International Journal of Engineering & Applied Sciences (IJEAS)
Vol.5, Issue 3(2013)26-37

PERTURBATION ANALYSIS OF FREE CONVECTIVE MHD FLOW OF A MICROPOLAR FLUID WITH OHMIC HEATING AND VISCOUS DISSIPATION OVER A CHEMICALLY REACTING PLATE SUBJECTED TO A CONSTANT HEAT FLUX AND CONCENTRATION GRADIENT

Hitesh Kumar

Department of Information Technology, IBRI College of Technology, Ibri, P.O. BOX 466, Oman
*E-mail address: hiteshrsharma@gmail.com
Received: September 2013, Accepted: November 2013

Abstract

In this paper the study of a chemically reacting free convection MHD micropolar flow, heat and mass transfer in with the effects of ohmic heating and viscous dissipation past an infinite vertical plate which is subjected to a constant heat flux and a concentration gradient. The non-linear coupled partial differential equations are solved by using multi parameter perturbation technique. The results for transverse velocity, angular velocity and temperature are obtained and illustrated graphically to observe the effects of various parameters on these functions. The numerical discussion with physical analysis of the influence of various parameters also presented.

Keywords: Heat transfer, mass transfer, heat generation, concentration, inclined wall

1. Introduction

We know that fluids in geothermal region are electrically conducting. Flows arising from temperature difference have great significance for the applications to the geophysics and engineering. There are many interesting aspects of such flows, so analytical solutions of such problem are presented by many authors. Gebhart and Pera [1], Sparrow et al. [2], Soundalgekar [3], Acharya et al. [4], Singh and Chand [5] are some of them. Investigations of the flow streaming into a porous and permeable medium, assuming velocity of the flow not small (Reynolds number is moderately high) were obtained by Yamamoto and Iwamura [6], Yamamoto and Yoshida [7], Brinkman [8], Raptis et al. [9, 10]. All above authors used generalized Darcy's law, and the generalized Darcy's law is derived without taking into account the angular velocity of the fluid particles. Raptis [11] in his research paper on a horizontal plate used flow equations with angular velocity. Such fluids are known as polar fluids in the literature. Raptis [12] in another research paper discussed magnetopolar fluid through a porous medium.

Analysis of the transport processes and their interaction with chemical reactions can be quite difficult and is intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes the core of chemical reaction engineering. Combined heat and mass transfer problems with chemical reaction are of importance in many processes. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agricultural drying, and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of printed circuitry, manufacture of pulp-insulated cables, etc. Diffusion rates can be altered tremendously by chemical reactions. The Effect of a chemical reaction depends whether the reaction is homogeneous or heterogeneous. This depends on whether they occur in an interface or as a single phase volume reaction. In a well-mixed system, the reaction is heterogeneous if the reactants are in multiple phase, and homogeneous if the reactants are in the same phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. For example, the formation of smog is a first-order homogeneous chemical reaction. Consider the emission of NO_2 from automobiles and other smoke stacks. This NO_2 reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxyacetylnitrate, which forms an envelope termed as the photochemical smog. Kandasamy and Devi [13] studied the effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection. Also, the studies of heat generation or absorption in moving fluids for problems involving chemical reactions and those concerned with dissociating fluids are equally important. Specifically, the effects of heat generation may alter the temperature distribution, consequently affecting the particle deposition rate in nuclear reactors, electronic chips, and semiconductor wafers. In fact, the literature is replete with examples of heat transfer in the laminar flow of viscous fluids. The problem of heat transfer in MHD boundary-layer flow and heat annihilation over a stretching sheet is considered by Kumar [14].

Recently, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer in for example metallurgical processing. Melt refining involves magnetic field application to control excessive heat transfer rates. Both laminar and turbulent flows are of interest. Many studies in MHD thermo-convection flows have been conducted. Asghar et. al. [15] investigated the MHD flow due to non-coaxial rotations of a porous disk, moving with uniform acceleration in its own plane and a second grade fluid at infinity. Chen [16] studied the problem of combined heat and mass transfer of an electrically conducting fluid in MHD natural convection, adjacent to a vertical surface with Ohmic heating.

In the present paper a study of steady free convection flow of a laminar, incompressible MHD micropolar fluid and thermal and mass diffusion in porous medium is carried out. The object of the paper is to analyze the effects of magnetic field, heat source and dissipation on the velocity and thermal transport in the boundary layer, when the wall is at prescribed heat flux.

2. Mathematical Analysis

Here in the paper it is considered to be a free convection flow of an incompressible and electrically conducting viscous thermo-micropolar fluid past an infinite vertical plate is considered. The vertical plate is assumed to be at a constant heat flux and a constant concentration gradient. A magnetic field (B_0) of uniform strength is applied transversely to the direction of the flow that is *y*-axis and the induced magnetic field is neglected. Taking the *x*-axis along the vertical porous plate in upward direction and *y* axis normal to it. Since the length of the plate is large and fluid flow extends to infinity, therefore all physical variables are independent of *x* and hence the functions of *y* only. The governing equations of continuity, momentum, concentration, angular velocity and energy for the flow in the presence of ohmic heating, heat generation, chemical reaction and viscous dissipation are:

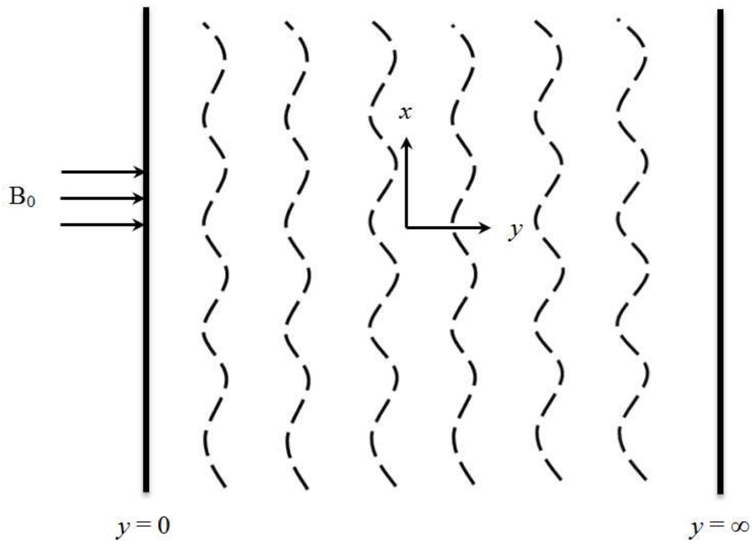

Fig.1:- Physical Model and Coordinate System

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$v^* = -V_0 \quad \text{(Constant)} \tag{2}$$

$$\frac{dp}{\partial y^*} = 0 \Rightarrow p \text{ is independent of } y^* \tag{3}$$

$$\rho v^* \frac{\partial u^*}{\partial y^*} = (\kappa + \mu)\frac{\partial^2 u^*}{\partial y^{*2}} + \rho g \beta_T (T - T_\infty) + \rho g \beta_c (c - c_\infty) + \kappa \frac{\partial \omega_a}{\partial y^*} - \sigma B_0^2 u^* \tag{4}$$

$$\rho j \left(v \frac{\partial \omega_a}{\partial y^*} \right) = \gamma \frac{\partial^2 \omega_a}{\partial y^{*2}} - 2\kappa \omega_a \tag{5}$$

$$\rho c_p v^* \frac{\partial T}{\partial y^*} = k \frac{\partial^2 T}{\partial y^{*2}} + \mu \left(\frac{\partial u^*}{\partial y^*} \right)^2 + Q(T - T_\infty) + \sigma B_0^2 u^{*2} \tag{6}$$

$$v^* \frac{\partial c}{\partial y^*} = D \frac{\partial^2 c}{\partial y^{*2}} - k_l (c - c_\infty) \tag{7}$$

with the boundary conditions:

$$u^* = 0,\ \omega_a = -\frac{1}{2}\frac{\partial u^*}{\partial y^*},\ \ \frac{\partial T}{\partial y} = -\frac{q}{k},\ -D\frac{\partial c}{\partial y^*} = m_w,\ at\quad y = 0$$
$$u^* \to 0,\ \ \omega_a \to 0,\ \ T \to T_\infty,\ \ c \to c_\infty,\ \ as\quad y \to \infty \tag{8}$$

here, $V_0 > 0$, $\gamma = \left(\mu + \frac{\kappa}{2}\right)j = \mu\left(1 + \frac{a}{2}\right)j$ and $j = \frac{\vartheta^2}{V_0^2}$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced as

$$y = \frac{V_0\, y^*}{\vartheta},\ u = \frac{u^*}{V_0},\ M = \frac{\sigma B_0^2 \vartheta}{\rho V_0^2},\ \Pr = \frac{\mu c_p}{k},\ \theta = \frac{T - T_\infty}{q\vartheta / k\, V_0},\ C = \frac{c - c_\infty}{m_w \vartheta / V_0\, D},\ Ec = \frac{k V_0^3}{q \vartheta c_p},$$
$$G_r = \frac{g\beta_T\, q\vartheta^2}{k V_0^4},\ G_c = \frac{g\beta_c\, m_w \vartheta^2}{V_0^4 D},\ S = \frac{Q\vartheta}{\rho c_p V_0^2},\ \omega = \frac{\vartheta\omega_a}{V_0^2},\ Sc = \frac{\vartheta}{D},\ K_c = \frac{\vartheta K_l}{v_w^2},\ a = \frac{\kappa}{\mu}\ .$$

Equations (4)-(7) change to

$$(1+a)\frac{d^2u}{dy^2} + \frac{du}{dy} - Mu + a\frac{d\omega}{dy} + G_r\,\theta + G_c\,C = 0 \tag{9}$$

$$\left(1 + \frac{a}{2}\right)\frac{d^2\omega}{dy^2} + \frac{d\omega}{dy} - 2a\omega = 0 \tag{10}$$

$$\frac{d^2\theta}{dy^2} + \Pr\frac{d\theta}{dy} + S\Pr\theta + \Pr Ec\left(\frac{du}{dy}\right)^2 + \Pr Ec\, M\, u^2 = 0 \tag{11}$$

$$\frac{d^2C}{dy^2} + Sc\frac{dC}{dy} - K_c\ Sc\ C = 0 \tag{12}$$

boundary conditions change to:

$$at\quad y = 0,\ \ u = 0,\ \ \frac{d\theta}{dy} = -1,\ \ \omega = -\frac{1}{2}\frac{du}{dy},\ \ \frac{dC}{dy} = -1$$
$$as\quad y \to \infty,\ \ u \to 0,\ \ \theta \to 0,\ \ \omega \to 0,\ \ C \to 0 \tag{13}$$

Solution of the equation (12) is

$$C = b_1 e^{-b_2 y} \tag{14}$$

In order to solve the coupled equations (9) to (11), for boundary conditions (13) we expand u, ω and θ in powers of the Eckert number Ec which is very small (Ec<< 1) for incompressible fluids.

$$u = u_0 + Ec\ u_1 + 0\left(Ec^2\right),$$
$$\omega = \omega_0 + Ec\ \omega_1 + 0\left(Ec^2\right),$$
$$\theta = \theta_0 + Ec\ \theta_1 + 0\left(Ec^2\right). \tag{15}$$

Thus on using the above series expansions in equations (9) to (11) and equating the coefficient of like powers of *Ec* to zero, the zeroth order and first order equations are solved to give

$$\omega_0 = b_3\, e^{-b_4 y} \tag{16}$$

$$\theta_0 = b_0\, e^{-b_5 y} \tag{17}$$

$$u_0 = b_7\, e^{-b_6 y} + b_3 b_8\, e^{-b_4 y} - b_9\, e^{-b_5 y} - b_{10}\, e^{-b_2 y} \tag{18}$$

$$\theta_1 = d_2\, e^{-b_5 y} + d_3\, e^{-2 b_6 y} + d_4\, e^{-2 b_4 y} + d_5\, e^{-2 b_5 y} + d_6\, e^{-2 b_2 y} + d_7\, e^{-(b_4 + b_6) y}$$
$$+ d_8\, e^{-(b_5 + b_6) y} + d_9\, e^{-(b_2 + b_6) y} + d_{10}\, e^{-(b_4 + b_5) y} + d_{11}\, e^{-(b_2 + b_4) y} + d_{12}\, e^{-(b_2 + b_5) y} \tag{19}$$

$$\omega_1 = f_1\, e^{-b_4 y} \tag{20}$$

$$u_1 = f_2\, e^{-b_6 y} + b_8\, f_1\, e^{-b_4 y} + f_3\, e^{-b_5 y} + f_4\, e^{-2 b_6 y} + f_5\, e^{-2 b_4 y} + f_6\, e^{-2 b_5 y} + f_7\, e^{-2 b_2 y}$$
$$+ f_8\, e^{-(b_4 + b_6) y} + f_9\, e^{-(b_5 + b_6) y} + f_{10}\, e^{-(b_2 + b_6) y} + f_{11}\, e^{-(b_4 + b_5) y} + f_{12}\, e^{-(b_2 + b_4) y}$$
$$+ f_{13}\, e^{-(b_2 + b_5) y} \tag{21}$$

Skin Friction: - The skin friction at the wall y = 0 is given by

$$\tau = (1+\alpha)\left(\frac{\partial u}{\partial y}\right)_{y=0} = -(1+a)\,[b_6 b_7 + b_3 b_4 b_8 + b_5 b_9 + b_2 b_{10} + Ec\,\{ f_2 b_6 + f_1 b_4 b_8 + f_3 b_5$$
$$+ 2 f_4 b_6 + 2 f_5 b_4 + 2 f_6 b_5 + 2 f_7 b_2 + f_8 (b_4 + b_6) + f_9 (b_5 + b_6) + f_{10} (b_2 + b_6)$$
$$+ f_{11}(b_4 + b_5) + f_{12}(b_2 + b_4) + f_{13}(b_2 + b_5)\}] \tag{22}$$

Recovery Factor: - The recovery factor at the wall y = 0 is given by

$$R_f = \theta(0) = b_0 + Ec[\, d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_8 + d_9 + d_{10} + d_{11} + d_{12}\,] \tag{23}$$

where all the constants are given in the Appendix.

3. Result and Discussion

The main findings of this paper is to study the effects magnetic field, viscous dissipation and Prandtl number on fluid temperature, effects of viscous dissipation, thermal Grashof number and magnetic field over the velocity. Effects of Prandtl number, dissipation and magnetic field on the skin friction and recovery factor.

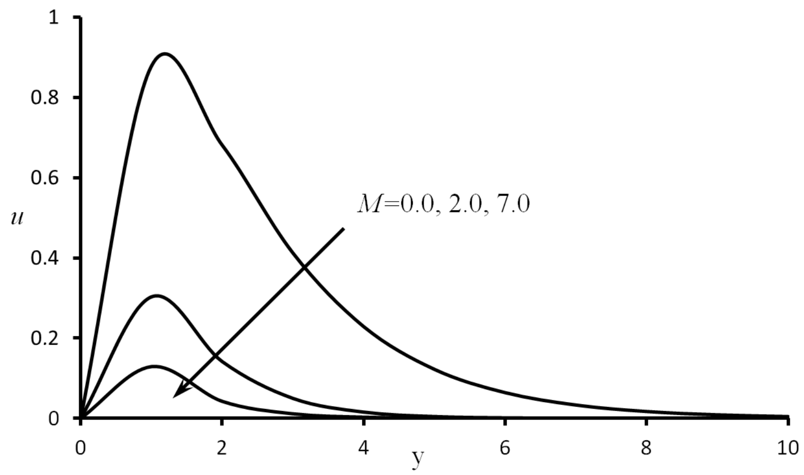

Fig.2:- Dimensionless transverse velocity against y for different values of M, when Sc = 0.6, K_c =1.0, a =0.5, Pr = 2.0, S = 0.4, G_r = 5.0, G_c = 0.5 and Ec =0.01

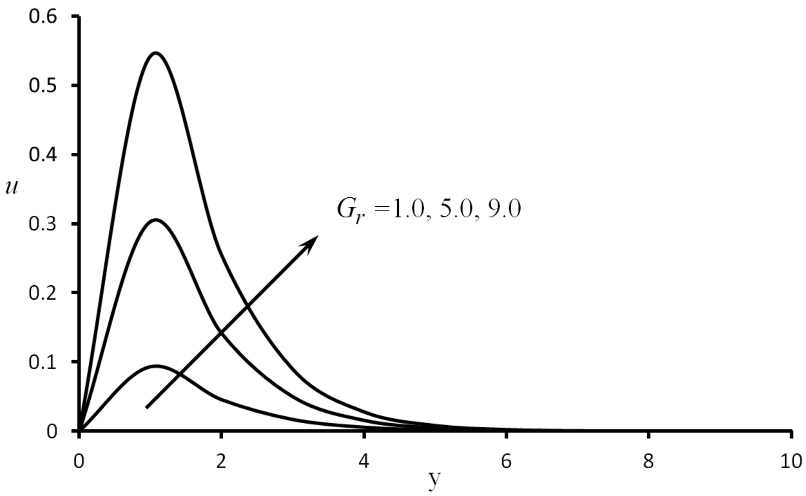

Fig.3:- Dimensionless transverse velocity against y for different values of G_r, when Sc = 0.6, K_c =1.0, a =0.5, Pr = 2.0, S = 0.4, M = 2.0, G_c = 0.5 and Ec =0.01

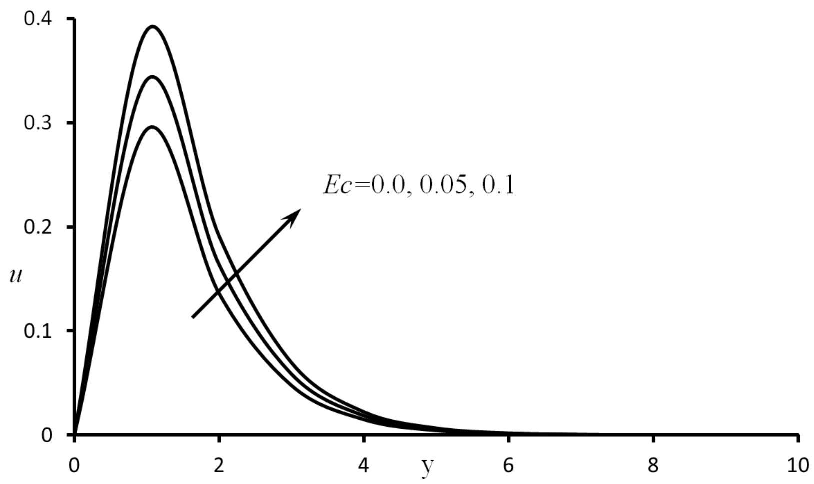

Fig.4:- Dimensionless transverse velocity against y for different values of *Ec*, when *Sc* = 0.6, K_c =1.0, *a* =0.5, Pr = 2.0, *S* = 0.4, *M* = 2.0, G_r = 5.0 and G_c = 0.5

Transverse velocity is presented in fig. 2, fig. 3 and fig. 4 for different variations in M, G_r and *Ec* respectively. The velocity decreases as M increases whereas it increases with an increase in G_r or *Ec*. Increasing magnetic field strength is to increase the retarding force and hence reduces the velocity, the thermal Grashof number G_r signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. The effect of *Ec* in the flow field is to increase the energy, yielding a greater buoyancy force, and hence the increase in buoyancy force due to increase in the dissipation parameter enhances the convective velocity.

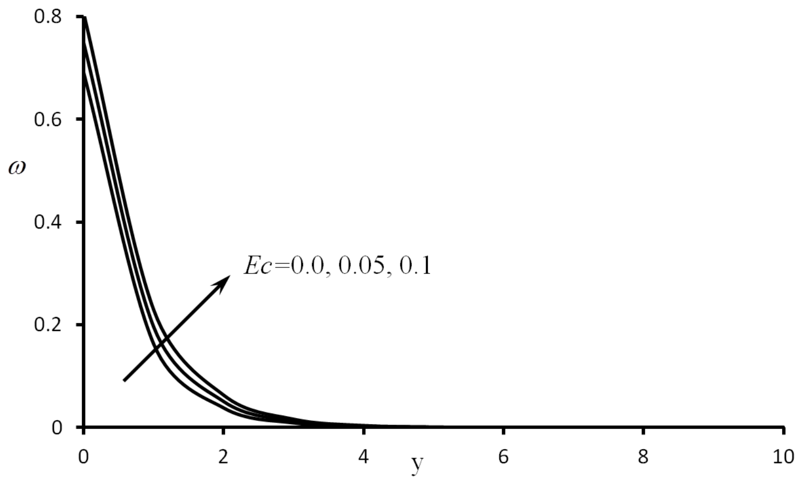

Fig.5:- Dimensionless angular velocity against y for different values of M, when *Sc* = 0.6, K_c =1.0, *a* =0.5, Pr = 2.0, *S* = 0.4, *M* = 2.0, G_r = 5.0 and G_c = 0.5

Figure 5 is drawn for the effects of *Ec* on angular velocity. *Ec* increases with ω. With increasing rotational velocity, the shear stress due to viscosity of the fluid, generates higher dissipation.

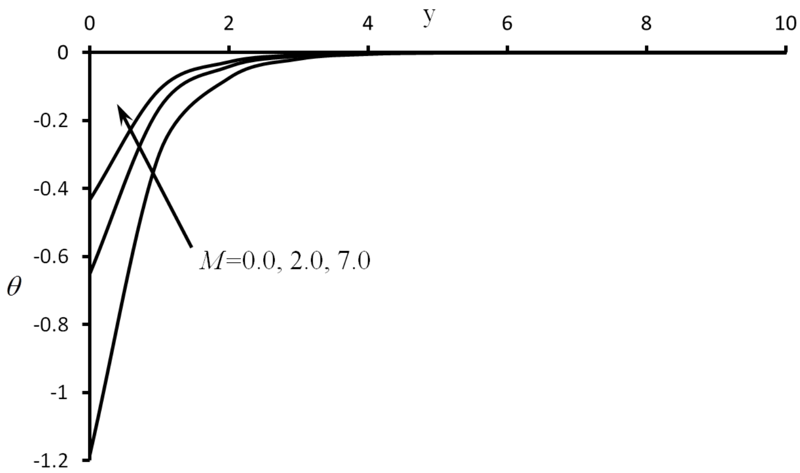

Fig.6:- Dimensionless temperature against y for different values of M, when Sc = 0.6, K_c = 1.0, a = 0.5, Pr = 2.0, S = 0.4, G_r = 5.0, G_c = 0.5 and Ec =0.01

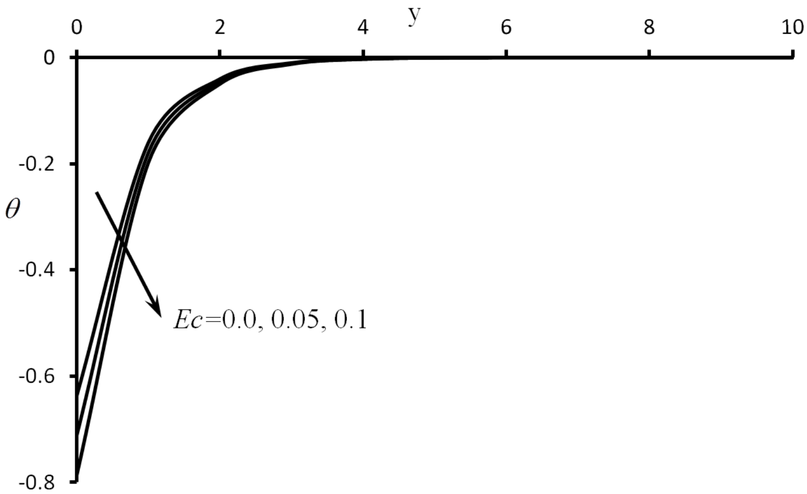

Fig.7:- Dimensionless temperature against y for different values of Ec, when Sc = 0.6, K_c = 1.0, a = 0.5, Pr = 2.0, S = 0.4, M = 2.0, G_r = 5.0 and G_c = 0.5

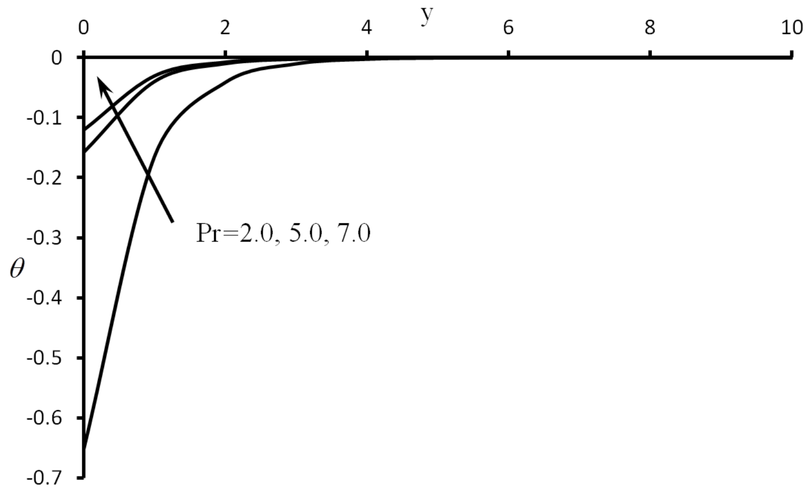

Fig.8:- Dimensionless temperature against y for different values of Pr, when *Sc* = 0.6, K_c =1.0, *a* = 0.5, *S* = 0.4, *M* = 2.0, G_r = 5.0, G_c = 0.5 and *Ec* =0.01

The temperature is drawn for various values of Pr, M and *Ec* in fig. 6, fig. 7 and fig. 8. θ increases with Pr or M, whereas decreases with an increase of *Ec*. As the wall is at prescribed heat flux the temperature increases with an increase in Pr, as the temperature rise due to heat flux impinging on the surface. Also, it is evident from fig. 7, to the fact that magnetic field increases the temperature of the fluid inside the boundary-layer because of excess heating. Figure 8 depicts that higher dissipative fluid has lower thermal boundary layer. Increasing *Ec* implies that dissipation of thermal energy is higher and that reduces the temperature.

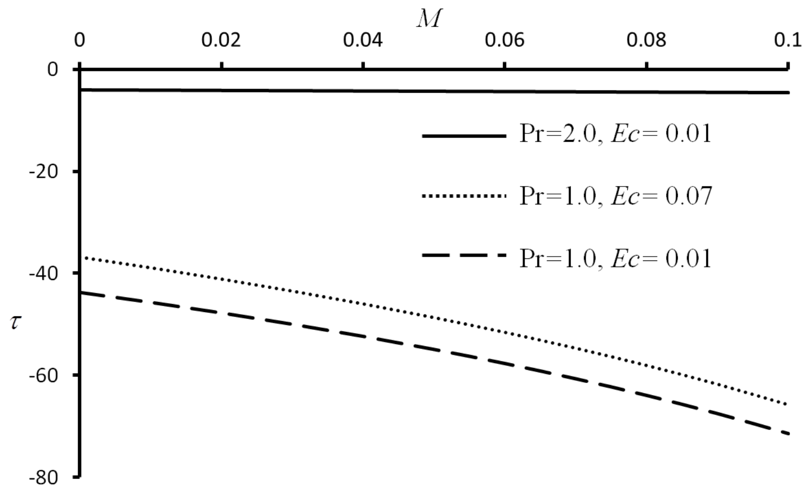

Fig.9:- Skin friction coefficient against M for different values of Pr and *Ec*, when *Sc* = 0.6, K_c =1.0, *a* =0.5, Pr = 2.0, *S* = 0.1, G_r = 5.0 and G_c = 0.5

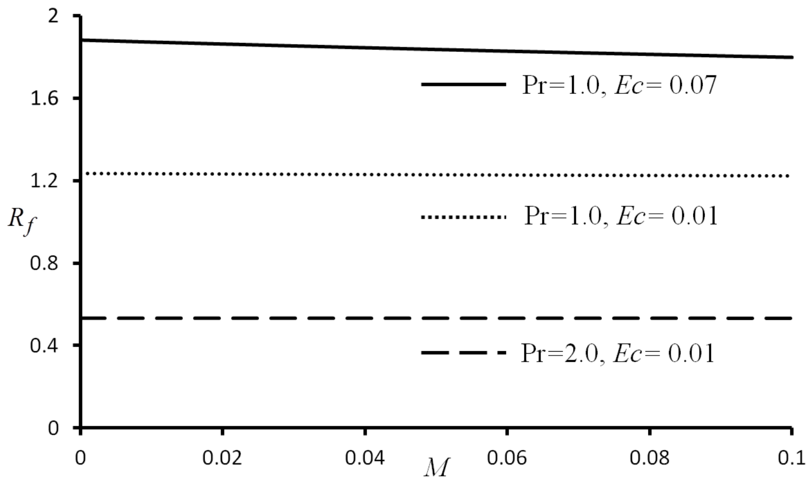

Fig.10:- Recovery factor against M for different values of Pr and Ec, when Sc = 0.6, K_c =1.0, a =0.5, Pr = 2.0, S = 0.1, G_r = 5.0 and G_c = 0.5

Figures 9 and 10 represent the skin friction coefficient and the recovery factor. The τ and R_f are plotted against magnetic field parameter, for the different values of Pr and Ec. It is noted that τ increases with Pr or Ec; and the phenomena reverses for M. R_f increases with Ec and decreases as Pr increases.

4. Conclusions

In this paper a problem of MHD free convective flow of a micropolar fluid with the effect of Ohmic heating and viscous dissipation over a chemically reacting plate is studied when the plate is at a constant heat flux. Increasing dissipation or thermal buoyancy, increases the transverse velocity and on the contrary effect of magnetic field is to decrease. The angular velocity found to be increased with dissipation. The fluid temperature increases as Prandtl number or magnetic field parameter increase; on the other hand for higher disspative fluid the temperature decreases. The skin friction coefficient increases with Prandtl number as well as with viscous dissipation; whereas decreases for Magnetic field parameter. The recovery factor increases with dissipation and decreases as Prandtl number increases.

Acknowledgment

The author is very much thankful to Prof. (Dr.) S. S. Tak, Jai Narain Vyas University, Jodhpur (India) for offering his valuable suggestions and assistance to improve the paper.

Nomenclature

y^* horizontal coordinate	(m)	k thermal conductivity	(W/m K)
u^* axial velocity	(m/s)	D mass diffusion coefficient	($m^2 s^{-1}$)
v^* transverse velocity	(m/s)	k_l rate of chemical reaction	(s^{-1})
ω_a angular velocity vector normal to the		q rate of heat transfer	(W/m^2)

xy –plane (rad/s)	m_w wall mass flux (mol/m^{2}s)
p^* pressure (Pa)	T_w wall temperature (K)
T^* temperature of the fluid (K)	V_0 suction velocity (m/s)
T_∞ far field temperature (K)	Q heat generation coefficient (W m^{-3} K^{-1})
c species concentration (mol/m^3)	a material parameter
c_∞ far field concentration (mol/m^3)	y dimensionless horizontal coordinate
ϑ kinematic viscosity (m^2/s)	u dimensionless axial velocity
ρ density (kg/m^3)	M magnetic field parameter
κ vortex viscosity (Pa.s)	Pr Prandtl number
μ dynamic coefficient of viscosity (Pa.s)	θ dimensionless temperature
g acceleration due to gravity (m/s^2)	C dimensionless species concentration
β_T Coefficient of thermal expansion (K^{-1})	Ec Eckert number
β_c coefficient of concentration expansion (m^3/mol)	G_r thermal Grashof number
	G_c solutal Grashof number
σ electrical conductivity (S/m)	S heat generation parameter
B_0 magnetic field coefficient (T)	ω dimensionless angular velocity
j micro inertia density (m^2)	Sc Schmidt number
γ spin gradient viscosity (kg.m/s)	K_c chemical reaction parameter
c_p specific heat (J kg^{-1} K^{-1})	K dimensionless permeability parameter

References: -

[1] Ghebhart, B. and Pera, L., The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion, *Int. J. Heat Mass Transfer*, 14, 2025-2050, 1971.

[2] Sparrow, E.M., Minkowycz, W.J. and Eckert, E.R.G., Transpiration – induced buoyancy and thermal diffusion in a helium air free convection boundary layer, *J. Heat Trans. ASME*, 86, 508-514, 1964.

[3] Soundalgekar, V.M., Effects of mass transfer on free convective flow of a dissipative incompressible fluid past a infinite vertical porous plate with suction, *Proc. Indian Acad. Sci.*, 84, 194-203, 1976.

[4] Acharya, M., Das, G.C. and Singh, L.P., Magnetic field effects on the free convection and mass transfer flow through porous medium with constant suction and constant heat flux, *Indian J. Pure Appl. Math.*, 31, 1-18, 2000.

[5] Singh, K.D. and Chand, K., Unsteady free convective MHD flow past a vertical porous plate with variable temperature, *Proc. Nat. Acad. Sci.*, 70, 49-58, 2000.

[6] Yamamoto, K. and Iwamura, N., Flow with convective acceleration through a porous medium, *J. Engin. Math.*, 10, 41-54, 1976.

[7] Yamamoto, K. and Yoshida, Z., Flow through a porous wall with convective acceleration, *J. Phys. Soc. Japan*, 37, 774-779, 1974.

[8] Brinkman, H.C., A calculation of a viscous force extend by a flowing fluid on a dense swarm of particles, *Appl. Sci. Res.*, 1, 27-34, 1947.

[9] Raptis, A., Perdikis, C. and Tzivanidis, G., Free convection flow through a porous medium bounded by a vertical surface, *J. Phys. D : Appl. Phys.*, 14, 99-102, 1981.

[10] Raptis, A. and Kafousias, N.G., Free convection and mass transfer flow through a porous medium under the action of a magnetic field, *Rev. Roum. Sci. Techn. - Mec. Appl.*, 27, 37-43, 1982.

[11] Raptis, A., Effects of couple stresses on the flow through a porous medium, *Rheol. Acta*, 21, 736-737, 1982.

[12] Raptis, A., Megnetohydrodynamic flow of a polar fluid through a porous medium, *Bull. de la classe des Sc.*, 69, 530-534, 1983.

[13] Kandasamy, R. and Devi, S.P.A., Effects of chemical reaction, heat and mass transfer on non-linear laminar boundary-layer flow over a wedge with suction or injection, *Journal of Computational and Applied Mechanics*, 5, 21-31, 2004.

[14] Kumar, H., Heat Transfer in MHD Boundary- Layer Flow through a Porous Medium, Due to a Non-Isothermal Stretching Sheet, with Suction, Radiation, and Heat Annihilation, *Chem Eng Comm* 200, 1-12, 2013.

[15] Asghar, S., Hanif, K., Hayat, T. and Khalique, C.M., MHD non-newtonian flow due to non-coaxial rotations of an accelerated disk and a fluid at infinity. *Communications in Nonlinear Science and Numerical Simulation*, 12, 465-485, 2007.

[16] Chen, C.H., Combined heat and mass transfer in MHD free convection from a vertical surface with Ohmic heating and viscous dissipation, *Int. J. Engineering Science*, 42, 699-713, 2004.