International Journal of Engineering and Applied Sciences (IJEAS)
Vol.3, Issue 4(2011)43-55

Finite Element Analysis of a Spherical Pressure Vessel under Simultaneous Thermal and Pressure Loadings

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Abstract: Pressure vessels have been in wide use for many years in chemical, petroleum, military industries as well as in nuclear power plants. They are usually subjected to high pressures and temperatures which may be constant or cycling. This paper studies finite element analysis of a spherical pressure vessel under simultaneous thermal and pressure loadings in transient state along with numeric analytical method. A new numeric-analytical method for calculating the transient stress and displacement is introduced. Since the FEM is used for determination of temperature distribution, stress and displacement, therefore, existence of some errors is possible, but because the length and volume size of elements and step time are selected small enough, so, the obtained results are very accurate. Since, the Riemann's integral method is used for calculation of displacement and stress; the created error will be slightly higher. So, by considering the approximate methods used in temperature and displacement calculations and the FEM used in temperature distribution computation, appearance of such errors seems to be normal, thus, if the elements are chosen relatively smaller and much precise methods are used, the errors would be negligible.

Keywords: Spherical pressure vessel; Finite element method, Stress; Temperature; Pressure; Von-mises stress

1. INTRODUCTION

According to the widespread applications of thin-walled and thick-walled pressure vessels in industrial units, study and analysis of these types of vessels would be one of the most important issues in mechanical engineering science. Factors such as vessel material, the shape, chemical composition and physical substances used in it, the environment of vessels and *etc.* all are factors which each can have different effects on performance of pressure vessels. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of pressure vessels means an explosion which may cause loss of life and property. The material of pressure vessels may be brittle such that cast iron or ductile such as mild steel. Cylindrical or spherical pressure vessels (e.g., hydraulic cylinders, gun barrels, pipes, boilers and tanks) are commonly used in industry to carry both liquids and gases under pressure. When the pressure vessel is exposed to this pressure, the material comprising the vessel is subjected to pressure loading, and hence stresses, from all directions. The normal stresses resulting from this pressure are functions of the radius of the element under consideration, the shape of the pressure vessel as well as the applied pressure [1].

Two types of analysis are commonly applied to pressure vessels. The most common method is based on a simple mechanics approach and is applicable to thin-walled pressure vessels which by definition have a ratio of inner radius, r, to wall thickness, t, of $r/t \geqslant 10$. The second method is based on elasticity solution and is always applicable regardless of the r/t ratio and can be referred to as the solution for thick-walled pressure vessels. Finite Element Analysis (FEA) is a practical tool in the study of pressure vessels, especially in determining stresses in local areas such as cavities, O-ring grooves and other areas difficult to analyze manually. Heckman [2] studied the application of different finite element methods in pressure vessel analysis. He tested three dimensional, symmetric and axi-symmetric models, and concluded that finite element analysis is an extremely powerful tool in analysis of pressure vessels when employed correctly. Chang et al. [3] presented application of ANSYS in stress analysis and optimization design of pressure vessels. They introduced new approach for stress analysis and optimization design of pressure vessels. Nath [4] studied stress analysis of thick-walled cylinders with variable internal pressure states using both theory (lame's formulae) and finite element method (ANSYS). Studies in the literature mostly involve analysis of temperature, stress and displacement of vessel's wall with transient boundary conditions and usually implement purely numerical methods that require complex calculations with powerful computers; on the other hand, if the boundary conditions are constant (the static condition), the temperature, stress and *etc*. can be calculated using analytical methods. In this paper, a new numeric-analytical method for calculating the transient stress and displacement is introduced, by which the calculation time is reduced compared to other numerical methods. For this purpose, the temperature variation with the time is calculated using finite difference method, and then by using stress and displacement equations as a function of temperature and radius of the wall, the wall static displacements and stresses are defined. In order to validate the results, those variables are also calculated by numerical "finite difference" method and MATLAB commercial software. Furthermore, finite element analysis of a thin-walled pressure vessel under simultaneous thermal and pressure loading is investigated using simulation-based method by FE-based computer code ANSYS. The Von-Mises yield criterion has been used to determine the distribution of stress intensity. The results are obtained and compared by both methods and a good agreement between them is noticed.

2. ASSUMPTIONS AND BOUNDARY CONDITIONS

In the present study, a spherical pressure vessel is taken into account. The inner radius and wall thickness of the vessel are assumed as 0.19 m and 10 mm, respectively. In the case of simultaneous thermal and pressure loadings, at t = 0, temperature of the inner wall is increased from 300 K to 500 K within 5 seconds. This temperature increase is linear with time as illustrated in EQ. (1). While the outer wall of the vessel at $t \geq 0$, exposing to convection with the environment, the environment temperature is constant at 300 K. In five seconds, the pressure inside the vessel is increased from 0 Mpa to 1 Mpa. It is also assumed to increase linearly with time.

$$T_{in} = 40t + 300 \tag{1}$$

$$P = 0.2t \tag{2}$$

3. FINITE ELEMENT ANALYSIS OF THIN-WALLED PRESSURE VESSEL

3.1. Thermal loading

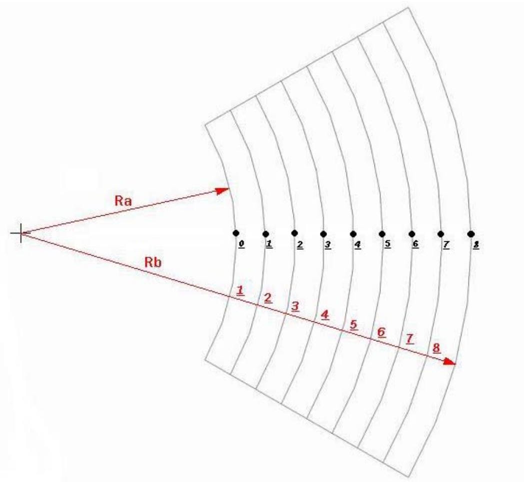
3.1.1. Finite difference method to obtain the temperature distribution with time

In the present study, we directly implemented finite element method to determine the approximate temperature distribution. For this purpose, we divided the wall into cylindrical elements and considered nodes between the elements. Then, the energy exchange equation is used for each element as shown in Figure 1. The black and red numbers represent node number and element number, respectively. Firstly, nodes 1 to 7 are selected to write the energy exchange equation. If we call these nodes as *m*, according to Figure 1 the energy exchange equation for the nodes will be as following:

The ΔV on each node of distance of $R_a + r_m$ from center is defined as:

$$\Delta V = 4\pi(R_a + m\Delta r)^2 \Delta r \tag{3}$$

$$-kA\frac{\Delta T}{\Delta r} + kA\frac{\Delta T}{\Delta r} = \Delta V \frac{\Delta T}{\Delta t} \tag{4}$$



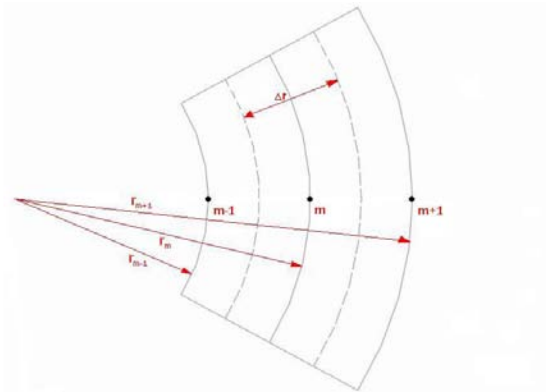


Figure 1. Configuration of the FVM study

$$\frac{4\pi k}{\Delta r}\left(R_a + m\Delta r - \frac{\Delta r}{2}\right)^2\left[T_{m-1}^i - T_m^i\right] + \frac{4\pi k}{\Delta r}\left(R_a + m\Delta r + \frac{\Delta r}{2}\right)^2\left[T_{m+1}^i - T_m^i\right] = \frac{4\pi}{\Delta t}(R_a + m\Delta r)^2 \Delta r \rho c_p\left[T_m^{i+1} - T_m^i\right] \tag{5}$$

The temperature of node *m* at T_m^{i+1} time is obtained according to other terms:

$$\begin{aligned} T_m^{i+1} = {} & \frac{\frac{4\pi k}{\Delta r}\left(R_a + m\Delta r - \frac{\Delta r}{2}\right)^2}{\frac{4\pi}{\Delta t}(R_a + m\Delta r)^2 \Delta r \rho c_p} T_{m-1}^i + \frac{\frac{4\pi k}{\Delta r}\left(R_a + m\Delta r + \frac{\Delta r}{2}\right)^2}{\frac{4\pi}{\Delta t}(R_a + m\Delta r)^2 \Delta r \rho c_p} T_{m+1}^i \\ & + \frac{\frac{4\pi(R_a + m\Delta r)^2 \Delta r \rho c_p}{\Delta t} - \frac{4\pi k}{\Delta r}\left(R_a + m\Delta r - \frac{\Delta r}{2}\right)^2 - \frac{4\pi k}{\Delta r}\left(R_a + m\Delta r + \frac{\Delta r}{2}\right)^2}{\frac{4\pi(R_a + m\Delta r)^2 \Delta r \rho c_p}{\Delta t}} T_m^i \end{aligned} \tag{6}$$

The above equation is used for nodes 1 to 7, whereas the convection boundary condition should be applied at the outer wall. For node 8 the heat exchange equation will be as following [5]:

$$4\pi R_b^2 h_0\left(T_\infty - T_8^i\right) + 4\pi k\left(R_b - \frac{\Delta r}{2}\right)^2 \frac{\partial T}{\partial r} = 4\pi R_b^2 \frac{\Delta r}{2}\rho c_p \frac{\partial T}{\partial t} \tag{7}$$

$$h_0 R_b^2\left(T_\infty - T_8^i\right) + k\left(R_b - \frac{\Delta r}{2}\right)^2 \frac{T_7^i - T_8^i}{\Delta r} = R_b^2 \frac{\Delta r}{2}\rho c_p \frac{T_8^{i+1} - T_8^i}{\Delta t} \tag{8}$$

The value of T_8^{i+1} is obtained based on other terms:

$$T_8^{i+1} = \frac{h_0 R_b^2}{R_b^2 \frac{\Delta r}{2\Delta t}\rho c_p} T_\infty + \frac{\frac{k}{\Delta r}\left(R_b - \frac{\Delta r}{2}\right)^2}{R_b^2 \frac{\Delta r}{2\Delta t}\rho c_p} T_7^i + \frac{R_b^2 \frac{\Delta r}{2\Delta t}\rho c_p - h_0 R_b^2 - \frac{k}{\Delta r}\left(R_b - \frac{\Delta r}{2}\right)^2}{R_b^2 \frac{\Delta r}{2\Delta t}\rho c_p} T_8^i \tag{9}$$

Where

$\Delta r = 0.00125m$ $\quad R_a = 0.19m$ $\quad R_b = 0.2m$ $\quad h_0 = 90\frac{W}{m^2K}$ $\quad T_\infty = 300K$ $\quad \Delta t = 0.05\sec$

With the initial condition of uniform temperature of 300 K, nine equations will be listed as following:

$$\begin{cases} T_0^{i+1} = T_0^i + 2 \\ T_1^{i+1} = 0.303858459T_0^i + 0.307856555T_2^i + 0.388284985T_1^i \\ T_2^{i+1} = 0.303871339T_1^i + 0.307843532T_3^i + 0.388285065T_2^i \\ T_3^{i+1} = 0.303884417T_2^i + 0.307830677T_4^i + 0.388285153T_3^i \\ T_4^{i+1} = 0.303896779T_3^i + 0.307817987T_5^i + 0.388285233T_4^i \\ T_5^{i+1} = 0.303909227T_4^i + 0.307805459T_6^i + 0.388285314T_5^i \\ T_6^{i+1} = 0.303921517T_5^i + 0.307793309T_7^i + 0.388285393T_6^i \\ T_7^{i+1} = 0.303933654T_6^i + 0.307780877T_8^i + 0.388285469T_7^i \\ T_8^{i+1} = 0.607891278T_7^i + 0.390197132T_8^i + 0.573476702 \end{cases} \tag{10}$$

3.1.2. Calculation of stress distribution

The Von-mises yield criterion is mainly used in the engineering analysis to study stress distribution and stress concentration. It is obtained through the absolute difference between the radial and tangential stresses [6]:

$$Vonmises = \left|\sigma_r - \sigma_\theta\right| = \frac{2\alpha E}{1-\upsilon}\left[\frac{-3a^3}{\left(b^3 - a^3\right)r^3}\int_a^b Tr^2 dr - \frac{1}{2r^3}\int_a^r Tr^2 dr - \frac{T}{2}\right] \tag{11}$$

If we write the above equation for nodes at time $i+1$:

$$Vonmises_m^{i+1} = \frac{2\alpha E}{1-\upsilon}\left[\frac{-3a^3}{\left(b^3-a^3\right)r_m^3}\int_a^b T_m^{i+1} r_m^2 dr - \frac{1}{2r_m^3}\int_a^r T_m^{i+1} r_m^2 dr - \frac{T_m^{i+1}}{2}\right] \tag{12}$$

Since we have temperature distribution only for nodes, therefore we cannot get the integrals exactly and the integral must get approximately by the following method:

$$\int_a^b T_m^{i+1} r_m^2 dr = 0.00125\left[\begin{array}{l} 0.190625^2\left(\frac{T_0^{i+1}+T_1^{i+1}}{2}\right)+0.191875^2\left(\frac{T_1^{i+1}+T_2^{i+1}}{2}\right)+0.193125^2\left(\frac{T_2^{i+1}+T_3^{i1}}{2}\right)+ \\ 0.194375^2\left(\frac{T_3^{i+1}+T_4^{i+1}}{2}\right)+0.195625^2\left(\frac{T_4^{i+1}+T_5^{i+1}}{2}\right)+0.196875^2\left(\frac{T_5^{i+1}+T_6^{i+1}}{2}\right)+ \\ 0.198125^2\left(\frac{T_6^{i+1}+T_7^{i+1}}{2}\right)+0.199375^2\left(\frac{T_7^{i+1}+T_8^{i+1}}{2}\right) \end{array}\right] \tag{13}$$

$$\int_a^r T_m^{i+1} r_m^2 dr = \sum_{m=1}^{m}\left(\frac{T_m^{i+1}+T_{m-1}^{i+1}}{2}\right)\left(\frac{r_m+r_{m-1}}{2}\right)^2 \tag{14}$$

The amount of the displacement of outer wall of the vessel can be defined from the radial and tangential stresses along with static equilibrium equation of the vessel wall:

$$u = \left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{\alpha}{r^2}\int_r^a Tr^2 dr + \left(\frac{2\alpha}{1-\upsilon}\left(\frac{1-2\upsilon}{b^3-a^3}\right)\int_a^b Tr^2 dr\right)r + \left(\frac{\alpha(1+\upsilon)a^3}{(1-\upsilon)\left(b^3-a^3\right)}\int_a^b Tr^2 dr\right)\frac{1}{r^2} \tag{15}$$

If we write the above equation for elements we have:

$$u_m^{i+1} = \left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{\alpha}{r_m^2}\int_a^r T_m^{i+1} r_m^2 dr + \left(\frac{2\alpha}{1-\upsilon}\left(\frac{1-2\upsilon}{b^3-a^3}\right)\int_a^b T_m^{i+1} r_m^2 dr\right)r_m + \left(\frac{\alpha(1+\upsilon)a^3}{(1-\upsilon)\left(b^3-a^3\right)}\int_a^b T_m^{i+1} r_m^2 dr\right)\frac{1}{r_m^2} \tag{16}$$

It is obvious that the radial displacement is maximum on outer wall, so the displacement at outer wall is calculated. By substituting numerical values for parameters, the following correlation is obtained for radial displacement of outer wall:

$$u_8 = 5.100788782\times 10^{-3}\times\int_a^b Tr^2 dr \tag{17}$$

It is written in finite difference form as:

$$u_8^{i+1} = 5.100788782 \times 10^{-3} \times \int_a^b T_m^{i+1} r_m^2 dr_m \tag{18}$$

Finally, the values of temperatures and stresses at nodes are obtained and the diagrams based on time are plotted.

3.2. Pressure loading

3.2.1. Calculation of stress and displacement distribution

The Von-mises stress is expressed as difference between radial and tangential stresses [7].

$$Vonmises = \left|\sigma_r - \sigma_\theta\right| = \left|\frac{Pa^3\left(2r^3 + b^3\right)}{2r^3\left(b^3 - a^3\right)} - \frac{Pa^3\left(b^3 - r^3\right)}{r^3\left(a^3 - b^3\right)}\right| = \left|\frac{3}{2}\frac{Pa^3}{\left(a^3 - b^3\right)}\left(\frac{b}{r}\right)^3\right| \tag{19}$$

It is clear that the Von-mises stress is inversely proportional to radius and directly proportional to pressure. Therefore, the value of it at inner and outer nodes is studied to obtain minimum and maximum values as following:

$$vonmises_8 = \left|\frac{3}{2}\frac{P(0.19)^3}{\left(0.19^3 - 0.2^3\right)}\left(\frac{0.2}{0.2}\right)^3\right| = 9.017090272P \tag{20}$$

$$vonmises_0 = \left|\frac{3}{2}\frac{P(0.19)^3}{\left(0.19^3 - 0.2^3\right)}\left(\frac{0.2}{0.19}\right)^3\right| = 10.51709027P \tag{21}$$

Hence, the value of displacement is maximum at outer wall compared to inner wall. Because, firstly, pressure inside of vessel is moving outwards and this displacement is along the radius and secondly, displacement in the outer wall equals to the relative displacement of the outer wall to each node plus displacement of the presumed node:

$$u_8 = u_m + u_{8/m} \Rightarrow u_8 > u_m$$

Also, the displacement can be extracted from the equations of stress-strain and displacement [7]:

$$u = re_{\theta} = \left[\frac{Pa^3}{(b^3 - a^3)E}\right](1 - 2\upsilon)r + \left[\frac{Pa^3}{(b^3 - a^3)E}\right](1 + \upsilon)\frac{b^3}{2r^2} \tag{22}$$

Its value at outer wall will be:

$$u_8 = \left[\frac{Pa^3}{(b^3 - a^3)E}\right](1 - 2\upsilon)b + \left[\frac{Pa^3}{(b^3 - a^3)E}\right](1 + \upsilon)\frac{b^3}{2b^2} = \left[\frac{Pa^3}{(b^3 - a^3)E}\right]\frac{3}{2}(1 - \upsilon)b = 6.596713409 \times 10^{-12} P \tag{23}$$

3.3. Simultaneous thermal and pressure loadings

Calculation of displacement and stresses in a vessel under simultaneous thermal and pressure loading is easily possible by using the principle of superposition. It can be done through summation of thermal and pressure loading effects.

4. FINITE ELEMENT ANALYSIS OF THIN-WALLED PRESSURE VESSEL USIING ANSYS

The ANSYS CAE (Computer-Aided Engineering) software program was used in conjunction with 3D CAD (Computer-Aided Design) solid geometry to simulate the behavior of pressure vessel under thermal and pressure loading conditions. The schematic of the vessel is depicted in Figure 2. Since the shape of vessel and the applied forces are symmetric about the vertical axis and horizontal plane passing from center of the vessel, it is enough to model 1/4 part of the vessel. Due to the nature of the vessel, the model was meshed with 125864 nodes three-dimensional shell structural elements (ANSYS Reference SHELL63). Each node has three translational and three rotational degrees of freedom. The material is assumed to be isotropic and linear elastic. The vessel is made of stainless steel and its characteristics are presented in Table 1.

Figure 2. Schematic of the spherical pressure vessel

Table 1. Characteristics of stainless steel

Density	$\rho = 7750 \frac{kg}{m^3}$
Young's Modulus	$190 \times 10^9\ pa$

Poisson's ratio	0.305
Thermal conductivity	$9.7\times10^{-6}\frac{1}{k}$
Specific heat	$486\frac{J}{kg.K}$

5. RESULTS AND DISCUSSION

The thin-walled pressure vessel under simultaneous thermal and pressure loading conditions is investigated at initial temperature of 300 K and at time t = 0. The wall is exposed to increased temperature boundary condition and its temperature is increased from 300 K to 500 K within 5 seconds and simultaneously, the inner pressure rises and reaches from 0 Mpa to 1 Mpa. After that the internal pressure and temperature remain constant. While the inner wall is exposed to the convection boundary condition with 300 K environment temperature and convection coefficient is $h_0 = 90\frac{W}{m^2K}$, reaction of vessel is investigated for t = 0 s to t = 50 s. The results for all of nodes are presented in Figure 3. It is evident from Figs. that the Von-mises stress reaches to its minimum value at node '3' during thermal shock stage and at this moment, stress has its maximum value at inner wall. Since finite volume method is used to obtain stress and temperature distribution and displacement, so some percentage of error is likely to appear. Because the size of volume and length of elements and step time are chosen small enough, thus the resulting answers are precisely obtained. The difference between the obtained temperatures at the nodes through two analytical and ANSYS methods does not exceed thousandth percent. But for the displacement and stress, since the Riemann integral method is used consequently, the approximate value of created error is slightly higher. However, temperature difference percentage does not exceed from two percent.

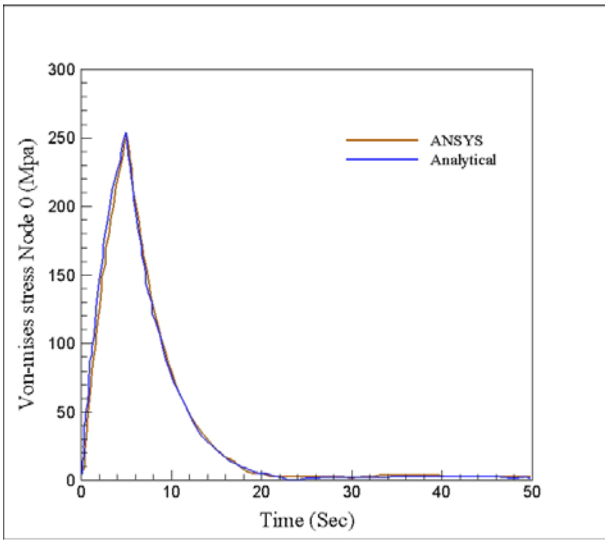
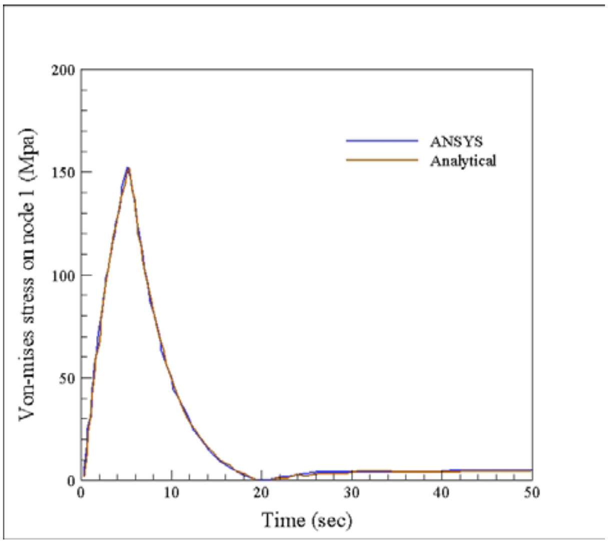
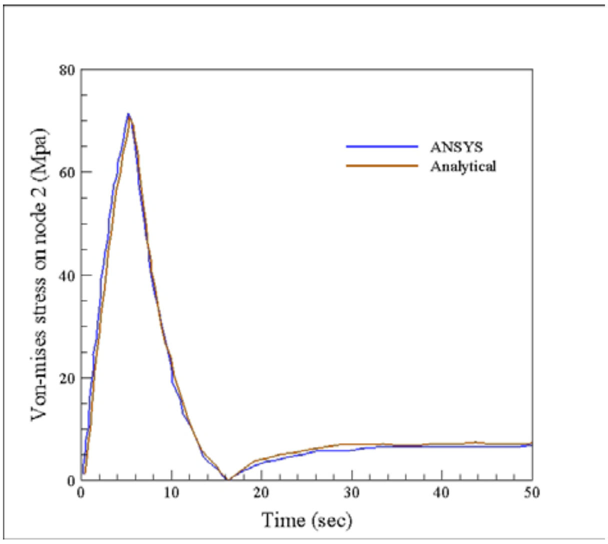


Figure 3. Comparison of effect of thermal and pressure loading on Von-mises at 8 nodes stress for two methods

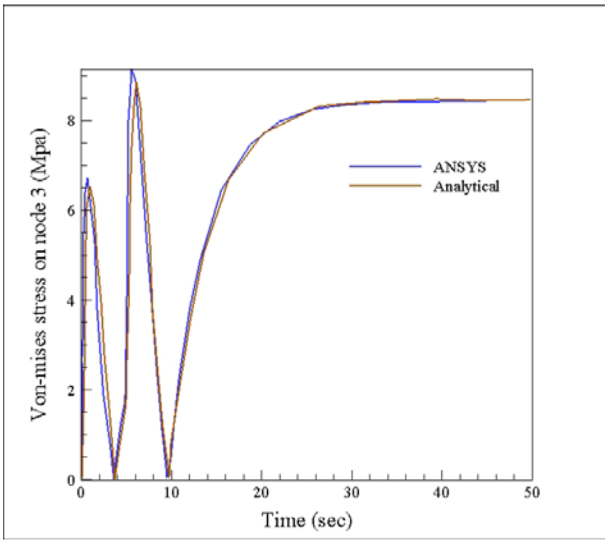
a)


Von-mises stress on node 1 (Mpa)
Time (sec)
ANSYS
Analytical
0
50
100
150
200
0
10
20
30
40
50

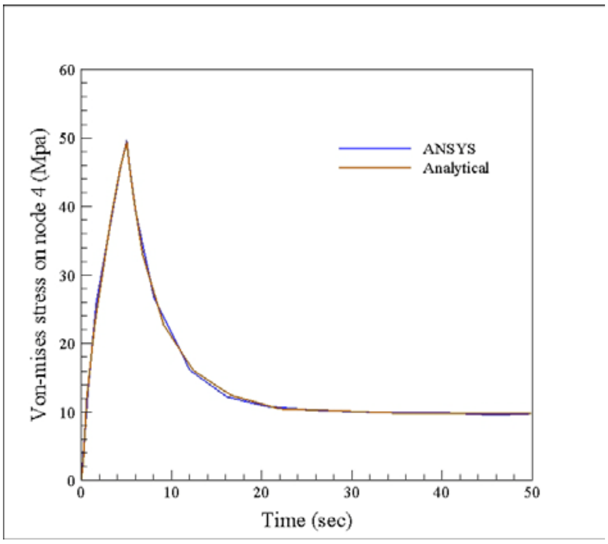
b)


Von-mises stress on node 2 (Mpa)
Time (sec)
ANSYS
Analytical
0
20
40
60
80
0
10
20
30
40
50

c)

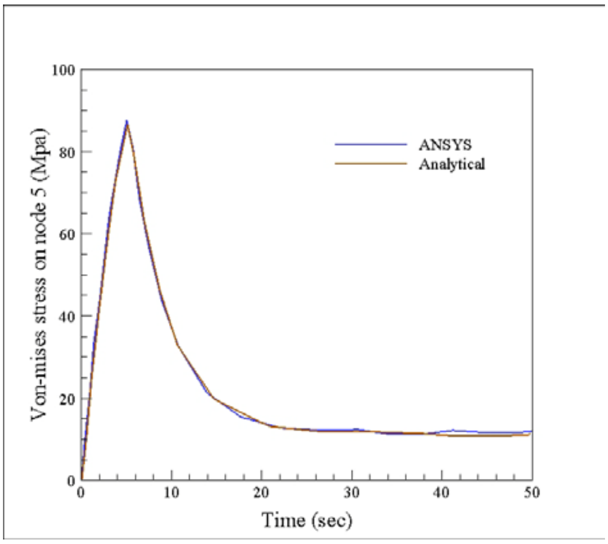

Von-mises stress on node 3 (Mpa)
Time (sec)
ANSYS
Analytical
0
2
4
6
8
0
10
20
30
40
50

d)



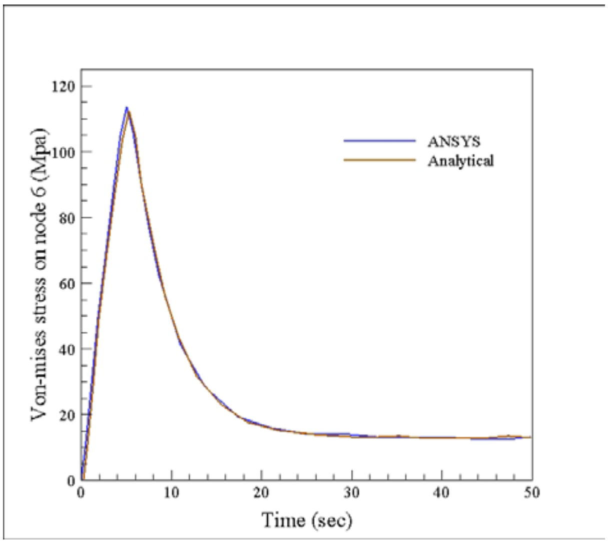
Von-mises stress on node 4 (Mpa)
Time (sec)
ANSYS
Analytical

e)



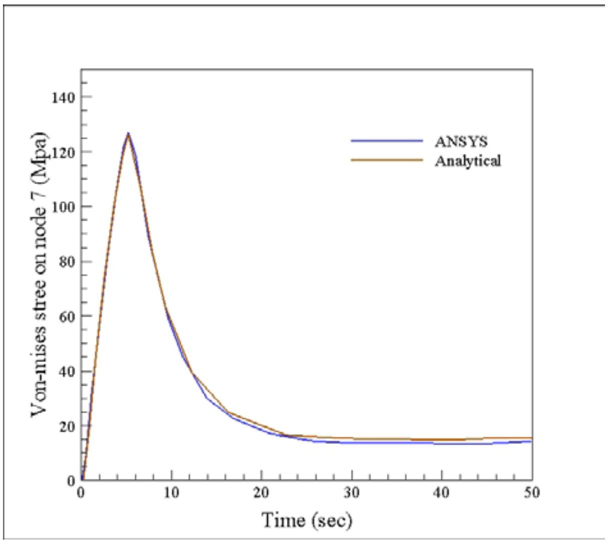
Von-mises stress on node 5 (Mpa)
Time (sec)
ANSYS
Analytical

f)

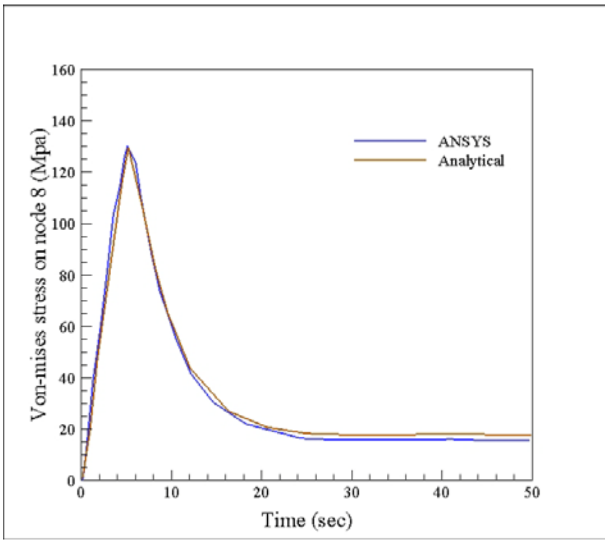


Von-mises stress on node 6 (Mpa)
Time (sec)
ANSYS
Analytical

g)



h)



i)

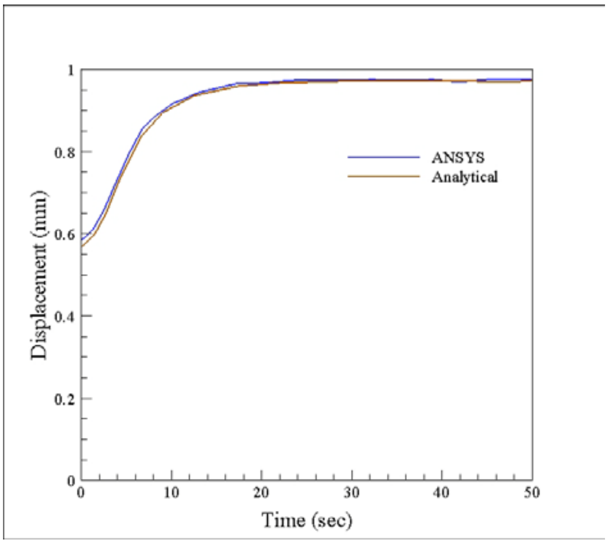


Figure 4. Comparison of effect of thermal and pressure loading on displacement for two methods

6. CONCLUSIONS

In this paper, finite element analysis of a thin-walled pressure vessel under simultaneous thermal and static loading is investigated using both analytical and simulation-based methods (ANSYS). The Von-Mises yield criterion has been used to determine the distribution of stress intensity. A new numeric-analytical method for calculation of transient temperature, stress and displacement of the vessel's wall is introduced. The variation of the wall temperature with time is calculated by numerical method to estimate the stress and displacement variations. The static equations are applied as functions of the temperature and radius of the wall. The results obtained by proposed method are compared by that of purely numerical "finite difference" method calculated by ANSYS. According to implementation of the approximate methods for calculation of integrals related to the temperature and displacement and use of finite difference method for calculation of temperature distribution, appearance of such errors seems natural. If elements are chosen smaller and more precise methods are used in calculation of approximate stress and displacement integrals, this error will be smaller and reaches to zero in the ideal case.

NOMENCLATURE

A	Area
E	Young's Modulus
F	Force
P	Pressure
T	Temperature
V	Volume
R_a	Inner radius
R_b	Outer radius
h	Convection heat transfer coefficient
i	Time moment
k	Conduction heat transfer coefficient
m	node
q	Heat flux per area unit
r	radius
t	time
u	Radial displacement
ΔV	Volume element
ΔA	Area element
Δt	Time element
$\Delta \theta$	Angle element
Δr	Radius element
T_∞	Environment temperature
ν	Poisson's ratio
α	Coefficient of thermal expansion
c_p	Specific heat capacity

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