International Journal of Engineering & Applied Sciences (IJEAS)
Vol.3, Issue 2(2011)1-12

ANALYTICAL SOLUTION FOR FIN WITH TEMPERATURE DEPENDENT HEAT TRANSFER COEFFICIENT

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Received Date: 04 December 2010; Accepted Date: 08 January 2011

Abstract

In this study, the differential transformation method (DTM) has been used for thermal characteristics of straight rectangular fin for all type of heat transfer and numerical comparison between DTM and adomain decomposition method (ADM) and exact analytical solution method as well as problem is solved numerically with fourth order Rang- Kutta method using shooting method because one of the boundary conditions is boundary value. Local heat transfer coefficient is assumed to vary power-law function of temperature. The obtained differential transform approximate analytic solution is in the form of an infinite power series, so that with obtained explicit form of temperature profile, the fin tip temperature, fin base heat transfer rate, and fin efficiency can be calculated directly from temperature profile easily, but in the exact analytical method obtained implicit form of temperature profile. Results showed that present results have excellent agreement with ADM results and exact analytical method results.

Keywords: DTM, ADM, analytical solution, fin, efficiency

1.Introduction

Fins are one of the most problems in the heat transfer to increase rate of heat transfer on a solid surface. For the cases of constant heat transfer coefficient, the analytical solution of temperature profile and rate of heat transfer can be easily obtained [1]. However in some problem such as boiling liquid, the heat transfer coefficient of fin no constant and varies with temperature difference between surface and the adjacent fluid in a nonlinear manner. The dependence of heat transfer coefficient on the local temperature difference can be governed by a power–law type form. Numerous studies have devoted to the analysis of fin performance of this type of problems due to its important application in engineering. Chang [2] solved a decomposition solution for temperature dependent surface heat flux. ADM results used for compared with results of the present study. Joneidi and Ganji [3] solved differential transformation method to determined fin efficiency of convective straight fins with temperature dependent thermal conductivity. Later Unal [4-7] made a series studies on an extended surface with nonuniform heat transfer coefficient and showed that the equation can be integrated analytically in a closed form for a limited number of cases. Liaw and Yeh [8] used the same model and further studies all possible type of heat transfer including the cases of film and transition boiling with and without heat transfer at fin tip.

They also conducted an analytical and experimental study for a fin with a various type of boiling occurring simultaneously at adjacent location on its surface [9].

Abbasbandy and shivanian [10] made exact analytical solution of a nonlinear equation arising in heat transfer that nonlinear equation same with equation that solved in this study with DTM. Of course, in the present study solution represented by an infinite series, but in the paper solved equation pure analytically. Sen Kou, Lee and Lai [11] made thermal analysis of a longitudinal fin with variable thermal properties by recursive formulation. Khani and abdul Aziz [12] made thermal analysis of a longitudinal trapezoidal fin with temperature dependent thermal conductivity and heat transfer coefficient, in this paper used of HAM for solved equation. In the present study for the comparison, problem solved numerically by Rang-Kutta fourth order method for N=1 and several assigned value of n. The concept of differential transformation method was first introduced by Zhou [13] in 1986 and it was used to solve both linear and nonlinear initial value problem in electric circuit analysis. One of most Advantages of this method reducing the size of computational work while the Taylor series method is computationally taken long time for large orders. Aziz and Hug [14] used the regular perturbation method and a numerical solution to compute a closed form solution for a straight convective fin with temperature-dependent thermal conductivity. The HAM was used by Domairry and Fazeli to solve rectangular purely convective fin with temperature dependent thermal conductivity [15]. Khani, ahmadzade Raji, and Hamidi Nejad [16] used HAM to evaluate the analytical approximate solution and efficiency of the nonlinear fin problem with temperature dependent thermal conductivity and heat transfer coefficient. Mustafa Inc [17] used HAM to evaluate the efficiency of straight fin with temperature dependent thermal conductivity and to determine temperature distribution within the fin. Arslanturk [18] and Rajabi [19] used the ADM and HPM to evaluate the efficiency of straight fins with temperature dependent thermal conductivity and to determine the temperature distribution within the fin. Lesnic and Heggs [20] applied the ADM to determine the temperature distribution within a single fin with a temperature dependent heat transfer coefficient. Ching-Huang and Chen [21] used to adomain decomposition method to evaluate fin efficiency and the optimal length of convective rectangular fin with variable thermal conductivity, and to determine the temperature distribution within the fin. Kundu and Das [22] made the thermal analysis and optimization of straight taper fins has been addressed. In this paper has been observed that the variable heat transfer coefficient has a strong influence over the fin efficiency. Mokheimer [23] investigated performance of annular fins of different subject to locally variable heat transfer coefficient, in this paper performance of fin expressed in terms of fin efficiency as a function of the ambient and fin geometry parameters.

Recently, differential transformation method has been used to solve a wide range of physical problem. This method provides a direct scheme for solving linear and nonlinear deterministic and stochastic equation without the need for linearization and yield rapidly convergent series solution. Rashidi and Erfani [24] used DTM for solved fin efficiency of convective straight fins with temperature dependent thermal conductivity and comparison results with HAM. Chiam [25] used of perturbation method for solve heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet. Their results of this study showed that the differential transformation method has many merits including fast convergence and high accuracy. In this study is to apply differential transformation method to investigate a straight fin governed by power-law type temperature dependent heat transfer coefficient. Base of DTM, temperature on the fin surface can be expressed explicitly as a function of position along the fin. The effect of exponent value and fin parameter on temperature profile as well as fin tip temperature can also be obtained quickly.

In addition to, heat transfer rate and fin efficiency are presented in detail. In the present study results are compared with [2, 10].

2. Fundamentals of differential transformation method

We suppose $y(t)$ to be analytic function in a domain *D* and $t = t_i$ represented any point in *D*. The function $y(t)$ is the represented by one power series whose center is located at t_i. The Taylor series expansion function of $y(t)$ is of the form

$$y(t) = \sum_{j=0}^{\infty} \frac{(t - t_i)^j}{j!} \left[\frac{d^j y(t)}{dt^j} \right]_{t=t_i} \quad \forall t \in D \tag{1}$$

The particular case of Eq. (1) when $t_i = 0$ is referred to as the Maclurin series of $y(t)$ and is expressed as:

$$y(t) = \sum_{j=0}^{\infty} \frac{t^j}{j!} \left[\frac{d^j y(t)}{dt^j} \right]_{t=0} \quad \forall t \in D \tag{2}$$

As expressed in that differential transformation method of the function $y(t)$ is defined as follow:

$$Y(j) = \sum_{j=0}^{\infty} \frac{H^j}{j!} \left[\frac{d^j y(t)}{dt^j} \right]_{t=0,} \tag{3}$$

Where $y(t)$ is the original function and $Y(j)$ is the transformed function. The differential spectrum of the $Y(j)$ is confined within the interval $t \in [0, H]$, where H is a constant. The differential inverse transform of $Y(j)$ is defined as follow:

$$y(t) = \sum_{j=0}^{\infty} (\frac{t}{H})^j Y(j) \tag{4}$$

Some of the original function and transformed function is shown in Table 1. It is clear that the concept of differential transformation is Taylor series expansion. For assigned solution with high accuracy may be calculated more number of series in Eq. (4).

3. Problem formulation with differential transformation method

Consider a straight fin of length *L* with a uniform cross-section area *A*. The fin surface is exposed to a convective environment at temperature T_∞ and the local heat transfer coefficient *h* along the fin surface is assumed to exhibit a power-law-type dependence on the local temperature difference between the fin and ambient fluid as

$$h = a(T - T_\infty)^n \tag{5}$$

Where a is dimensionless constant, T is the local temperature on the fin surface, and exponent n depends to heat transfer mode. In this paper the value of n varies between -3 and 5. For example, the exponent n may take the value -0.25, 0, 2 and 3, indicating the fin subject to laminar film boiling or condensation, convection, nucleate boiling, and radiation into to free space at zero absolute temperature, respectively. For one dimensional steady state heat conduction, the equation in term of dimensionless variables $X=x/L$ and $\theta = (T-T_\infty)/(T_b - T_\infty)$ can be written as

$$\frac{d^2\theta}{dX^2} - N^2\theta^{n+1} = 0; \tag{6}$$

Where axial distance X is measured from the fin tip, T_b is the fin base temperature, and N is the convective-conductive parameter of the fin defined as:

$$N = (\frac{h_b PL^2}{KA})^{\frac{1}{2}} = \left[\frac{aPL^2}{KA}(T_b - T_\infty)^n\right]^{\frac{1}{2}} \tag{7}$$

In the above equation, h_b, P, and K represent heat transfer coefficient at fin base, periphery of fin cross-section, and the conductivity of the fin, respectively. Boundary condition to Eq. (6) can be expressed as

$$X = 0 \qquad \frac{d\theta}{dX} = 0 \tag{8}$$

$$X = 1 \qquad \theta = 1 \tag{9}$$

In the transformation method power of variables should be integer and positive [24]. So that Eq. (6) solved with DTM for any value of n separately, for difference n taking the one-dimensional transform of Eq. (6) by using related definition that, is showed in Table 1, we have

$$(j+1)(j+2)F(j+2) - N^2F(j) = 0 \qquad n = 0 \tag{10}$$

$$(j+1)(j+2)F(j+2) - N^2\sum_{i=0}^{j}F(i)F(j-i) = 0 \qquad n = 1 \tag{11}$$

$$(j+1)(j+2)F(j+2) - N^2\sum_{i=0}^{j}\sum_{l=0}^{j-i}F(l)F(i)F(j-i-l) = 0 \qquad n = 2 \tag{12}$$

$$\begin{aligned}&(j+1)(j+2)F(j+2) - N^2\sum_{i=0}^{j}\sum_{l=0}^{j-i}\sum_{p=0}^{j-i-l}F(p)F(l)F(i)\times \\ &F(j-i-l-p) = 0 \qquad n = 3\end{aligned} \tag{13}$$

$$(j+1)(j+2)F(j+2)-N^2\sum_{i=0}^{j}\sum_{l=0}^{j-i}\sum_{p=0}^{j-i-l}\sum_{s=0}^{j-i-l-p}F(s)F(p)F(l)\times F(i)F(j-i-l-p-s)=0 \qquad n=4 \tag{14}$$

$$(j+1)(j+2)F(j+2)-N^2\sum_{i=0}^{j}\sum_{l=0}^{j-i}\sum_{p=0}^{j-i-l}\sum_{s=0}^{j-i-l-p}\sum_{z=0}^{j-i-l-p-s}F(z)F(s)F(p)\times F(l)F(i)F(j-i-l-p-s-z)=0 \qquad n=5 \tag{15}$$

$$\sum_{i=0}^{j}(j-i+1)(j-i+2)(j+1)(j+2)F(j+2)F(j-i+2)-N^4\sum_{i=0}^{j}\sum_{l=0}^{j-i}F(l)F(i)F(j-i-l)=0 \qquad n=0.5 \tag{16}$$

$$\sum_{i=0}^{j}(j-i+1)(j-i+2)(j+1)(j+2)F(j+2)F(j-i+2)-N^4F(j)=0 \qquad n=-0.5 \tag{17}$$

$$(j+1)(j+2)F(j+2)-N^2\delta(j)=0 \qquad n=-1 \tag{18}$$

$$\sum_{i=0}^{j}(j-i+1)(j-i+2)F(i)F(j-i+2)-N^2\delta(j)=0 \qquad n=-2 \tag{19}$$

$$\sum_{i=0}^{j}\sum_{l=0}^{j-i}(j-i-l+1)(j-i-l+2)F(l)F(i)\times F(j-i-l+2)-N^2\delta(j)=0 \qquad n=-3 \tag{20}$$

In the Eqs. (18), (19) and (20), $\delta(j)$ can be expressed as

$$\delta(j)=\begin{Bmatrix}1 & \text{for} \quad j=0\\ 0 & \text{otherwise}\end{Bmatrix} \tag{21}$$

Table 1 The fundamental operations of differential transform method.

Original function	Transformed function

$f(x)=\alpha g(x)\pm\beta h(x)$	$F(k)=\alpha G(k)\pm\beta H(k)$
$f(x)=g(x)h(x)$	$F(k)=\sum_{i=0}^{k}G(i)H(k-i)$
$f(x)=g(x)^{(n)}$	$F(k)=(k+1)(k+2)\ldots(k+n)G(k+n)$
$f(x)=x^n$	$F(k)=\delta(k-n)=\begin{Bmatrix}1 & k=n\\0 & k\neq n\end{Bmatrix}$
$f(x)=\exp(\alpha x)$	$F(k)=\dfrac{\alpha^k}{k!}$
$f(x)=(1+x)^n$	$F(k)=\dfrac{k(k-1)\ldots(k-m-1)}{k!}$

Now, dimensionless temperature is given by

$$\theta(X)=\sum_{j=0}^{\infty}F(j)X^j \tag{22}$$

Furthermore, applying the DTM to Eqs. (8) and (9) the boundary condition is given as follow

$$F(1)=0, \tag{23}$$
$$\sum_{j=0}^{\infty}F(j)=1,$$

By assuming

$$F(0)=\beta, \tag{24}$$

So that, substituting Eqs. (23) and (24) in Eq. (10) to (20) and by recursive method we can calculating another values of $F(j)$, in this paper for example we calculate another values of $F(j)$ for n=1, and some results are listed as

$$\begin{aligned}
&F(j)=0, \quad \text{for} \quad \mathrm{j}=3,5,7,\ldots\\
&F(2)=\frac{N^2\beta^2}{2},\\
&F(4)=\frac{N^4\beta^3}{12},\\
&F(6)=\frac{N^6\beta^4}{72},\\
&F(8)=\frac{N^8\beta^5}{504},\\
&\vdots
\end{aligned} \tag{25}$$

For another value of n assigned $F(j)$ similarity, by applying Eq. (23), we can obtain β. Hence substituting all $F(j)$ in to (22) we have series solution as below

$$\theta(X)=\beta+\frac{N^2\beta^2}{2}X^2+\frac{N^4\beta^3}{12}X^4+\frac{N^6\beta^4}{72}X^6+\frac{N^8\beta^5}{504}X^8+\dots \tag{26}$$

4. Result and discussion

In this study has been used of 50 terms of Eq. (22) which is sufficient for assigned good accuracy. Relationship between N and $\theta(0)=\theta_e$ for $-1\le n\le 5$ is shown in Fig. 1, but $0<n\le 5$ is usually used in this paper. Fig. 1 shows that, while the value of N increases then the fin base temperature decreases and with increasing the exponent value n the fin base temperature at the same value of N increase. Comparison between the present results and other reported results is shown in Table 2. The results are depicted in Table 2 show that have an excellent agreement between present and other reported results. Note that there is no solution for integer value of n less than -1 at N=1. Present method has a fast convergence and good accuracy. The temperature profile for several assigned values of n for N=1, as shown in Fig. 2. For validation of present results, these results are compared with ADM results [2] that have been shown in Fig. 2. According to Eq. (22), the temperature along the fin is expressed function of position X, so that temperature profile for any value of n assigned easily.

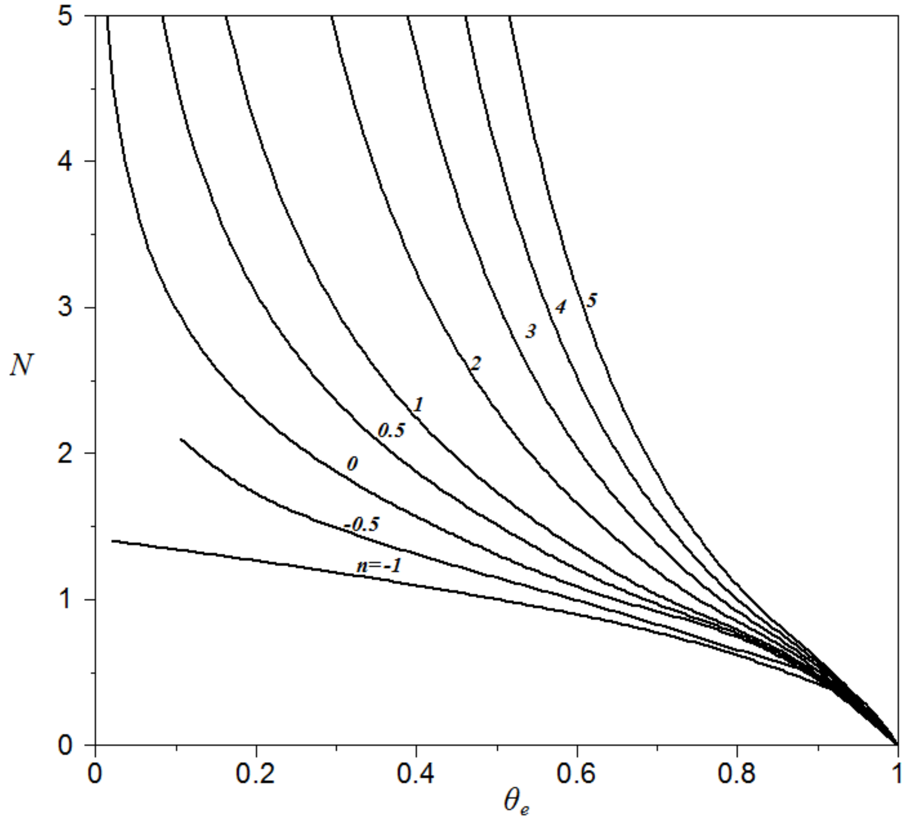

Fig. 1. The variational relationships of N and θ_e for several assigned values of n.

Table 2 Comparison of fin base temperature between present study, ADM, numerical solution, and exact analytical solution for N=1 and N=0.5 for several assigned values of n.

	N=1				*N*=0.5			
n	Present	ADM [2]	Numerical solution	Exact solution [10]	Present	ADM [2]	Numerical solution	Exact solution [10]
-2	-	-	-	-	0.858211	0.858168	0.85635	0.85878
-1	0.5	0.5	0.495008	0.5	0.875	0.875	0.87375	0.875
-0.5	0.594447	0.594458	0.59186	0.594446	0.881358	0.881358	0.880285	0.881358
0	0.648054	0.648054	0.647887	0.648054	0.886819	0.886819	0.88588	0.886819
0.5	0.684781	0.684779	0.68466	0.684781	0.891586	0.891586	0.89076	0.891586
1	0.712258	0.712263	0.712166	0.712256	0.895804	0.895804	0.895061	0.895804
2	0.751635	0.75166	0.751568	0.751622	0.902974	0.902974	0.90237	0.902974
3	0.779177	0.779217	0.77911	0.779145	0.908889	0.90889	0.908385	0.908889
4	0.799894	0.79994	0.79982	0.79984	0.913888	0.91389	0.91346	0.913887
5	0.816226	0.816268	0.816135	0.816149	0.918191	0.918194	0.91782	0.91819

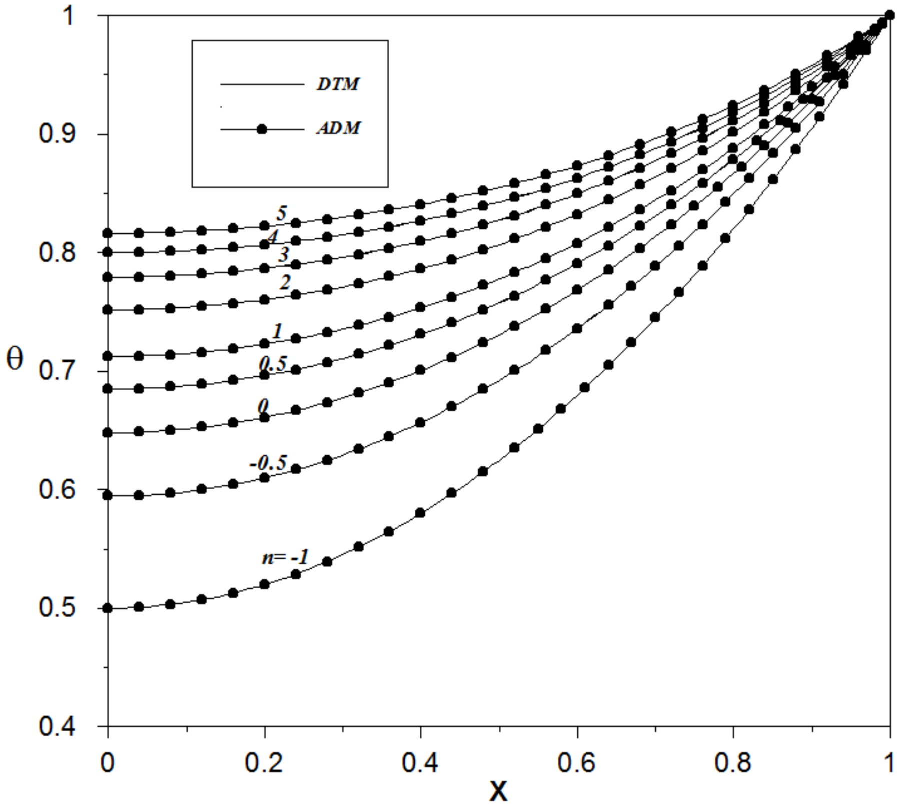

Fig. 2. Temperature profile for several assigned value of *n* at *N*=1

Abbasbandy and Shivanian [10] assigned temperature profile for difference value of *N* and *n*, they presented analytic exact solution in implicit form from for each value of parameters of equation, but in this method assigned complicated formula implicitly that should be solve with one of the mathematics software.

One of the most problems in the fins are heat transfer rate of fin base, and can be indicate by the dimensionless temperature gradient Q_b with the definition $Q_b = \frac{d\theta(1)}{dX}$. Fig. 3 shows dependence of Q_b and N for several assigned values of n, in this study variation of Q_b with N for $-3 \le n \le 3$ has been shown in Fig. 3. Fig. 3 illustrates that Q_b for negative value of n is more than positive value of n. The value of Q_b can be simply obtained via the direct differentiation of Eq. (22). It is found that the parameter Q_b increases with N for all the values n considered.

One of the most characteristics in the fins is consisted fin efficiency and fin effectiveness that study in heat transfer problems in the engineering. In this study, the fin efficiency can be obtained easily, if we define the fin efficiency η the usual way as the ratio of total heat transfer to that of fin at the base temperature, with the above definition, the fin efficiency can be expressed as,

$$\eta = \frac{\int_0^L Ph(T - T_\infty)\,dx}{PLh_b(T_b - T_\infty)} = \int_0^1 \theta^{n+1} dX. \tag{27}$$

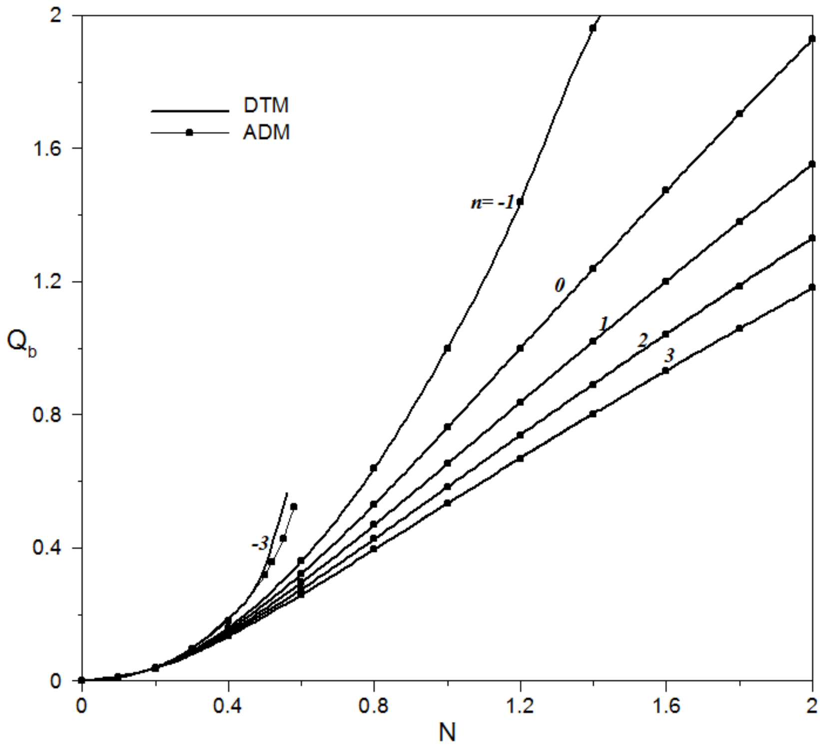

Fig. 3. the variation of Q_b with N for several assigned values of n.

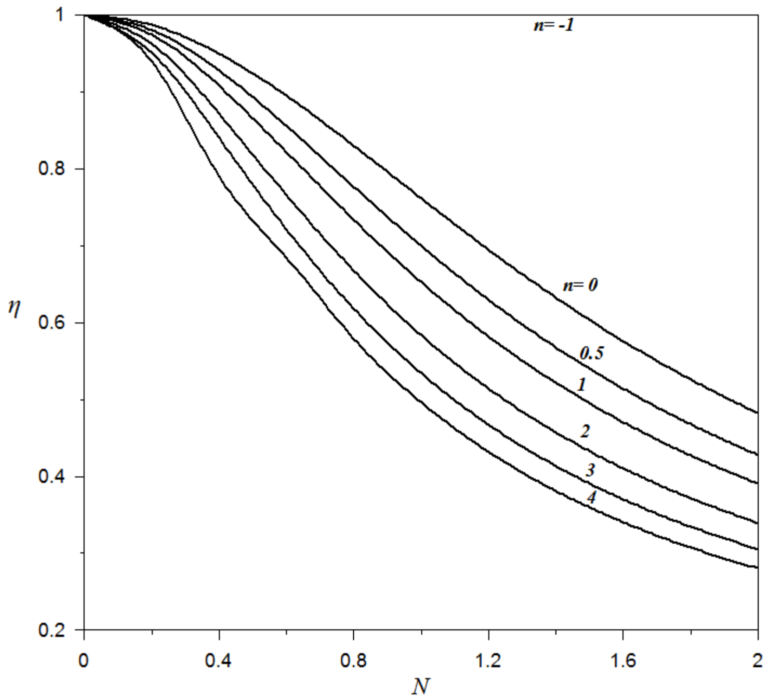

Fig. 4. variation of fin efficiency with N for several assigned values of n

Variation of efficiency with N for several assigned values of n is shown in Fig. 4. The efficiency of fin for n=-1 is equal 1, that this result is not physically in the real world. So that Eq. (27) is valid only for $n \geq -1$ as shown in Fig. 4, It is found that the fin efficiency decreases with N as $n > -1$, and while the value of n increases, the value of η decreases in the same value of N.

5. Conclusion

The differential transformation method has been used to solution of the heat conduction problem for fin with temperature dependent heat transfer coefficient, in this paper heat transfer coefficient varying as a power-law function of temperature. This method is not a exact analytical solution because final solution of this method is infinite power series function, but this method without any assumption and linearization similar ADM, this character is very important for system with strong nonlinearities which extremely sensitive to small changes parameters. One of the other advantages of this method is that fast convergence and good accuracy. In this paper were compared DTM, ADM [2], numerical and exact analytical solution [10] together. With comparison of four methods together conclude that present result are high accuracy relatively ADM and numerical with exact analytical solution method, but difference between DTM and ADM results are not sensibility. The fin efficiency and heat transfer rate of fin base can be obtained from explicit form of temperature profile quickly, but in the exact analytical method [10] have a complicated implicit form for temperature profile related to position of X that should be used of mathematics software. From DTM can be applied to nonlinear heat conduction problem and highly nonlinear problem in engineering.

References

[1] Q.D. Kern, D.A. Kraus, Extended Surface Heat Transfer, McGraw-Hill, New York, 1972.
[2] Chang M. H., A decomposition solution for fins with temperature dependent surface heat flux, Int. J. Heat Mass Transfer, 48, 1819-1824, 2005.
[3] Joneidi A. A., Ganji D. D. and Babaelahi M., Differential transformation method to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity, Int. Commun. Heat Mass Transfer, 36, 757-762, 2009.
[4] Unal H. C., Determination of the temperature distribution in an extended surface with a non-uniform heat transfer coefficient, Int. J. Heat Mass Transfer, 28, 2279-2284, 1986.
[5] Unal H. C., A simple method of dimensioning straight fins for nucleate pool boiling, Int. J. Heat Mass Transfer, 29, 640-644, 1987.
[6] Unal H. C., An analytic study of boiling heat transfer from a fin, Int. J. Heat Mass Transfer, 30, 341-349, 1987.
[7] Unal H. C., The effect of the boundary at a fin tip on the performance of the fin with and without internal heat generation, Int. J. Heat Mass Transfer, 31, 1483-1496, 1988.
[8] Liaw S. P. and Yeh R. H., Fins with temperature dependent surface heat flux –I: Single heat transfer mode, Int. J. Heat Mass Transfer, 37, 1509-1515, 1994.
[9] Liaw S. P. and Yeh R. H., Fins with temperature dependent surface heat flux –II: Multi-boiling heat transfer, Int. J. Heat Mass Transfer, 37, 1517-1524, 1994.
[10] Abbasbandy S. and Shivanian E., Exact analytical solution of a nonlinear equation arising in heat transfer, Phys. Lett, A 374, 567-574, 2010.
[11] Kou H. S., Lee J. J. and Lai C. Y., Thermal analysis of a longitudinal fin with variable thermal properties by recursive formulation, Int. J. Heat Mass Transfer, 48, 2266-2277, 2005.
[12] Khani F. and Aziz A., Thermal analysis of a longitudinal trapezoidal fin with temperature-dependent thermal conductivity and heat transfer coefficient, Communication in Nonlinear Science Numerical Simulation, 15, 590-601, 2010.
[13] Zhou J. K., Differential Transformation method and its Application for Electrical Circuits, Hauzhang Univ. press, Wuhan, China, 1986.
[14] Aziz A. A. and Hug E., Perturbation solution for convecting fin with variable thermal conductivity, J. Heat Transfer, 91, 300-310, 1995.
[15] Domairry G. and Fazeli M., Homotopy analysis method to determine the fin efficiency of convective straight fins with temperature dependent thermal conductivity, Communication in Nonlinear Science and Numerical, Simulation 14, 489-499, 2009.
[16] Khani F., Ahmadzade Raji M. and Hamidi Nejad H., Analytical solution and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat Transfer coefficient, Communication in Nonlinear Science Numerical Simulation, 14, 3327-3338, 2009.
[17] Mostafa Inc., Application of homotopy analysis method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity, Mathematics and Computers in Simulation, 79, 189-200, 2008.
[18] Arslanturk C., A decomposition method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity, Int. Commun. Heat Mass Transfer, 32, 831-841, 2005.
[19] Rajabi A., Homotopy perturbation method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity, Phys. Lett. A 364, 33-37, 2007.
[20] Lesnic D. and Heggs P. J., Adomain decomposition method for power-law fin-type problems, Int. Commun. Heat Mass Transfer, 31, 673-682, 2004.

[21] Chiu C. H. and Chen C. K., A decomposition method for solving the convective longitudinal fins with variable thermal conductivity, Int. J. Heat Mass Transfer, 45, 2067-2075, 2002.
[22] Kundu B. and Das P. K., Performance analysis and optimization of straight taper fins with variable heat transfer coefficient, Int. J. Heat Mass Transfer. 45, 4739-4751, 2002.
[23] Mokhimer M. A., Performance of annular fins with different profiles subject to variable heat transfer coefficient, Int. J. Heat Mass Transfer, 45, 3631-3642, 2002.
[24] Rashidi M. M. and Erfani E., New analytical method for solving Burgers' and nonlinear heat transfer equation and comparison with HAM, Comput. Phys. Commun, 180, 1539-1544, 2009.
[25] Chiam T. C., Heat transfer in a fluid with variable thermal conductivity over a linearly stretching sheet, Acta Mechanica, 129, 63-72, 1998.