DESIGN OF A KNOWLEDGE-BASED MANUFACTURING FLOW SHOP SEQUENCING SYSTEM USING FUZZY LOGIC

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Abstract

This study investigated an approach for incorporating statistics with fuzzy sets in the flowshop sequencing problem. It considers a flow shop problem with imprecise processing times with the objective to minimize the makespan. This work is based on the assumption that the precise value for the processing time of each job is unknown, but that some sample data are available. A combination of statistics and fuzzy sets provides a powerful tool for modeling and solving this problem. The processing times are described by triangular fuzzy numbers. The issue that arises is how to rank the constructed job sequences with respect to their obtained makespans, which are fuzzy numbers. A new distance measure between fuzzy makespans is introduced which includes an optimism/pessimism indicator and a function related to λ-levels of fuzzy sets, enabling the decision maker to express his/her preference. Our work intends to extend the crisp flowshop sequencing problem into a generalized fuzzy model that would be useful in practical situations. In this study, we constructed a fuzzy flow shop sequencing model based on statistical data, which uses level (1-α, 1-β) interval-valued fuzzy numbers to represent the unknown job processing time.

Keywords: Flowshop Sequencing Problem, Fuzzy Flowshop Model, Interval-Valued Fuzzy Number.

1. Introduction

The selection of an appropriate order for a series of jobs to be done on a finite number of service facilities is called sequencing. If work centers are lightly loaded and if jobs all require the same amount of processing time, sequencing presents no particular difficulties. However, for heavily loaded work centers especially in situations where relatively lengthy jobs are involved, the order of processing can be very important in terms of cost associated with jobs waiting for processing and in terms of idle time at work centers. High volume systems are often referred to as flow system. Flow shop problem concerns the sequencing of a given number of jobs through a series of machines in the exact same order on all machines with the aim to satisfy a set of constraints as much as possible, and optimize a set of objectives. The commonly studied objectives include: makespan, mean flow time, tardiness etc. Among those objectives, the makespan, defined as the time when the last job completes on the last machine, the total time needed to complete a group of jobs from the beginning of the first job to the completion of the last job, is the most frequently studied one. A large number of deterministic scheduling algorithms have been proposed by Pinedo [11] in last decades to deal with flow shop scheduling problems with various objectives and constraints. However, it is often difficult to apply those algorithms to real-life flow shop problems. The processing times of jobs could be uncertain due to incomplete knowledge or uncertain environment which implies that there exist various external sources and types of uncertainty. Fuzzy sets and logic can be used to tackle uncertainties inherent in actual flow shop scheduling problems as in Slowinski and Hapke [13]. Majority of approaches consider fuzzy processing time and/or fuzzy due dates as in McCahon and Lee [8], Petrovic and Song [10], Sakawa and Kubota [12]. The fuzzy processing times are described by triangular membership functions. The objective considered is the minimization of makespan. Because of the fuzziness of processing times, the obtained makespans are also fuzzy numbers. In order to select the job sequence with the "minimum" makespan, sophisticated fuzzy ranking techniques which are capable of describing all the possible relationships between fuzzy numbers and allow the modeling of preferences of the decision maker (DM) are required.

2. Preliminaries

Interval valued fuzzy set: The fundamental reason of introducing an extension of the concept of a fuzzy set e.g. a probabilistic set as in Hirota [6], Czogala [2], Czogala and Pedrycz [3], or a fuzzy set of type n as in Zadeh [18] is connected with the fact that the formal, fuzzy set-representation of verbal expressions occurring in a verbal model of a phenomenon, object or process (in a verbal decisional procedure) is not often sufficiently adequate Hirota [6]. As a rule, the membership functions of fuzzy sets representing particular verbal expressions cannot be defined unequivocally on the basis of available information. Therefore, it is not always possible for a membership function of the type μ : $X \rightarrow [0,1]$ to assign precisely one point from the interval [0,1] to each element $x \in X$ without the loss of at least a part of information. The definitions of a fuzzy set of type 2 Zadeh [18] and of a probabilistic set Hirota [6] suggest at this point assigning a fuzzy set from a family $f([0, 1])^{1*}$ or a probability distribution respectively, instead of a point from [0, 1]. It should be emphasized, however, that from the point of view of practice, this type of formal representation of a generalized membership function is characterized by some 'redundancy' in relation to reality. We do not have, as a rule, such extensive and precise information that it could be possible to define precisely a group of fuzzy sets from *f* ([0, 1]) being the generalized membership function of a fuzzy set of type 2, or a probability distribution for a probabilistic set. On the other hand (which is very important), the introduction of the above generalizations of the concept of a fuzzy set may considerably complicate the formal apparatus of approximate inference by Dubois and Prade [4]. This fact has a decisively negative influence on the effectiveness of its application. In the light of the above considerations, a proposal is put forward to apply the extension of the concept of fuzzy set represented by a generalized membership function having the form of 'band'.

Some definitions of fuzzy numbers and an interval-valued fuzzy set are provided with some relevant operations found in Gorzalezang [5], Kaufmann and Gupta [7].

Definition 1: A fuzzy set $\tilde{b}_\lambda$ defined on $R= (-\infty, \infty)$, which has the following membership function, is called a level λ fuzzy point, $0 < A < 1$:

$$\mu_{\tilde{b}_\lambda}(x) = \begin{cases} \lambda, x = b, \\ 0, otherwise. \end{cases} \quad (1)$$

Definition 2: A fuzzy set $[\alpha_\alpha, \beta_\alpha]$, where $0 \le a \le 1$ and defined on *R*, which has the following membership function is called a level a fuzzy interval,

$$\mu_{[\alpha_\alpha,\beta_\alpha]}(x)=\begin{cases}\alpha, a\le x\le b\\ 0, otherwise.\end{cases} \quad (2)$$

When $a = b$ and $[a_\alpha, b_\alpha] = [b_\alpha, b_\alpha] = \tilde{b}_\lambda$, the level a fuzzy interval becomes a level a fuzzy point.

Definition 3: The level λ triangular fuzzy number $\tilde{A}$, $0<\lambda<1$, is a fuzzy set defined on R with a membership function as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\lambda(x-a)/(b-a), a\le x\le b,\\ \lambda(c-x)/(c-b), b\le x\le c,\\ 0, \quad otherwise\end{cases} \quad (3)$$

For convenience, the fuzzy num in Definition 3 is denoted by $\tilde{A}, = (a, b, c; \lambda)$.

Definition 4: A fuzzy set $\tilde{B}$ defined on $R= (-\infty, \infty)$ which has the following property is called an interval-valued fuzzy set, $\tilde{B}=\{(x,[\mu_{\tilde{B}^L}(x),\mu_{\tilde{B}^U}(x)])\}, x\in R$, Where $0\le\mu_{\tilde{B}^L}(x)\le\mu_{\tilde{B}^U}(x)\le 1$. Symbolically $\tilde{B}$ is denoted by $[\tilde{B}^L,\tilde{B}^U]$.

Let $\tilde{A}^L=(a,b,c;\lambda)$ and $\tilde{A}^U=(p,b,r;\rho), 0<\lambda\le\rho\le 1$. We have $\tilde{A}=[\tilde{A}^L,\tilde{A}^U]=[(a,b,c;\lambda),(p,b,r;\rho)]$, $p<a<b<c<r$, where A is the level (λ, ρ) interval–valued fuzzy number from Fig.1. The membership function of $\tilde{A}$ can be expressed as

$$\mu_{\tilde{A}^L}(x)=\begin{cases}\lambda(x-a)/(b-a), a\le x\le b,\\ \lambda(c-x)/(c-b), b\le x\le c,\\ 0, \quad otherwise\end{cases} \quad (4)$$

$$\mu_{\tilde{A}^U}(x)=\begin{cases}\rho(x-p)/(b-p), p\le x\le b,\\ \rho(r-x)/(r-b), b\le x\le r,\\ 0, \quad otherwise\end{cases} \quad (5)$$

Let the family of all level (λ, ρ) interval-valued fuzzy numbers denoted by $F_{IV} = \{[(a, b, c; \lambda), (p, b, r; \rho)] \,|\, \forall p<b<c<r, a, b, c, p, r\in R\}, 0<\lambda\le\rho\le 1$. Before defining the ranking of level (λ, ρ) interval-valued fuzzy numbers on $F_{IV}(\lambda, \rho)$, we provide a definition of the signed distance, which is similar to that in Yao and Wu [17], on R.

Definition 5: Let $d(b,0)=b, b,0\in R$, denote the signed distance of b measured from the origin 0.

Remark 1: Geometrically, $0<b$ represents that b goes to the right-hand side of the origin 0, and that the distance between b and 0 is denoted by $b=d(b,0)$. Similarly, $b<0$ represents that b goes to the left-hand side of 0 and the distance between b and 0 is denoted by $-b=-d(b,0)$.

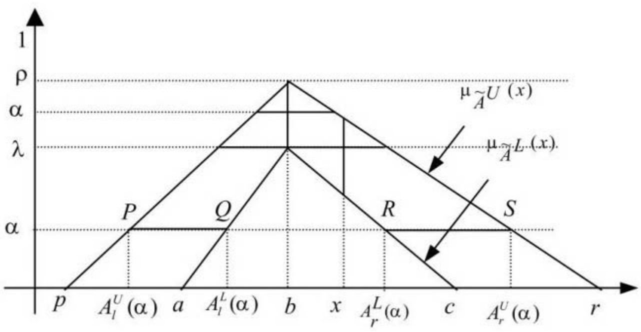

Fig. 1. An α–cut of level (λ, ρ) interval-valued fuzzy number $\tilde{A}$.

Consider the ordering of level (λ, ρ) interval-valued fuzzy numbers on $F_{IV}(\lambda, \rho)$. Let $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a,b,c;\lambda),(p,b,r;\rho)] \in F_{IV}(\lambda,\rho)$. Fig.1 shows a α-cut of level (λ, ρ) interval-valued fuzzy number $\tilde{A}$. From Fig. 1, we can see that a α-cut of $\tilde{A}$, $0 \le \alpha \le \lambda$, is $[A_l^U(\alpha), A_l^L(\alpha)] \cup [A_r^U(\alpha), A_r^L(\alpha)]$. From Eq.(4) and (5), we have $A_l^L(\alpha) = a + (b-a)\dfrac{\alpha}{\gamma}$, $A_r^L(\alpha) = c - (c-b)\dfrac{\alpha}{\lambda}$, $A_l^U(\alpha) = p + (b-p)\dfrac{\alpha}{\rho}$ and $A_r^U(\alpha) = r + (r-b)\dfrac{\alpha}{\rho}$.

Definition 6: Let $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a,b,c;\lambda),(p,b,r;\rho)] \in F_{IV}(\lambda,\rho), 0 < \lambda < \rho \le 1$. The signed distance from $\tilde{0}_1$ to $\tilde{A}$ is defined by $d(\tilde{A}_1, \tilde{0}_1) = \dfrac{1}{8}(6b + a + c + 4p + 4r + 3(2b - p - r)\dfrac{\lambda}{\rho}$.

Definition 7: Let $\tilde{A} = [(a_1, b_1, c_1; \lambda), (p_1, b_1, r_1; \rho)]$ and $\tilde{B} = [(a_2, b_2, c_2; \lambda), (p_2, b_2, r_2; \rho)] \in F_{IV}(\lambda, \rho)$, $0 < \lambda \le \rho \le 1$. The rankings of interval-valued fuzzy numbers on $F_{IV}(\lambda, \rho)$ are defined by

$$\tilde{B} \prec \tilde{A} \text{ iff } d(\tilde{B},\tilde{0}_1) < d(\tilde{A},\tilde{0}_1),$$
$$\tilde{B} \approx \tilde{A} \text{ iff } d(\tilde{B},\tilde{0}_1) = d(\tilde{A},\tilde{0}_1).$$

Definition 8: Let $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a_1, b_1, c_1; \lambda), (p_1, b_1, r_1; \rho)]$ and $\tilde{B} = [\tilde{B}^L, \tilde{B}^U] = [(a_2, b_2, c_2; \lambda), (p_2, b_2, r_2; \rho)] \in F_{IV}(\lambda, \rho)$ be interval-valued fuzzy numbers. The binary operator $\oplus$ is defined by $\tilde{A} \oplus \tilde{B} = [\tilde{A}^L \oplus \tilde{B}^L, \tilde{A}^U + \tilde{B}^U]$.

Remark 2: According to Definition 6 if $\tilde{I} = [(b,b,b;\lambda),(b,b,b;\rho)]$ is a member of $F_{IV}(\lambda, \rho)$. Then we have $d(\tilde{I}, \tilde{0}_1) = 2b$.

Finally, consider the problem for estimating the mean of a population by sampling in inferential statistics. It is clear that the sample mean $\bar{x}$ is just a point estimate for the population mean μ. Because of sampling error, we cannot expect $\bar{x}$ to be precisely equal to μ. This leads to the use of confidence interval estimation instead. The $(1-\alpha)\times 100\%$ confidence interval estimate for the population mean is expressed as follows:

$$\left[\bar{x} - t_{n-1}(\alpha_1)\frac{s}{\sqrt{n}}, \bar{x} + t_{n-1}(\alpha_2)\frac{s}{\sqrt{n}}\right] \tag{6}$$

Where $0<\alpha_j<1$, $j = 1, 2$, $\alpha_1+\alpha_2=\alpha$, and $0<\alpha<1$. The sample mean is $\bar{x} = (1/n)\sum_{j=1}^{n} x_j$ and the sample variance is $s^2 = [1/(n-1)]\sum_{j=1}^{n}(x_j - \bar{x})^2$, where $x_1, x_2, \ldots, x_n$ are sample data. Let T be a *t–distribution* with $n-1$ degree of freedom, and also let $t_{n-1}(\alpha_j)$, $j=1, 2$, be the constant that satisfies $P(T > t_{n-1}(\alpha_j)) = \alpha_j$.

Since the *t–distribution* with $n-1$ degree of freedom is symmetrical on the y-axis, we have

$$\begin{aligned}
P\left(\bar{x} - t_{n-1}(\alpha_1)\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{n-1}(\alpha_2)\frac{s}{\sqrt{n}}\right) &= P\left(-t_{n-1}(\alpha_1) \le \frac{\sqrt{n}(\mu - \bar{x})}{s} \le t_{n-1}(\alpha_2)\right) \\
&= P\left(-t_{n-1}(\alpha_1) \le \frac{\sqrt{n}(\mu - \bar{x})}{s} \le 0\right) + P\left(0 \le \frac{\sqrt{n}(\mu - \bar{x})}{s} \le t_{n-1}(\alpha_2)\right) \\
&= \frac{1}{2} - \alpha_1 + \frac{1}{2} - \alpha_2 \\
&= 1 - \alpha
\end{aligned}$$

3. Construction a Fuzzy Flowshop Sequencing Model

3.1 Flowshop Problem

Let us consider a simple flow shop problem with *n* jobs and *m* machines as found in Song and Petrovic [14]. An *n*-dimensional permutation of integers $(1,...,k,...,n)$ is used to represent a job sequence $(x_1,...,x_k,...,x_n)$ which is the same on all machines, where x_k denotes the *k-th* job, $k=1,...,n$. Let $\tilde{P}_{x_k,j}$ and $\tilde{C}_{x_k,j}$ be the fuzzy processing time and the fuzzy completion time of job x_k on machine *j*, respectively. The task is to find such a sequence of jobs which has minimum makespan. The makespan namely the completion time of job x_n on machine *m* can be calculated using formulae (7) to (10), where $\tilde{+}$ and $m\tilde{a}x$ represent fuzzy addition and fuzzy max operation, respectively.

$$\tilde{C}_{x_1,1} = \tilde{P}_{x_1,1}, \tag{7}$$

$$\tilde{C}_{x_k,1} = \tilde{C}_{x_{k-1},1} + \tilde{P}_{x_k,1}, \; for\; k = 2,....,n \tag{8}$$

$$\tilde{C}_{x_1,j} = \tilde{C}_{x_1,j-1} + \tilde{P}_{x_1,j}, \; for\; j = 2,....,m \tag{9}$$

$$\tilde{C}_{x_k,j} = m\tilde{a}x(\tilde{C}_{x_{k-1},j}, \tilde{C}_{x_k,j-1}) + \tilde{P}_{x_k,j}, \; for\; k = 2,....,n,\; j = 2,....,m \tag{10}$$

In the calculation of fuzzy makespan two basic operations are applied: fuzzy addition $\tilde{+}$ and fuzzy maximum $m\tilde{a}x$. Given two triangular fuzzy numbers $\tilde{P}_1 = (a_1, b_1, c_1)$ and $\tilde{P}_2 = (a_2, b_2, c_2)$, where $a_1(a_2)$ and $c_1(c_2)$ are lower and upper bounds, while b_1 (b_2) is the modal value of the triangle. We employ the following fuzzy addition and fuzzy maximum in order to preserve the triangular form of the obtained result:

$$\tilde{P}_1 + \tilde{P}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$$

$$m\tilde{a}x(\tilde{P}_1, \tilde{P}_2) = (max(a_1, a_2), max(b_1, b_2), max(c_1 + c_2))$$

Let us suppose that a number of job sequences are constructed. The question is how to evaluate their fuzzy makespans.

3.2 A Flowshop Sequencing Model

Obviously, point estimates have the limitation that they do not provide information about the precision of an estimate as Yao and Lin [17]. Therefore, when we use sample data to estimate the unknown time parameter, it is unreasonable to expect that a sample mean $\bar{t}_{jk}$ will be exactly equal to the population mean t_{jk}. In other words, some sampling error is to be anticipated. Because of this sampling error, it is essential for us to provide information about the accuracy of an estimate. This thus leads to the use of a confidence interval estimation of the population mean.

With a confidence interval estimate for the unknown t_{jk}, the $(1-\alpha)\times 100\%$ confidence interval estimate of t_{jk} is obtained from Eg.(6) as

$$\left[\bar{t}_{jk} - t_{n-1}(\alpha_1)\frac{s_{jk}}{\sqrt{n}}, \bar{t}_{jk} + t_{n-1}(\alpha_2)\frac{s_{jk}}{\sqrt{n}}\right], \tag{11}$$

where $0<\alpha<1$, $0<\alpha_j<1$, $j=1, 2$, and $\alpha_1+\alpha_2=\alpha$.

Note that the interval estimation in Eq.(11) is not a value but an interval. However, the confidence level of the confidence interval estimation signifies the confidence of the estimate, which has the same property as that of a membership grade for fuzzy numbers in a fuzzy set. We propose an approach for incorporating statistics with fuzzy sets for the unknown time parameter problem. Our approach is described as follows.

First, we choose an estimate of t_{jk} in Eq.(11) based on the fuzzy viewpoint. Then the evaluation for the accuracy of that estimate is conducted. When the estimate is exactly equal to $\bar{t}_{jk}$ in Eq.(11), the error rate is zero and the confidence level is $1-\alpha$ (the maximum value). In contrast, when the estimate approaches one of the two ends of the interval, $\bar{t}_{jk} - t_{n-1}(\alpha_1)\frac{s_{jk}}{\sqrt{n}}$ or $\bar{t}_{jk} + t_{n-1}(\alpha_2)\frac{s_{jk}}{\sqrt{n}}$, the error rate becomes larger and the confidence level decreases to zero. Since the membership grade of fuzzy numbers is equivalent to the confidence level of the confidence interval estimate, it is reasonable for the $(1-\alpha)\times 100\%$ confidence interval to be transformed to the following fuzzy number,

$$\tilde{t}_{jk}^{L} = \left(\bar{t}_{jk} - t_{n-1}(\alpha_1)\frac{s_{jk}}{\sqrt{n}}, \bar{t}_{jk}, \bar{t}_{jk} + t_{n-1}(\alpha_2)\frac{s_{jk}}{\sqrt{n}}; 1-\alpha \right). \tag{12}$$

Note that the membership grade of $\bar{t}_{jk}$ would not always be $1-\alpha$ during the job j process operation on machine k. Since the membership grade of $\bar{t}_{jk}$ also has an accuracy problem, we introduce another $(1-\beta)\times 100\%$ confidence interval estimate for t_{jk},

$$\left[\bar{t}_{jk} - t_{n-1}(\beta_1)\frac{s_{jk}}{\sqrt{n}}, \bar{t}_{jk} + t_{n-1}(\beta_2)\frac{s_{jk}}{\sqrt{n}} \right],$$

where $0<\beta<1$, $0<\beta_j<1$, $j=1, 2$, and $\beta_1+\beta_2=\beta$. Similarly, the $(1-\beta)\times 100\%$ confidence interval can be transformed to the level $(1-\beta)$ fuzzy number as follows:

$$\tilde{t}_{jk}^{U} = \left(\bar{t}_{jk} - t_{n-1}(\beta_1)\frac{s_{jk}}{\sqrt{n}}, \bar{t}_{jk}, \bar{t}_{jk} + t_{n-1}(\beta_2)\frac{s_{jk}}{\sqrt{n}}; 1-\beta \right), \tag{13}$$

Where $0<\beta<\alpha<1$, $0<\beta_j<\alpha_j<1$, $j=1, 2$, $t_{n-1}(\alpha_1)\left(s_{jk}/\sqrt{n}\right) < \bar{t}_{jk}$ and $t_{n-1}(\beta_1)\left(s_{jk}/\sqrt{n}\right) < \bar{t}_{jk}$. Then we have

$$0 < \bar{t}_{jk} - t_{n-1}(\beta_1)\frac{s_{jk}}{\sqrt{n}} < \bar{t}_{jk} - t_{n-1}(\alpha_1)\frac{s_{jk}}{\sqrt{n}} < \bar{t}_{jk} < \bar{t}_{jk} + t_{n-1}(\alpha_2)\frac{s_{jk}}{\sqrt{n}} < \bar{t}_{jk} + t_{n-1}(\beta_2)\frac{s_{jk}}{\sqrt{n}}.$$

Finally, according to Eqs.(12) and (13), the $(1-\alpha, 1-\beta)$ interval-valued fuzzy number is obtained as follows:

$$\bar{t}_{jk} = \left[\tilde{t}_{jk}^{L}, \tilde{t}_{jk}^{U}\right], \quad j = 1,2,\dots,n \quad and \quad k = 1,2,\dots,m.$$

According to definition 6, the signed distance of level $(1-\alpha, 1-\beta)$ interval-valued fuzzy $\tilde{t}_{jk}$ is

$$d\left(\tilde{t}_{jk}, \tilde{0}_1\right) = 2\bar{t}_{jk} + \frac{1}{8}\left\{ \left(t_{n-1}(\alpha_2) - t_{n-1}(\alpha_1)\right)\frac{s_{jk}}{\sqrt{n}} + \left(4 - \frac{3(1-\alpha)}{1-\beta}\right)\left(t_{n-1}(\beta_2) - t_{n-1}(\beta_1)\right)\frac{s_{jk}}{\sqrt{n}} \right\}.$$

Let, $t_{jk}^* = \frac{1}{2} d(\tilde{t}_{jk}, \tilde{0}_1)$

Then we have

$$t_{jk}^* = \bar{t}_{jk} + \frac{1}{16}\left\{ (t_{n-1}(\alpha_2) - t_{n-1}(\alpha_1))\frac{s_{jk}}{\sqrt{n}} + \left(4 - \frac{3(1-\alpha)}{1-\beta}\right)(t_{n-1}(\beta_2) - t_{n-1}(\beta_1))\frac{s_{jk}}{\sqrt{n}} \right\}. \quad (14)$$

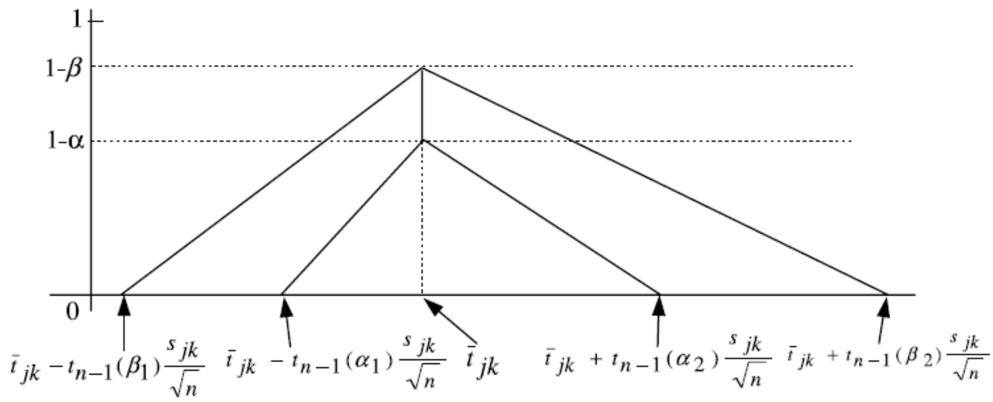

Fig. 2. The level *(1-α, 1-β)* interval-valued fuzzy number $\bar{t}_{jk}$

Note that t_{jk}^* represents an estimate of the job processing time in the fuzzy sense, which is derived from the level *(1-α,1-β)* interval-valued fuzzy number $\tilde{t}_{jk}$, using the signed distance ranking method in Definitions 6 and 7. The Decision Maker (DM) then can use t_{jk}^*, as an estimate of t_{jk} for solving the known flow-shop sequencing problem.

Remark 3: When $\alpha_1 = \alpha_2 = 1/2$ we have *α=1*. From equation (14) it clear that $t_{jk}^* = \bar{t}_{jk} + \frac{1}{4}[t_{n-1}(\beta_2) - t_{n-1}(\beta_1)]\frac{s_{jk}}{\sqrt{n}}$. Model uses the level *(1-β)* fuzzy numbers that corresponds to the $(1-\beta)\times 100\%$ confidence interval, $\left[\bar{t}_{jk} - t_{n-1}(\beta_1)\frac{s_{jk}}{\sqrt{n}}, \bar{t}_{jk}, \bar{t}_{jk} + t_{n-1}(\beta_2)\frac{s_{jk}}{\sqrt{n}}\right]$, to represent the unknown job processing times.

4. Ranking of Schedules with Uncertain Makespan

To rank makespan for the flow shop problem, the approach is based on the concept of distance measure which is an adaption of Tran's [15]. The fuzzy makespan of job sequencing are ranked by their distances to the fuzzy reference. To allow for more subjectivity of the DM, a parameter ρ introduced to enable expressing the subjective attitude of the DM and a function $f(\lambda)$ to consider the impact of different λ-levels of makespan on the DM.

4.1 A Distance Measure between Intervals

Given two intervals, $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$, and let $a_1 \le a_2$, $b_1 \le b_2$ we define the distance between I_1 and I_2 as:

$$d(I_1,I_2)=\int_0^1[(a_2+(b_2-a_2)x)-(a_1+(b_1-a_1)x)]dx$$
$$=\frac{1}{2}(a_2-a_1)+\frac{1}{2}(b_2-b_1)$$

Inspired by Campos & Munoz's [1] work, we modify (7) by including the parameter ñ to allow the decision maker to introduce his/her subjectivity into distance measure:

$$d(I_1,I_2)=\rho(a_2-a_1)+(1-\rho)(b_2-b_1), \qquad 0<\rho<1$$

The higher ρ means that more importance is put on the low bound of the interval, and less importance is given on the upper bound of the interval at the same time. In ranking of makespans, the higher value of ñ will give more importance on the low bound of the makespan and its distance from the reference. Thus, ρ can be used to reflect the optimism-pessimism attitude of the decision maker by putting different importance on the low bound and upper bound of the makespan. We call ρ optimism-pessimism indicator.

4.2 A Distance Measure between Fuzzy Numbers

Suppose the triangular fuzzy reference $\tilde{R}$ is represented as $\tilde{R}=(x_s,y_s,z_s)$ and that fuzzy makespan $\tilde{M}_i$ of schedule i is represented by a triangular membership function, i.e., $\tilde{M}_i=(x_i,y_i,z_i)$. It can be proved that $x_s\le x_i, y_s\le y_i, z_s\le z_i$. At any λ -level sets $\tilde{R}$ and $\tilde{M}_i$,

$$\tilde{R}^{\lambda}=[x_s+\lambda(y_s-x_s), z_s-\lambda(z_s-y_s)], \qquad \tilde{M}_i^{\lambda}=[x_i+\lambda(y_i-x_i), z_i-\lambda(z_i-y_i)].$$

Since $x_s+\lambda(y_s-x_s)\le x_i+\lambda(y_i-x_i)$ and $z_s+\lambda(z_s-y_s)\le z_i+\lambda(z_i-y_i)$ always hold, we can use the interval distance measure discussed in section 4.1 to define a new distance measure between two fuzzy numbers. Hence, the distance measure between two fuzzy numbers $\tilde{R}$ and $\tilde{M}_i$ can be define as:

$$D(\tilde{R},\tilde{M}_i)=\int_0^1 d(\tilde{R}^{\lambda},\tilde{M}_i^{\lambda})f(\lambda)d\lambda$$

Where $f(\lambda)$ is continuous positive function defined on [0,1], which serves to subjectively express the importance of λ-levels. For example, when $f(\lambda)=1$, it means that all λ-levels have the same importance of λ-level increases with higher values of λ.

Suppose we want to rank two fuzzy makespan, $\tilde{M}_i=(x_i,y_i,z_i)$ and $\tilde{M}_j=(x_j,y_j,z_j)$ assume that the fuzzy reference is $\tilde{R}=(x_s,y_s,z_s)$.

To obtain the rank, we calculate:

$$\Delta=D(\tilde{R},\tilde{M}_i)-D(\tilde{R},\tilde{M}_j) \qquad (15)$$

If < 0, then we consider that $\tilde{M}_i<\tilde{M}_j$, which means that sequence *i* is better than *j*;

If = 0, then we consider that $\tilde{M}_i=\tilde{M}_j$, which means that sequence *i* and *j* are the same with respect to makespan;

If > *0*, then we consider that $\tilde{M}_i>\tilde{M}_j$, which means that sequence *i* is worse than sequence *j*;

From formula (15) the following is derived:

$$\Delta=\left[\rho x_i\int_0^1(1-\lambda)f(\lambda)d\lambda+y_i\int_0^1\lambda f(\lambda)d\lambda+(1-\rho)z_i\int_0^1(1-\lambda)f(\lambda)d\lambda\right]$$
$$-\left[\rho x_j\int_0^1(1-\lambda)f(\lambda)d\lambda+y_j\int_0^1\lambda f(\lambda)d\lambda+(1-\rho)z_j\int_0^1(1-\lambda)f(\lambda)d\lambda\right] \qquad (16)$$

Let M_i be the crisp value that approximates fuzzy set $\tilde{M}_i$ (we will denote it as proxy value).

$$M_i=\rho x_i\int_0^1(1-\lambda)f(\lambda)d\lambda+y_i\int_0^1\lambda f(\lambda)d\lambda+(1-\rho)z_i\int_0^1(1-\lambda)f(\lambda)d\lambda$$

Similarly, M_j is proxy value of $\tilde{M}_j$

$$M_j=\rho x_j\int_0^1(1-\lambda)f(\lambda)d\lambda+y_j\int_0^1\lambda f(\lambda)d\lambda+(1-\rho)z_j\int_0^1(1-\lambda)f(\lambda)d\lambda$$

Then $\Delta = M_i - M_j$

From formula (16), we notice that the ranking result of two makespans has no direct relationship with fuzzy reference, and it only requires the comparison of proxy values of two makespan. To rank more than two makespans, i.e., $\{\tilde{M}_k | k = 1,.....,n\}$, the proxy value M_k of each $\tilde{M}_k$, can be ordered to get the ranking of

$$\tilde{M}_k,\ k = 1,.....,n.$$

5. m-Machine n-Job Flowshop Sequencing Algorithm

Given n jobs to be processed on m machines in the same order, the process time of job *i* on machine *j* being t_{ij} $(i = 1,2 ..... n;\ j = 1, 2..... m)$, find the sequence of jobs such that the total elapsed time (makespan) is minimized.

The flow-shop sequencing problem as presented above is a combinational search problem with *n*! possible sequences. If one could enumerate all n! sequences, the sequences with minimum total completion time could be identified, but this procedure is quite expensive and impractical for large n. The proposed algorithm is based on the assumption that a job with more total process time on all the machines should be given higher priority than a job with less total process time. An overview of the proposed algorithm can be stated as follows. The two jobs with the highest total process times are selected from the n-jobs. The best partial sequence for these two jobs is found by an 'exhaustive search', i.e. considering the two possible partial schedules. The relative positions of these two jobs with respect to each other are fixed in the remaining steps of the algorithm. Next, the job with the third highest total process time is selected and the three partial sequences are tested in which this job is placed at the beginning, middle and end of the partial sequence found in the first step. The best partial sequence will fix the relative positions of these three jobs for the remaining steps. This process is repeated until all jobs are fixed and a complete sequence is found.

The number of enumerations in the algorithm is $\frac{n(n+1)}{2}-1$, of which *n* enumerations are complete sequences and the rest are partial sequences.

The following is the step by step procedure for the curtailed-enumeration algorithm:

1) For each job i calculated as: $T_i = \sum_{j=0}^{m} t_{i,j}$, where $t_{i,j}$ is the process time of job *i* on machine *j*.
2) Arrange the jobs in descending order of T_i.
3) Pick the two jobs from the first and second position of the list of Step 2, and find the best sequence for these two jobs by calculating makespan for the two possible sequences. Do not change the relative positions of these two jobs with respect to each other in the remaining steps of the algorithm. Set $i = 3$.
4) Pick the job in the *i*th position of the list generated in Step 2 and find the best sequence by placing it at all possible *i* positions in the partial sequence found in the previous step, without changing the relative positions to each other of the already assigned jobs. The number of enumerations at this step equals *i*.
5) If $n = i$, STOP, otherwise set $i = i+1$ and go to Step 4.

5.1 Numerical Illustration

The four jobs, five machines flow-shop problem given in Table 1 is solved.

Table 1. Operation time matrix

		Machines (m)				
		1	2	3	4	5
Jobs (n)	1	5	9	8	10	1
	2	9	3	10	1	8
	3	9	4	5	8	6
	4	4	8	8	7	2

Step 1: $T_1 = 5+9+8+10+1 = 33$.
$T_2 = 9+3+10+1+8 = 31$.
$T_3 = 9+4+5+8+6 = 32$.
$T_4 = 4+8+8+7+2 = 29$.
Step 2: 1.3.2.4
Step 3: Pick job 1 and job 3, and find the optimal partial sequence for these two jobs.

Table 2. Makespan for partial sequence 1-3

		Machines (m)				
		1	2	3	4	5
Jobs	1	5/5	9/14	8/22	10/32	1/33
	3	9/14	4/18	5/27	8/40	6/46

Table 3. Makespan for partial sequence 3-1

		Machines (m)				
		1	2	3	4	5
Jobs	3	9/9	4/13	5/18	8/26	6/32
	1	5/14	4/23	8/31	10/41	1/42

This is done in Table 2 and 3, where it is clear that sequence 3-1 is the best with makespan = 42. In the next steps, the relative position of job 1 and 3 should always be 3-1, i.e. job 1 and 3. Set $i = 3$.

Step 4: Take the job in the third position of the list of step 2 (job 2) and find the optimal sequence by placing job 2 at all three possible position in the partial sequence 3-1 obtain in the last step. The makespan of these partial sequences are given in table- 4, 5 and 6.

Table 4. Makespan for partial sequence 3-1-2

		Machines (m)				
		1	2	3	4	5
Jobs	3	9/9	4/13	5/18	8/26	6/32
	1	5/14	9/23	8/31	10/41	1/42
	2	9/23	3/26	10/41	1/42	8/50

Table 5. Makespan for partial sequence 3-2-1

		Machines (m)				
		1	2	3	4	5
Jobs	3	9/9	4/13	5/18	8/26	6/32
	2	9/18	3/31	10/31	1/32	8/40
	1	5/23	9/32	8/40	10/50	1/51

Table 6. Makespan for partial sequence 2-3-1

		Machines (m)				
		1	2	3	4	5
Jobs	2	9/9	3/12	10/22	1/23	8/31
	3	9/18	4/22	5/27	8/35	6/41
	1	5/23	9/32	8/40	10/50	1/51

There possible combinations tested above show that sequences 3-2-1 (Table 4) is the best with makespan = 50.

Step 5: *i* not equal to n, hence $i = 3+1 = 4$, and go to step 4

Step 4: Pick the job in the fourth position of the list of step 2 (job 4) and find the optimal sequence by placing job 4 in the last step 4. Calculations of the makespans are given in table 7 to 10.

Table 7. Makespan for complete sequence 3-1-2-4

		Machines (m)				
		1	2	3	4	5
Jobs	3	9/9	4/13	5/18	8/26	6/32
	1	5/14	9/23	8/31	10/41	1/42
	2	9/23	3/26	10/41	1/42	8/50
	4	4/27	8/35	8/49	7/56	2/58

Table 8 .Makespan for complete sequence 3-1-4-2

		Machines (m)				
		1	2	3	4	5
Jobs	3	9/9	4/13	5/18	8/26	6/32
	1	5/14	9/23	8/31	10/41	1/42
	4	4/18	8/31	8/39	7/48	2/50
	2	9/27	3/34	10/49	1/50	8/58

Table 9. Makespan for complete sequence 3-4-1-2

		Machines (m)				
		1	2	3	4	5
Jobs	3	9/9	4/13	5/18	8/26	6/32
	4	4/13	8/21	8/29	7/36	2/38
	1	5/18	9/30	8/38	10/48	1/49
	2	9/27	3/33	10/48	1/49	8/57

Table 10. Makespan for complete sequence 4-3-1-2

		Machines (m)				
		1	2	3	4	5
Jobs	4	4/4	8/12	8/20	7/27	2/29
	3	9/13	4/17	5/25	8/35	6/41
	1	5/18	9/27	8/35	10/45	1/46
	2	9/27	3/30	10/45	1/46	8/54

The sequence 4-3-1-2 yields the minimum makespan of 54 (Table 10).

Step 5: $i = n$, hence STOP.

This problem was solved for the optimal makespan by evaluating all n! = 24 sequences and the sequence 4-3-1-2 is optimal. The proposed algorithm made nine enumerations of which four were complete sequences and five were partial sequences. It should be noted that the number of enumerations will increase if ties exist in partial sequences and each tied sequence is examined.

5.2 Triangular Fuzzy Number (Fuzzyfication)

Triangular fuzzy number $\tilde{t}_{jk}$ is represented as:

$$\tilde{t}_{jk} = \left[\bar{t}_{jk} - t_{n-1}(\beta_1)\frac{s_{jk}}{\sqrt{n}}, \bar{t}_{jk}, \bar{t}_{jk} + t_{n-1}(\beta_2)\frac{s_{jk}}{\sqrt{n}} \right];$$

Where, S^2 is Variance and S_{jk} is Standard deviation.

$$S^2 = \frac{\sum_{k=1}^{n}(x_k - \bar{x})^2}{n-1}$$

Let, $\beta_1 = 0.005; \beta_2 = 0.045; \beta = \beta_1 + \beta_2 = 0.05;$
From students t-distribution chart: (for 9 degree of freedom)

$$t_{n-1}(\beta_1) = 3.250,\ t_{n-1}(\beta_2) = 1.9188\ ;\ n = 10$$

5.3 Defuzzyfication

$$M_i = \rho x_i \int_0^1 (1-\lambda) f(\lambda) d\lambda + y_i \int_0^1 \lambda f(\lambda) d\lambda + (1-\rho) z_i \int_0^1 (1-\lambda) f(\lambda) d\lambda$$

Where $f(\lambda)$ is a continuous positive function defined on [0,1], which serve to subjectively express the importance of λ-levels, it means that all λ-levels have the same importance; when $f(\lambda)=\lambda$, the importance of λ-level increases with higher value of λ.
Now putting $f(\lambda) = 1$.

$$M_i = \rho x_i \left[\lambda - \lambda^2/2\right]_0^1 + y_i \left[\lambda^2/2\right]_0^1 + (1-\rho) z_i \left[\lambda - \lambda^2/2\right]_0^1$$
$$= \rho\, x_i (0.5) + y_i (0.5) + (1-\rho)\, z_i (0.5)$$

Fuzzy makespan are calculated for the possible job sequences and represented by triangular fuzzy numbers. The job sequences S_1= *(3-2-1-4)*, S_2= *(4-3-2-1)*, S_3= *(3-4-2-1)* and S_4= *(3-2-4-1)* have makespan $\tilde{M}_1 = (115.05, 128.70, 139.59)$, $\tilde{M}_2 = (125.88, 139.82, 151.06)$, $\tilde{M}_3 = (121.81, 139.32, 150.64)$, $\tilde{M}_4 = (118.95, 135.32, 146.71)$ respectively and they are given in Fig. 3.

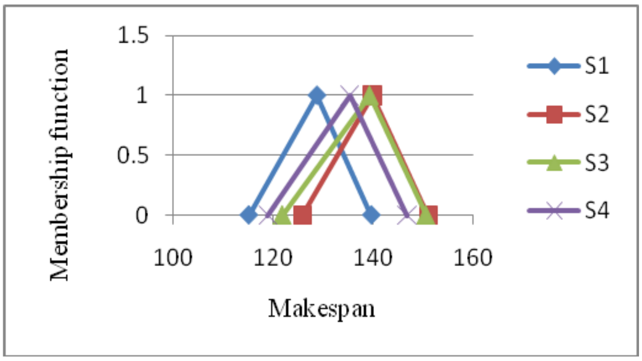

Fig. 3. Fuzzy makespan of S_1, S_2, S_3 and S_4

Suppose the decision maker sets $\rho = 0.3$ and $f(\lambda) = 1$ for this problem. The proxy values M_k of each $\tilde{M}_k$ are calculated as: $M_1 = 130.464$, $M_2 = 141.663$, $M_3 = 140.650$, $M_4 = 136.851$. Thus

$\tilde{M}_1$ has the makespan with respect to the decision maker's preference for ρ, and S_1 is the best job sequence.

The value of ρ affects the result in the sequencing process. The value of M_1 M_2 M_4 and M_4 are calculated for $\rho=0$, $\rho=0.7$, and $\rho=1$are shown in Table 11.

Table 11. Values of M_k, $k = 1, 2, 3, 4$ for ρ =0, 0.7, and 1.

For ρ = 0	For ρ = 0.7	For ρ = 1
M_1 = 134.145	M_1 = 125.556	M_1 = 121.875
M_2 = 145.440	M_2 = 136.627	M_2 = 132.850
M_3 = 144.980	M_3 = 134.889	M_3 = 130.565
M_4 = 141.015	M_4 = 131.299	M_4 = 127.135

The results are shown in Figure 4 shows how M_1, M_2, M_4, and M_4 change when ρ vary from 1 to 0. No matter what value ρ takes, M_2 is always the largest number. Through the analysis of what will happen with changing ρ, a clear knowledge can be obtained how to set ρ to achieve the desired result.

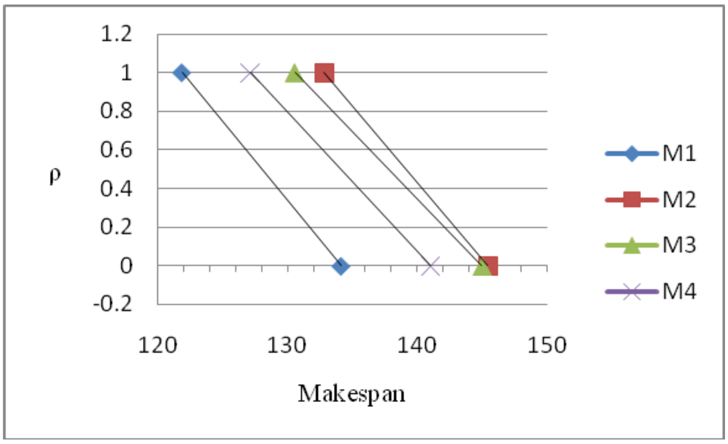

Fig. 4. Change of M_1, M_2, M_3 and M_4 with varying ρ

6. Conclusion

Fuzzy control is supposed to be a good and attractive method in the sense that it can imitate human reasoning and thinking. The control process can automatically be done by a computer to replace human beings if such a fuzzy controller can be designed with good performance. Besides, control knowledge can be accumulated and improved in the course of time.

The fuzzy control structure looks very general and simple, consisting of four units. However, very concrete decisions should be taken in each unit by choosing one alternative from numerous choices. It is extremely difficult, if not impossible, to say which choice in each unit is absolutely better than the others because all units of a fuzzy controller are in one way or another interrelated. The choice preferences are obtained by empirical studies and analysis, which are helpful for designing a reasonable or workable fuzzy controller. But it cannot be said that an optimal control solution can be found even given a very concrete case because of the lack of systematic design guidelines. One who is involved in the application of fuzzy control concepts often feels frustrated by the current status of fuzzy control theory.

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