International Journal of Engineering and Applied Sciences (IJEAS)
Vol.3, Issue 4(2011)7-20

FREE VIBRATION ANALYSIS OF WALL-FRAME STRUCTURES BY DIFFERENTIAL QUADRATURE METHOD

Kanat Burak Bozdogan
Department of Civil Engineering
Cumhuriyet University, 35540
Sivas, Turkey
kbbozdogan@yahoo.com.tr

Abstract- Differential Quadrature Method (DQM) has very wide applications in the field of structural vibration. The main advantages of the Differential Quadrature Method are its inherent conceptual simplicity and the fact that easily programmable. In this paper free vibration analysis of wall-frame structures were studied. A wall-frame structure was modeled as an cantilever beam in this study. The governing differential equation of wall-frame structures were solved using Differential Quadrature Method (DQM). At the end of the study, a sample taken from literature was solved and the results were evaluated in order to test the convenience of the method.

Key Words- Free Vibration, Wall-Frame Structures, Differential Quadrature Method.

1.INTRODUCTION

A number of methods, such as finite element method, has been developed for analyses of buildings. The continuum model is a very simple and efficient method used in static and dynamic analysis of shear wall-frame buildings. There are numerous studies [1-44] in the literature regarding the continuum method. In continuum model a wall-frame structure was modeled as an cantilever beam. The governing differential equations of equivalent shear flexure cantilever beam are formulated using the continuum approach.

The Differential Quadrature Method (DQM) was initially presented by Bellman et al. [45-46] as an efficiently and accurate numerical method to solve differential equations. Afterwards many researchers demonstrated their successful applications of the method in mechanics [47-64]. In this study free vibration analysis of wall-frame structures were studied using Differential Quadrature Method (DQM). The following assumptions are made in this study; the behavior of the material is linear elastic, small displacement theory is valid, P-delta effects and axial deformation in columns and walls are negligible, the structures are regular (i.e their characteristics do not vary over the height), the floor slabs of the buildings have great in-plane and small out-of- plane stiffness, torsion effects are negligible.

2.ANALYSIS

2.1. Physical Model

The behavior of the wall-frame structures ignoring axial deformations of wall and columns may be presented by combination of flexural cantilever beam and shear cantilever beam deforming in bending and shear configurations (Fig. 1.) [22-42].

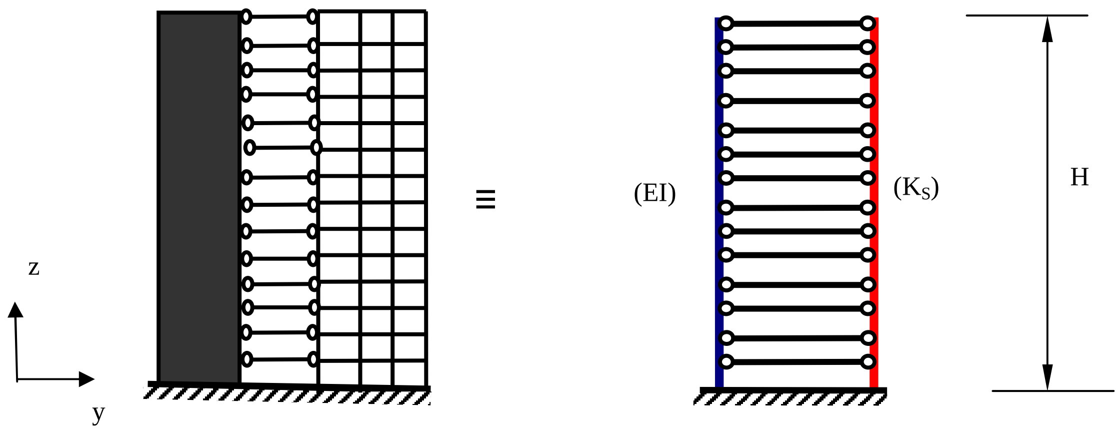

Figure 1. Physical model of equivalent sandwich beam

In the Figure 1. EI are the total bending rigidities of shear walls, (K_s) are the equivalent shear rigidity of the storey for framework. For frame elements which consists of n columns and n-1 beams , Ks can be calculated as follows [7, 23] :

$$K_S = \frac{12E}{h_i(1/\sum_1^n I_c/h + 1/\sum_1^{n-1} I_g/l)} \quad (1)$$

where $\sum I_c / h$ represents the sum of moments of inertia of the columns per unit height of frame , and $\sum I_g / l$ represents the sum of moments of inertia of each beam per unit span across one floor of frame.

2.2. Exact Solution of Governing Equation of Wall-Frame Structures

The governing equation for free vibration of wall-frame structures can be written as [22-42].

$$\frac{\rho}{EI}\frac{\partial^2 u(\xi,t)}{\partial t^2}+\frac{1}{H^4}\frac{\partial^4 u(\xi,t)}{\partial \xi^4}-H^2\frac{K_s}{EI}\frac{\partial^2 u(\xi,t)}{\partial \xi^2}=0 \tag{2}$$

where ρ is the mass per unit length in the model, H is the total height of the building u(ξ, t) is the lateral displacements at non-dimensional height ξ=z/H (varying between zero at the base of the building and one at roof level) at time t.
If a sinusoidal variation of u with circular frequency ω is assumed then

$$u(\xi,t) = y(\xi)\sin(\omega t) \tag{3}$$

Where y(ξ) is the amplitude of the sinusoidally varying displacement.

Substituting Eq.(3) in Eq. (2) results

$$\frac{d^4 y}{dz^4}-k^2\frac{d^2 y}{dz^2}-\alpha y=0 \tag{4}$$

in which $k = H\sqrt{\frac{K_s}{EI}}$ and $\alpha = \frac{\rho}{EI}H^4\omega^2$

The boundary conditions of a problem are;

$$y(0) = 0 \tag{5a}$$

$$\frac{dy(0)}{d\xi} = 0 \tag{5b}$$

$$\frac{EI}{H^2}\frac{d^2y(1)}{d\xi^2}=0 \tag{5c}$$

$$\frac{d^3y(1)}{d\xi^3}-k^2\frac{dy(1)}{d\xi}=0 \tag{5d}$$

Under the boundary conditions the exact solution of Equation (4), the circular frequencies are obtained as

$$\omega=\frac{2\pi\eta}{H^2}\sqrt{\frac{EI}{\rho}} \tag{6}$$

For the first three modes values for frequency parameter (η) are given in Table 1, Table 2 and Table 3 as a function of parameter k [41].

Table 1 Frequency parameter (η) for the first natural frequency

k	H	k	η	k	η	k	η	k	η
0	0.5596	4.5	1.465	9.5	2.680	14.5	3.913	20	5.278
0.1	0.5606	5.0	1.586	10	2.803	15.0	4.036	30	7.769
0.5	0.5851	5.5	1.706	10.5	2.926	15.5	4.160	40	10.26
1.00	0.6542	6.0	1.827	11.0	3.049	16.0	4.284	50	12.76
1.50	0.7511	6.5	1.949	11.5	3.172	16.5	4.408	60	15.26
2.00	0.8628	7.0	2.070	12.0	3.295	17.0	4.532	70	17.76
2.50	0.9809	7.5	2.192	12.5	3.418	17.5	4.656	80	20.26
3.00	1.1014	8.0	2.313	13.0	3.542	18.0	4.781	90	22.76
3.5	1.2226	8.5	2.435	13.5	3.665	18.5	4.905	100	25.26
4.00	1.3437	9.0	2.558	14.0	3.789	19.0	5.029	>10000	k/4

Table 2 Frequency parameter (η) for the second natural frequency

k	η	k	η	k	η	k	η	k	η
0	3.507	4.5	5.290	9.5	8.643	14.5	12.19	20	16.18
0.1	3.508	5.0	5.606	10	8.992	15.0	12.55	30	23.54
0.5	3.536	5.5	5.929	10.5	9.342	15.5	12.91	40	30.97
1.00	3.622	6.0	6.257	11.0	9.694	16.0	13.27	50	38.43
1.50	3.760	6.5	6.590	11.5	10.05	16.5	13.63	60	45.90
2.00	3.943	7.0	6.926	12.0	10.40	17.0	13.99	70	53.38
2.50	4.165	7.5	7.266	12.5	10.76	17.5	14.35	80	60.86
3.00	4.416	8.0	7.607	13.0	11.11	18.0	14.72	90	68.35
3.5	4.691	8.5	7.951	13.5	11.47	18.5	15.08	100	75.84
4.00	4.984	9.0	8.296	14.0	11.83	19.0	15.45	>10000	3k/4

Table 3 Frequency parameter (η) for the third natural frequency

k	η	k	H	k	η	k	η	k	η
0	9.819	4.5	11.65	9.5	16.27	14.5	21.72	20	28.06
0.1	9.820	5.0	12.04	10	16.79	15.0	22.28	30	40.01
0.5	9.844	5.5	12.44	10.5	17.32	15.5	22.85	40	52.21
1.00	9.919	6.0	12.87	11.0	17.85	16.0	23.42	50	64.53
1.50	10.04	6.5	13.32	11.5	18.39	16.5	23.99	60	76.90
2.00	10.21	7.0	13.78	12.0	18.94	17.0	24.57	70	89.31
2.50	10.42	7.5	14.26	12.5	19.49	17.5	25.15	80	101.74
3.00	10.68	8.0	14.75	13.0	20.04	18.0	25.72	90	114.19
3.5	10.97	8.5	15.25	13.5	20.59	18.5	26.30	100	126.65
4.00	11.30	9.0	15.75	14.0	21.15	19.0	26.89	>10000	5k/4

2.3. Solution Governing Equation of Wall-Frame Structures via Differential Quadrature Method (DQM)

As shown in Figure 2 we consider a one- dimensional problem. It is assumed that a function y(ξ) is smooth over the whole domain.

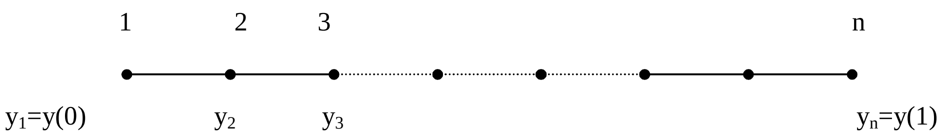

Figure 2 Discretization of one Dimensional Problem

If it assumed that y is a polynomial of degree n-1, y function can be written as;

$$y = \sum_{i=1}^{n} a_{i-1} * \xi^{i-1} \qquad (7)$$

where a_{i-1} are the coefficients, n is the number of nodes.

Using Equation (7), the first second, third and fourth derivatives of y function respect to ξ can be written as

$$\frac{dy}{d\xi} = \sum_{i=1}^{n} (i-1) * a_{i-1} * \xi^{i-2} \tag{8}$$

$$\frac{d^2y}{d\xi^2} = \sum_{i=1}^{n} (i-1) * (i-2) * a_{i-1} * \xi^{i-3} \tag{9}$$

$$\frac{d^3y}{d\xi^3} = \sum_{i=1}^{n} (i-1) * (i-2) * (i-3) * a_{i-1} * \xi^{i-4} \tag{10}$$

$$\frac{d^4y}{d\xi^4} = \sum_{i=1}^{n} (i-1) * (i-2) * (i-3) * (i-4) * a_{i-1} * \xi^{i-5} \tag{11}$$

Functional values in the whole domain can be written as;

$$y_1 = y(0) = a_0 \tag{12.1}$$

$$y_1 = y(h) = a_0 + a_1 h \tag{12.2}$$

.

.

$$y_n = y(n*h-h) = a_0 + a_1 * (h*n-h) + ... + a_n * (h*n-h)^n \tag{12.n}$$

Where h is the distance of two grid points and is equal to 1/(n-1).

Equation (13) shows the matrix form of equations (12.1),(12.2)….. (12.n)

$$\begin{bmatrix} y_1 \\ y_2 \\ . \\ . \\ . \\ y_{n-1} \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & 0 & . & . & . & . & 0 \\ 1 & h & . & . & . & . & h^n \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ 1 & (n-2)*h & . & . & . & . & (n-2)^n*h^n \\ 1 & (n-1)*h & . & . & . & . & (n-1)^n*h^n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} = A \begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} \tag{13}$$

Using Equations (11), (9) and (7) Equation (4) can be written as

$$\sum_{i=1}^{n}(i-1)*(i-2)*(i-3)*(i-4)*a_{i-1}*\xi^{i-5}-\sum_{i=1}^{n}(i-1)*(i-2)*a_{i-1}*\xi^{i-3}-\sum_{i=1}^{n}a_{i-1}*\xi^{i-1}=0 \tag{14}$$

When consider the boundary conditions Equations (5a), (5b),(5c) and (5d) can be written as

$$y_1 = y(0) = a_0 = 0 \tag{15}$$

$$\frac{dy(0)}{d\xi} = a_1 = 0 \tag{16}$$

$$\frac{d^2y(1)}{d\xi^2} = \sum_{i=1}^{n}(i-1)*(i-2)*a_{i-1}*1^{i-3} = 0 \tag{17}$$

$$\frac{d^3y(1)}{d\xi^3} - k^2\frac{dy(1)}{d\xi} = \sum_{i=1}^{n}(i-1)*(i-2)*(i-3)*a_{i-1}*1^{i-4} - k^2\sum_{i=1}^{n}(i-1)*(i-2)*a_{i-1}*1^{i-3} = 0 \tag{18}$$

Using Equations (14),(15),(16),(17) and (18) the matrix equation (19) can be written

$$B\begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ . \\ a_{n-1} \\ a_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

The elements of B matrix can be written as follows;

$$B(1,1) = 1 \tag{20}$$

$$B(1,i) = 0 \qquad i = 2.....n \tag{21}$$

$$B(2,1) = 0 \tag{22}$$

$$B(2,2)=1 \tag{23}$$

$$B(2,i)=0 \qquad i=3.....n \tag{24}$$

$$B(n-1,i)=(i-1)*(i-2)*1 \qquad i=1...n \tag{25}$$

$$B(n,i)=(i-1)*(i-2)*(i-3) \qquad i=1...n \tag{26}$$

$$B(j,i)=(i-1)*(i-2)*(i-3)*(i-4)*(j*h-h)^{i-4}-k^2(i-1)*(i-2)*(j*h-h)^{i-2}-\alpha(j*h-h)^i \qquad j=3....n-2 \tag{27}$$

When coefficients vector is solved out from Equation (19) and is substituted to the Equation (13), Equation (28) is obtained.

$$\begin{bmatrix}0\\0\\.\\.\\.\\0\\0\end{bmatrix}=BA^{-1}\begin{bmatrix}y_1\\y_2\\.\\.\\.\\y_{n-1}\\y_n\end{bmatrix}=C\begin{bmatrix}y_1\\y_2\\.\\.\\.\\y_{n-1}\\y_n\end{bmatrix} \tag{28}$$

The values of ω which set the determinant of C matrix to zero are the circular frequencies.

3. A VALIDITY OF THE METHOD

In this part of the study for a different values of k parameter, is obtained frequency parameter (η) using Differential Quadrature method and compared with the exact values in Table 4, Table 5 and Table 6. A program for Differantial Quadrature Method was prepared in MATLAB.

Table 4 Comparison of Frequency parameter (η) for the first natural frequency

k	Analytical	DQM (n=8)	DQM (n=10)	DQM (n=12)
0	0.5596	0.5598	0.5596	0.5596
2	0.8628	0.8630	0.8627	0.8628
5	1.5860	1.5836	1.5852	1.5856
10	2.803	2.7940	2.7904	2.7992
15	4.036	4.0721	4.0119	4.0151
20	5.278	5.4038	5.2769	5.2418
50	12.76	13.5890	13.2292	13.0107
100	25.26	27.2447	26.5671	26.1624

Table 5 Comparison of **f**requency parameter (η) for the second natural frequency

k	Analytical	DQM (n=8)	DQM (n=10)	DQM (n=12)
0	3.5070	3.5610	3.5062	3.5070
2	3.9430	4.0228	3.9430	3.9434
5	5.6060	5.8078	5.6175	5.6073
10	8.9920	9.5460	9.1260	9.0219
15	12.5500	13.5613	12.9505	12.6929
20	16.1800	17.6965	16.9295	16.5389
50	38.4300	43.1441	41.4873	40.5542
100	75.8400	85.9639	82.7547	80.9567

Table 6 Comparison of **f**requency parameter (η) for the third natural frequency

k	Analytical	DQM (n=8)	DQM (n=10)	DQM (n=12)
0	9.8190	8.327	10.3306	9.7589
2	10.2100	8.715	10.7197	10.1496
5	12.04	10.642	12.5208	11.9811
10	16.79	16.1474	17.2422	16.7856
15	22.28	22.6529	22.9011	22.3987
20	28.06	29.3597	29.0266	28.3935
50	64.53	70.3910	68.5277	67.1584
100	126.65	139.7029	135.9768	133.4607

4. CONCLUSIONS

In this paper free vibration analysis of wall-frame structures were studied. A wall-frame structure was modeled as an cantilever beam in this study. The governing differential equation of wall-frame structures were solved using Differential Quadrature Method (DQM). At the end of the study, it was observed from the literature that the presented method gave results sufficient. For n=12, in the first mode the error of the DQM is shown to be less than 4%. Because of the main advantages of the Differential Quadrature Method are its inherent conceptual simplicity and the fact that easily programmable it can be used at the concept design stage.

5. REFERENCES

1. Balendra T, Swaddiwudhipong S, Quek S.T (1984), "Free Vibration of Asymmetric Shear Wall-Frame Buildings", Earthquake Engineering and Structural Dynamics, 12,629-650
2. Basu A, Nagpal AK, Bajaj RS, Guliani A (1979), "Dynamic Characteristics of Coupled Shear Walls", ASCE Journal of Structural Division, 105,1637-1651
3. Bilyap S (1979), "An Approximate Solution For High- Rise Reinforced Concrete Panel Buildings with Combined Diaphragms", International Journal for Housing Science, 3(6),477-481
4. Boutin C, Hans S, Ibraim E, Roussilon P (2005)," In Situ Experiments and Seismic Analysis of Existing Buildings", Stability of Braced, Partially Braced and Unbraced Frames; Classical Approach, Earthquake Engineering and Structural Dynamics, 12,1531-1546
5. Bozdogan KB (2009), "An Approximate Method For Static and Dynamic Analyses of Symmetric Wall-Frame Buildings", The Structural Design of Tall and Special Buildings, 2009,18(3),279-290
6. Georgoussis KG (2006)," A Simple Model for Assessing and Modal Response Quantities in Symmetrical Buildings" , The Structural Design of Tall and Special Buildings, 15, 139-151.
7. Heidebrecht AC, Stafford Smith B (1973)," Approximate Analysis of Tall Wall-Frame Buildings", ASCE Journal of Structural Division, 99(2),199-221
8. Hoenderkamp DCJ (2000)," Approximate Analysis of High-Rise Frames with Flexible Connections" , The Structural Design of Tall Buildings, 9, 233-248.
9. Hoenderkamp DCJ (2001)," Elastic Analysis of Asymmetric Tall Buildings", The Structural Design of Tall Buildings, 10, 245-261.
10. Hoenderkamp DCJ (2002), "A Simplified Analysis of High-Rise Structures with Cores" , The Structural Design of Tall Buildings, 11, 93-107.
11. Kaviani P, Rahgozar R.,Saffari H (2008), "Approximate Analysis of Tall Buildings Using Sandwich Beam Models with Variable Cross-Section", The Structural Design of Tall Buildings, 17(2),401-418.
12. Kuang JS, Ng SC (2000)," Coupled Lateral Vibration of Asymmetric Shear Wall Structures", Thin Walled Structures,38(2), 93-104
13. Laier JE (2008)," An Improved Continuous Medium Technique For Structural Frame Analysis", The Structural Design of Tall Buildings, 17,25-38
14. Lee J, Bang M, Kim JY (2008), "An Analytical Model For High- Rise Wall-Frame Structures with Outriggers" ,The Structural Design of Tall and Special Buildings,17 (4), 839-851
15. Li GQ, Choo BS (1996)," A Continuous Discrete Approach to the Free Vibration Analysis of Stiffened Pierced Walls on Flexible Foundations" , International Journal of Solids and Structures,33(2),249-263
16. Mancini E, Savassi W (1999)," Tall Buildings Structures Unified Plane Panels Behaviour", The Structural Design of Tall Buildings ,8,155-170

17. Meftah SA, Tounsi A (2008),” Vıbration Characteristics of Tall Buildings Braced by Shear Walls and Thin-Walled Open-Section Structures”, The Structural Design of Tall Buildings, 17,203-216
18. Michel C, Hans S, Guegen P, Boutin C (2006), “In Situ Experiment and Modeling of RC Structure Using Ambient Vibration and Timoshenko Beam”, First European Conference on Earthquake Engineering and Seismology Geneva-Switzerland
19. Miranda E (1999),” Approximate Lateral Drift Demands in Multi-Story Buildings Subjected to Earthquakes”, ASCE Journal of Structural Division, 125(4),417-425
20. Miranda E, Reyes JC (2002),” Approximate Lateral Drift Demands in Multi-Story Buildings with Nonuniform Stiffness”, ASCE Journal of Structural Division, 128(7),840-849
21. Miranda E, Taghavi S (2005),” Approximate Floor Acceleration Demands in Multistorey Buildings I Fourmulation”, ASCE Journal of Structural Division, 131(2),203-211
22. Miranda, E., Akkar,S.,Generalized Interstory Drift Spectrum, ASCE Journal of Structural Engineering, 132 (6), 840-852, 2006.
23. Murashev V,Sigalov E,Baikov V (1972), Design of Reinforced Concrete Structures,Mir Publisher,Moscow
24. Ng,SC, Kuang JS,(1999),”Triply Coupled Vibration of Asymmetric Wall-Frame Buildings”, ASCE Journal of Structural Division, 126(8), 982-987
25. Nollet JM, Stafford Smith B (1993),” Behavior of Curtailed Wall-Frame Structures”, ASCE Journal of Structural Division, 119(10), 2835-2853
26. Paknahad M, Noorzaei J, Jaafar MS, Waleed (2007),” Analysis of Shear Wall Structure Using Optimal Membrane Triangle Element”, Finite Elements ın Analysis and Design,861-869
27. Potzta G, Kollar LP (2003), “Analysis of Building Structures by Replacement Sandwich Beams”, International Journal of Solids and Structures, 40, 535-553
28. Rafezy B, Zare A, Howson PW (2007),” Coupled Lateral –Torsional Frequencies of Asymmetric, Three Dimensional Frame Structures”,International Journal of Solids and Structures,44,128-144
29. Rafezy B,Howson, WP (2008),” Vibration Analysis of Doubly Asymmetric,Three Dimensional Structures Comprising Wall and Frame Assemblies with Variable Cross Section”, Journal of Sound and Vibration, 318 (1-2),247-266
30. Reinoso E, Miranda E (2005), “Estimation of Floor Acceleration Demands in High Rise Buildings During Earthquakes”, The Structural Design of Tall and Special Buildings, 14,107-130
31. Rosman R (1964).” Approximate Analysis of Shear Walls Subject to Lateral Loads”, Proceedings of the American Concrete Institute, 61(6),717-734
32. Savassi W, Mancini E (2004),” One-Dimensional Finite Element Solution for Tall Building Structures Unified Plane Panels Formulation”, The Structural Design of Tall and Special Buildings,13(4),315-333

33. Savassi,W, Mancini E (2009),” One-Dimensional Finite Element Solution for Non-Uniform Tall Building Structures and Loading”, The Structural Design of Tall and Special Buildings,18 ,441-453
34. Stafford Smith B, Crowe E (1986), “Estimating Periods of Vibration of Tall Buildings”, ASCE Journal of Structural Division, 112(5),1005-1019
35. Swaddiwudhipong S,Lee LS, Zhou Q (2001),” Effect of The Axial Deformation on Vibration of Tall Buildings”, The Structural Design of Tall Buildings, 10,79-91
36. Taghavi S, Miranda E (2005),” Approximate Floor Acceleration Demands in Multistorey Buildings II: Applications”, ASCE Journal of Structural Division, 131(2),212-220
37. Tarjan G, Kollar PL (2004),” Approximate Analysis of Building Structures with Identical Stories Subjected to Earthquakes”,International Journal of Solids and Structures,41(5-6),1411-1433
38. Toutanji H (1997),”The Effect of Foundation Flexibility on The Interaction Of Walls and Frames”, Engineering Structures,19(12),1036-1042
39. Wang Y, Arnaouti C, Guo S (2000), “A Simple Approximate Formulation for The First Two Frequencies of Asymmetric Wall-Frame Multi-Storey Building Structures”, Journal of Sound and Vibration, 236(1),141-160
40. Zalka K (1994), “Mode Coupling in The Torsional Flexural Buckling of Regular Multistorey Buildings”, The Structural Design of Tall Buildings ,3,227-245
41. Zalka K (2000), Global Structural Analysis,E-FN Spon Taylor and Francis Group,341 p
42. Zalka K (2001), “A Simplified Method For Calculation of Natural Frequencies of Wall-Frame Buildings”, Engineering Structures, (23),1544-1555
43. Zalka K (2002), “Buckling Analysis of Buildings Braced by Frameworks, Shear Walls and Cores”, The Structural Design of Tall Buildings, 11,197-219.
44. Zalka K (2003), “A Hand Method for Predicting the Stability of Regular Buildings, Using Frequency Measurements”, The Structural Design of Tall and Special Buildings, 12, 273-281.
45. Zalka K (2009), “A Simple Method For The Deflection Analysis of Tall-Wall-Frame Building Structures Under Horizontal Load”, The Structural Design of Tall and Special Buildings, 18 (3), 291-311

46. Bellman, R.E., Casti, J., Differential Quadrature and Long Term Integration, J Math Anal Appl,(34), 235-238, 1971
47. Bellman, R.E., Kashef, B.G., Casti, J., Differential Quadrature: a Technique for the Rapid Solution of Nonlinear Partial Differential Equations, J Comput Phys, (10), 40-52,1972
48. Bert, C.W., Wang, Z., Striz, A. G., Differential Quadrature for Static and Free Vibration Analysis of Anisotropic Plates, International Journal of Solids and Structures, 30 (13), 1737–1744, 1993

49. Bert, C.W., Malik, M., Free Vibration Analysis of Tapered Rectangular Plates by Differential Quadrature method: a Semi- Analytical Approach, Journal of Sound and Vibration, 190 (1), 41–63, 1996
50. Bert, C.W., Malik, M., Differential Quadrature Method in Computational Mechanics: a Review, Applied Mechanics Review, 49 (1), 1–28.,1996
51. Bert,C.W.,. Jang, S.K., Striz, A.G., Two New Approximate Methods for Analyzing Free Vibration of Structural Components, American Institute of Aeronautics and Astronautics Journal 26 (5), 612–618, 1987
52. Liew, K.M., Teo, T.M., Han, J.B., Comparative Accuracy of DQ and HDQ Methods for Three-Dimensional Vibration Analysis of Rectangular Plates, International Journal for Numerical Methods in Engineering, 45 , 1831–1848, 1999.
53. Liew, K.M., Teo, T.M., Han, J.B, Three Dimensional Static Solutions of Rectangular Plates by Variant Differential Quadrature Method, International Journal of Mechanical Sciences, 43 (2001), 1611–1628, 2001
54. Liew, K.M., Teo, T.M., Three Dimensional Vibration Analysis of Rectangular Plates Based on Differential Quadrature Method, Journal of Sound and Vibration, 220 (4), 577–599, 1999
55. Shu, C., Richards, B.E., Application of Generalized Differential Quadrature to Solve Two-Dimensional Incompressible Navier–Stokes Equations, International Journal for Numerical Methods in Fluids, 15 ,791–798, 1992
56. Shu, C., Xue, H., Explicit Computations of Weighting Coefficients in the Harmonic Differential Quadrature, Journal of Sound and Vibration, 204 (3),. 549–555, 1997.
57. Striz, A.G, Jang, S.K, Bert, C.W, Nonlinear Bending Analysis of Thin Circular Plates by Differential Quadrature, Thin-Walled Structures, 6, 51–62, 1988
58. Striz, A.G., Wang, X., Bert, C.W , Harmonic Differential Quadrature Method and Applications to Analysis of Structural Components, Acta Mechanica,111,. 85–94, 1995.
59. Civalek, Ö., Application of Differential Quadrature (DQ) and Harmonic Differential Quadrature (HDQ) for Buckling Analysis of Thin Isotropic Plates and Elastic Columns, Engineering Structures, 26, 171-186, 2004.
60. Civalek, Ö., Ülker M., Harmonic Differential Quadrature (HDQ) for Axisymmetric Bending Analysis of Thin Isotropic Circular Plates, International Journal of Structural Engineering and Mechanics, 17 (1), 1–14, 2004.
61. Civalek, Ö., Geometrically Nonlinear Dynamic Analysis of Doubly Curved Isotropic Shells Resting on Elastic Foundation by a Combination of HDQ–FD Methods, International Journal of Pressure Vessels and Piping, 82 (6), 470–479, 2005.
62. Kaya, B., Solution of Advection-Diffusion Equation Using The Differential Quadrature Method, KSCE Journal of Civil Engineering, 14 (1), 69-75, 2010.

63. Karami, G., Malekzadeh, P., Application of a New Differential Quadrature Methodology for Free Vibration Analysis of Plates, Int. J. Numer. Meth. Eng., 56 , 847–867,2003

64. Malekzadeh, P., Farid, M., A DQ Large Deformation Analysis of Composite Plates on Nonlinear Elastic Foundations, Composite Structure, 79 (2007), 251–260, 2007
65. Eftekhari, S. A.,Jafari , A.A., A New Mixed Finite Element-Differential Quadrature Formulation for Forced Vibration of Beams Carrying Moving Loads, Journal of Applied Mechanics, 2010 (In press).