A MATHEMATICAL THEOREM ON THE ONSET OF INSTABILITIES IN THE FLOW OF COUPLE-STRESS FLUID HEATED AND SOLUTED FROM BELOW SATURATING A POROUS MEDIUM

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Abstract

In this paper, the effect of suspended particles on double-diffusive convection in couple-stress fluid saturating a porous medium is considered. By applying linear stability theory and normal mode analysis method, a mathematical theorem is derived which states that the onset of instability at marginal state, cannot manifest as stationary convection if the thermal Rayleigh number R, the medium permeability parameter P_l, *the couple-strtess parameter F, the stable solute gradient S and suspended particles parameter B, satisfy the inequality*

$$R \le \frac{4\pi^4 F}{BP_l} + \frac{SB'}{B}$$

.

This result clearly verifies the stabilizing character of couple-stress parameter and stable solute gradient while destabilizing character of suspended particles and medium permeability.

Keywords: Couple-Stress fluid, Porous medium, Suspended particles, Double-diffusive convection.

1. Introduction

The problem of double-diffusive convection in porous media has attracted considerable interest during the last few decades, because it has various applications in geophysics, food processing, soil sciences, ground water hydrology and nuclear reactors etc. The thermal instability of a Newtonian fluid, under various assumptions of hydrodynamics and hydromagnetics has been discussed in detail by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium. Wooding [3] has discussed the Rayleigh instability of a thermal boundary layer saturating a porous medium whereas Scanlon and Segel [4] have considered the effect of suspended particles on the onset of Bénard convection and found that suspended particles destabilize the layer.
In all the above studies, the fluid is considered to be Newtonian. Although the problem of double-diffusive convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to this problem with non-Newtonian fluids. Non-Newtonian fluids with suspended particles find many applications in modern technology and industries. One such type of non-Newtonian fluid is couple-stress fluid. Stokes [5] proposed the theory of couple-stress fluid. According to the theory of Stokes [5], couple-stresses are found to appear in fluids of very large molecules. The long chain hylauronic acid molecules are found as additives in synovial fluid. Walicki and Walicka [6] modeled synovial fluid as

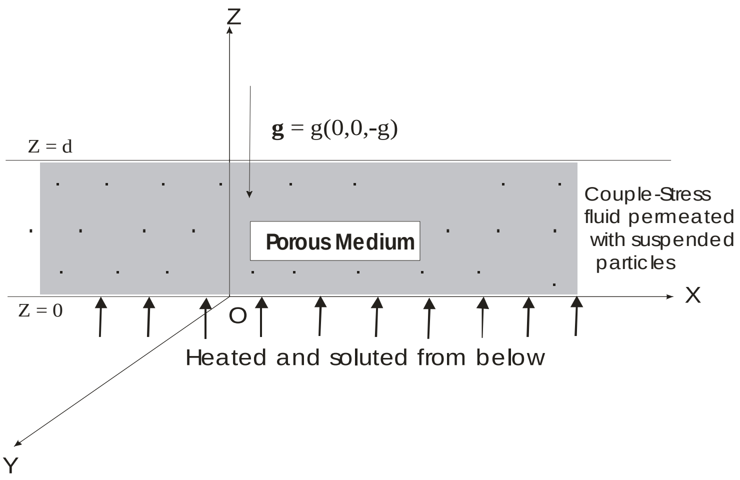
couple-stress fluid in human joints. Sharma and Sharma [7] have studied the couple-stress fluid heated from below in porous medium.

One of the applications of couple-stress fluid is its use in the study of the mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing which has articular cartilage as the bearing and synovial fluid as lubricant. When fluid film is generated, squeeze film action is capable of providing considerable protection to the cartilage surface. The shoulder, knee, hip and ankle joints are the loaded-bearing synovial joints of human body and these joints have low-friction coefficient and negligible wear. Normal synovial fluid is clear or yellowish and is a viscous, non- Newtonian fluid.

The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [8], Ingham and Pop [9], Nield and Bejan [10], Sharma and Rana [11, 12] and Rana and Kumar [13]. Recently, Kumar [14] studied the hydromagnetic stability of stratified couple-stress fluid in the presence of suspended particles through porous medium whereas Rana and Sharma [15] studied thermosolutal instability of compressible Walters' (model B') rotating fluid in the presence of suspended particles and magnetic field in porous medium and found that suspended particles and medium permeability have destabilizing effects on the system.
More recently, Rana and Thakur [16] derived a mathematical theorem on the onset of couple-stress fluid permeated with suspended particles saturating a porous medium while Pap and Vivona [17] discussed the applications of pseudo analysis in the theory of fluid mechanics. In the present paper, a mathematical theorem is derived on the onset of instabilities in the flow of incompressible couple-stress elastico-viscous fluid heated and soluted from below saturating a porous medium.

2. **Mathematical Model and Perturbation Equations**

Consider an infinite, horizontal, incompressible couple-stress viscoelastic fluid layer of depth d, bounded by the planes z = 0 and z = d in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity **g**(0, 0, -g) as shown below in the schematic sketch of physical situation. This layer is heated and soluted from below such that a steady adverse temperature gradient $\beta = \left(\left| \frac{dT}{dz} \right| \right)$ and a uniform solute gradient $\beta' = \left(\left| \frac{dC}{dz} \right| \right)$ are maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.



Schematic Sketch of Physical Situation
Fig.1.

The equations expressing the conservation of momentum, mass, temperature, concentration and equation of state for couple-stress fluid in a porous medium (Chandrasekhar [1], Sharma and Sharma [11], Kumar [14] and Rana and Sharma [15]) are

$$\frac{1}{\varepsilon}\left[\frac{\partial \mathbf{v}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{v}.\nabla)\mathbf{v}\right]=-\frac{1}{\rho_0}\nabla p+\mathbf{g}\left(1+\frac{\delta\rho}{\rho_0}\right)-\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)\mathbf{v}+\frac{K'N}{\rho_0\varepsilon(\mathbf{v}_d-\mathbf{v})}, \tag{1}$$

$$\nabla.\mathbf{v}=0, \tag{2}$$

$$\rho C_f\left(\frac{\partial}{\partial t}+\mathbf{v}.\nabla\right)T+mNC_{pt}\left(\varepsilon\frac{\partial}{\partial t}+\mathbf{v}_d.\nabla\right)T=k_T\nabla^2 T, \tag{3}$$

$$\rho C'_f\left(\frac{\partial}{\partial t}+\mathbf{v}.\nabla\right)C+mNC'_{pt}\left(\varepsilon\frac{\partial}{\partial t}+\mathbf{v}_d.\nabla\right)C=k_T\nabla^2 C, \tag{4}$$

$$\rho=\rho_0\left[1-\alpha(T-T_0)+\alpha'(C-C_0)\right], \tag{5}$$

where ρ, ν, μ_c, p, ε, T, α, v(0, 0, 0), denote respectively, the density, kinematic viscosity, couple-stress viscosity, pressure, medium porosity, temperature, thermal coefficient of expansion, velocity of the fluid, $v_d(\bar{x},t)$, and $N(\bar{x},t)$, denote the velocity and number density of the particles respectively, $K' = 6\pi\rho\nu\eta$, where η is particle radius, is the Stokes drag coefficient, $v_d(q,r,s)$, and $\bar{x}=(x,y,z), C_f, C_{pt}, k_T$ denote, respectively, the heat capacity of the pure fluid, heat capacity of particles, 'effective thermal conductivity' of pure fluid and C'_f, C'_{pt} denote heat capacities analogous to solute. The suffix zero refers to values at the reference level z = 0.

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid as appeared in the equation of momentum (1). The buoyancy force on the particles is neglected as the particles are very small in size. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. Thus, if mN is the mass of particles per unit volume, then the equations of momentum and mass for the particles are

$$mN\left[\frac{\partial \mathbf{v}_d}{\partial t}+\frac{1}{\varepsilon}(\mathbf{v}_d.\nabla)\mathbf{v}_d\right]=K^{'}N(\mathbf{v}-\mathbf{v}_d), \tag{6}$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla.(N\mathbf{v}_d)=0, \tag{7}$$

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of momentum for the particles (6).

The initial state of the system is taken to be quiescent layer (no settling) with a uniform particle distribution number. The initial state is

$$\mathbf{v}=(0,0,0), \mathbf{v}_d=(0,0,0),\ T=-\beta z+T_0, \rho=\rho_0(1+\alpha\beta z). \tag{8}$$

is an exact solution to the governing equations. Let **v**(u,v,w), θ, δp and $\delta\rho$ denote, respectively, the perturbations in fluid velocity ***v***(0,0,0), temperature T, pressure p and density ρ .The change in density $\delta\rho$ caused by perturbation θ in temperature is given by

$$\delta\rho=-\rho_0(\alpha\theta-\alpha^{'}\gamma).. \tag{9}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\frac{\partial \mathbf{v}}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p-\mathbf{g}\frac{\delta\rho}{\rho_0}-\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)\mathbf{v}+\frac{K^{'}N}{\rho_0\varepsilon(\mathbf{v}_d-\mathbf{v})}, \tag{10}$$

$$\nabla.\mathbf{v}=0, \tag{11}$$

$$(1+b\varepsilon)\frac{\partial\theta}{\partial t}=\beta(w+bs)+\kappa\nabla^2\theta, \tag{12}$$

$$(1+b^{'}\varepsilon)\frac{\partial\gamma}{\partial t}=\beta(w+bs)+\kappa^{'}\nabla^2\gamma, \tag{13}$$

where $b=\frac{mNC_{pt}}{\rho_0C_f}$, $b^{'}=\frac{mNC^{'}_{pt}}{\rho_0C^{'}_f}$ and w, s are the vertical fluid and particles velocity, $\kappa=\frac{k_T}{\rho_0C_f}$, is the thermal diffusivity and $\kappa^{'}=\frac{k_T}{\rho_0C^{'}_f}$, is the solute diffusivity.

In the Cartesian form, equations (10)-(13) with the help of equation (9) can be expressed as

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t}=-\frac{1}{\rho_0}\frac{\partial}{\partial x}(\delta p)-\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)u+\frac{mN}{\rho_0\varepsilon\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)}\frac{\partial u}{\partial t}, \tag{14}$$

$$\frac{1}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial y}(\delta p) - \frac{1}{k_1}\left(\nu - \frac{\mu_c}{\rho_0}\nabla^2\right)v + \frac{mN}{\rho_0\varepsilon\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\frac{\partial v}{\partial t}, \tag{15}$$

$$\frac{1}{\varepsilon}\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}(\delta p) + g(\alpha\theta - \alpha'\gamma) - \frac{1}{k_1}\left(\nu - \frac{\mu_c}{\rho_0}\nabla^2\right)w + \frac{mN}{\rho_0\varepsilon\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\frac{\partial w}{\partial t}, \tag{16}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \ , \tag{17}$$

$$(1+b\varepsilon)\frac{\partial \theta}{\partial t} = \beta(w+bs) + \kappa\nabla^2\theta \, , \tag{18}$$

$$(1+b'\varepsilon)\frac{\partial \gamma}{\partial t} = \beta(w+bs) + \kappa'\nabla^2\gamma \, , \tag{19}$$

Operating equation (14) and (15) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding and using equation (17), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) = -\frac{1}{\rho_0}\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\delta p - \frac{1}{k_1}\left(\nu - \frac{\mu_c}{\rho_0}\nabla^2\right)\left(\frac{\partial w}{\partial z}\right) + \frac{mN}{\rho_0\varepsilon\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right), \tag{20}$$

Operating equation (16) and (20) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding to eliminate δp between equations (16) and (20), we get

$$\frac{1}{\varepsilon}\frac{\partial}{\partial t}(\nabla^2 w) = -\frac{1}{k_1}\left(\nu - \frac{\mu_c}{\rho_0}\nabla^2\right)\nabla^2 w + g\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\alpha\theta - \alpha'\gamma) - \frac{mN}{\rho_0\varepsilon\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)}\frac{\partial}{\partial t}(\nabla^2 w), \tag{21}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}$

3. **Normal Mode Analysis**

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w, \theta, \gamma] = [W(z), \Theta(z), \Gamma(z)]\exp(ilx + imy + nt), \tag{22}$$

where l and m are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.

Using expression (22) in equations (21), (18) and (19) become

$$\frac{n}{\varepsilon}\left(\frac{d^2}{dz^2}-k^2\right)W=-\frac{1}{k_1}\left(\nu-\frac{\mu_c}{\rho_0}\nabla^2\right)\left(\frac{d^2}{dz^2}-k^2\right)W-gk^2\left(\alpha\Theta-\alpha^{'}\Gamma\right)-\frac{mN}{\rho_0\varepsilon\left(\frac{m}{K^{'}}\frac{\partial}{\partial t}+1\right)}\left(\frac{d^2}{dz^2}-k^2\right)W, \qquad (23)$$

$$\left(1+b\varepsilon\right)\frac{\partial\Theta}{\partial t}=\beta\left(w+bs\right)+\kappa\left(\frac{d^2}{dz^2}-k^2\right)\Theta, \qquad (24)$$

$$\left(1+b^{'}\varepsilon\right)\frac{\partial\Gamma}{\partial t}=\beta\left(w+bs\right)+\kappa^{'}\left(\frac{d^2}{dz^2}-k^2\right)\Gamma, \qquad (25)$$

Equation (23) and (25) in non-dimensional form, become

$$\left[\left(1+\frac{M}{1+\tau_1\sigma}\right)\frac{\sigma}{\varepsilon}+\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W=-\frac{g\alpha a^2 d^2\Theta}{\nu}\backslash+\frac{g\alpha^{'}a^2 d^2\Gamma}{\nu}, \qquad (26)$$

$$\left(D^2-a^2-EP_r\sigma\right)\Theta=-\frac{\beta d^2}{\kappa}\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \qquad (27)$$

$$\left(D^2-a^2-E^{'}P_r^{'}\sigma\right)\Gamma=-\frac{\beta d^2}{\kappa^{'}}\left(\frac{B^{'}+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \qquad (28)$$

where we have put

$\tau=\frac{m}{K^{'}},\tau_1=\frac{\tau\nu}{d^2},E=1+b\varepsilon,B=1+b,B^{'}=1+b^{'},E^{'}=1+b^{'}\varepsilon\ a=kd,\sigma=\frac{nd^2}{\upsilon},,P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $P_r=\frac{\nu}{\kappa}$, is the thermal Prandtl number, $P^{'}{}_r=\frac{\nu}{\kappa^{'}}$, is the Schmidt number, $F=\frac{\mu_c}{\mu d^2}$, is the couple-stress parameter and $D^*=d\frac{d}{dz}$ and the superscript * is suppressed for convenience.

Substituting $W=W^*,\Theta=\frac{\beta d^2}{\kappa}\Theta^*$ and $\Gamma=\frac{\beta^{'}d^2}{\kappa^{'}}\Gamma^*$ in equations (26), (27) and (28) respectively and dropping * for convenience, we obtain

$$\left[\left(1+\frac{M}{1+\tau_1\sigma}\right)\frac{\sigma}{\varepsilon}+\frac{1-F\left(D^2-a^2\right)}{P_l}\right]\left(D^2-a^2\right)W=-Ra^2\Theta+Sa^2\Gamma, \qquad (29)$$

$$\left(D^2-a^2-EP_r\sigma\right)\Theta=-\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \qquad (30)$$

$$\left(D^2-a^2-E^{'}P_r^{'}\sigma\right)\Gamma=-\left(\frac{B^{'}+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \qquad (31)$$

where $R=\frac{g\alpha\beta d^4}{\nu\kappa}$, is the thermal Rayleigh number

and $S=\frac{g\alpha^{'}\beta^{'}d^4}{\nu\kappa^{'}}$, is the analogous solute Rayleigh number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are (Chandrasekhar [1])

$$W = D^2 W = \Gamma = \Theta = 0 \text{ at } z = 0 \text{ and } 1. \quad (32)$$

Then, we prove the following theorem:

Theorem: If R > 0, F > 0, B = 1+b, b > 0, $B' = 1 + b', b > 0$ and $\sigma = 0$, then the necessary condition for the existence of non-trivial solution (W, Θ, Γ) of equations (29)-(31) together with the boundary conditions (32) is that

$$R > \frac{F}{BP_l} \frac{\pi^2 (\pi^2 + a^2)^2}{a^2} + \frac{SB'}{B}.$$

Proof: If the instability sets in stationary convection and 'principle of exchange of stability' is valid, the neutral or marginal state will be characterized by $\sigma = 0$. Thus the relevant governing equations (29)-(31) are reduced to

$$\left[\frac{1 - F(D^2 - a^2)}{P_l}\right](D^2 - a^2)W = -Ra^2\Theta + Sa^2\Gamma, \quad (33)$$

$$(D^2 - a^2)\Theta = -BW, \quad (34)$$

$$(D^2 - a^2)\Gamma = -B'W, \quad (35)$$

together with the boundary conditions (26).

Multiplying equation (33) by W^* (the complex conjugate of W) throughout and integrating the resulting equation over the vertical range of z, we get

$$\frac{1}{P_l}\int_0^1 W^*(D^2 - a^2)W dz - \frac{F}{P_l}\int_0^1 W^*(D^2 - a^2)^2 W dz = -Ra^2\int_0^1 W^*\Theta dz + Sa^2\int_0^1 W^*\Gamma dz. \quad (36)$$

Taking complex conjugate on both sides of equations (34) and (35), we get

$$(D^2 - a^2)\Theta^* = -BW^*, \quad (37)$$

$$(D^2 - a^2)\Gamma^* = -B'W^*, \quad (38)$$

Using equations (37) and (38) in the right hand side of equation (36), we obtain

$$\frac{1}{P_l}\int_0^1 W^*(D^2 - a^2)W dz - \frac{F}{P_l}\int_0^1 W^*(D^2 - a^2)^2 W dz = \frac{Ra^2}{B}\int_0^1 \Theta^*(D^2 - a^2)\Theta dz - \frac{Sa^2}{B'}\int_0^1 \Gamma^*(D^2 - a^2)\Gamma dz. \quad (39)$$

Integrating term by term on both sides of equation (39) for an appropriate number of times by making use of boundary conditions (32), we obtain

$$\frac{1}{P_l}\int_0^1 \left(|DW|^2 + a^2|W|^2\right)dz - \frac{F}{P_l}\int_0^1 \left(|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right)dz = \frac{Ra^2}{B}\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz - \frac{Sa^2}{B'}\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz. \quad (40)$$

Since W, Θ and Γ satisfy $W(0) = 0 = W(1), \Theta(0) = 0 = \Theta(1)$, and $\Gamma(0) = 0 = \Gamma(1)$, we have by Rayleigh-Ritz inequality

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz, \tag{41}$$

$$\int_0^1 |D\Theta|^2 dz \geq \pi^2 \int_0^1 |\Theta|^2 dz, \tag{42}$$

$$\int_0^1 |D\Gamma|^2 dz \geq \pi^2 \int_0^1 |\Gamma|^2 dz, \tag{43}$$

and

$$\int_0^1 \left|D^2 W\right|^2 dz \geq \pi^4 \int_0^1 |W|^2 dz. \tag{44}$$

Further, multiplying equation (34) by Θ^* (the complex conjugate of Θ), integrating by parts each term of resulting equation on the right hand side for an appropriate boundary condition, namely $\Theta(0) = 0 = \Theta(1)$, it follows that

$$\begin{aligned}
\frac{1}{B}\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz &= \text{Real part of } \left(\int_0^1 \Theta^* W dz\right) \\
&\leq \left|\int_0^1 \Theta^* W dz\right|, \\
&\leq \int_0^1 \left|\Theta^* W\right| dz, \\
&\leq \int_0^1 \left|\Theta^*\right||W| dz, \\
&\leq \int_0^1 |\Theta||W| dz, \\
&\leq \left(\int_0^1 |\Theta|^2 dz\right)^{1/2} \left(\int_0^1 |W|^2 dz\right).
\end{aligned} \tag{45}$$

(by using Cauchy-Schwartz inequality)

Thus, inequalities (42) can be written as

$$\frac{\pi^2 + a^2}{B}\left(\int_0^1 |\Theta|^2 dz\right)^{1/2} \leq \left(\int_0^1 |W|^2 dz\right)^{1/2}. \tag{46}$$

Combining inequalities (45) and (46), we obtain

$$\frac{1}{B}\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz \leq \frac{B}{\pi^2 + a^2}\int_0^1 |W|^2 dz. \tag{47}$$

Using inequality (42) in (47), we get

$$\frac{1}{B}\int_0^1 \left(|D\Theta|^2 + a^2|\Theta|^2\right)dz \leq \frac{B}{\pi^2\left(\pi^2 + a^2\right)}\int_0^1 |DW|^2 dz. \tag{48}$$

Similarly, we can write

$$\frac{1}{B'}\int_0^1 \left(|D\Gamma|^2 + a^2|\Gamma|^2\right)dz \leq \frac{B'}{\pi^2\left(\pi^2 + a^2\right)}\int_0^1 |DW|^2 dz. \tag{49}$$

Thus, if R > 0, F > 0, B = 1+b, b. 0, using the inequalities (44), (48) and (49), the equation (40) becomes

$$\left(1+a^2\right)I+\left[\frac{F}{P_l}\left(\pi^2+a^2\right)-\frac{RBa^2}{\pi^2\left(\pi^2+a^2\right)}+\frac{SB^{'}a^2}{\pi^2\left(\pi^2+a^2\right)}\right]\int_0^1\left|D^2W\right|^2dz<0\ , \qquad (50)$$

where $I=\int_0^1\left(\left|DW\right|^2+a^2\left|W\right|^2\right)dz$, is positive definite.

Therefore, we must have

$$\left[\frac{F}{P_l}\left(\pi^2+a^2\right)\right]<\left[\frac{RBa^2}{\pi^2\left(\pi^2+a^2\right)}+\frac{SB^{'}a^2}{\pi^2\left(\pi^2+a^2\right)}\right],$$

which implies that

$$R>\frac{F}{BP_l}\frac{\pi^2\left(\pi^2+a^2\right)^2}{a^2}+\frac{SB^{'}}{B}. \qquad (51)$$

Since minimum value of $\frac{\pi^2\left(\pi^2+a^2\right)}{a^2}$ is $4\pi^4$ at $a^2=\pi^2>0$, hence, we necessarily have

$$R>\frac{4\pi^4F}{BP_l}+\frac{SB^{'}}{B}, \qquad (52)$$

which completes the proof of the theorem.
From physical point of view, the above theorem states that the onset of instability at marginal state in a couple-stress fluid heated and soluted from below permeated with suspended particles in porous medium cannot manifest as stationary convection, if the thermal Rayleigh number *R*, the couple-stress parameter *F*, stable solute gradient *S*, medium permeability P_l and suspended particles number density *B*, satisfy the inequality

$$R\leq\frac{4\pi^4F}{BP_l}+\frac{SB^{'}}{B}. \qquad (53)$$

4. Conclusion

The effect of suspended particles on double-diffusive convection in couple-stress fluid in a porous medium has been investigated. From the above theorem, the main conclusions are as follows:

(i) The necessary condition for the onset of instability as stationary convection for couple-stress elastico-viscous fluid is

$$R>\frac{4\pi^4F}{BP_l}+\frac{SB^{'}}{B}.$$

(ii) The sufficient condition for non-existence of stationary convection at marginal state is

$$R \le \frac{4\pi^4 F}{BP_l} + \frac{SB'}{B}.$$

(iii)In the inequality (52), the thermal Rayleigh number R > 0 , is directly proportional to the couple-stress parameter F and stable solute gradient S. Thus, couple-stress parameter and stable solute gradient have stabilizing effects on the system as derived by Sharma and Sharma [7], Kumar [14] and Rana and Sharma [15].

(iv)In the inequality (52), the thermal Rayleigh number R > 0 , is inversely proportional to the suspended particles number density parameter B, which mathematically established the destabilizing effect of suspended particles number density parameter on the system as derived by Scanlon and Segel [4], Rana and Kumar [12], Rana and Sharma [15] and Kumar [14].

(v) The medium permeability has a destabilizing effect on the system as can be seen from inequality (52), which is an agreement with the earlier work of Sharma and Sharma [7], Rana and Kumar [13], Kumar [14] and Rana and Sharma [15].

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