International Journal of Engineering & Applied Sciences (IJEAS)
Vol.3, Issue 3(2011)60-71

VIBRATION CHARACTERISTICS OF FUNCTIONALLY GRADIENT SHELLS WITH AN EXPONENTIAL LAW DISTRIBUTION USING WAVE PROPAGATION APPROACH RESTED ON TWO PARAMETERS FOUNDATIONS

M. Rafiee, A. Nezamabadi, S. Mareishi

Mechanical Engineering Department, Islamic Azad University ,Arak Branch, Arak, Iran.

Received Date: 22February 2011; Accepted Date: 03 June 2011

Abstract

The aim of this paper is to deal with the dynamic behaviour and vibration characteristics of thin functionally graded circular cylindrical shells. Material properties in the shell thickness direction are graded in accordance with the exponential law. Expressions for the strain-displacement and curvature-displacement relationships are taken from Love's thin shell theory. The Rayleigh-Ritz approach is used to derive the shell eigenfrequency equation. Axial modal dependence is assumed in the characteristic beam functions. Natural frequencies of the shells are observed to be dependent on the constituent volume fractions. The results are compared with those available in the literature for the validity of the present methodology.

Keywords: Elastic Shell, functionally gradient material, exponential law.

1. Introduction

Functionally graded materials (FGMs) have gained wide application in a variety of industries due to their distinctive material properties that vary continuously and smoothly through certain dimensions. Compared with common composites, FGMs avoid the inter-laminar stress gaps that are caused by mismatches in the properties of two different materials, and can be adjusted appropriately according to practical requirements.

In the last years, some researchers have analyzed various characteristics of functionally graded structures (Ng et al., [1]; Yang et al., [2]; Della Croce and Venini, [3]; Liewet al., [4];Wu and Tsai, [5]; Elishakoff et al., [6-7]; Patel et al., [8]; Pelletier and Vel, [9]; Zenkour, [10]; Arciniega and Reddy, [11]; Nie and Zhong, [12]; Roque et al., [13]; Najafizadeh and Isvandzibaei[14]. Yang and Shen, [15]).In addition to FGPs, functionally graded material shells have attracted research attention. Heetal. [16] proposed a finite element model for a doubly curved FGM shell with piezoelectric sensors and actuators, and demonstrated that effective static and dynamic control of FGM shells could be achieved b yappropriately varying the displacement and velocity feedback gains. Liewetal. [17] examined the thermal stress behavior of functionally graded hollow circular cylinders, and Jabbari et al. [18] analyzed the mechanical and thermal stresses in a functionally graded hollow thick cylinder subjected to a non-axisymmetric steady-state load. Jacob and Vel [19] investigated the steady-state thermoelastic response of functionally graded isotropic and orthotropic cylindrical shells subjected to thermal and mechanical load susing the Flugge and Donnell shell theories. Hosseini Kord kheili and Naghdabadi [20] developed a finite element formulation for a geometrically non-linear thermo elastic FGM shell using an updated Lagrangian approach, and Arciniega and Reddy [21] derived a tensor-based finite element model for large deformation analysis of FGM shells. Other studies have investigated the thermomechanical postbuckling, generalized coupled thermoelasticity, and vibration of FGM shells [22–24].

In this paper, a wave propagation approach is applied to study the vibration characteristics of FGM circular cylindrical shells. The axial modal dependence is approximated by exponential functions. This approach has been developed by Zhang et al. [25]. This avoids a large amount of algebraic manipulations. Employing this approach, the vibration characteristics of FGM cylindrical shells are studied for simply supported-simply supported, clamped-simply supported and clamped–clamped boundary conditions. Validity and accuracy of the present method are verified by comparing the present results with those available in the literature. A good agreement is observed between the two sets of the results.

2. Functionally gradient materials

FGMs are basically composite materials, which are made by mixing two or more different materials. Most of the FGMs are being used in a high temperature environment and their material properties are temperature dependent. A typical material property Pi is expressed as a function of environment temperature T.K. by Touloukian [27]. If P_i represents a material property of the *i*th constituent material of an FGM consisting of *k* constituent materials, then the effective material property *P* of the FGM is written as

$$P = \sum_{i=1}^{k} P_i V_i \tag{1}$$

where V_i is the volume fraction of the *i*th constituent material. Also, the sum of volume fractions of the constituent materials is equal to 1, i.e.,

$$\sum_{i=1}^{k} V_i = 1. \tag{2}$$

The volume fraction depends on the thickness variable and is defined as

$$V_i = \left(\frac{z - R_i}{R_o - R_i}\right)^p \tag{3}$$

for a cylindrical shell. R_i and R_o denote inner and outer radii of the shell, respectively, and *z* is the thickness variable in the radial direction. *p* is known as the power law exponent. It is a non-negative real number and lies between zero and infinity. For a cylindrical shell, the volume fraction is assumed as

$$V_i = \left(\frac{z + 0.5h}{h}\right)^p, \tag{4}$$

where *h* is the shell uniform thickness. When the shell is considered to consist of two materials, the effective Young's modulus *E*, the Poisson ratio *ν*, and the mass density *ρ* are given by

$$
\begin{cases}
E = (E_1 - E_2)(\frac{z + 0.5h}{h})^p + E_2 \\
\nu = (\nu_1 - \nu_2)(\frac{z + 0.5h}{h})^p + \nu_2 \\
\rho = (\rho_1 - \rho_2)(\frac{z + 0.5h}{h})^p + \rho_2
\end{cases}
\tag{5}
$$

From these relationships, the following facts are noted: at the inner surface, the FGM properties are those of the constituent material 2; at the outer surface, they are those of the material 1. Thus, the FGM properties change continuously from the material 2 at the inner surface to the material 1 at the outer surface. Arshad et al.[26] amended the volume fraction law (4) and assumed it in the exponential form,

$$
V_i = 1 - \exp(-(z / h + 0.5))^p. \tag{6}
$$

This law depends only on one base e (= 2.718 · · ·). Now, this law is further modified and extended to a general base (b > 0) written as

$$
V_i = 1 - b^{-(z/h+0.5)^p}. \tag{7}
$$

In this case, the effective Young's modulus E, the Poisson ratio ν, and the mass density ρ are expressed as

$$
\begin{cases}
E = (E_1 - E_2)(1 - b^{-(z/h+0.5)^p}) + E_2 \\
\nu = (\nu_1 - \nu_2)(1 - b^{-(z/h+0.5)^p}) + \nu_2 \\
\rho = (\rho_1 - \rho_2)(1 - b^{-(z/h+0.5)^p}) + \rho_2
\end{cases}
\tag{8}
$$

In these expressions, it is noted that, when $z = -0.5h$, $E = E2$, $\nu = \nu2$, and $\rho = \rho2$; when $z = 0.5h$, $E = (E_1-E_2)(1-b^{-1})+E_2$, $\nu = (\nu_1-\nu_2)(1-b^{-1})+\nu_2$, and $\rho = (\rho_1-\rho_2)(1-b^{-1})+\rho_2$.This shows that, at the inner surface of the shell, the material properties of the constituent material 2 are dominant, whereas, at its outer surface, the material properties are the resultant ones of the constituent materials 1 and 2.

3. Analytical model and formulation

The structure is an isotropic thin elastic cylindrical shell with Young's modulus E, Poisson's ratio ν, radius of the middle surface R, thickness h, and length L. The foundation is represented by continuous elastic (axial, circumferential, radial, and rotational) springs and distributed on a limited arc. The axial, circumferential, radial, and rotational spring coefficients are denoted by Ku, Kv, Kw, and Kb, respectively. In the analysis, all the spring coefficients are assumed to be constant along the enclosed arc and the angles that define the enclosed arc are denoted by 41 and 42. The geometry and generalized model of the structure are shown in Fig. 1.

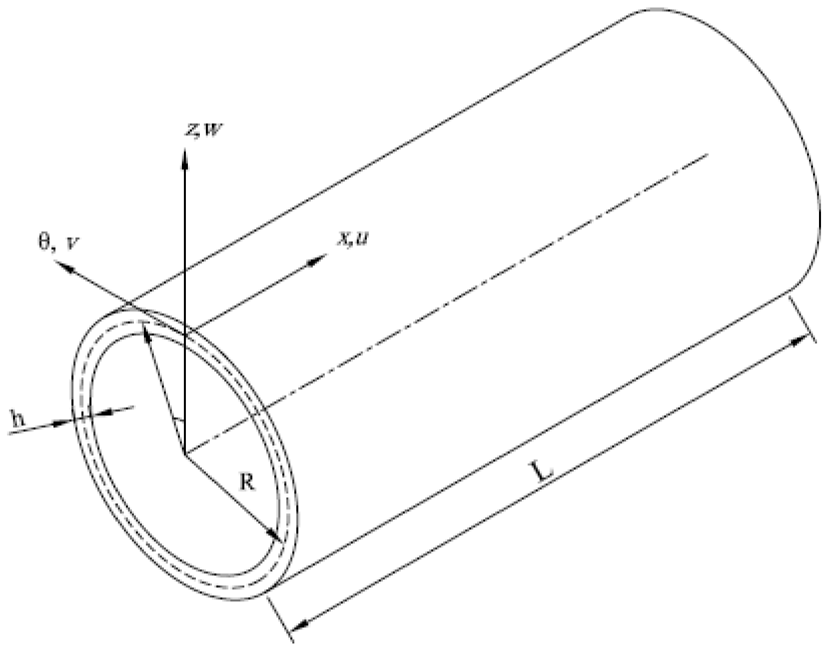

Fig. 1. Geometry and model of a circular cylindrical shell.

Constitutive relation for a thin cylindrical shell under plane stress condition is given by

$$\{\sigma\} = [Q]\{e\} \tag{9}$$

where $\{\sigma\}$ and $\{e\}$ represent stress and strain vectors and $[Q]$ is the reduced stiffness matrix. The stress vector is defined as

$$\{\sigma\}^T = \{\sigma_x \quad \sigma_y \quad \sigma_{x\theta}\} \tag{10}$$

where σ_x and σ_θ are the normal stresses in x and θ directions and $\sigma_{x\theta}$ is the shear stress in the $x\theta$ plane. Similarly, the strain vector is defined,

$$\{e\}^T = \{e_x \quad e_y \quad e_{x\theta}\} \tag{11}$$

where e_x and e_y are the normal strains in x and θ directions and $e_{x\theta}$ is the shear strain in the $x\theta$ plane. The reduced sti€ness matrix $[Q]$ is defined as

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{12}$$

For isotropic materials the reduced stiffness Qij are defined as

$$Q_{11} = \frac{E}{1-\nu^2} \tag{13}$$

$$Q_{22} = \frac{E}{1-\nu^2} \tag{14}$$

$$Q_{12} = \frac{\nu E}{1-\nu^2} \tag{15}$$

$$Q_{66} = \frac{E}{2(1+\nu)} \tag{16}$$

where E and ν represent Young's modulus and Poisson ratio, respectively.Using Love's shell theory [28], the strain components are defined as

$$\begin{aligned} e_x &= e_1 - zk_1 \\ e_\theta &= e_2 - zk_2 \\ e_{x\theta} &= \gamma - 2z\tau \end{aligned} \tag{17}$$

where e_1, e_2 and are the reference surface strains and k_1, k_2 and τ are the surface curvatures. These surface strains and curvatures are defined as

$$\{e_1 \quad e_2 \quad \gamma\} = \left\{ \frac{\partial u}{\partial x}, \quad \frac{1}{R}(\frac{\partial v}{\partial \theta} + w), \quad \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta} \right\} \tag{18}$$

$$\{k_1 \quad k_2 \quad \tau\} = \left\{ \frac{\partial^2 w}{\partial x^2}, \quad \frac{1}{R^2}(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}), \quad \frac{1}{R}(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}) \right\} \tag{19}$$

For a thin cylindrical shell the force and moment resultants are defined as

$$\{N_x \quad N_\theta \quad N_{x\theta}\} = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{x\theta}\} dz \tag{20}$$

$$\{M_x \quad M_\theta \quad M_{x\theta}\} = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{x\theta}\} zdz \tag{21}$$

Substituting Eqs. (17) and (9) into Eqs. (20) and (21) following constitutive equation is obtained

$$\{N\} = [S]\{\varepsilon\} \tag{22}$$

where $\{N\}$, $\{\varepsilon\}$ and $[S]$ are defined as

$$\{N\}^T = \{N_x \quad N_\theta \quad N_{x\theta} \quad M_x \quad M_\theta \quad M_{x\theta}\} \tag{23}$$

$$\{\varepsilon\} = \{e_1 \quad e_2 \quad \gamma \quad k_1 \quad k_2 \quad \tau\} \tag{24}$$

$$
[S]=\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \tag{25}
$$

Aij, Bij and Dij are the extensional, coupling and bending stiffnesses, respectively. These matrices are defined as

$$
\left\{ A_{ij} \quad B_{ij} \quad D_{ij} \right\} = \int_{-h/2}^{h/2} Q_{ij} \left\{ 1 \quad z \quad z^2 \right\} dz \tag{26}
$$

For a vibrating thin cylindrical shell the expression for the strain energy, U is given by

$$
\begin{aligned}
U = \frac{1}{2}\int_0^L \int_0^{2\pi} [& A_{11}e_1^2 + A_{22}e_2^2 + 2A_{12}e_1e_2 + A_{66}\gamma^2 + 2B_{11}e_1k_1 + 2B_{12}(e_1k_2 + e_2k_1) \\
& + 2B_{22}e_2k_2 + 2B_{66}\gamma\tau + D_{11}k_1^2 + D_{22}k_2^2 + 2D_{12}k_1k_2 + D_{66}\tau^2] R d\theta dx,
\end{aligned} \tag{27}
$$

After substituting the expressions for the surface strains $e11$, $e22$ and $e12$ and the curvatures $k11$, $k22$ and $k12$ from the relations (4) and (5), the expression for the shell strain energy is transformed to the following form:

$$
\begin{aligned}
U = \frac{1}{2}\int_0^L \int_0^{2\pi} [& A_{11}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{A_{22}}{R^2}\left(\frac{\partial v}{\partial \theta} + w\right)^2 + \frac{2A_{12}}{R}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial \theta} + w\right) + A_{66}\left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right) \\
& + 2B_{11}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial x^2}\right) + 2B_{12}\left\{\frac{1}{R^2}\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) + \frac{1}{R}\left(\frac{\partial v}{\partial \theta} + w\right)\frac{\partial^2 w}{\partial x^2}\right\} \\
& + \frac{2B_{22}}{R^3}\left(\frac{\partial v}{\partial \theta} + w\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) + \frac{2B_{66}}{R}\left(\frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial u}{\partial \theta}\right)\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right) + D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 \\
& + \frac{D_{22}}{R^4}\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right)^2 + \frac{2D_{12}}{R^2}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial v}{\partial \theta}\right) + \frac{D_{66}}{R^2}\left(\frac{\partial^2 w}{\partial x \partial \theta} - \frac{\partial v}{\partial x}\right)^2] R d\theta dx.
\end{aligned} \tag{28}
$$

Without the rotatory inertia the kinetic energy, T, for a thin-walled cylindrical shell is given by

$$
T = \frac{1}{2}\int_0^L \int_0^{2\pi} \rho_T \left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 \right] R d\theta dx, \tag{29}
$$

where t denotes the time and ρ_T is the mass density per unit length and is defined as

$$
\rho_T = \int_{-h/2}^{h/2} \rho dz \tag{30}
$$

where ρ is the mass density of the shell material. The Lagrangian energy functional F is defined by thedifference between the two energies given by the expressions (8) and (9) and is given by

$$F = K - S \tag{31}$$

Substituting the expressions for strain and kinetic energies of the shell from Eqs. (8) and (9), respectively,into the expression (11) and then employing Hamilton's principle [29], the governing equations for the shell dynamical behavior are obtained in the following partial differential equation forms:

$$\begin{gathered}
A_{11}\frac{\partial^2 u}{\partial x^2} + A_{66}\frac{\partial^2 u}{\partial \theta^2} + \left(\frac{A_{12}+A_{66}}{R} + \frac{B_{11}+2B_{66}}{R^2}\right)\frac{\partial^2 v}{\partial x \partial \theta} + \frac{A_{12}}{R}\frac{\partial w}{\partial x} - B_{11}\frac{\partial^3 w}{\partial x^3} \\
-\left(\frac{B_{11}+2B_{66}}{R^2}\right)\frac{\partial^3 w}{\partial x^2 \partial \theta} = \rho_T \frac{\partial^2 u}{\partial t^2} \\
\\
\left(\frac{A_{12}+A_{66}}{R} + \frac{B_{11}+2B_{66}}{R^2}\right)\frac{\partial^2 u}{\partial x \partial \theta} + \left(\frac{A_{22}}{R^2} + \frac{2B_{66}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^2 v}{\partial \theta^2} + \left(A_{66} + \frac{4B_{66}}{R^2} + \frac{4D_{66}}{R^2}\right)\frac{\partial^2 v}{\partial x^2} \\
+\left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial w}{\partial \theta} - \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3 w}{\partial \theta^3} - \left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}}{R^2} + \frac{4D_{66}}{R^2}\right)\frac{\partial^3 w}{\partial x^2 \partial \theta} = \rho_T \frac{\partial^2 v}{\partial t^2} \qquad (32) \\
\\
\frac{A_{12}}{R}\frac{\partial u}{\partial x} - B_{11}\frac{\partial^3 u}{\partial x^3} - \left(\frac{B_{11}+2B_{66}}{R^2}\right)\frac{\partial^3 u}{\partial x \partial \theta^2} + \left(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3}\right)\frac{\partial v}{\partial \theta} - \left(\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4}\right)\frac{\partial^3 v}{\partial \theta^3} \\
-\left(\frac{B_{12}+2B_{66}}{R} + \frac{D_{12}}{R^2} + \frac{4D_{66}}{R^2}\right)\frac{\partial^3 v}{\partial x^2 \partial \theta} + \frac{A_{22}}{R^2}w - \frac{2B_{12}}{R}\frac{\partial^2 w}{\partial x^2} - \frac{2B_{22}}{R^3}\frac{\partial^2 w}{\partial \theta^2} + \left(\frac{D_{22}}{R^4}\right)\frac{\partial^4 w}{\partial \theta^4} \\
+D_{11}\frac{\partial^4 w}{\partial x^4} + \left(\frac{2D_{12}}{R^2} + \frac{4D_{66}}{R^2}\right)\frac{\partial^4 w}{\partial x^2 \partial \theta^2} = -\rho_T \frac{\partial^2 w}{\partial t^2} + kw - G\nabla^2 w
\end{gathered}$$

G represents the shear modulus of the material used for the elastic foundation and K for the Winkler foundation
modulus, and the expression for the differential operator ∇^2 is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} \tag{33}$$

The Winkler model is a special case of the Pasternak model when G=0.

4 .Solution procedure

The wave propagation approach is employed to analyze the vibration characteristics of functionally graded cylindrical shells with Pasternak-type foundations. This approach is very simple and easily applicable to determine the shell frequencies. This has been successfully applied by a number of researchers, see Zhang et al. [25,30]. For separating the spatial and temporal variables, the following forms of modal displacement deformations are assumed:

$$\begin{aligned} u(x,\theta,t) &= Ae^{i(n\theta+\omega t-k_m x)}, \\ v(x,\theta,t) &= Be^{i(n\theta+\omega t-k_m x)}, \\ w(x,\theta,t) &= Ce^{i(n\theta+\omega t-k_m x)}, \end{aligned} \tag{34}$$

in the axial, circumferential and radial directions, respectively. The coefficients *A*, *B* and *C* denote the waveamplitudes, respectively, in the *x*, θ and *z* directions, respectively, *n* is the number of circumferential waves and *km* is the axial wave number that has been specified in [25] for a number of boundary conditions. These axial wave numbers *km* are chosen to satisfy the required boundary conditions at the two ends of the cylindrical shell. ω is the natural circular frequency for the cylindrical shell. On substituting the expressions for *u, v* and *w* from Eq. (12) into Eq. (10) and simplifying the algebraic expressions and rearranging the terms, the frequency equation is written in the following eigenvalue form:

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{12} & C_{22} & C_{23} \\ C_{13} & C_{23} & C_{33} \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \rho_T \omega^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} \tag{35}$$

where *Cij (i, j* = 1, 2, 3*)* are some matrix coefficients depending on the shell parameters and the type of boundary conditions specified at the ends of a cylindrical shell and are given in Appendix B. Equation (13) is solved for shell frequencies and mode shapes using some computer software. The three frequencies are obtained corresponding to the axial, circumferential and radial displacements. The smallest frequency is associated with the radial direction and dominant.

5. Results and discussion

A number of comparisons of numerical results for cylindrical shells are presented to verify the validity of the present approach and accuracy of the results. In Table 1, the frequency parameters _ for an isotropic cylindrical shell are compared with those evaluated by Pradhan et al. [5] for clamped–clamped edge conditions. In this case the shell parameters are taken to be *L/R* = 20, *h/R* = 0.002, *m* = 1. Table 2 represents the frequency parameters, Ω for a clamped– simply supported cylindrical shell, and a comparison is made with those values evaluated in Loy et al. [24]. The shell parameters are chosen to be *m* = 1, *L/R* = 20, *h/R* = 0.01 and *ν*=0.3. Tables 3 give the comparisons of natural frequencies (Hz) of two types of functionally graded cylindrical shells. In Table 3, the shell is assumed to be composed of stainless steel at the outer surface and nickel at the inner surface of the shell.

5.1. Vibration frequency analysis based on elastic foundations

A vibration frequency analysis for functionally graded cylindrical shells based on elastic foundations is performed where the material configurations are composed of two constituent materials viz. stainless steel and nickel. An FGM cylindrical shell may be classified into two types.

Table 1 Comparison of frequency parameters $\Omega = \omega R\sqrt{\rho(1-\upsilon^2)/E}$ for a cylindrical shellwith the clamped–clamped boundary conditions *(m* = 1*, ν* = 0.3*, L/R* = 20*, h/R* = 0.002*)*

n	Ref. [5]	Present
1	0.0342	0.0348
2	0.0119	0.0119
3	0.0072	0.0072
4	0.0089	0.0091
5	0.0136	0.0137

Table 2 Comparison of frequency parameters $\Omega = \omega R\sqrt{\rho(1-\upsilon^2)/E}$ for a cylindrical shell with the clamped–simply supported boundary conditions *(m* = 1*, L/R* = 20*, h/R* = 0.01*, ν* = 0.3*)*

n	Ref. [8]	Present
1	0.023974	0.024718
2	0.011225	0.011265
3	0.022310	0.022319
4	0.042139	0.042132
5	0.068024	0.068044

Table 3 Comparison of natural frequencies (Hz) for a cylindrical shell with the clamped–simply supported boundary conditions *(m* = 1*, L/R* = 20*, h/R* = 0.01*, ν* = 0.3*)*

n	*p=0.7*		*p=0.7*		*p=0.7*	
	Ref. [4]	Present	Ref. [4]	Present	Ref. [4]	Present
1	13.269	13.270	13.103	13.104	12.914	12.915
2	4.4994	4.4986	4.4435	4.4430	4.3765	4.3761
3	4.1749	4.1741	4.1235	4.1230	4.0576	4.0570
4	7.0691	7.0687	6.9820	6.9818	6.8726	6.8723
5	11.290	11.291	11.151	11.152	10.978	10.979
6	16.527	16.528	16.323	16.324	16.071	16.072
7	22.735	22.736	22.454	22.455	22.108	22.109
8	29.903	29.903	29.533	29.534	29.078	29.079
9	38.028	38.029	37.559	37.560	36.981	36.981
10	47.111	47.112	46.529	46.530	45.813	45.814

Table 4 Variation of natural frequencies (Hz) against circumferential wave number *n* Type I (FG cylindrical shell on elastic foundation) *(m* = 1*, h/R* = 0.002*, L/R* = 20*)*

n	p^{ss}=0	p^{N}=0	*p=0.5*	*p=0.7*	*p=1*	*p=2*	*p=5*	*p=15*	*p=30*
1	193.91	185.70	191.06	190.41	189.67	188.32	186.99	186.18	185.95
2	414.72	397.20	408.63	407.21	405.67	402.79	399.96	398.23	397.73
3	634.75	607.97	625.45	623.32	620.94	616.53	612.21	609.56	608.79
4	853.21	817.21	840.69	837.83	834.63	828.7	822.9	819.34	818.31
5	1,070.71	1,025.6	1,055.0	1,051.4	1,047.4	1,039.9	1,032.7	1,028.2	1,026.9
6	1,287.71	1,233.4	1,268.8	1,264.5	1,259.7	1,250.8	1,242.0	1,236.6	1,235.1
7	1,504.42	1,441.0	1,482.3	1,477.3	1,471.7	1,461.2	1,451.0	1,444.7	1,442.9
8	1,720.91	1,648.4	1,695.7	1,689.9	1,683.5	1,671.5	1,659.8	1,652.6	1,650.6
9	1,937.32	1,855.6	1,908.8	1,902.4	1,895.1	1,881.7	1,868.5	1,860.4	1,858.5
10	2,153.62	2,062.8	2,121.9	2,114.8	2,106.7	2,091.8	2,077.1	2,068.1	2,065.6

Table 5 Variation of natural frequencies (Hz) against circumferential wave number *n* Type II (FG cylindrical shell on elastic foundation) *(m* = 1, *h/R* = 0.002, *L/R* = 20*)*

n	p^{ss}=0	p^{N}=0	*p=0.5*	*p=0.7*	*p=1*	*p=2*	*p=5*	*p=15*	*p=30*
1	194.0	185.7	188.32	188.95	189.67	191.05	192.46	193.35	193.62
2	414.73	397.2	402.79	404.14	405.67	408.62	411.63	413.55	414.11
3	634.77	607.98	616.53	618.59	620.94	625.44	630.05	632.98	633.84
4	853.21	817.22	828.71	831.48	834.63	840.68	846.87	850.81	851.96
5	1,070.7	1,025.6	1,040.0	1,043.5	1,047.4	1,055.0	1,062.8	1,067.7	1,069.2
6	1,287.7	1,233.4	1,250.7	1,254.9	1,259.7	1,268.8	1,278.2	1,284.1	1,285.8
7	1,504.4	1,441.0	1,461.2	1,466.1	1,471.7	1,482.3	1,493.3	1,500.2	1,502.2
8	1,720.9	1,648.4	1,671.5	1,677.1	1,683.5	1,695.7	1,708.1	1,716.1	1,718.4
9	1,937.3	1,855.6	1,881.7	1,887.9	1,895.1	1,908.8	1,922.9	1,931.8	1,934.1
10	2,153.6	2,062.8	2,091.8	2,098.7	2,106.7	2,121.9	2,137.6	2,147.5	2,150.4

In type I FG cylindrical shell material properties vary continuously from those of nickel on its inner surface to stainless steel on its outer surface. The second is termed as a type II FG cylindrical shell. It has properties that vary continuously from stainless steel on its inner surface to nickel on its outer surface. Tables 4 and 5 show the variation of natural frequencies (Hz) of a functionally graded cylindrical shell with the Winkler and Pasternak foundations. The simply supported boundary conditions are specified at the ends of the shell. In Table 4 values of natural frequencies (Hz) are given for a functionally graded cylindrical shell type I. The frequency increases with increasing the circumferential wave number, *n*, and the vibration becomes the beam-type. But it decreases with increasing the values of the power law exponent *p*. In Table 5 values of natural frequencies (Hz) are listed for a functionally graded cylindrical shell type II. In this case they increase with the power law exponents, *p*, but increase with the circumferential wave number, *n*. Thus the influence of the constituent volume fractions on the frequencies for type I and type II functionally graded cylindrical shells is different based on elastic foundations. It is observed that the natural frequency of a functionally graded cylindrical shell on an elastic foundation increases continuously with increasing values of circumferential *n*.

6. Concluding remarks

In this study the vibration characteristics of a functionally graded cylindrical shell are analyzed based on the Winkler and Pasternak foundations. The shell dynamical equations are solved by using the wave propagation approach. The influence of these elastic foundations is pronounced and this effect converts the shell vibration into beam-type. This analysis can be extended to study the influence of boundary conditions on shell vibrations based on the Winkler and Pasternak foundations.

References

[1] Ng, T.Y., Lam, K.Y., Liew, K.M., 2000. Effect of FGM materials on the parametric resonance of plate structures. Comput. Methods Appl. Mech. Eng. 190, 953–962.
[2] Yang, J., Kitipornchai, S., Liew, K.M., 2003. Large amplitude vibration of thermoelectro-mechanically stressed FGM laminated plates. Comput. Methods Appl. Mech. Eng. 192, 3861–3885.

[3] Della Croce, L., Venini, P., 2004. Finite elements for functionally graded Reissner-Mindlin plates. Comput. Methods Appl. Mech. Eng. 193, 705–725.
[4] Liew, K.M., He, X.Q., Kitipornchai, S., 2004. Finite element method for the feedback control of FGM shells in the frequency domain via piezoelectric sensors and actuators. Comput. Methods Appl. Mech. Eng. 193, 257–273.
[5] Wu, C.P., Tsai, Y.H., 2004. Asymptotic DQ solutions of functionally graded annular spherical shells. Eur. J. Mech. A – Solids 23, 283–299.
[6] Elishakoff, I., Gentilini, C., Viola, E., 2005a. Forced vibrations of functionally graded plates in the three-dimensional setting. AIAA J. 43, 2000–2007.
[7] Elishakoff, I., Gentilini, C., Viola, E., 2005b. Three-dimensional analysis of an all around clamped plate made of functionally graded materials. Acta Mech. 180, 21–36.
[8] Patel, B.P., Gupta, S.S., Loknath, M.S., Kadu, C.P., 2005. Free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory. Compos. Struct. 69, 259–270.
[9] Pelletier, J.L., Vel, S.S., 2006. An exact solution for the steady-state thermoelastic response of functionally graded orthotropic cylindrical shells. Int. J. Solids Struct. 43, 1131–1158.
[10] Zenkour, A.M., 2006. Generalized shear deformation theory for bending analysis of functionally graded plates. Appl. Math. Model. 30, 67–84.
[11] Arciniega, R.A., Reddy, J.N., 2007. Large deformation analysis of functionally graded shells. Int. J. Solids Struct. 44, 2036–2052.
[12] Nie, G.J., Zhong, Z., 2007. Semi-analytical solution for three dimensional vibration of functionally graded circular plates. Comput. Methods Appl. Mech. Eng. 196, 4901–4910.
[13] Roque, C.M.C., Ferreira, A.J.M., Jorge, R.M.N., 2007. A radial basis function for the free vibration analysis of functionally graded plates using refined theory. J. Sound Vib. 300, 1048–1070.
[14] Najafizadeh, M.M., Isvandzibaei, M.R.: Vibration of functionally graded cylindrical shells based on higher order shear deformation plate theory with ring support. Acta Mech. **191**, 75–91 (2007)
[15] Yang, J., Shen, H.S., 2007. Free vibration and parametric resonance of shear deformable functionally graded cylindrical panels. J. Sound Vib. 261, 871–893.
[16] He XQ, Liew KM, Ng TY, Sirvashanker S. A FEM model for the active control of curved FGM shells using piezoelectric sensor/actuator layers. International Journal for Numerical Methods in Engineering 2002;54:853–70.
[17] Liew KM, Kitipornchai S, Zhang XZ, Lim CW. Analysis of the thermal stress behavior of functionally graded hollow circular cylinders. International Journal of Solids and Structures 2003;40:2355–80.
[18] Jabbari M, Sohrabpour S, Eslami MR. General solution for mechanical and thermal stresses in a functionally graded hollow cylinder due to nonaxisym- metric steady-state loads. Journal of Applied Mechanics 2003;70:111–18.
[19] Jocob LP, Vel SS. An exact solution for the steady-state thermoelastic response of functionally graded orthotropic cylindrical shells. International Journal of Solids and Structures 2006;43:1131–58.
[20] Hosseini Kordkheili SA, Naghdabadi R. Geometrically non-linear thermo- elastic analysis of functionally graded shells using finite element method. International Journal for Numerical Methods in Engineering 2007 964–86.
[21] Arciniega RA, Reddy JN. Large deformation analysis of functionally graded shells. International Journal of Solids and Structures 2007;44:2036–52.
[22] Woo J, Merguid SA, Stranata JC, Liew KM. Thermomechanical postbuckling analysis of moderately thick functionally graded plates and shallow shells. International Journal of

Mechanical Sciences 2005;47:1147–71. [23] Bahtui A, Eslami MR. Generalized coupled thermoelasticity of functionally graded cylindrical shells. International Journal for Numerical Methods in Engineering 2007;69:676–97.
[24] Loy CT, Lam KY, Reddy JN. Vibration of functionally graded shells. Interna- tional Journal of Mechanical Sciences 1999;41:309–24.
[25] Zhang, X.M., Liu, G.R., Lam, K.Y. 2001: Vibration analysis of thin cylindrical shells using wave propagation approach. J. Sound Vib. 239, 397–403.
[26] Arshad, S. H., Naeem, M. N., and Sultana, N. Frequency analysis of functionally graded material cylindrical shells with various volume fraction laws. *Proceedings of the Institution of Mechanical Engineers, Part C*: *Journal of Mechanical Engineering Science* 221, 1483–1495 (2007)
[27] Touloukian YS. Thermophysical properties of high temperature solid materials. New York: Mac- millian, 1967.
[28] Love AEH. A treatise on the mathematical theory of elasticity. 4th Ed. Cambridge: Cambridge University Press, 1952.
[29] Bhimaraddi A. A higher order theory for free vibration analysis of circular cylindrical shells. International Journal of Solids and Structures 1984;20:623±30.
[30] Zhang, X.M., Liu, G.R., Lam, K.Y.: Vibration analysis of cylindrical shells using the wave propagation approach. J. Sound Vib. 239, 397–401 (2001).