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SIMULATION OF BED CHANGES IN RIVERS WITH FINITE VOLUME METHOD BY KINEMATIC WAVE MODEL

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Abstract

Predict of sediment particles movement is important mater, that can be help engineers to control flow structures, dams reservoirs, irrigation systems and etc. Simplification and sufficient sensible resolute cased to select one dimensional (1-D) approach for simulation of sediment transports. In this study we tried to simulate 1-D phenomena of sediment transport with explicit and implicit schemes in equilibrium conditions. In order to solve government equations used finite volume method by upwind scheme. Finally the model has been verified by Laboratory research and the results are generally fitted well in with the measurements.

Keywords: Sediment transport, 1-D model, Finite volume method, Kinematic wave model, Explicit, Implicit

1. Introduction

Sediment transport is one of the complicated phenomena in nature that influence in human life continuously. In previous years, scientists tried to understand and predict this phenomen, so they introduced various empirical relations [Guy et al., 1966; Langbein and Leopold, 1968; Soni, 1981a; Wathen and Hoey, 1998; Lisle et al., 1997, 2001], but most of the time the result of this relations were not satisfactory. In recent years by developing in computer science, researchers able to compute more complicated problems fast, so they interested to developed different numerical methods in order to predict flow and sediment movement.

The continuity equations of water and sediment can be solved by different methods, approaches and schemes. In this study, Finite Volume Method with Kinematic Wave Model approaches has been used. Moreover Lax and upwind scheme has been selected. The most important advantage of Finite Volume Method is its ability to conservative of quantities such as mass, momentum, energy, and species in solution. Most of investigators tried to solve sediment transport equations by finite difference and finite element methods. Fuladipanah et al (2010) developed one dimensional implicit finite difference method for calculating flow and suspended load. Seo et al (2009) selected finite element method by applied the Galerkin Method in order to determined suspended sediment transport in rivers. Fang et al.(2008) for discretisation of the sediment transport equations used the Preissmann implicit four-point Finite Difference Method. In their solutions, flow, sediment transport and change of bed in rivers and channels can be predicted. Tayfur and Singh (2006) solved de Saint Venant equations by Kinematic Wave Model that described the evolution and movement of bed profiles in alluvial channels under the equilibrium conditions. They used explicit finite difference method in order to discretised relations. Paquier (1998) solved de Saint Venant equations by finite difference method with second-order Godunov-type explicit scheme. De

Vries (1965) used explicit finite difference scheme to simulations of water and bed level changes in one dimension. Wu and Wang (2008) solved one-dimensional explicit finite-volume model for sediment transport with transient flows over movable beds.In addition to conventional methods, in recent years, new methods are used. Kaya and Tayfur (2011) suggested a method that can be predict sediment movements with differential quadrature method by using Kinematic Wave Model.

2. Governing Equations

A general view of wide rectangular alluvial channel with both layers is given in figure 1. The one dimensional equations for equilibrium sediment transport processes in unsteady flow conditions can be expressed as follow

Continuity equation for water is

$$\frac{\partial h(1-c)}{\partial t}+\frac{\partial hu(1-c)}{\partial x}+p\frac{\partial z}{\partial t}=q_{lw} \tag{1}$$

Continuity equation for sediment is

$$\frac{\partial hc}{\partial t}+\frac{\partial huc}{\partial x}+(1-p)\frac{\partial z}{\partial t}+\frac{\partial q_{bs}}{\partial x}=q_{ls} \tag{2}$$

Where '*h*' is the flow height (*L*), '*u*' is the velocity of flow (*L/T*), '*c*' is the volumetric concentration of sediment in suspension (L^3/L^3), '*p*' is the porosity of sediment in bed level(L^3/L^3), '*z*' is the movable bed layer elevation (*L*), 'q_{lw}' is the lateral water flux (*L/T*), 'q_{bs}' is the sediment flux in the movable bed layer (L^2/T), 'q_{ls}' is the lateral sediment flux (L/T), 'ρ_s' is the sediment mass density (M/L^3).
In laboratory flume of Dokuz Eylül University that studies carried out there is no influence of lateral sediment or flow, so 'q_{lw}' and 'q_{ls}' are equal to zero. It must be attention that there are five unknown in relations (1) and (2) (h, u, c, z, and q_{bs}). So other three equations must be used to solve the set of relations above. Instead of third equation can be used momentum equation. Because of simplification in kinematic wave model can be considered frictional slope as a bed slope. By using Manning or Chezy equations can be written as

$$So=Sf \tag{3}$$

$$u=\frac{1}{n}R^{\frac{2}{3}}S_0^{\frac{1}{2}} \tag{4}$$

$$u=C\sqrt{RS_0} \tag{5}$$

where' S_0 'is the bed slope, 'S_f' is the friction slope, 'n' is Manning coefficient, R is hydraulic radius and C is Chezy coefficient.
The forth equation obtains from volumetric concentration of sediments transported by water flow (c) (Ching and Cheng, 1964; de Vries, 1965; Lai, 1991; Pianese, 1994)

$$c = \delta u^{\eta} h^{\xi} \tag{6}$$

where δ , η and ξ depends to water flow and sediment characteristics. In this research we used Velikanov [1954] relations

$$c = \frac{\kappa u^3}{g v_f h} \tag{7}$$

where $\eta = 3$ and $\xi = -1$. With paying attention to equations (6) and (7) can be obtain the value of δ as below

$$\delta = \frac{\kappa}{g v_f} \tag{8}$$

Where 'v_f' is the average fall velocity of sediment particles (L/T) and 'κ' is the coefficient of sediment transport capacity. Ching and Cheng (1964) by considering field measurements suggested the amount of 'κ' as ($\kappa = 0.756 \times 10^{-4}$).

As a fifth equation can be used sediment flux in the movable bed layer (q_{bs}). In order to compute the value of sediment flux researchers proposed different kind of empirical relations. This study employed the equation of Engelund and Fredsoe(1976):

$$\mathrm{q_{bs}} = 18.74\sqrt{\Delta.\mathrm{g}.\mathrm{D}_{50}^{3}}\,(\tau_* - \tau_{*cr})^{3/2} \tag{9}$$

Where 'D_{50}' is the bed material size where 50% of the material is finer in mm,'Δ' is the relative specific gravity and,' τ_* ' and ' τ_{*cr} 'are the dimensionless shear stress and dimensionless critical bed shear stresses. The amount of relative specific gravity can be computed by

$$\Delta = \frac{(\gamma_s - \gamma)}{\gamma} \tag{10}$$

Where ' γ_s ' and ' γ ' are the specific weights of sediment and water and value of dimensionless shear stress can be computed by

$$\tau_* = \frac{u_*^2}{g.\Delta.ds} \tag{11}$$

Where ds is the diameter of sediment particle that can be use d_{50} of mixture of sediment particles and u_* is shear velocity that defined as

$$u_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{g.h.S_0} \tag{12}$$

Where 'τ_0' is the shear stress, 'ρ' is the density of water,' g ' is the gravity acceleration, 'h'

is the flow depth and 'S_0' is the bed slope and. They suggested value of critical bed shear stress as $\tau_{*cr} = 0.05$.

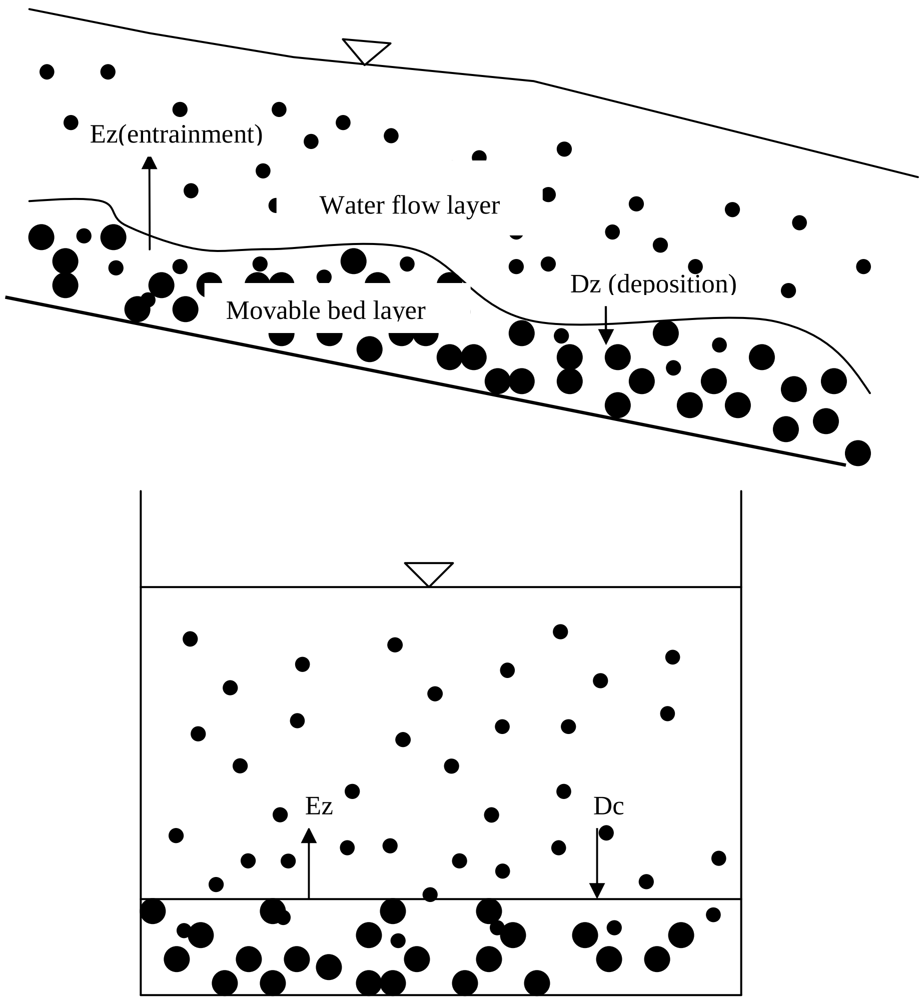


Fig 1. Schematic representation of two layer system

3. Numerical method

The finite volume method is a method for representing and evaluating partial differential equations in the form of algebraic equations [LeVeque, 2002; Toro, 1999].

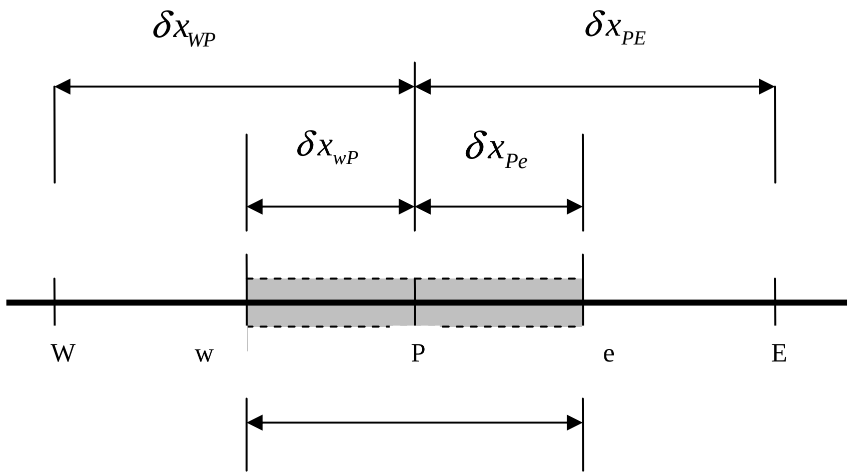


Fig. 2. Illustration of finite volume method in one dimensional

Figure 2 shows a one dimensional finite volume network. In this method, channel's length is discreted into N parts and the center of each fragment is a grid point (P). The minuscules (e and w) refers to face of control volume for point P. The value of (P) can be considered as our unknowns like depth of water (h) or velocity (V).

In order to solve governing equations by finite volume method the following notation can be used.

$$W_t + F_x - S = 0 \tag{13}$$

Where $W = \begin{bmatrix} h(1-c)+pz \\ hc+(1-p)z \end{bmatrix}$ $F = \begin{bmatrix} hu(1-c) \\ huc+q_{bs} \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

By consider the integration of equation (12) can be seen that

$$\int_{CV}\int_{t}^{t+\Delta t} \frac{\partial W}{\partial t} dt.dV = -\int_{CV}\int_{t}^{t+\Delta t} \frac{\partial F}{\partial x} dt.dV + \int_{CV}\int_{t}^{t+\Delta t} Sdt.dV \tag{14}$$

By assumption that the values of '*P*' is govern to the whole of control volume, the left hand side of equation (14) can be written as

$$\int_{CV}\left[\int_{t}^{t+\Delta t} \frac{\partial W}{\partial t} dt\right] dV = (W_p - W_p^o)\Delta V \tag{15}$$

In this equation the letters with superscribed '*0*' refers to value at time 't' but values at time '$t+\Delta t$' don't have superscripted. With substitute equation (15) into equation (18) and with integration of 'F'order to location (x) equation (15) can be rearranged as follows:

$$(W_p - W_p^o)\Delta V = -\left(A_e.F_e - A_w.F_w\right)\Delta t + S.\Delta t.\Delta V \tag{16}$$

Where' A' is the face area of the control volume, 'ΔV' is the volume of control volume. The amount of control volume can be determined by product of face area of the control volume to width of the control volume ($A\Delta x$).

According to selected method of solution, values of unknowns can be used at time 't' or time '$t+\Delta t$'. The general form of unknown with respect to weighting parameter 'θ' between 0 and 1 can be written as below:

$$I_T = \int_t^{t+\Delta t} W_p dt = \left[\theta W_p + (1-\theta) W_p^0\right]\Delta t \tag{17}$$

Where

θ	0	$1/2$	1
I_T	$W_p^0 \Delta t$	$\frac{1}{2}\left(W_p + W_p^0\right)\Delta t$	$W_p \Delta t$

By taking into account of equation (16) can be rewritten equation (15) as follow.

$$(W_p - W_p^o)\Delta V = \theta\left[-\left(A^0{}_e.F^0{}_e - A^0{}_w.F^0{}_w\right)\Delta t\right] + (1-\theta)\left[-\left(A_e.F_e - A_w.F_w\right)\Delta t\right] + S.\Delta t.\Delta V \tag{18}$$

Note that by selecting 'θ' as zeros the known values at old time level 't' are used; this scheme called as explicit method. When 'θ' selected in the range of zero and one ($0<\theta\le 1$) the values of unknown at the new time level are used; the resulting schemes are called implicit method. The extreme case of $\theta=1$ is termed fully implicit method and the name of scheme that used $\theta=1/2$ is Crank-Nicolson scheme.

In this study explicit and fully implicit scheme has been employed. Moreover, in order to solve the finite volume method, Upwind and Lax scheme has been used. Explain of them can be found in below. The main idea of upwind scheme is that the value of calculated node 'ϕ' at a cell is taken to be equal to the value at the upstream node. Because of considering the effect of flux direction, the result of this scheme is satisfactory. Imagine of upwind scheme can be found in figure 3.

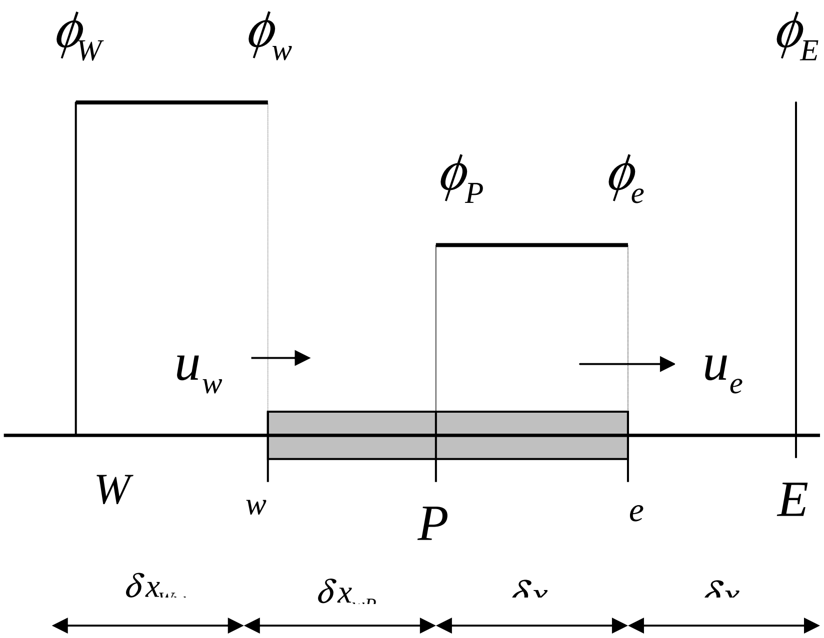


Fig. 3. Upwind scheme

It is suggested in use of Lax scheme that average values of neighbor pointes instead of point p can be used in values old time level.

$$\phi_P^0 = \frac{\phi_E^0 + \phi_W^0}{2} \tag{19}$$

By considering upwind scheme and by using fully implicit method can be rewrite equation (18) as below

$$(W_p - W_p^o)\Delta V = \left[-\left(A_P.F_P - A_W.F_W\right)\Delta t\right] + S.\Delta t.\Delta V \tag{20}$$

The final form of continuity equations of water and sediment for Kinematic wave model in equilibrium condition with some simplification and manipulation and by employed of upwind and Lax schemes can be written as follow.

3.1.Explicit

Continuity equation of water is

$$ah_p - ma.h_p^{1.5} + P.a.z_p + s1 = 0$$
$$s1 = -h_p^0.a + mah_p^{o1.5} - P.a.\frac{z_E^0 + z_W^0}{2} + (((\frac{h^0{}_P + h^0{}_E}{2})^{2.5}) - ((\frac{h^0{}_P + h^0{}_W}{2})^{2.5})) - ((n(\frac{h^0{}_P + h^0{}_E}{2})^3) - (n(\frac{h^0{}_P + h^0{}_W}{2})^3))$$

Continuity equation of sediment is

$$m.a.h_P^{1.5} + (1-p).a.z_p + s2 = 0$$
$$s2 = -m.a.h_P^{01.5} - (1-p).a.\frac{z_E^0 + z_W^0}{2} + (\,(n(\frac{h^0{}_P + h^0{}_E}{2})^3) - (n(\frac{h^0{}_W + h^0{}_P}{2})^3))$$
$$+(\frac{h^0{}_P + h^0{}_E}{2}.qbs^0{}_e - \frac{h^0{}_W + h^0{}_P}{2}.qbs^0{}_w)$$

3.2.Fully implicit

Continuity equation of water is

$$ah_p - ma.h_p^{1.5} + P.a.z_p + ((\frac{h_P + h_E}{2})^{2.5}) - ((\frac{h_P + h_W}{2})^{2.5}) - ((n(\frac{h_P + h_W}{2})^3) - (n(\frac{h_P + h_W}{2})^3)) + s1 = 0$$
$$s1 = -h_p^0.a + ma(\frac{h_E^0 + h_W^0}{2})^{1.5} - P.a.\frac{z_E^0 + z_W^0}{2}$$

Continuity equation of sediment is

$$m.a.h_P^{1.5} + (1\text{-p}).\text{a}.\text{z}_\text{p} + (\,(n(\frac{h_P + h_E}{2})^3) - (n(\frac{h_W + h_P}{2})^3)) + (\frac{h_P + h_E}{2}).(\frac{qbs_P + qbs_P}{2}) - (\frac{h_W + h_P}{2}).(\frac{qbs_P + qbs_W}{2}) + s2 = 0$$
$$s2 = -m.a.(\frac{h_E^0 + h_W^0}{2})^{1.5} - (1\text{-p}).\text{a}.\frac{z_E^0 + z_W^0}{2}$$

Where' $m = \delta.\alpha^3$ ',' $n = \delta.\alpha^4$ 'and ' $a = \frac{\Delta A}{\Delta t}$ '

4. Boundary and initial conditions

Flow regimes may be either subcritical or supercritical, or mixed in the channel flow. If subcritical flow occurs, only flow discharge should be imposed in inlet, and if supercritical flow occurs, both flow discharge and water level should be specified. In the outlet, if the flow is subcritical, the value of water level should be given, and if the flow is supercritical, no boundary condition is needed. Boundary conditions for sediment discharge are always specified at the inlet. In the case of non-uniform sediment transport, the size composition of inflowing sediment should be provided too (Wu and Wang 2008).

In laboratory flume used in this study, the first three meter is fixed bed and after that movable bed with sediment particles started. In the inlet of channel value of flow rate and velocity of flow was measured. In addition, in the length of channel depth of water was measured.

5. Laboratory flume

In this research a rectangular channel with 18.6 m length and 80 cm width was used. The slope of channel was 0.005 from horizontal. The general view of flume that used in study is given in Figure (5.a and b).

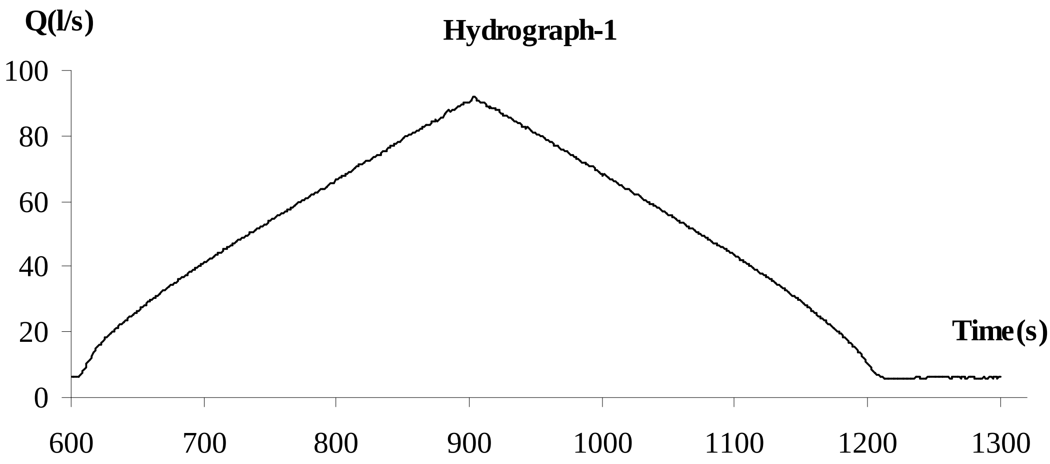
Fig 5a. View of flume from downstream

Fig 5b. General view of the flume

6. Model tests

In this research kinematic wave model was tested by two different hydrographs. These hydrographs can be found in figures below. Before begin to pass the hydrographs a smooth surface of sediments in vertical and stream-wise direction was made. The elevation of this bed level measured in longitude and latitude directions times. One dimensional initial bed profile was given at Figure 7. The developed models can be determined depth and velocity of flow and bed load changes depending to time. The comparison between results of experimental study and models prediction was given in figure 8 and 9 for Hydrograph 1.

Comparison between the results of experimental and the numerical studies at various distance form upstream of channel (x=5, 8, 11 and 15m) given in figure 8. It must be noted that the results of the study depends to the selected empirical sediment flux, explain of initial and boundary conditions, the selected friction slope and the sediment properties that influence in sediment particles movement. With consider the applied relations and parameters differences of experimental and numerical results for bed elevations are reasonable (Figure 9).



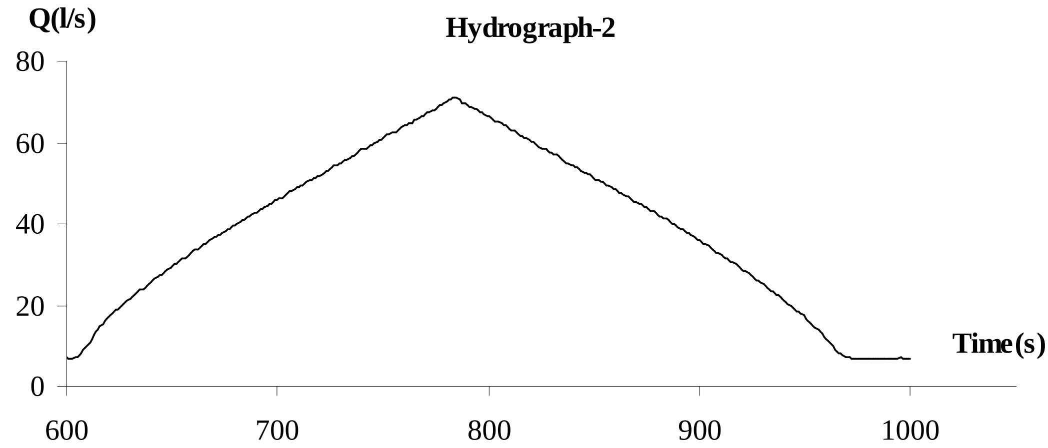


Fig. 6. Inflow hydrographs

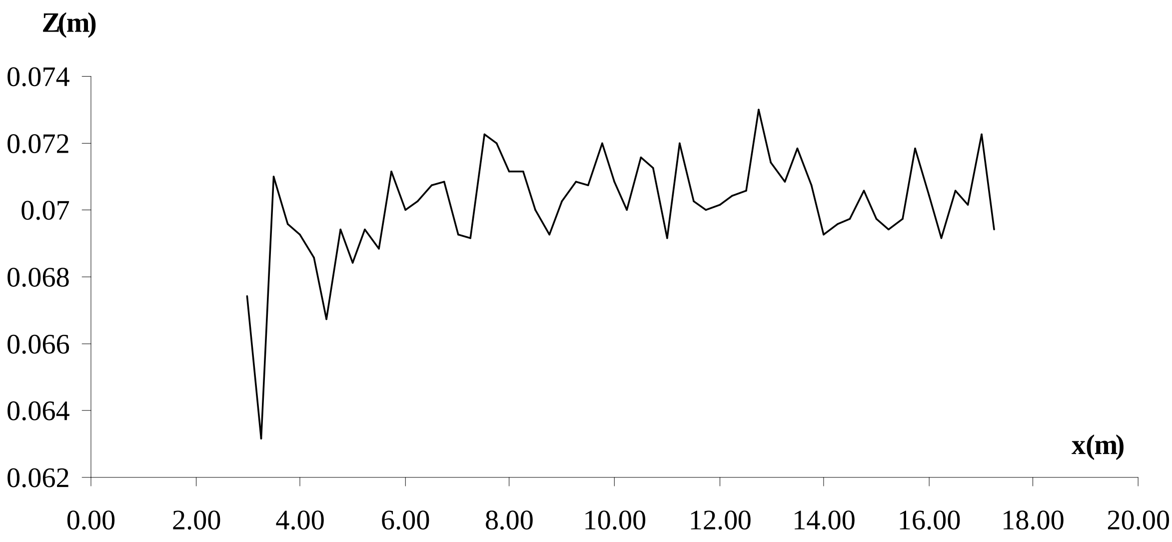


Fig.7. Initial bed profile

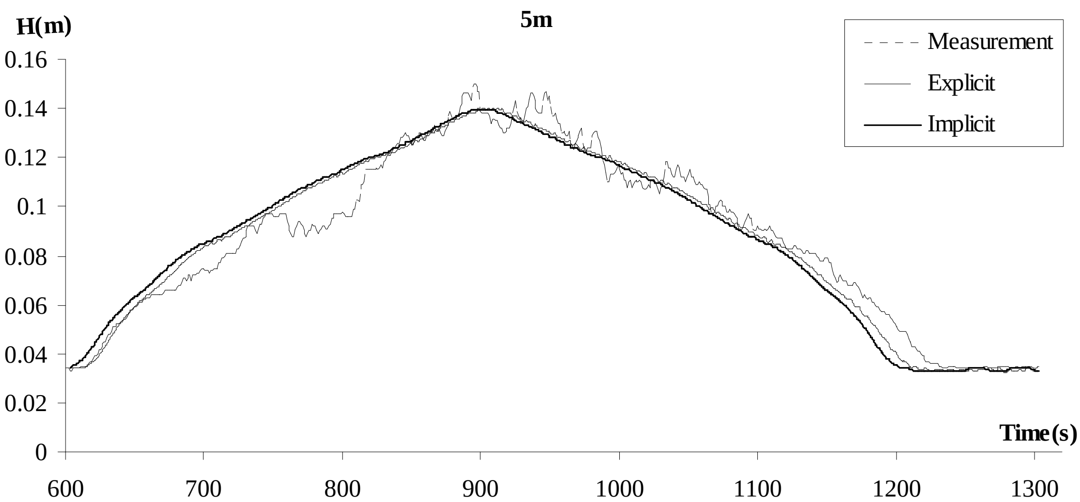


Fig. 8a. Experimental and numerical results for flow depth in x=5m and Hydrograph 1

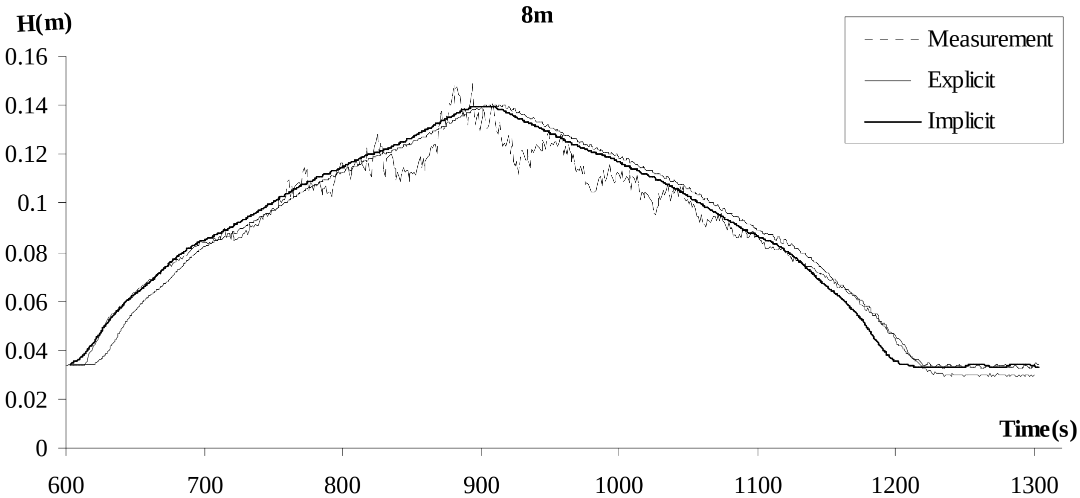


Fig. 8b. Experimental and numerical results for flow depth in x=8m and Hydrograph 1

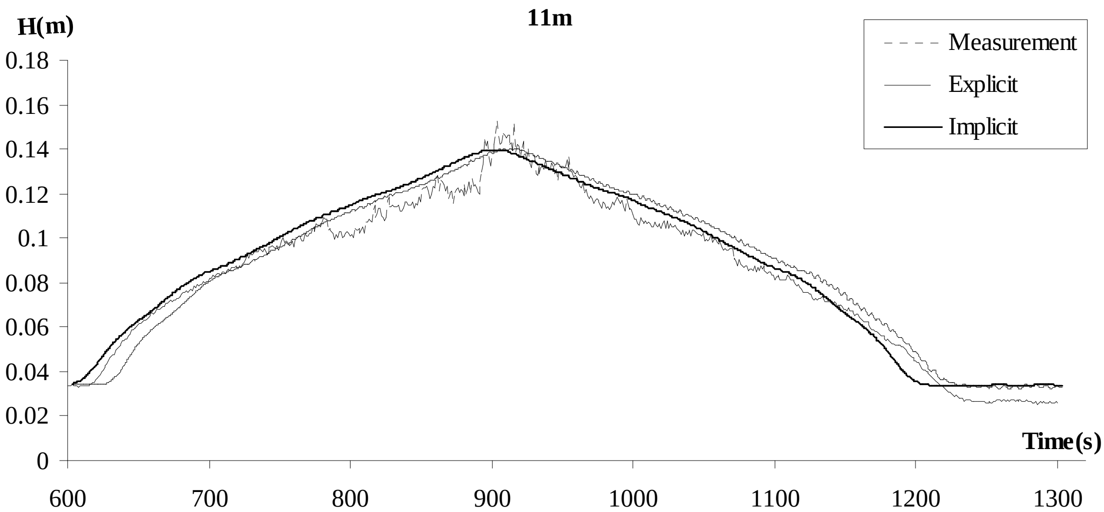


Fig. 8c. Experimental and numerical results for flow depth in x=11m and Hydrograph 1

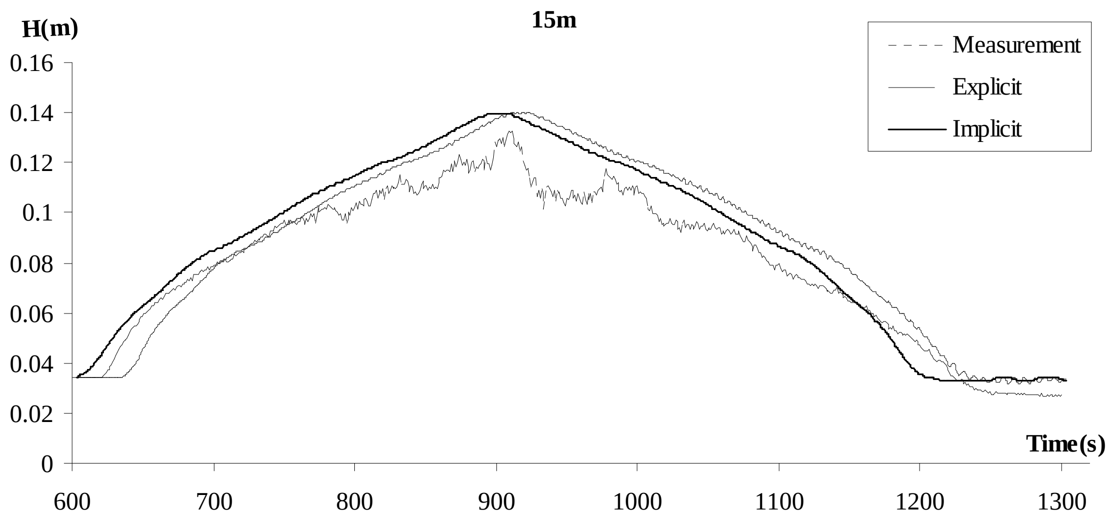


Fig. 8d. Experimental and numerical results for flow depth in x=15m and Hydrograph 1

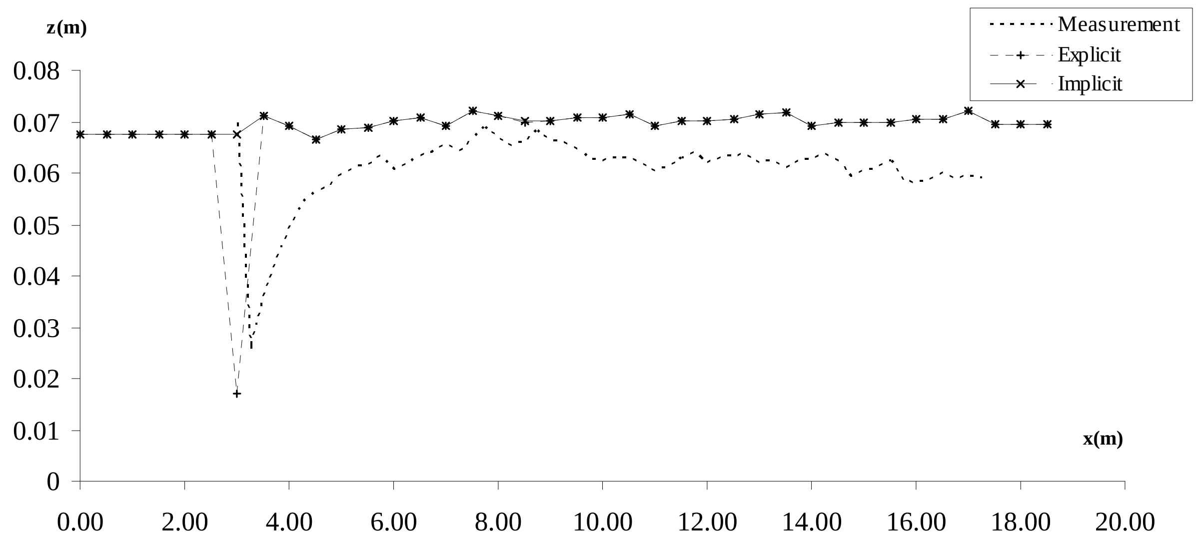


Fig. 9. Comparison of experimental and numerical results for bed profile in Hydrograph 1

For the hydrograph 2, experimental and numerical results was given in figure 10 and 11. Calculated flow depths for Hydrograph 2 are seen in Figure 10. The numerical results are compatible to experimental results at x=5, 8, 11 and 15m (Figure 10). In the compared between experimental and numerical results for bed elevations, the differences are the satisfactory degree (Figure 11).

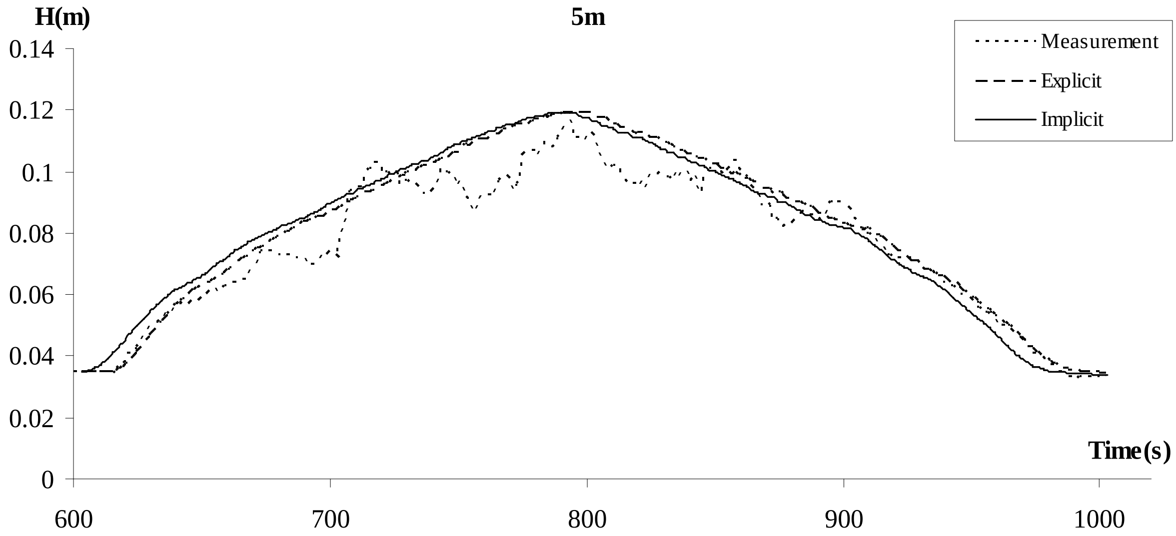


Fig. 10a. Comparison of flow depth for hydrograph 2 in x=5m

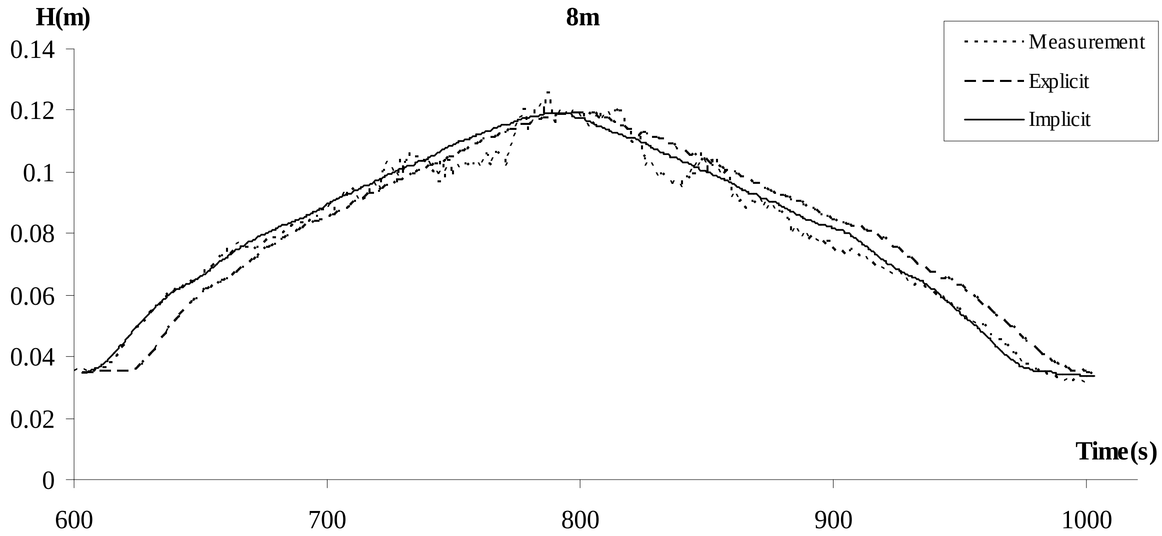


Fig. 10b. Comparison of flow depth for hydrograph 2 in x=8m

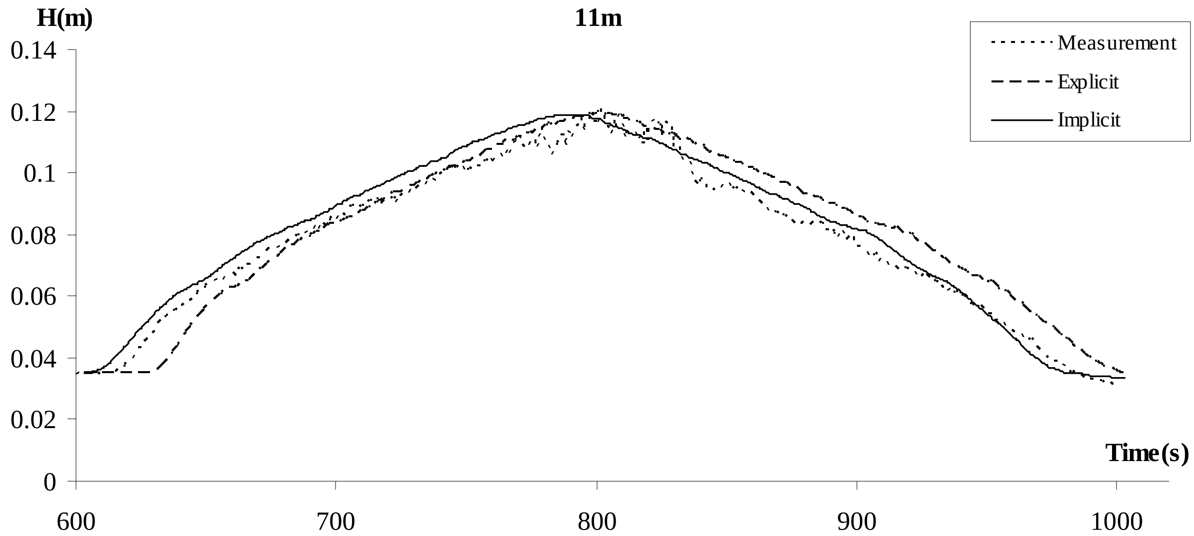


Fig. 10c. Comparison of flow depth for hydrograph 2 in x=11m

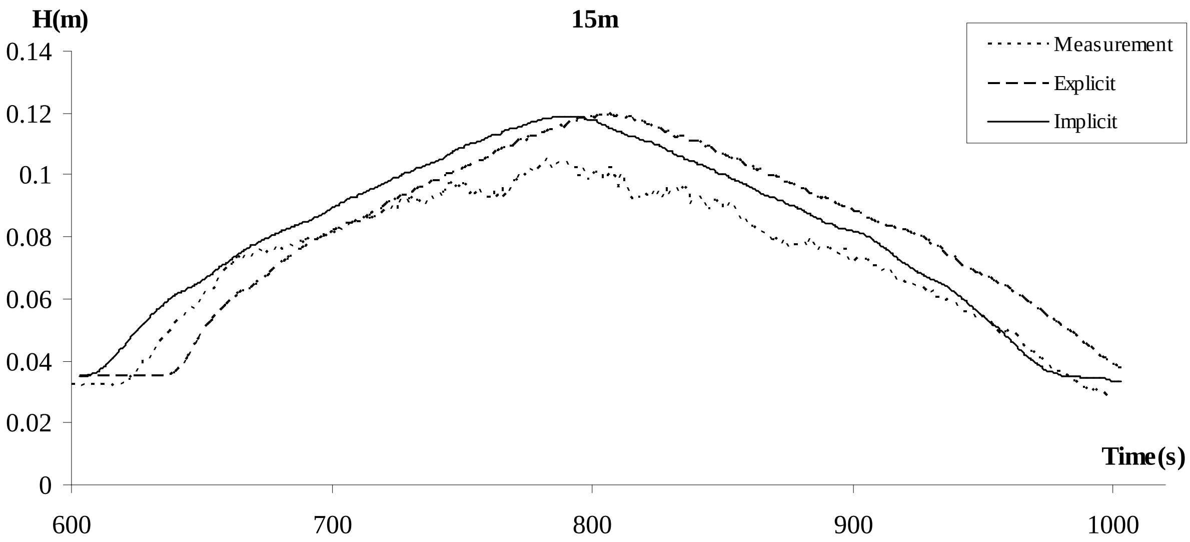


Fig. 10d. Comparison of flow depth for hydrograph 2 in x=15m

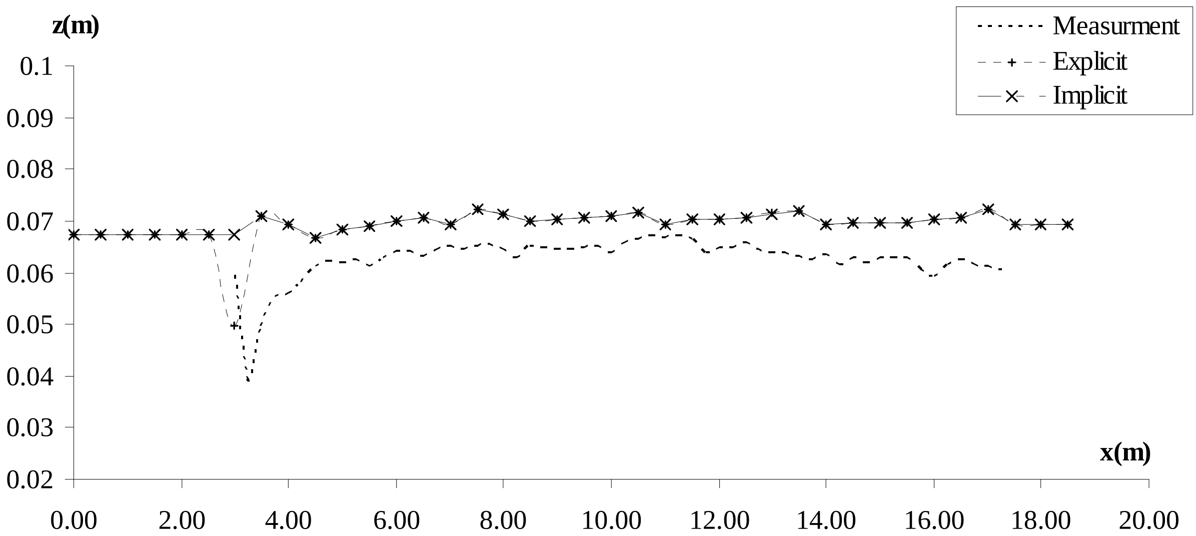


Fig. 11. Comparison of experimental and numerical results for bed profile in Hydrograph 2

7. Conclusions

In the modeling of one dimensional flow and sediment transport, in this study, the Kinematic Wave Model was used. The evolution and movement of bed profiles can be described by the kinematic wave theory employing a functional relation between sediment transport rate and sediment concentration (Tayfur and Singh 2006). For the numerical solution, one dimensional finite-volume models has been developed to simulate the transported of sediment over movable beds. In the models explicit and implicit approximations was used. These models can simulate flow, sediment transport and bed level change in equilibrium conditions. The models were tested by two different hydrographs. The results of models were satisfactory.
Finally, tests in field cases are needed to enhance the reliability of the established model. In the future studies, we will try to develop models with less simplifications like dynamic wave model. Moreover, the investigation of the influence of the sediment and flow parameters and empirical relations can be useful.

8. Acknowledgments

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