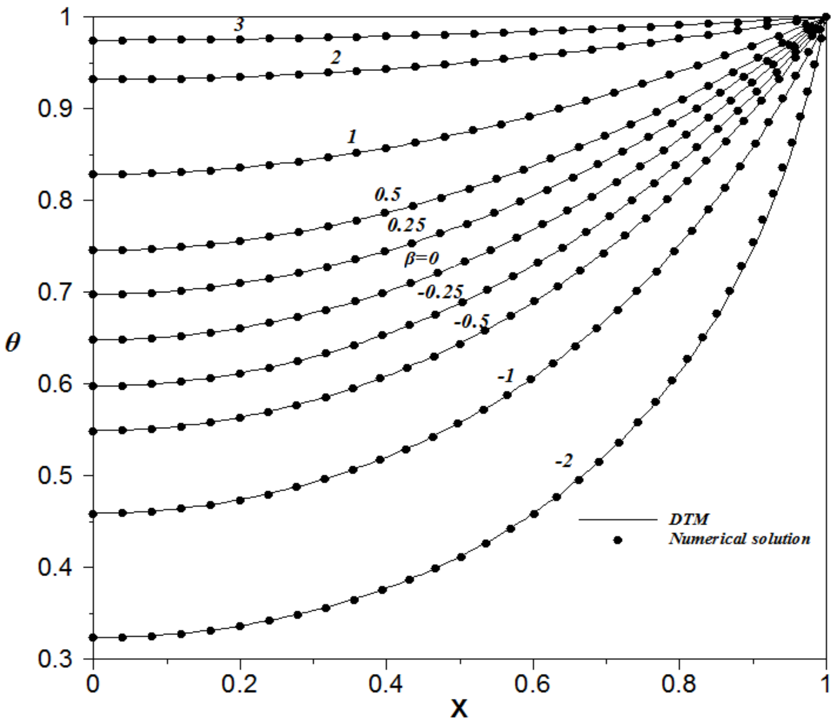

Fig. 1. Temperature profile for several assigned values of β with N=1

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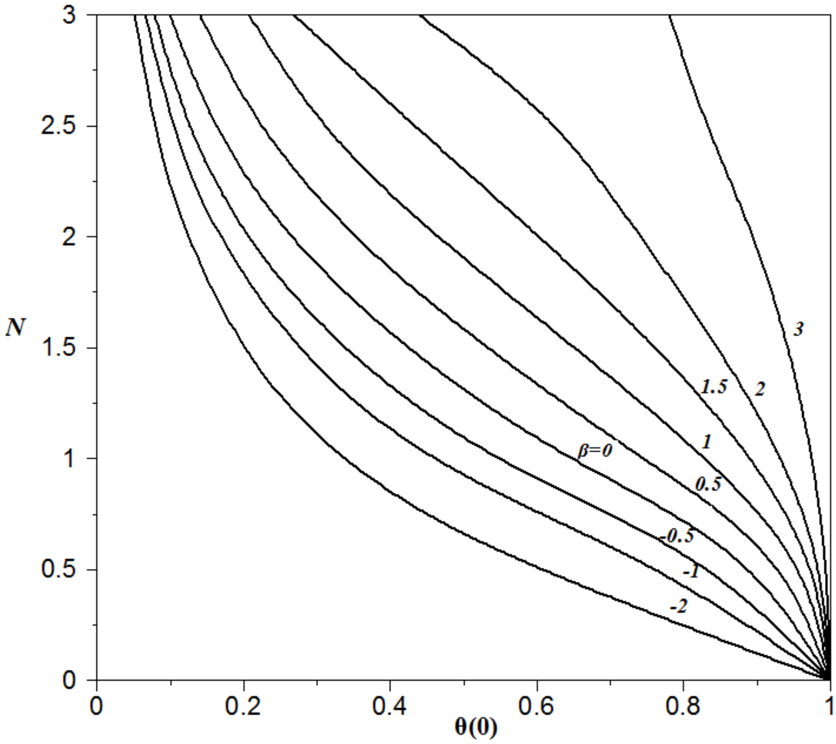

Fig. 2. The variational relationships of N and $\theta(0)$ for several assigned values of β

One of the most characteristics in the fins is consisted fin efficiency and fin effectiveness that study in heat transfer problems in the engineering. In the present study, fin efficiency can be obtained easily, if we define the fin efficiency η the usual way as the ratio of total heat transfer to that of fin at the base temperature, with the above definition, the fin efficiency can be expressed as,

$$\eta = \frac{\int_0^L Ph\left(T - T_\infty\right) dx}{PLh(T_b - T_\infty)} = \int_0^1 \theta dX. \tag{20}$$

Efficiency of the fin for several assigned value of β at $N = 1$ is shown in Fig. 4, that results showed that for positive value of β the value of efficiency greater than negative value of β. One of the characteristics of fin that usage in engineering problem is fin base heat transfer rate that can be expressed $Q_b = \dfrac{d\theta(1)}{dX}$, and variation of the Q_b with N for several assigned value of β is shown in Fig. 5, that results showed that for negative value of β, the value of Q_b is greater than positive value of β.

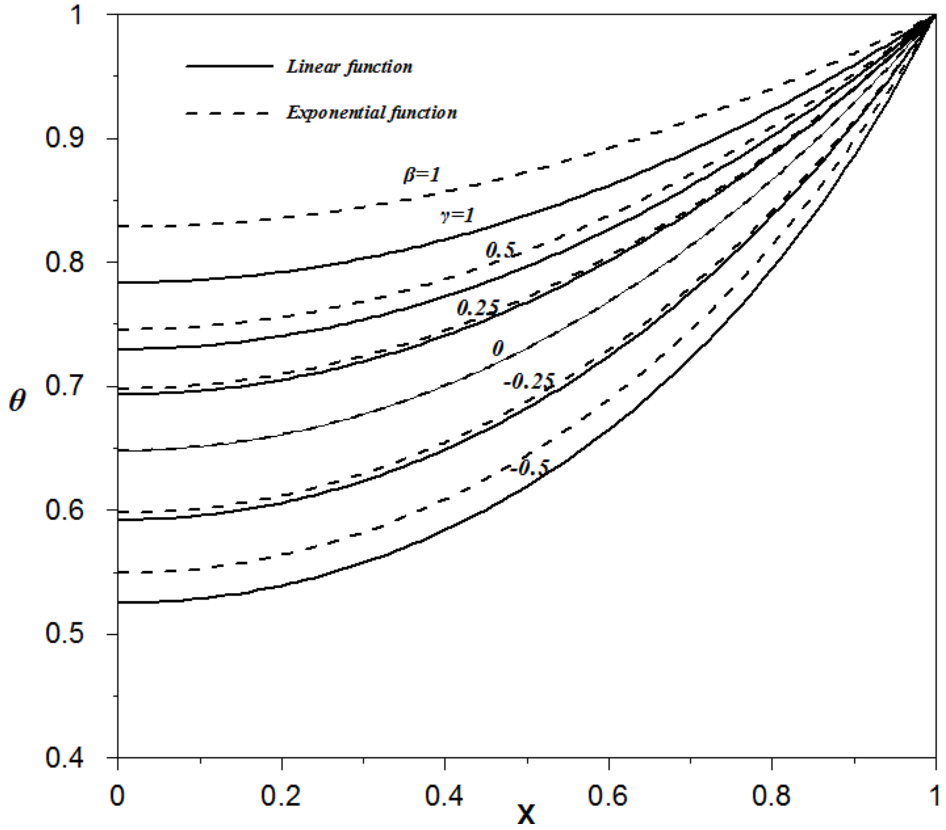

Fig.3. Effect of variable thermal conductivity on fin temperature profile for two types of temperature dependent conductivity for several assigned value of $\beta = \gamma$ with $N = 1$. 1) Linear function [3]. 2) Exponentially function.

Results showed that thermal conductivity is very important in heat conduction problem and variation of thermal conductivity with temperature has direct effect on distribution of temperature and characteristics of fin such as fin efficiency, fin base heat transfer rate and effectiveness of fin.

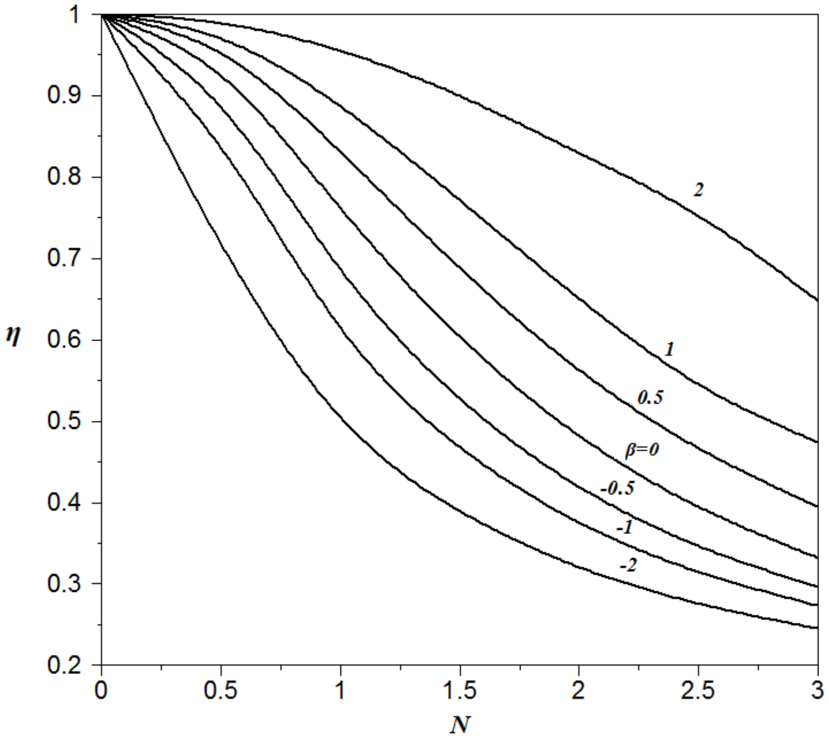

Fig. 4. The variation of η with N for several assigned values of β.

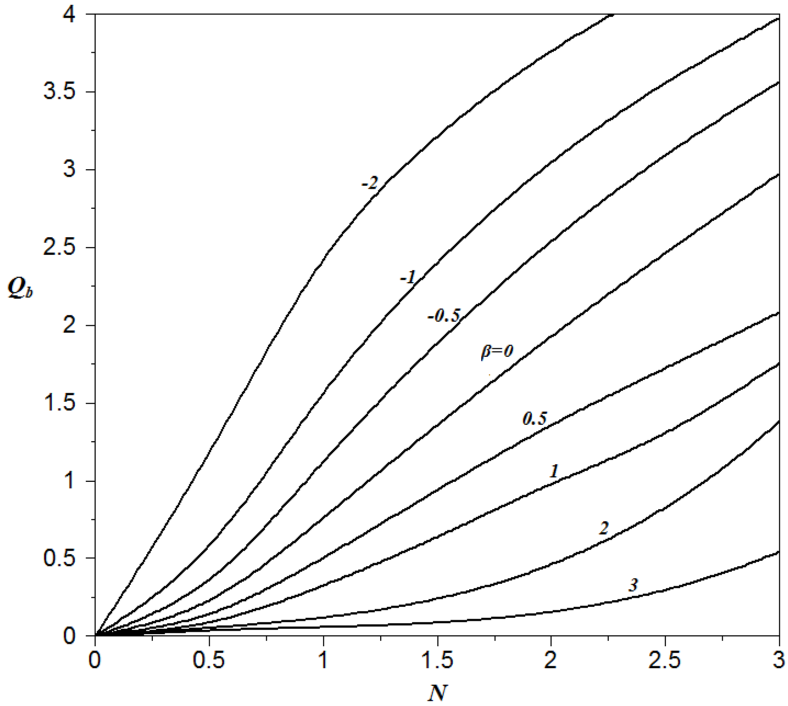

Fig. 5. The variations of Q_b with N for several assigned values of β.

6. Conclusion

In this study, has been used of differential transformation method (DTM) for solving heat conduction problem. Fin efficiency and heat transfer rate of fin base can be obtained from explicit form of temperature profile quickly, this method has applied for linear and nonlinear differential equation and solution of this method is an infinite power-series form and it has high accuracy and fast convergence, and can be observed that existing excellent agreement between DTM and numerical results and for numerical solution used of fourth-order Range-Kutta method. Overall DTM is good approximate analytical method for solving linear and nonlinear engineering problems without any assumption and linearization, in this paper, present results compared with [3] for investigation of variable thermal conductivity on characteristics of fin.

Appendix A. Differential transformed of $f(\theta) = e^{a\theta}$

In this section, we will introduce an efficient method to calculate differential transformation for nonlinear function, the transformed function of $f(\theta) = e^{a\theta}$, the a is constant. From the definition of transform [8] can be expressed follow as:

$$F(0) = [e^{a\theta(X)}]_{X=0} = e^{a\theta(0)} = e^{a\Theta(0)} \tag{A.1}$$

Now, take a differential of $f(\theta) = e^{a\theta}$ with respect to X, we get:

$$\frac{df(\theta)}{dX} = ae^{a\theta}\frac{d\theta(X)}{dX} = af(\theta)\frac{d\theta(X)}{dX} \tag{A.2}$$

Used of differential transformation to Eq. (A.2) gives:

$$(j+1)F(j+1) = a\sum_{i=0}^{j}(i+1)\Theta(i+1)F(j-i) \tag{A.3}$$

Replacing $j+1$ by j gives:

$$F(j) = a\sum_{i=0}^{j-1}\frac{(i+1)\Theta(i+1)F(j-i-1)}{j}, \qquad j \geq 1 \tag{A.4}$$

Combining of Eq. (A.1) and Eq. (A.4), we obtain the recursive relationship for calculating the exponential function of $f(\theta) = e^{a\theta}$

$$F(j)=\begin{cases} e^{a\Theta(0)} & j=0 \\ a\sum_{i=0}^{j-1}\frac{(i+1)\Theta(i+1)F(j-i-1)}{j}, & j\geq 1 \end{cases} \tag{A.5}$$

Nomenclature

A	cross-section area of the fin (m^2)	F	transformed exponential function
Θ	transformed temperature	λ	dimensional constant in Eq. (12)
β	dimensional constant in Eq. (6)	X	non-dimensional space coordinate
h	heat transfer coefficient (W m^{-2} K^{-1})	x	dimensional space coordinate (m)
k	thermal conductivity of the fin (W m^{-1} K^{-1})	θ	dimensionless temperature
T	Temperature (K)		
η	fin efficiency		
γ	constant in Eq. (13) (K)		Subscripts
L	the length of the fin (m)	∞	refer to the ambient property
N	dimensionless fin parameter defined in Eq. (7)	b	refer to the fin base
P	periphery of the fin cross-section area (m)		
Q_b	dimensionless temperature gradient at fin base		

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