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STATIC ANALYSIS OF TWO DIRECTIONAL FUNCTIONALLY GRADED CIRCULAR PLATE UNDER COMBINED AXISYMMETRIC BOUNDARY CONDITIONS

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Abstract

This paper is concerned with the theoretical analysis of static behavior of the two-directional functionally graded circular plate embedded on two parameter elastic foundation (Winkler- Pasternak type) under axisymmetric transverse and shear loads by using semi-analytical method. This method gives an analytical solution in the thickness and approximate solution in the radius directions by employing the state-space based differential quadrature method. The governing state equations are derived based on 3D theory of elasticity, and assuming the material properties of the plate except the Poisson's ratio varies continuously throughout the thickness and radius directions in the form of an exponential function. The stresses and displacements distribution are obtained by expanding the state variables and solving these state equations. The effects of foundation stiffnesses, material heterogeneity indices, loads ratio and the plate geometric parameter on the deformations and stresses distribution of the FG circular plate are investigated in numerical examples. The main foundings are: i- the mechanical behavior of the plate with the softer (metal rich) surface supported by elastic foundation differ significantly from that of the plate with the harder (ceramic rich) surface subjected to the same foundation.ii- the effecte of shear interaction on in-plain stresses is much more than the other stress components.The results are reported for the first time and are discussed in detail.

Key words: FG Circular Plate, Elastic Foundation, Elasticity, State - Space, DQ Method.

1. Introduction

A new class of materials known as "two-directional functionally graded materials" (2D-FGMs) has found many applications in modern engineering fields such as aerospace, mechanical, civil, nuclear and so on. These materials can be designed to achieve particular desired properties and the gradition in properties of the material and can be optimize stress distribution in various directions. There for, they are a convenient selection to use in structures and machine elements of modern industries such as spacecrafts, advanced combustion engines, power plants and high temprature turbines which usually subjected to multi-directional thermal and mechanical loads.

The analysis of static and dynamic behavior of uni and multi-directional FGMs circular plate under thermal and mechanical loads have gained more attension by the researchers in recent years. For instance, Nemat-Alla [1] introduced the concept of adding a third material constituent to the conventional FGMs material in order to significantly reduce the thermal stresses in machine elements that subjected to sever thermal loading, and his investigation on 2D-FGMs has shown that it is more capable of reducing thermal and residual stresses than one-directional FGMs. Nie and Zhong [2] investigated the axisymmetric bending of 2D-FGM circular and annular plates based on the three-dimensional theory of elasticity using semi-analytical and ANSYS software. Lu et al. [3] presented a semi-analytical solution for the static analysis of multidirectional FG rectangular plate.
Nie and Zhong [4] investigated the dynamic behavior of multi-directional FGM annular plates based on the three-dimensional theory of elasticity using the state- space method combined with the one dimensional differential quadrature rule (DQM). Alibeigloo [5] discussed bending behavior of FGM rectangular plate with integrated surface piezoelectric layers resting on elastic foundation.Shariyat and Alipour [6] analyzed the free vibration and modal stress of two-directional functionally graded circular plate embedded on two-parameter elastic foundations by employing the differential transform method. Yun et al. [7] investigated the axisymmetric bending of FG circular plates as analytically by using direct displacement method. Golmakani and Kadkhodayan [8] analyzed the axisymmetric nonlinear bending of an annular functionally graded plate under mechanical loading based on FSDT

and TSDT by using the dynamic relaxation (DR) method combined with the finite difference technique.Akgoz and Civalek [9] applied the discrete singular convolution method to investigate the nonlinear vibration behavior of geometrically nonlinear thin laminated plates resting on non-linear elastic foundation. Malekzadeh et al. [10] studied the free vibration of teperature-dependent functionally graded annular plates on elastic foundations by using DQ method. Yas and Tahouneh [11] investigated the free vibration of functionally graded annular plates on elastic foundation based on the three-dimensional theory of elasticity and using the differential quadrature method. Jodaei et al. [12] obtained the natural frequencies of FG annular plates by using semi-analytical approach and comparative behavior modeling with artificial neural network (ANN). Ponnusamy and Selvamani [13] studied the wave propagations in a thermoelastic homogeneous circular plate embedded in an elastic medium based on generalized two dimensional theory of thermoelasticity. Eftekhari and Jafari [14] employed the combined application of the finite element method (FEM) and the differential quadrature method (DQM) to analysis the vibration and buckling problems of rectangular plates. The non-linear free and forced vibration of moderately thick annular FGM plate was studied based on the first-order shear deformation plate theory and von Kármán-type equation by Amini et al. [15]. Golmakani and Kadkhodayan [16] investigated the axisymmetric bending and stretching of circular and annular functionally graded plates with variable thicknesses under combined thermal-mechanical loading based on the first-order shear deformation theory and employing the dynamic relaxation (DR) method to solve the governing equations. Recently author discussed the static behavior of unidirectional FG circular plate resting on elastic foundation under the effect of axisymmetric transverse load by using semi-analytical method [17].

To the best of author knowledge, no work has been reported till date which concerns the static analysis of bi-directional functionally graded circular plate supported by elastic foundations and subjected to compound axisymmetric transverse and shear loads. In this work, the material properties of the plate except the Poisson's ratio (ν) are assumed to be graded in the thickness and radial directions according to the exponential distribution of the constituent. The formulations are based on the three-dimensional theory of elasticity and a semi-analytical approach, which makes use of the state space method and the one-dimensional differential quadrature rule is employed to extract the numerical results. Finally the effects of the gradient indices, thickness to radius ratio of the plate, loads ratio and foundation elastic coeficients on the displacement and stress fields are investigated.

2. Problem formulation

2.1 Geometry and properties of the plate

Consider a 2D-functionally graded circular plate with radius a and height h, subject to uniform transverse (p) and shear (q) loads on the top surface and supported by an elastic medium in the bottom surface, as shown in Figure 1.

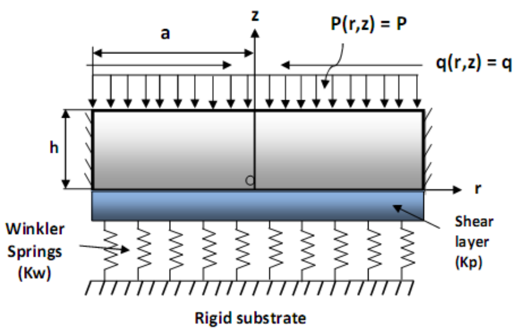


Fig. 1 Geometry of 2D-FGMs circular plate resting on elastic foundation

Since the plate geometry, distribution of material properties, applied loads, and boundary conditions are independent from circumferential direction, the problem is axisymmetric. The material properties of the plate except the Poisson's ratio ν, varies continuously throughout the thickness and radius directions in accordance with the exponential distribution as follow:

$$C_{ij}(r,z) = C_{ij}^{0}(0,0)\, e^{\lambda_1(\frac{z}{h})+\lambda_2(\frac{r}{a})} \tag{1}$$

where $C_{ij}^{0}(0,0)$ are the elastic constants of the plate material in the center of bottom surface, λ_1 and λ_2 are the parameters indicating the trens of gradient.

2.2 Basic equations of 3D elasticity theory

When referred to the cylindrical coordinates (r, θ, z), the basic equations for the axisymmetric problem of a transversely isotropic FGM body are

$$\sigma_{r,r} + \tau_{rz,z} + r^{-1}(\sigma_r - \sigma_\theta) = 0 \quad , \quad \tau_{rz,r} + \sigma_{z,z} + r^{-1}\tau_{rz} = 0 \tag{2}$$

$$\varepsilon_r = u_{,r} \,,\, \varepsilon_\theta = r^{-1} u \,,\, \varepsilon_z = w_{,z} \,,\, \gamma_{rz} = u_{,z} + w_{,r} \tag{3}$$

$$\begin{aligned}\sigma_r &= C_{11}\varepsilon_r + C_{12}\varepsilon_\theta + C_{13}\varepsilon_z \\ \sigma_\theta &= C_{12}\varepsilon_r + C_{11}\varepsilon_\theta + C_{13}\varepsilon_z \\ \sigma_z &= C_{13}\varepsilon_r + C_{13}\varepsilon_\theta + C_{33}\varepsilon_z \\ \tau_{rz} &= C_{44}\gamma_{rz}\end{aligned} \tag{4}$$

where $\sigma_r, \sigma_\theta, \sigma_z$ and τ_{rz} are the stress components; u and w are the displacements in r-direction and z-direction, respectively. The comma denotes differentiation with respect to the indicated variable.

Substitution the eq. (1, 3, 4) in to eq. (2) can be leads to the following differential equations in terms of displacement components in the bottom surface of the plate

$$u_{,zz} = -\frac{C_{11}^0}{C_{44}^0}\left(u_{,rr} + (\frac{1}{r} + \frac{\lambda_2}{a})u_{,r} - r^{-2}u\right) - \frac{C_{12}^0}{C_{44}^0}\frac{\lambda_2}{a}\frac{u}{r} - \frac{\lambda_1}{h} w_{,r} - \frac{\lambda_1}{h}u_{,z} + (-1-\frac{C_{13}^0}{C_{44}^0})\, w_{,rz} + (\frac{C_{23}^0 - C_{13}^0}{C_{44}^0})\frac{1}{r}\, w_{,z}$$
$$-\frac{C_{13}^0}{C_{44}^0}\frac{\lambda_2}{a}\, w_{,z}$$

$$w_{,zz} = -\frac{\lambda_1}{h}\frac{C_{13}^0}{C_{33}^0}(u_{,r} + \frac{u}{r}) - \frac{C_{44}^0}{C_{33}^0}(w_{,rr} + \frac{1}{r}w_{,r} + \frac{\lambda_2}{a}w_{,r}) - \frac{C_{13}^0 + C_{44}^0}{C_{33}^0}(u_{,rz} + \frac{1}{r}u_{,z}) - \frac{C_{44}^0}{C_{33}^0}\frac{\lambda_2}{a}u_{,z} - \frac{\lambda_1}{h}w_{,z} \tag{5}$$

3. The solution procedure

In this study, a semi-analytical approach is employed to solve the governing differentials equations appeared in Eq. (5). This method combines the state space method (SSM) in the z- direction of cicular plate to obtain an analytical solution and uses the one-dimentional differential quadrature rule in the radial direction to express the static behavior of the plate. By using this method a linear eigenvalue system in terms of the displacements is established and by solving the resulted eigenvalue system, the stresses and displacements in variouse points of the plate are obtained.

3. 1 State –space method

By taking the state variables as $u, w, u_{,z}, w_{,z}$ the state space notation of equations (5) can be written as

$$\frac{\partial}{\partial z}[\delta] = \begin{bmatrix} D1 \\ D2 \end{bmatrix}[\delta] \tag{6}$$

where δ is the state vector and $\delta = \begin{bmatrix} u & w & u_{,z} & w_{,z} \end{bmatrix}^T$.

For the sake of transform from physical domain to a normalized computational domain the elements of the matrix D2 must normalized, for this purpose the following dimensionless quantities are introduced:

$$\eta = \frac{z}{h}, \quad R = \frac{r}{a}, \quad U = \frac{u}{h}, \quad W = \frac{w}{h}, \quad \bar{C}_{ij}^{0} = \frac{C_{ij}^{0}}{C_{33}^{0}} \tag{7}$$

By considering these quantities the eq. (6) can be rewritten as

$$\left[\delta\right]_{,\eta} = \begin{bmatrix} D1 \\ D2(R) \end{bmatrix} [\delta] \tag{8}$$

where the elements of matrix D1 are constant and the elements of matrix D2 are functions from variable R.

In order to solve the eq. (8), the special derivatves are discretized by applying the one dimensional differential quadrature method as an efficient and accurate numerical tool.

3.2 Differential –quadrature method and its application

The differential quadrature method is a numerical solution technique for initial and/or boundary problems. The DQ method approximates the derivative of a function at any discrete point by a weighted linear summation of the functional values in the whole domain. According to this rule, the nth derivative of a function $\Phi(r)$ at discretized point r_i can be approximated as [18]

$$\left.\frac{\partial^{(n)}\Phi(r)}{\partial r^{n}}\right|_{r_i} = \sum_{j=1}^{N} A_{ij}^{(n)}\,\Phi(r_j) \tag{9}$$

where N denotes the total number of discrete points, $\Phi(r)$ includs u, w and $\Phi(r_j)$ is the function value at any discrete point. The weighting coefficients for the frist-order derivative in the radius direction can be determined as follow [18].

$$A_{ik}^{(1)} = \frac{\prod_{k=1,k\neq i}^{N} (r_i - r_k)}{(r_i - r_j)\prod_{k=1,k\neq j}^{N} (r_j - r_k)}, \quad A_{ii}^{(1)} = -\sum_{j=1,j\neq i}^{N} A_{ij}^{(1)}, \quad i \neq j, \quad i,j = 1,2,3,\cdots,N \tag{10}$$

The weighting coefficients of the nth-order derivatives can be obtained from the following relationships

$$A_{ij}^{(n)} = n\left[A_{ii}^{(n-1)}A_{ij}^{(1)} - \frac{A_{ij}^{(n-1)}}{r_i - r_j}\right], \quad A_{ii}^{(n)} = -\sum_{j=1,j\neq i}^{N} A_{ij}^{(n)}, \quad n = 2,3,\cdots,N-1, \quad i \neq j, \quad i,j = 1,2,3,\cdots,N \tag{11}$$

In the present study, the grid points are taken non-uniformly spaced and are given by the following equation named Richard-Shu criterion.

$$r_i = \frac{a}{2}\left[1 - \cos\left(\frac{(i-1)\pi}{N-1}\right)\right], \quad i = 1,2,3,\cdots,N \tag{12}$$

By implementation the one dimensional DQ rule to the derivatives of unknown functions u, w the following relations can be obtained.

$$U_{,R}\Big|_{R_i} = \sum_{j=1}^{N} A_{ij}^{(1)} U_j, \quad W_{,R}\Big|_{R_i} = \sum_{j=1}^{N} A_{ij}^{(1)} W_j, \quad U_{,RR}\Big|_{R_i} = \sum_{j=1}^{N} A_{ij}^{(2)} U_j, \quad W_{,RR}\Big|_{R_i} = \sum_{j=1}^{N} A_{ij}^{(2)} W_j$$

$$U_{,RZ}\Big|_{R_i}=\sum_{j=1}^{N}A_{ij}^{(1)}U_{j,Z}\quad,\quad W_{,RZ}\Big|_{R_i}=\sum_{j=1}^{N}A_{ij}^{(1)}W_{j,Z} \tag{13}$$

4. The boundary and edges conditions

The edges and boundary conditions for solid circular plate with clamped and simply supported edge are defined as follow:

The regularity conditions of the plate on the centeral point are

$$r=0\,,\ u=0\,,\ w_{,r}=0 \tag{14}$$

Clamped:

$$r=a\ ,\ u=0\,,\ w=0 \tag{14-a}$$

Simply supported:

$$r=a\ ,\ \sigma_r=0\,,\ w=0 \tag{14-b}$$

The boundary conditions in the bottom surface of the plate are

$$\tau_{rz}=0\ ,\ \sigma_z=f_{zo}(r)\qquad \text{at } z=0 \tag{15}$$

where $f_{zo}(r)$ is the interaction between the plate and foundation. In the referred coordinate system this interaction can be expressed as follow

$$f_{zo}=k_w w_o-k_p(w_{o,rr}+r^{-1}w_{o,r}) \tag{16}$$

where f_{zo} denotes the foundation reaction per unit area and k_w, k_p are the Winkler and Pasternak coeficients, respectively.

The boundary conditions in the top surface of the plate are

$$\tau_{rz}=-q\ ,\ \sigma_z=-p\qquad \text{at } z=h \tag{17}$$

The discretized form of equations appeared in Eqs. 14-17 can be written as

$$R=0\,,U_1=0\,,W_1=-\sum_{j=2}^{N}\frac{A_{1j}}{A_{11}}W_j \tag{18}$$

$$R=1\,,\ U_N=0\,,\ W_N=0 \tag{18-a}$$

$$R=1\,,\ \sigma_{RN}=0\,,\ W_N=0 \tag{18-b}$$

At $\eta=0$

$$\frac{\partial U_i}{\partial\eta}+\frac{h}{a}\Big(\sum_{j=2}^{N-1}A_{ij}^{(1)}W_j-\frac{A_{i1}^{(1)}}{A_{11}^{(1)}}\sum_{j=2}^{N-1}A_{ij}^{(1)}W_j\Big)=0$$

$$\frac{\partial W_i}{\partial\eta}+\frac{h}{a}\bar{C}_{13}^{0}\Big(\sum_{j=1}^{N}A_{ij}^{(1)}U_j+\frac{U_i}{R_i}\Big)=\frac{1}{e^{R_i\lambda_2}}\Big(K_W W_i-K_P\big(\sum_{j=1}^{N}A_{ij}^{(2)}W_j+\frac{1}{R_i}\sum_{j=1}^{N}A_{ij}^{(1)}W_j\big)\Big)\ ,\ i=1,2,3,\cdots,N \tag{19}$$

where $K_W=\frac{k_W h}{\bar{C}_{33}^{0}C_{33}^{0}}$, $K_P=\frac{k_p}{\bar{C}_{33}^{0}C_{33}^{0}}\frac{h}{a^2}$ are dimensionless coeficients of foundation.

At $\eta=1$

$$\frac{\partial U_i}{\partial\eta}+\frac{h}{a}\Big(\sum_{j=2}^{N-1}A_{ij}^{(1)}W_j-\frac{A_{i1}^{(1)}}{A_{11}^{(1)}}\sum_{j=2}^{N-1}A_{ij}^{(1)}W_j\Big)=0$$

$$\frac{\partial W_i}{\partial\eta}+\frac{h}{a}\bar{C}_{13}^{0}\Big(\sum_{j=1}^{N}A_{ij}^{(1)}U_j+\frac{U_i}{R_i}\Big)=\frac{-P}{\bar{C}_{33}^{0}C_{33}^{0}e^{(\lambda_1+R_i\lambda_2)}}\ ,\ i=1,2,3,\cdots,N \tag{20}$$

By implement the boundary conditions in Eq. (13), the solution to Eq. (8) can be written as:

$$\delta_i(\eta)=\exp(M_i\cdot\eta)\delta_i(0) \tag{21}$$

where

$$\delta_i(\eta)=\big[U_i(\eta)\quad W_i(\eta)\quad U_{i,\eta}(\eta)\quad W_{i,\eta}(\eta)\big]^T,\ \delta(0)=\big[U_i(0)\quad W_i(0)\quad U_{i,\eta}(0)\quad W_{i,\eta}(0)\big]^T$$

$i = 2,3,\cdots,N-1$

and $M_i = \begin{bmatrix} D1_i \\ D2(R_i) \end{bmatrix}_{4(N-2)\times 4(N-2)}$. In Eq. (21), $\exp(M_i \cdot \eta)$ is the matrix exponential function, $\delta_i(\eta)$ and $\delta_i(0)$ are the values of the state variables at an arbitrary plane η and the bottom plane $\eta = 0$, respectively. The elements of matrix M_i are

$$D1_i = \begin{bmatrix} 0 & 0 & [\delta_{ij}]_{(N-2)\times(N-2)} & 0 \\ 0 & 0 & 0 & [\delta_{ij}]_{(N-2)\times(N-2)} \end{bmatrix}_{2(N-2)\times 4(N-2)}, \quad \delta_{ij} = 0 (i \neq j); \delta_{ii} = 1 \tag{21-a}$$

$$D2(R_i) = \begin{bmatrix} [b_{ij}^{11}]_{(N-2)\times(N-2)} & [b_{ij}^{12}]_{(N-2)\times(N-2)} & [b_{ij}^{13}]_{(N-2)\times(N-2)} & [b_{ij}^{14}]_{(N-2)\times(N-2)} \\ [b_{ij}^{21}]_{(N-2)\times(N-2)} & [b_{ij}^{22}]_{(N-2)\times(N-2)} & [b_{ij}^{23}]_{(N-2)\times(N-2)} & [b_{ij}^{24}]_{(N-2)\times(N-2)} \end{bmatrix}_{2(N-2)\times 4(N-2)} \tag{21-b}$$

$$b_{ij}^{11} = -\frac{C_{11}^0}{C_{55}^0}\left(\frac{h}{a}\right)^2 \left(\sum_{j=2}^{N-1} A_{ij}^{(2)} + \left(\frac{1}{R_i} + \lambda_2\right)\sum_{j=2}^{N-1} A_{ij}^{(1)} - \frac{1}{R_i^2}\right) - \frac{C_{12}^0}{C_{55}^0}\left(\frac{h}{a}\right)^2 \frac{\lambda_2}{R_i}, \qquad b_{ij}^{12} = -\frac{\lambda_1 h}{a}\sum_{j=2}^{N-1} A_{ij}^{(1)}$$

$$b_{ij}^{13} = 0\ (i \neq j),\ b_{ii}^{13} = -\lambda_1, \qquad b_{ij}^{14} = -\left(\frac{h}{a}\right)\left(1 + \frac{C_{13}^0}{C_{55}^0}\right)\sum_{j=2}^{N-1} A_{ij}^{(1)} + \left(\frac{h}{a}\right)\left(\frac{C_{23}^0 - C_{13}^0}{C_{55}^0}\right)\frac{1}{R_i} - \frac{C_{13}^0}{C_{55}^0}\left(\frac{h}{a}\right)\lambda_2$$

$$b_{ij}^{21} = -\frac{\lambda_1 h}{a}\frac{C_{13}^0}{C_{33}^0}\left(\sum_{j=2}^{N-1} A_{ij}^{(1)} + \frac{1}{R_i}\right), \qquad b_{ij}^{22} = -\frac{C_{55}^0}{C_{33}^0}\left(\frac{h}{a}\right)^2 \left(\sum_{j=2}^{N-1} A_{ij}^{(2)} + \left(\frac{1}{R_i} + \lambda_2\right)\sum_{j=2}^{N-1} A_{ij}^{(1)}\right)$$

$$b_{ij}^{23} = -\frac{C_{13}^0 + C_{55}^0}{C_{33}^0}\left(\frac{h}{a}\right)\left(\sum_{j=2}^{N-1} A_{ij}^{(1)} + \frac{1}{R_i}\right) - \frac{C_{55}^0}{C_{33}^0}\frac{h}{a}\lambda_2, \quad b_{ij}^{24} = 0\ (i \neq j),\ b_{ii}^{24} = -\lambda_1, \quad i = 2,3,\cdots,N-1$$

The array of Eq. (21) in the bottom and top surfaces of the plate can be written as

$$\delta_i(1) = \exp(M_i)\delta_i(0) \tag{22}$$

where $\exp(M_i)$, $\delta_i(1)$, $\delta_i(0)$ are the global transfer matrix and the state vectors in the bottom and top surfaces of the plate respectively.

By substitution the boundary conditions discused in Eq. (19-20) in to Eq. (22) the following algebraic equations can be obtained

$$GT = Q \tag{23}$$

where G is a $4(N-2)\times 4(N-2)$ matrix, Q is a traction force vector and

$$T = \begin{bmatrix} U_i(0) & W_i(0) & U_i(1) & W_i(1) \end{bmatrix}^T, \quad i = 2,3,\cdots,N-1 \tag{24}$$

By solving Eq. (24) all displacements at $\eta = 0$, $\eta = 1$ are obtained, and then all mechanical quantities are obtained along the thickness of the 2D-FGMs circular plate by using Eqs. (21) and (4).

5. Numerical results and discussions

5.1 Code validation

A computer code has been developed to study the static response of 2D-FGMs circular plate embedded on elastic medium to axisymmetric transverse and shear loads. Since there are no results available in the open litrature for 2D- FGM circular plate with compound boundary conditions. The results of the prepared code are compared with the results for 2D- FGM circular plate without elastic foundation deformed due to tansverse mechanical load with those of Nie and Zhong [2]. A clamped circular plate with Young’s moduli 380 Gpa and Poisson’s ratio $\nu = 0.3$ at center point of the bottom surface of the plate simlar as Ref. [2] is considered. Valuse of other parameters are:

$a=1.0m$, $h=0.1a$, $\lambda_1=\lambda_2=1$, $\tau_{rz}=0$, $\sigma_z=0$ at $\eta=0$ and $\tau_{rz}=0$, $\sigma_z=-1GPa$ at $\eta=1$

The results of present code and Ref. [2] are shown in Table 1. It can be observed from Table 1 that the obtained results agree well.

Table1. Dimensionless deflection of 2D functionally graded circular plate

		R							
		0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875
$\eta=0$	Ref. [2]	-1.513	-1.451	-1.289	-1.052	-0.775	-0.495	-0.250	-0.075
	present	-1.521	-1.460	-1.295	-1.054	-0.776	-0.493	-0.248	-0.074

For numerical illustration a clamped 2D-FGM circular plate consisting of Titanium and Zirconium studied earlier by Yun et al. [7] is considered. The properties of plate constituents are as table 2.

Table2. Mechanical properties of FGMs plate constituents

Materials	Titanum	Zirconium
Young's modulus, E(GPa)	110.25	278.41
Poisson's ratio, ν	0.288	0.288

Valuse of other parameters are:

$a=1.0m$, $h=0.02a$, $E_{(0,0)}=110.25GPa$, $E_{(a,h)}=278.41GPa$ (25)

The boundary conditions are:

$\tau_{rz}=0$, $\sigma_z=f_{zo}$ at $\eta=0$ and $\tau_{rz}=-1GPa$, $\sigma_z=-1GPa$ at $\eta=1$ (26)

5.2 Covergence of the solution method

In order to extract the numerical results and to show the effect of number of discretized points on the solution method, the convergence of DQ rule in the radius direction is investigated and is used as an evaluation criterion. For a clamped circular plate with parameters explained in Eqs. (25, 26) and $k_p=k_w=0.1$, $\lambda_1=\lambda_2=1$ the non-dimensional deflection of the plate vs. number of discrete points N at a location $R=0.5$ is plotted in Fig.2. It is seen from Fig. 2 that the value of W_o approaches to constant value as N increases. This figure confirms that the convergence of this method is high, relative to other numerical methods in engineering and science. The number of grid points in the next sections is nine (N=9).

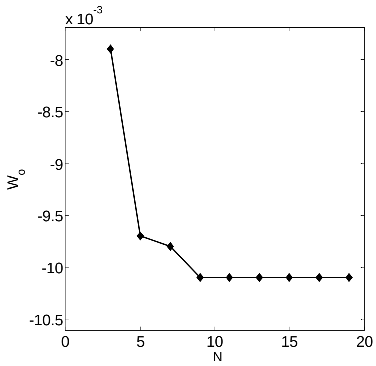
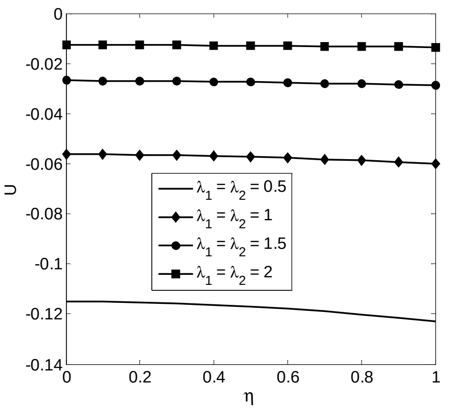


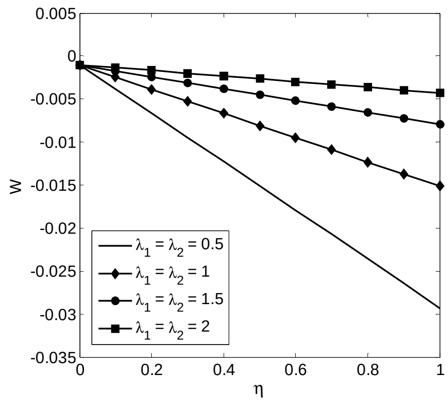
Fig.2 Convergence of non-dimensional deflection of the plate at a location ($R=0.5$)

5.3 **Effect of material heterogeneity indices**

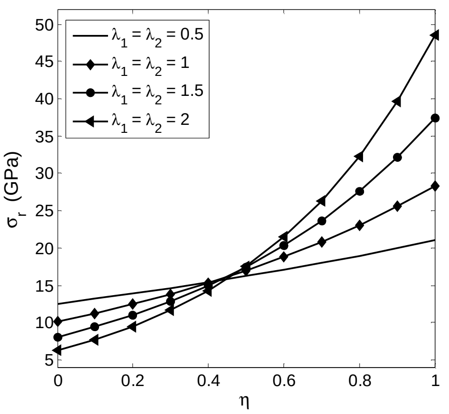
The effect of material heterogeneity indices on variation of mechanical quantities along the plate thickness with parameters defined in Eqs. (25, 26) and $k_p = k_w = 0.1$ at a point $R = 0.96$ is depicted in Fig.3. It is observed from from Fig. 3 that the displacements in the plate thickness direction decrease as heterogeneity indices increase. The value of in-plane stresses decrease gradually along the thickness of the plate when η is less than 0.45 and then increase as gradient parameters increase. The value of σ_z and τ_{rz} decreases in the thickness direction with heterogeneity indices increasing. The in-plane stresses take high values relative to other stress components due to shear force at top surface of the plate.



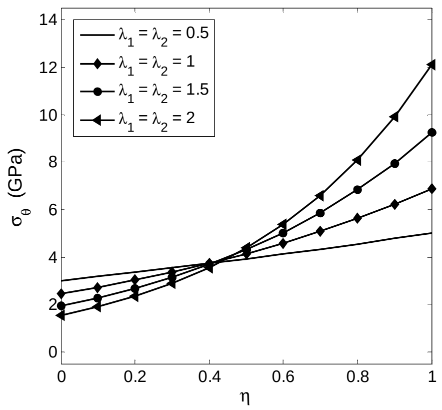
a) Radial displacement



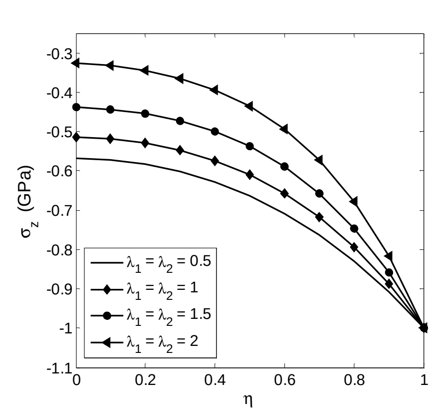
b) Transverse displacement



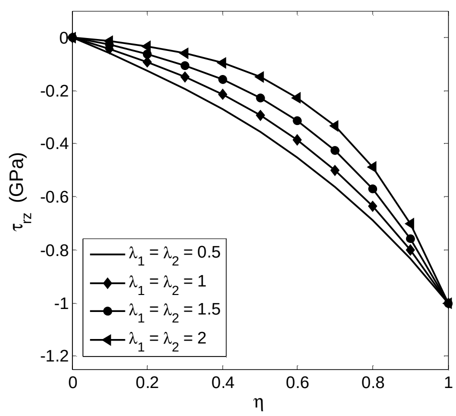
c) Radial stress



d) Tangential stress



e) Transverse normal stress

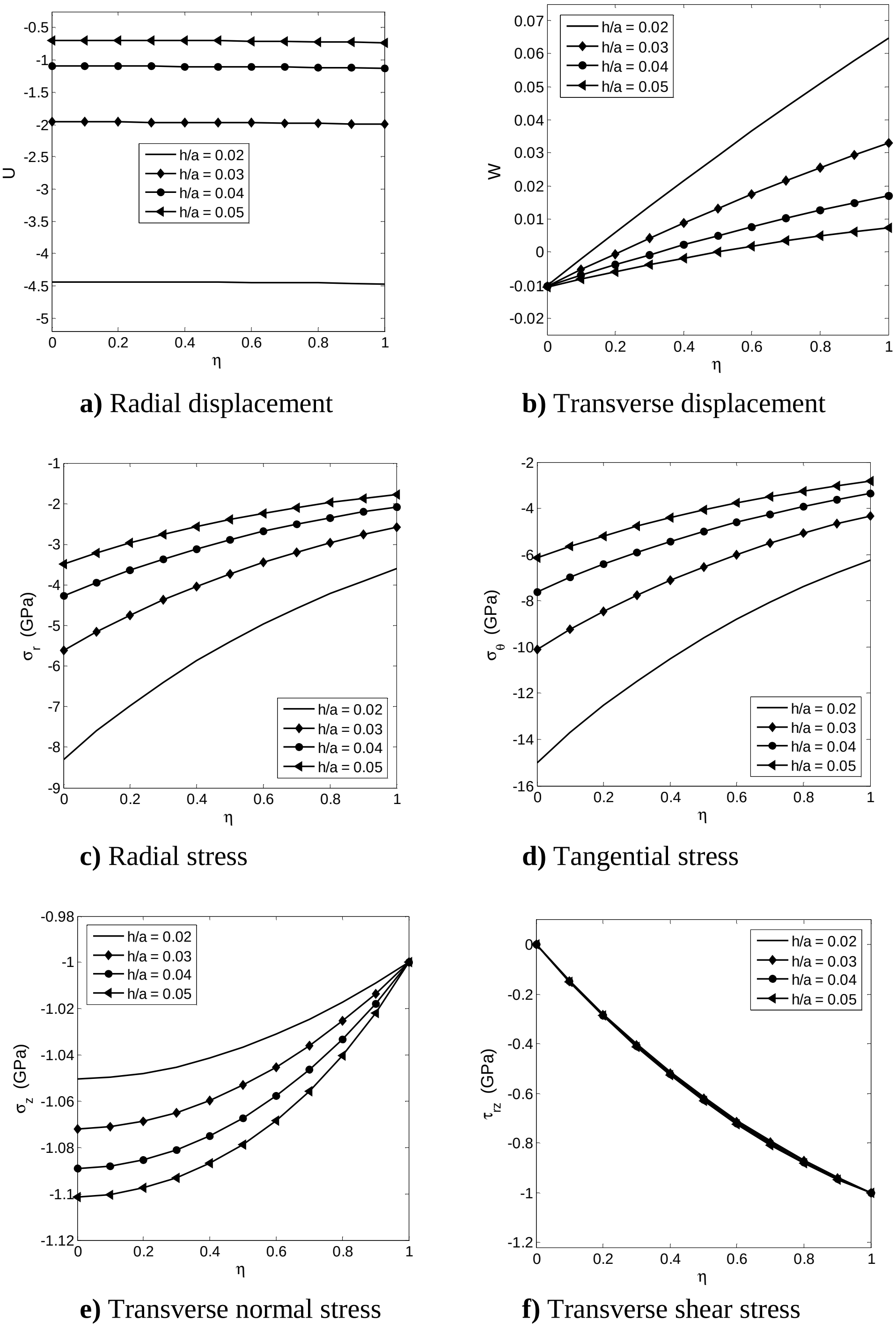


f) Transverse shear stress

Fig.3 Effect of the material heterogeneity indices on physical quantities across the plate thickness

5.4 Effect of thickness to radius ratio

Fig.4 shows the effect of thickness to radius ratio on variation of mechanical quantities along the thickness direction with parameters discussed in Eqs. (25, 26), $k_p = k_w = 0.1$, $\lambda_1 = \lambda_2 = Ln(E_T / E_Z)$ at radius mid point. It is evident from Fig.4 that the displacements, in-plane stresses decrease and the transverse normal stress increase as the aspect ratio of the plate increase. The plate aspect ratio hasn't a considerable effect on transverse shear stress variation through the thicknbess direction.



a) Radial displacement

b) Transverse displacement

c) Radial stress

d) Tangential stress

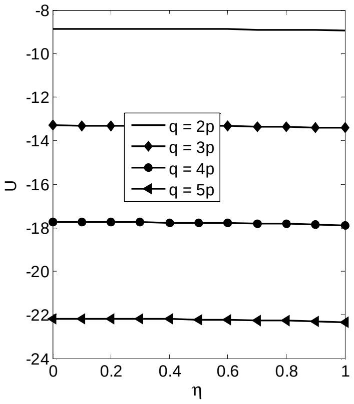
e) Transverse normal stress

f) Transverse shear stress

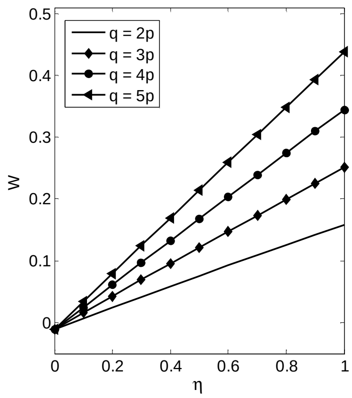
Fig.4 Effect of the geometric parameter (thickness to radius ratio) on physical quantities across the plate thickness

5.5 Effect of loads ratio

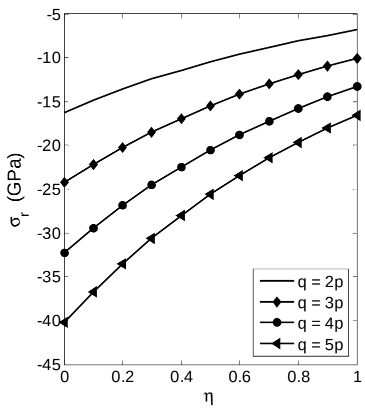
The effect of loads ratio on variation of mechanical quantities through the thickness of the plate with parameters discussed in Eqs. (25, 26) and $k_p = k_w = 0.1$, $\lambda_1 = \lambda_2 = Ln(E_T / E_Z)$ at a location $R = 0.5$ is plotted in Fig. 5. It is seen from Fig. 5 that all displacements and stresses increase as loads ratio increase. The radial displacement is affected by additional compression due to shear intraction therewith the in-plain stresses increases.



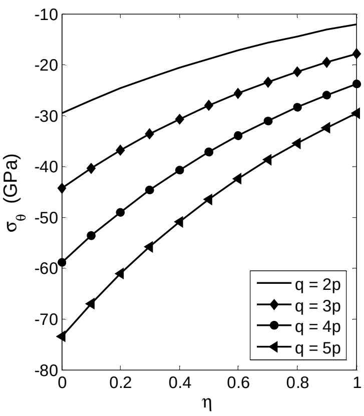
a) Radial displacement



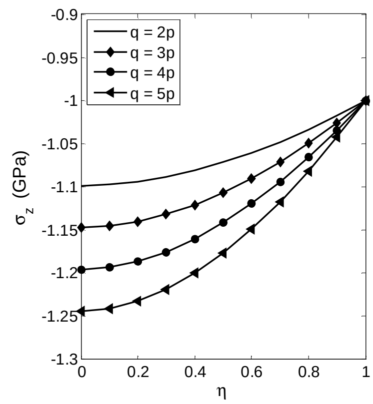
b) Transverse displacement



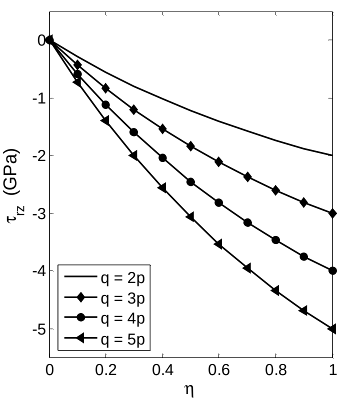
c) Radial stress



d) Tangential stress



e) Transverse normal stress



f) Transverse shear stress

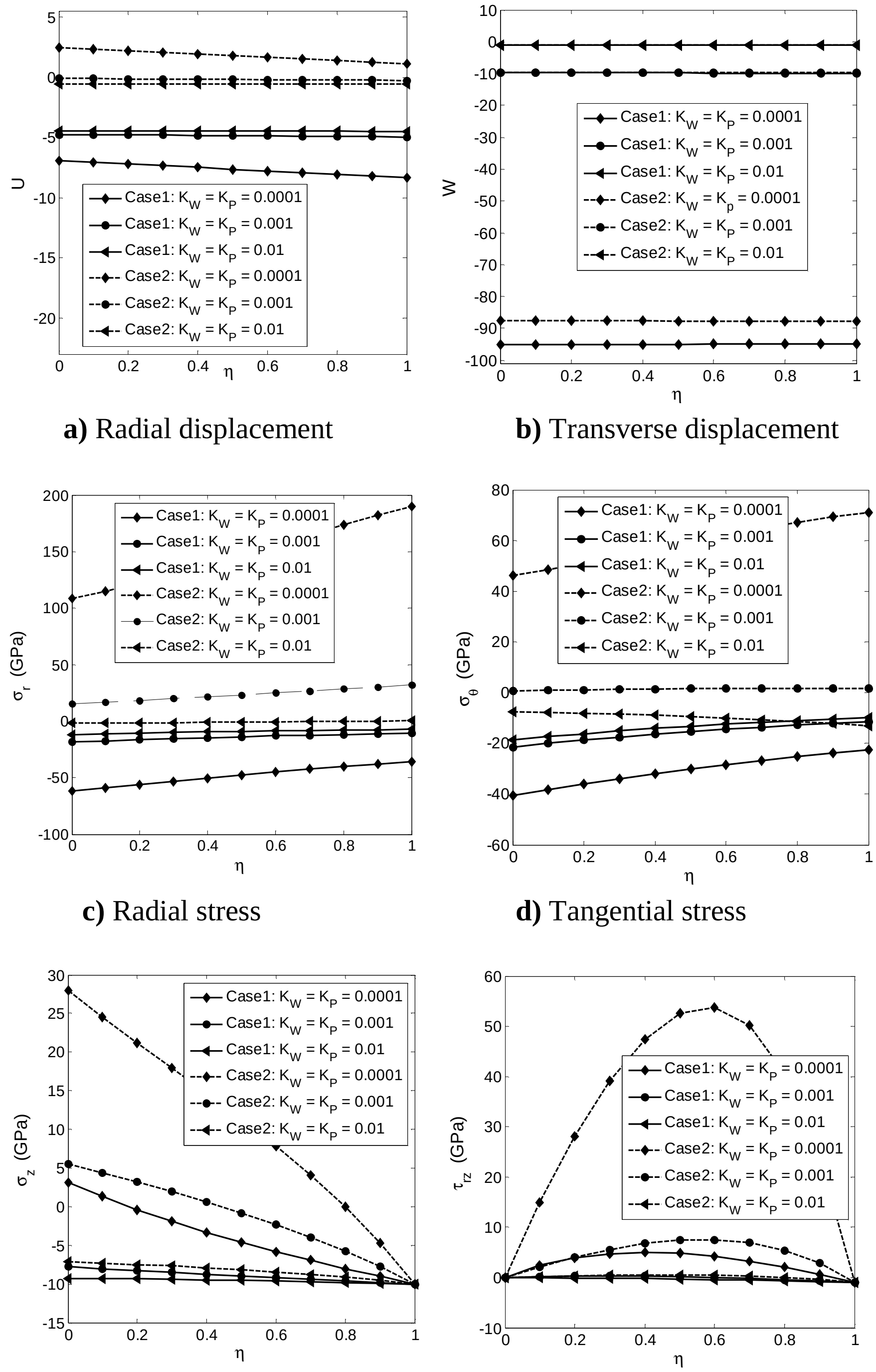
Fig.5 Effect of the loads ratio on physical quantities across the plate thickness

5.6 Effect of two parameter foundation

In order to illustrate the effect of two parameter foundation coefficients and soft/ hard surface of the plate supported by elastic foundation the following cases are considered in this study

Case 1: the hard (ceramic rich) surface resting on elastic foundation with gradient parameters $\lambda_1=\lambda_2=\mathrm{Ln}(E_T/E_Z)$

Case 2: the soft (metal rich) surface supported by elastic foundation with gradient indices $\lambda_1=\lambda_2=\mathrm{Ln}(E_Z/E_T)$



a) Radial displacement **b)** Transverse displacement

c) Radial stress **d)** Tangential stress

e) Transverse normal stress **f)** Transverse shear stress

Fig.6 Effect of the foundation stiffnesses coefficients on physical quantities across the plate thickness

Fig. 6 illustrates the displacements and stresses due to compound loading for different values of foundation coefficients by considering the parameters of Eq. (25, 26) and $P=10$, $q=1$Gpa. It can be seen from Fig.6 that in both cases the displacements and stresses dcreases with increasing foundation stiffnesses. The pick of transverse shear stress decreases as k_p, k_w increase. The variations of mechanical quantities for a known case differ significantly from the other case.

6. Conclusions

Based on the results and discussion presented in this paper, the following main conclusions may be drawn.

- The static behavior of the plate with the softer (metal rich) surface supported by elastic foundation differ significantly from that of the plate with the harder (ceramic rich) surface subjected to the same foundation.
- The rigidity of the plate increases with the increasing of elastic foundation stiffnesses
- The surface buckling increase as shear interaction increase

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