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VIBRATION CONTROL OF A NONLINEAR DYNAMICAL SYSTEM WITH TIME VARYING STIFFNESS SUBJECTED TO MULTI EXTERNAL FORCES

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Abstract *The dynamical system with time varying stiffness subjected to multi excitation forces studied. The system is written as two degree of freedom consists of the main system and absorber. The multiple time scale perturbation method is applied to get the approximate solution up to the third approximation. The stability of the system at the simultaneous primary resonance is investigated using both frequency response equations and phase-plane methods. The effects of different parameters are studied numerically.*

Keywords: Perturbation method, Response, Chaotic response stability**,** Vibration control, Time varying stiffness.

1.Introduction

This problem made the vibration is the one of a non-desired phenomenon in our life. One of the most common methods of vibration control is the dynamic absorber. It has the advantages of low cost and simple operation at one modal frequency. In the domain of many mechanical vibration systems the coupled non-linear vibration of such systems can be reduced to non-linear second order differential equations which are solved analytically and numerically.

Nayfeh and Mook [1, 2] studied the nonlinear systems with linear natural frequencies which were commensurate or nearly commensurate. Internal resonance provides coupling and energy exchange among the vibration modes. Jain [3] shows the solutions of differential equations using Runge-Kutta fourth-order method. Asfar et. al. [4] studied the response of self excited two-degree-of-freedom system to multi-frequency excitations. Queini et. al. [5] studied the regulation of a two-degree-of-freedom structure using internal resonance. They introduce a controller taking the form of a second-order system that is coupled to the plant. Eissa [6] investigated the non-linear mechanical oscillator subjected to parametric and external excitation forces. Queini and Nayfeh [7] proposed a non-linear control law to suppress the vibrations of the first mode of a cantilever beam when subjected to a principal parametric excitation. The method of multiple scales is applied throughout. The analysis revealed that cubic velocity feedback reduced the amplitude of the response.

Eissa [8] reported that when using a dynamic absorber, its damping coefficient should be kept minimal for better system performance. Pai and Schilz [9] designed a refined non-linear vibration absorber was using a quadratic velocity coupling term in the controller and adding a negative velocity feedback to the system. Olkan [10] studied the basic absorption action of auto-parametric system under sinusoidal excitation numerically and experimentally. Ji and Hansen [11] studied an experimental investigation of the non-linear response of clamped beam subjected to a harmonic axial load. Ashour and Nayfeh [12] also studied non-linear adaptive control of flexible structures using the saturation phenomenon. This phenomenon was utilized to suppress high-amplitude bending and torsional vibration modes of rectangular cantilever plates. Sayam et. al. [13] studied numerical simulations of the response of a uniform, cantilever beam subjected to a base excitation. Song et. al. [14] presented a study of the vibration response of the spring-mass-damper system with a parametrically excited pendulum hinged to the mass using the harmonic balance method. The results were verified by numerical calculation. The third order approximation was used to analyze the response characteristic and the stability of the system. Attilio [15] applied an asymptotic perturbation method based on Fourier expansion and time rescaling. Eissa and Amer [16] simulated the vibration of a second order system to the first mode of a cantilever beam subjected to both external and parametric excitation at primary and sub-harmonic resonance. They reported that the vibration of the system can be controlled by adding a feedback cubic velocity non-linear term. They reported also that there is a threshold value for the linear damping coefficient which can be applied to control the system vibration. Eissa et. al. [17, 18] studied the passive and active control in some non-linear differential equations describing the vibration of the aircraft wing subjected to: multi-excitation force, multi-parametric excitations are considered. The same system is considered with 1:2 internal resonances, 1:3 internal resonance and 1:2:4 internal resonance active controllers.

Amer [19] investigated the coupling of two non-linear oscillators of the main system and absorber representing ultrasonic cutting process. The multiple time scale perturbation technique is applied throughout. A threshold value of linear damping has been obtained, where the system vibration can be reduced dramatically. Amer and EL-Sayed [20] studied the non-linear dynamics of a two-degree-of freedom vibration system with non-linear damping and non-linear spring stiffness analytically using the method of multiple scales perturbation technique up to the third order approximation. The system consists of the main one and an absorber. Amer and El-emam [21] investigated the nonlinear dynamical system with time varying stiffness subjected to multi-excitation forces without control, and studied the effects of different parameters. Eissa and et. al. [22, 23] studied the vibration reduction of nonlinear dynamical system described the nonlinear spring pendulum under multi parametric and multi external excitations. El- Gohary and El-Ganini [24, 25] applied active control for suppressing the vibration of a non-linear plant when subjected to external and parametric excitation in the presence of 1:2 and 1:3 internal resonance.

In this paper, the coupled non-linear differential equations of the non-linear dynamical two-degree-of-freedom vibrating system including quadratic and cubic non-linearties are studied. The system consists of the main system and the absorber. The system subject to multi external excitation forces is considered with simultaneous primary resonance case passive control absorber. The method of multiple scales perturbation technique is applied throughout to determine the solution up to third order approximations. The different resonance cases are reported and studied numerically. Stability of the system is studied applying both frequency response functions and phase-plane methods.

2. Mathematical Modeling

Using a linear tuned mass absorber (TMA) connected to the system, equations of motions can be written in the following form:

$$\ddot{U}_2 + 2\varepsilon\zeta\omega\dot{U}_2 + \omega^2 U_2 + \varepsilon\alpha U_2^2 + \varepsilon(\beta_1 + \beta_2 \cos St)U_2^3 + \varepsilon\zeta_1(\dot{U}_2 - \dot{U}_1) + \varepsilon\gamma(U_2 - U_1)$$
$$= \varepsilon\sum_{i=1}^{N} F_j \cos\Omega_j t \qquad (1a)$$

$$\ddot{U}_1 + 2\varepsilon\zeta_2\omega_1(\dot{U}_1 - \dot{U}_2) + \omega_1^2(U_1 - U_2) = 0 \qquad (1b)$$

where U_1 donates the response of the second-order controller, U_2 represents one of the model co-ordinates of a structure, ω and ω_1 are the natural frequencies, ζ, ζ_1 and ζ_2 are the damping coefficients, α, β_1 and β_2 are non-linear coefficients of the wings, γ is the linear coefficient , ε is a small perturbation parameter, F_j the forcing amplitudes and Ω_j are the excitation frequencies, j=1, 2, 3, 4 for simplicity.

2.1. Perturbation analysis.

The method of multiple time scale is applied to determine a first order uniform expansion for the solution of equations (1a) and (1b) as in the form:

$$U_n(t,\varepsilon) = u_{no}(T_o, T_1) + \varepsilon u_{n1}(T_o, T_1) + ..., \qquad (n=1,2) \tag{2}$$

where ε is a small perturbation parameter, $T_o=t, T_1=\varepsilon t$ are fast and slow time scales respectively, and the time derivatives became

$$\frac{d}{dt} = D_o + \varepsilon D_1 + \varepsilon^2 D_2 \;, \frac{d^2}{dt^2} = D_o^2 + 2\varepsilon D_o D_1 + \varepsilon^2 (D_1^2 + 2 D_o D_2) \tag{3}$$

Substituting equations (2) and (3) in to equations (1a) and (1b) and equating the coefficients of the same power of ε in both sides, we obtain

$$(D_o^2 + \omega^2) u_{2o} = 0 \tag{4a}$$

$$(D_o^2 + \omega_1^2) u_{1o} = \omega_1^2 u_{2o} \tag{4b}$$

$$(D_o^2 + \omega^2) u_{21} = -2 D_o D_1 u_{2o} - 2\zeta\omega D_o u_{2o} - \alpha u_{2o}^2 - (\beta_1 + \beta_2 \cos St) u_{2o}^3 - \zeta_1 (D_o u_{2o} - D_o u_{1o})$$
$$-\gamma(u_{2o} - u_{1o}) + \sum_{j=1}^{N} F_j \cos \Omega_j T_o \tag{5a}$$

$$(D_o^2 + \omega_1^2) u_{11} = \omega_1^2 u_{21} - 2 D_o D_1 u_{1o} - 2\zeta_2 \omega_1 (D_o u_{1o} - D_o u_{2o}) \tag{5a}$$

$$(D_o^2 + \omega^2) u_{22} = -2 D_o D_1 u_{21} - D_1^2 u_{2o} - 2\zeta\omega (D_o u_{21} + D_1 u_{2o}) - 2\alpha u_{2o} u_{21} - 3(\beta_1 + \beta_2 \cos St) u_{2o}^2 u_{21}$$
$$-\zeta_1 (D_o u_{21} + D_1 u_{2o} - D_o u_{11} - D_1 u_{1o}) - \gamma(u_{21} - u_{11}) \tag{6a}$$

$$(D_o^2 + \omega_1^2) u_{12} = \omega_1^2 u_{22} - 2 D_o D_1 u_{11} - D_1^2 u_{1o} - 2\zeta_2 \omega_1 (D_o u_{11} + D_1 u_{1o} - D_o u_{21} - D_1 u_{2o}) \tag{6b}$$

$$(D_o^2 + \omega^2) u_{23} = -2 D_o D_1 u_{22} - D_1^2 u_{21} - 2\zeta\omega (D_o u_{22} + D_1 u_{21}) - \alpha (u_{21}^2 + 2 u_{2o} u_{22}) - \gamma (u_{22} - u_{12})$$
$$-3(\beta_1 + \beta_2 \cos St)(u_{2o} u_{21}^2 + u_{2o}^2 u_{22}) - \zeta_1 (D_o u_{22} + D_1 u_{21} - D_o u_{12} - D_1 u_{11}) \tag{7a}$$

$$(D_o^2 + \omega_1^2) u_{13} = \omega_1^2 u_{23} - 2 D_o D_1 u_{12} - D_1^2 u_{11} - 2\zeta_2 \omega_1 (D_o u_{12} + D_1 u_{11} - D_o u_{22} - D_1 u_{21}) \tag{7b}$$

The general solution of equations (4a) and (4b) are given by

$$u_{2o}(T_o, T_1) = A_o(T_1) e^{i\omega T_o} + cc \tag{8a}$$

$$u_{1o}(T_o, T_1) = B_o(T_1) e^{i\omega_1 T_o} + C_o(T_1) e^{i\omega T_o} + cc \tag{8b}$$

where A_o, B_o, C_o are complex function in T_1 and cc are represents the complex conjugate of the preceding terms. Substituting equations (8a) and (8b) in to equations (5a) and (5b), and eliminating the secular terms, then the general solution obtained as:

$$u_{21}(T_o,T_1) = K_1 e^{i\omega T_o} + K_2 e^{2i\omega T_o} + K_3 e^{3i\omega T_o} + K_4 e^{i(S+\omega)T_o} + K_5 e^{i(S-\omega)T_o} + K_6 e^{i(S+3\omega)T_o} + K_7 e^{i(S-3\omega)T_o} + K_8 e^{i\omega_1 T_o} + K_9 e^{i\Omega_1 T_o} + K_{10} e^{i\Omega_2 T_o} + K_{11} e^{i\Omega_3 T_o} + K_{12} e^{i\Omega_4 T_o} + K_{13} + cc \tag{9a}$$

$$u_{11}(T_o,T_1) = F_1 e^{i\omega_1 T_o} + F_2 e^{i\omega T_o} + F_3 e^{2i\omega T_o} + F_4 e^{3i\omega T_o} + F_5 e^{i(S+\omega)T_o} + F_6 e^{i(S-\omega)T_o} + F_7 e^{i(S+3\omega)T_o} + F_8 e^{i(S-3\omega)T_o} + F_9 e^{i\Omega_1 T_o} + F_{10} e^{i\Omega_2 T_o} + F_{11} e^{i\Omega_3 T_o} + F_{12} e^{i\Omega_4 T_o} + F_{13} + cc \tag{9b}$$

Where K_i and F_i $(i = 1,2,...,13)$ are complex functions in T_1, cc are complex conjugate. Similarly, substituting from Eqs. (8a), (8b), (9a) and(9b) in to Eqs. (6a) and (6b) we get

$$u_{22}(T_o,T_1) = R_1 e^{i\omega T_o} + R_2 e^{2i\omega T_o} + R_3 e^{3i\omega T_o} + R_4 e^{4i\omega T_o} + R_5 e^{5i\omega T_o} + R_6 e^{i\omega_1 T_o} + R_7 e^{i(\omega_1+\omega)T_o} + R_8 e^{i(\omega_1-\omega)T_o} + R_9 e^{i(\omega_1+2\omega)T_o} + R_{10} e^{i(\omega_1-2\omega)T_o} + \sum_{j=1}^{4}\Big(R_{(10+j)} e^{i\Omega_j T_o} + R_{(14+j)} e^{i(\Omega_j+\omega)T_o} + R_{(18+j)} e^{i(\Omega_j-\omega)T_o} + R_{(22+j)} e^{i(\Omega_j+2\omega)T_o} + R_{(26+j)} e^{i(\Omega_j-2\omega)T_o}\Big) + R_{31} e^{isT_o} + R_{32} e^{i(S+\omega)T_o} + R_{33} e^{i(S-\omega)T_o} + R_{34} e^{i(S+2\omega)T_o} + R_{35} e^{i(S-2\omega)T_o} + R_{36} e^{i(S+3\omega)T_o} + R_{37} e^{i(S-3\omega)T_o} + R_{38} e^{i(S+4\omega)T_o} + R_{39} e^{i(S-4\omega)T_o} + R_{40} e^{i(S+5\omega)T_o} + R_{41} e^{i(S-5\omega)T_o} + R_{42} e^{i(2S+\omega)T_o} + R_{43} e^{i(2S-\omega)T_o} + R_{44} e^{i(2S+3\omega)T_o} + R_{45} e^{i(2S-3\omega)T_o} + R_{46} e^{i(2S+5\omega)T_o} + R_{47} e^{i(2S-5\omega)T_o} + R_{48} e^{i((\omega_1+S)+2\omega)T_o} + R_{49} e^{i((\omega_1-S)+2\omega)T_o} + R_{50} e^{i((\omega_1+S)-2\omega)T_o} + R_{51} e^{i((\omega_1-S)-2\omega)T_o} + R_{52} e^{i(\omega_1+S)T_o} + R_{53} e^{i(\omega_1-S)T_o} + \sum_{j=1}^{4}\Big(R_{(53+j)} e^{i((\Omega_j+S)+2\omega)T_o} + R_{(57+j)} e^{i((\Omega_j-S)+2\omega)T_o} + R_{(61+j)} e^{i((\Omega_j-S)+2\omega)T_o} + R_{(65+j)} e^{i((\Omega_j-S)-\omega)T_o} + R_{(69+j)} e^{i(\Omega_j+S)T_o} + R_{(73+j)} e^{i(\Omega_j-S)T_o}\Big) + R_{78} + cc \tag{10a}$$

$$u_{12}(T_o,T_1) = L_1 e^{i\omega T_o} + L_2 e^{2i\omega T_o} + L_3 e^{3i\omega T_o} + L_4 e^{4i\omega T_o} + L_5 e^{5i\omega T_o} + L_6 e^{i\omega_1 T_o} + L_7 e^{i(\omega_1+\omega)T_o} + L_8 e^{i(\omega_1-\omega)T_o} + L_9 e^{i(\omega_1+2\omega)T_o} + L_{10} e^{i(\omega_1-2\omega)T_o} + \sum_{j=1}^{4}\Big(L_{(10+j)} e^{i\Omega_j T_o} + L_{(14+j)} e^{i(\Omega_j-\omega)T_o} + L_{(18+j)} e^{i(\Omega_j+\omega)T_o} + L_{(22+j)} e^{i(\Omega_j+2\omega)T_o} + L_{(26+j)} e^{i(\Omega_j-2\omega)T_o}\Big) + L_{31} e^{iST_o} + L_{32} e^{i(S+\omega)T_o} + L_{33} e^{i(S-\omega)T_o} + L_{34} e^{i(S+2\omega)T_o} + L_{35} e^{i(S-2\omega)T_o} + L_{36} e^{i(S+3\omega)T_o} + L_{37} e^{i(S-3\omega)T_o} + L_{38} e^{i(S+4\omega)T_o} + L_{39} e^{i(S-4\omega)T_o} + L_{40} e^{i(S+5\omega)T_o} + L_{41} e^{i(S-5\omega)T_o} + L_{42} e^{i(2S+\omega)T_o} + L_{43} e^{i(2S-\omega)T_o} + L_{44} e^{i(2S+3\omega)T_o} + L_{45} e^{i(2S-3\omega)T_o} + L_{46} e^{i(2S+5\omega)T_o} + L_{47} e^{i(2S-5\omega)T_o} + L_{48} e^{i((\omega_1+S)+2\omega)T_o} + L_{49} e^{i((\omega_1-S)+2\omega)T_o} + L_{50} e^{i((\omega_1+S)-2\omega)T_o} + L_{51} e^{i((\omega_1-S)-2\omega)T_o} + L_{52} e^{i(\omega_1+S)T_o} + L_{53} e^{i(\omega_1-S)T_o} + \sum_{j=1}^{4}\Big(L_{(53+j)} e^{i((\Omega_j+2\omega)+S)T_o} + L_{(57+j)} e^{i((\Omega_j+2\omega)-S)T_o} + L_{(61+j)} e^{i((\Omega_j-2\omega)+S)T_o} + L_{(65+j)} e^{i((\Omega_j-2\omega)-S)T_o} + L_{(69+j)} e^{i(\Omega_j+S)T_o} + L_{(73+j)} e^{i(\Omega_j-S)T_o}\Big) + L_{78} + cc \tag{10b}$$

where R_m and L_m $(m=1,2,...,78)$ are complex functions in T_1, cc are complex conjugates. From the above analysis the general solutions of u_2 and u_1 is given by

$$U_2 = u_{2o} + \varepsilon u_{21} + \varepsilon^2 u_{22} + \varepsilon^3 u_{23} + o(\varepsilon^4) \tag{11a}$$

and $$U_1 = u_{1o} + \varepsilon u_{21} + \varepsilon^2 u_{12} + \varepsilon^3 u_{13} + o(\varepsilon^4) \tag{11b}$$

From above-proposed solution, the reported resonance cases are:

(i) Trivial resonance: $\Omega_j \cong \omega \cong \omega_1 \cong S = 0$

(ii) Primary resonance: $\Omega_1 = \omega,\ \omega_1 \cong \omega, \omega_1 \cong S$

(iii) Sub-harmonic resonances: $\omega_1 \cong n\omega, n = 2,3,4,5...,7,\ \omega \cong 2S$

(iv) Super-harmonic resonances: $\Omega_j \cong \omega/2, \Omega_j \cong \omega_1/2, j = 1,2,3,4$

(v) Combined resonances:

(1) $\omega_1 \cong \pm(\pm\omega \pm S)$, (2)$\omega_1 \cong \pm(\pm\omega \pm 2S)$, (3) $\omega_1 \cong \pm(\pm\Omega_1 \pm 2\omega)$, (4) $\omega_1 \cong \pm\frac{1}{2}(\pm 2\omega \pm S)$,

(5) $\omega_1 \pm \omega \cong \pm(\pm\Omega_2 \pm \Omega_1)$, (6)$\omega_1 \pm \omega \cong \pm(\pm\Omega_2 \pm \Omega_1 \pm S)(7)\omega \cong \pm(2\omega_1 \pm S)$,

(8)$\omega \pm S \cong \pm\frac{1}{2}(\pm\Omega_4 \pm \Omega_2)$

(vi) Simultaneous resonance: any combination of the above resonance cases is considered as simultaneous resonance

2.2. Stability of the system

We study the stability of the system at the simultaneous primary resonance $\Omega_1 \cong \omega, \Omega_2 \cong \omega_1$ and $S \cong 2\omega$. Using the detuning parameters σ_1, σ_2 and σ such that

$$\Omega_1 \cong \omega + \varepsilon\sigma_1 \quad ,\Omega_2 = \omega_1 + \varepsilon\sigma_2 \text{ and } S \cong 2\omega + \varepsilon\sigma \tag{12}$$

Eliminating the secular terms of equations (9a) and (9b),leads to the solvability conditions for the first order approximation and noting that A_o and B_o are functions in T_1 only, we get

$$[-2i\,\omega(D_1A_o + \zeta\omega A_o) - 3\beta_1 A_o^2\bar{A}_o - i\,\omega\zeta_1 A_o - \gamma A_o + \frac{\omega_1^2 A_o}{(\omega_1^2 - \omega^2)}(i\,\omega\zeta_1 + \gamma)]e^{i\omega T_o}$$
$$-\frac{3\beta_2}{2}\bar{A}_o^2 A_o e^{i(S-\omega)T_o} + \frac{1}{2}F_1 e^{i\Omega_1 T_o} = 0 \tag{13}$$

$$-[2i\,\omega_1(D_1B_o + \zeta_2\omega_1 B_o) + \frac{\omega_1^2 B_o}{(\omega_1^2 - \omega^2)}(i\,\omega_1\zeta_1 + \gamma)]e^{i\omega_1 T_o} + \frac{\omega_1^2 F_2}{2(\omega^2 - \Omega_2^2)}e^{i\Omega_2 T_o} = 0 \tag{14}$$

Putting the polar form

$$A_o = \frac{1}{2}a_1(T_1)e^{i\mu_1(T_1)} \tag{15a}$$

and $$B_o = \frac{1}{2}a_2(T_1)e^{i\mu_2(T_1)} \tag{15b}$$

where a_1, a_2, μ_1 and μ_2 are real. Substituting Equations (15a) and (15b) in to equations (13), (14), and separating real and imaginary parts we get the following

$$a_1' = -a_1\zeta\omega - \frac{1}{2}\zeta_1 a_1 + \frac{\omega_1^2 a_1\zeta_1}{2(\omega_1^2 - \omega^2)} - \frac{3}{16}\beta_2 a_1^3 \sin\theta + \frac{F_1}{2\omega}\sin\theta_1 \tag{16a}$$

$$a_1\mu_1' = \frac{3\beta_1 a_1^3}{8\omega} + \frac{\gamma a_1}{2\omega} - \frac{\gamma a_1 \omega_1^2}{2\omega(\omega_1^2-\omega^2)} + \frac{3\beta_2 a_1^3}{16\omega}\cos\theta - \frac{F}{2\omega}\cos\theta_1 \tag{16b}$$

$$a_2' = -a_2\zeta_2\omega_1 - \frac{a_2\omega_1^2\zeta_1}{2(\omega_1^2-\omega^2)} + \frac{\omega_1 F_2}{2(\omega^2-\Omega_2^2)}\sin\theta_2 \tag{16c}$$

$$a_2\mu_2' = \frac{a_2\gamma\omega_1}{2(\omega_1^2-\omega^2)} + \frac{\omega_1 F_2}{2(\omega^2-\Omega_2^2)}\cos\theta_2 \tag{16d}$$

Where $\theta = \sigma T_1 - 2\mu_1, \theta_1 = \sigma_1 T_1 - \mu_1,$ and $\theta_2 = \sigma_2 T_1 - \mu_2$. For steady-state solutions, $a_1' = a_2' = \theta' = \theta_1' = \theta_2' = 0,$ and equations (16a), (16b), (16c) and (16d) becomes

$$\frac{F_1}{2a_1\omega}\sin\theta_1 = \zeta\omega + \frac{1}{2}\zeta_1 - \frac{\omega_1^2\zeta_1}{2(\omega_1^2-\omega^2)} + \frac{3}{16}\beta_2 a_1^2\sin\theta \tag{17a}$$

$$-\frac{F_1}{2\omega}\cos\theta_1 = \frac{(\sigma_1+\sigma)}{3} - \frac{3\beta_1 a_1^2}{8\omega} - \frac{\gamma}{2\omega} + \frac{\gamma\omega_1^2}{2\omega(\omega_1^2-\omega^2)} - \frac{3\beta_2 a_1^2}{16\omega}\cos\theta \tag{17b}$$

$$\frac{\omega_1 F_2}{2a_2(\omega^2-\Omega_2^2)}\sin\theta_2 = \zeta_2\omega_1 + \frac{\omega_1^2\zeta_1}{2(\omega_1^2-\omega^2)} \tag{17c}$$

$$\frac{\omega_1 F_2}{2a_2(\omega^2-\Omega_2^2)}\cos\theta_2 = \sigma_2 - \frac{\gamma\omega_1}{2(\omega_1^2-\omega^2)} \tag{17d}$$

Squaring equations (17a), (17b) and adding the result, we get the corresponding frequency response equations (FRE)

$$\sigma_1^2 - \left(\frac{9\beta_1 a_1^2}{4\omega} - 2\sigma - \frac{3\gamma\omega_1^2}{\omega(\omega_1^2-\omega^2)} + \frac{3\gamma}{\omega}\right)\sigma_1 + \Big[\sigma^2 - \frac{9\beta_1 a_1^2\sigma}{4\omega} + \frac{3\gamma\omega_1^2\sigma}{\omega(\omega_1^2-\omega^2)} - \frac{3\gamma\sigma}{\omega}$$
$$+9\Big(\frac{9\beta_1^2 a_1^4}{64\omega^2} + \frac{\gamma^2}{4\omega^2} + \frac{\omega_1^2\gamma^2}{2\omega^2(\omega_1^2-\omega^2)} + \frac{\omega_1^4\gamma^2}{4\omega^2(\omega_1^2-\omega^2)^2} - \frac{3\beta_1 a_1^2\gamma\omega_1}{8\omega(\omega_1^2-\omega^2)} + \frac{9\beta_1 a_1^2\gamma}{8\omega}$$
$$+\frac{\zeta_1^2\omega_1^4}{4(\omega_1^2-\omega^2)^2} - \frac{\zeta_1^2\omega_1^2}{2(\omega_1^2-\omega^2)} + \zeta^2\omega^2 + \frac{1}{4}\zeta^2 + \zeta\zeta_1\omega - \frac{\zeta\zeta_1\omega_1^3}{(\omega_1^2-\omega^2)} - \frac{9\beta_2^2 a_1^4}{256\omega^2}$$
$$-\frac{F_1^2}{4a_1^2\omega^2} + \frac{9\beta_2 a_1}{16\omega^2}\Big)\Big] = 0 \tag{18}$$

Similarly, from equation (17c) and (17d), we get

$$\sigma_2^2 - \frac{\gamma\omega_1}{(\omega_1^2-\omega^2)}\sigma_2 + \Big[\frac{\gamma^2\omega_1^2}{4(\omega_1^2-\omega^2)^2} + \zeta_2^2\omega_1^2 + \frac{\omega_1^4\zeta_1^2}{4(\omega_1^2-\omega^2)^2} + \frac{\zeta\zeta_1\omega_1^3}{(\omega_1^2-\omega^2)}$$
$$-\frac{\omega_1^2 F_2^2}{4a_2^2(\omega^2-\Omega_2^2)^2}\Big] = 0 \tag{19}$$

Now to determine the stability of the steady –state linear solution, let A_o and B_0 Expressed in cartizian form as following

$$A_0(T_1) = \frac{1}{2}(p_1 - iq_1)e^{i\sigma_1 T_1} \tag{20a}$$

and
$$B_0(T_1) = \frac{1}{2}(p_2 - iq_2)e^{i\sigma_2 T_1} \tag{20b}$$

where p_n and q_n, (n=1, 2) are real values. Inserting equations (20a) and (20b) in to the linear form of equations (14a) ,(14b) and separating real and imaginary parts, the following system of equations is obtained as:

$$p_1' + \upsilon_1 p_1 + \eta_1 q_1 = 0 \tag{21a}$$

$$q_1' + \upsilon_1 q_1 - \eta_1 p_1 = 0 \tag{21b}$$

$$p_2' + \upsilon_2 p_2 + \eta_2 q_2 = 0 \tag{21c}$$

$$q_2' + \upsilon_2 q_2 - \eta_2 p_2 = 0 \tag{21d}$$

where,
$$\upsilon_1 = (\zeta\omega + \frac{1}{2}\zeta_1 - \frac{\omega_1^2\zeta_1}{2(\omega_1^2 - \omega^2)}), \quad \upsilon_2 = (\zeta_2\omega_1 + \frac{\omega_1^2\zeta_1}{2(\omega_1^2 - \omega^2)})$$

$$\eta_1 = (\sigma_1 + \frac{\gamma\omega_1^2}{2\omega(\omega_1^2 - \omega^2)} - \frac{\gamma}{2\omega}), \quad \eta_2 = (\sigma_2 - \frac{\gamma\omega_1}{2(\omega_1^2 - \omega^2)})$$

The stability of linear solution is investigated from the zero characteristics matrix

$$\begin{vmatrix} \lambda + \upsilon_1 & \eta_1 & 0 & 0 \\ -\eta_1 & \lambda + \upsilon_1 & 0 & 0 \\ 0 & 0 & \lambda + \upsilon_2 & \eta_2 \\ 0 & 0 & -\eta_2 & \lambda + \upsilon_2 \end{vmatrix} = 0$$

The eigen values are given by

$$\lambda^4 + r_1\lambda^3 + r_2\lambda^2 + r_3\lambda + r_4 = 0$$

Where,
$$r_1 = 2(\upsilon_1 + \upsilon_2), \quad r_2 = \upsilon_1^2 + \upsilon_2^2 + 4\upsilon_1\upsilon_2 + \eta_1^2 + \eta_2^2$$

$$r_3 = 2\upsilon_1\upsilon_2(\upsilon_1 + \upsilon_2) - 2\upsilon_2\eta_1^2 + 2\upsilon_1\eta_2^2, r_4 = (\upsilon_1^2 + \eta_1^2)(\eta_2^2 + \upsilon_2^2)$$

According to the Routh-Hurwitz criterion, the linear solution is stable if the following are satisfied

$$r_1 > 0, \quad r_1r_2 - r_3 > 0, \quad r_3(r_1r_2 - r_3) - r_1^2 r_4 > 0, \quad r_4 > 0$$

3. Numerical Result

The main system response and the phase plane for a non-resonant case at some practical values of equations parameters are shown in Fig.1. It can be seen from figure that the maximum steady state amplitude is about 0.007 (1.4% of the maximum excitation amplitude F_1). The phase-plane shows approximately fine limit cycle denoting the system is free of chaos.

3 .1 Effects of parameters

The effects of different parameters are studied as in Fig. 2. From this figure the amplitudes of the system and the absorber are monotonic decreasing functions on the damping coefficient ζ and the nonlinear parameter γ as shown in Figs (2a, 2b) From Fig 2c the amplitude of the absorber is monotonic decreasing function in the damping coefficient ζ_2. The amplitudes of the system and absorber have maximum value at resonance case $\Omega \cong \omega$ as show in Fig 2d. From Fig (2e, 2f) the amplitude are monotonic increasing functions of the excitation amplitudes F_j .

3.2. Resonance cases

The system without absorber is studied numerically at simultaneous primary resonance case ($\Omega_1 \cong \omega$, $S \cong 2\omega$) as in Fig (3) it can be seen that the amplitude increases to about 700% of the basic case in Fig .1.

3.2. Effects of control

1. The system with absorber is solved numerically at non resonance case as shown in Fig. 4. We find that the amplitude of the main system is about 57% of the basic case in Fig.1 which mean that the control is active and reduced the amplitude of the system.
2. Fig.5. illustrates the system with absorber at the simultaneous primary resonance $\Omega_1 \cong \omega \cong \omega_1, S = 2\omega$; it can be shown that the amplitude of the main system to about 14% compared with the basic case shown in Fig .1.
3. The effect of the control on the other resonance cases are studied also as shown in

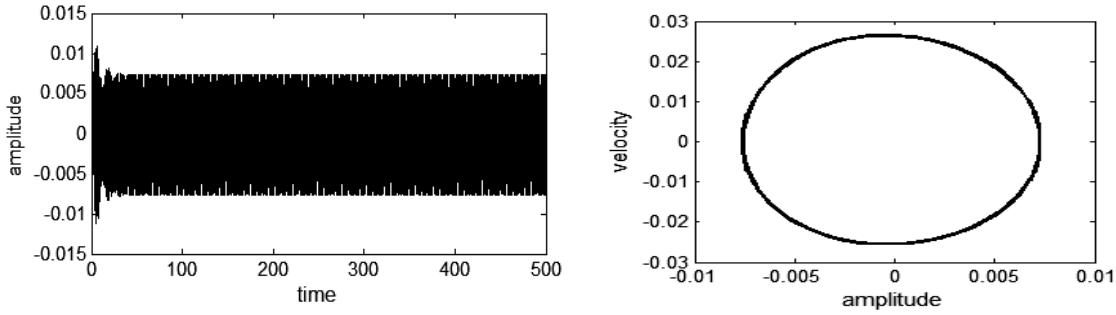


Figure1. Non-resonant case (without absorber)

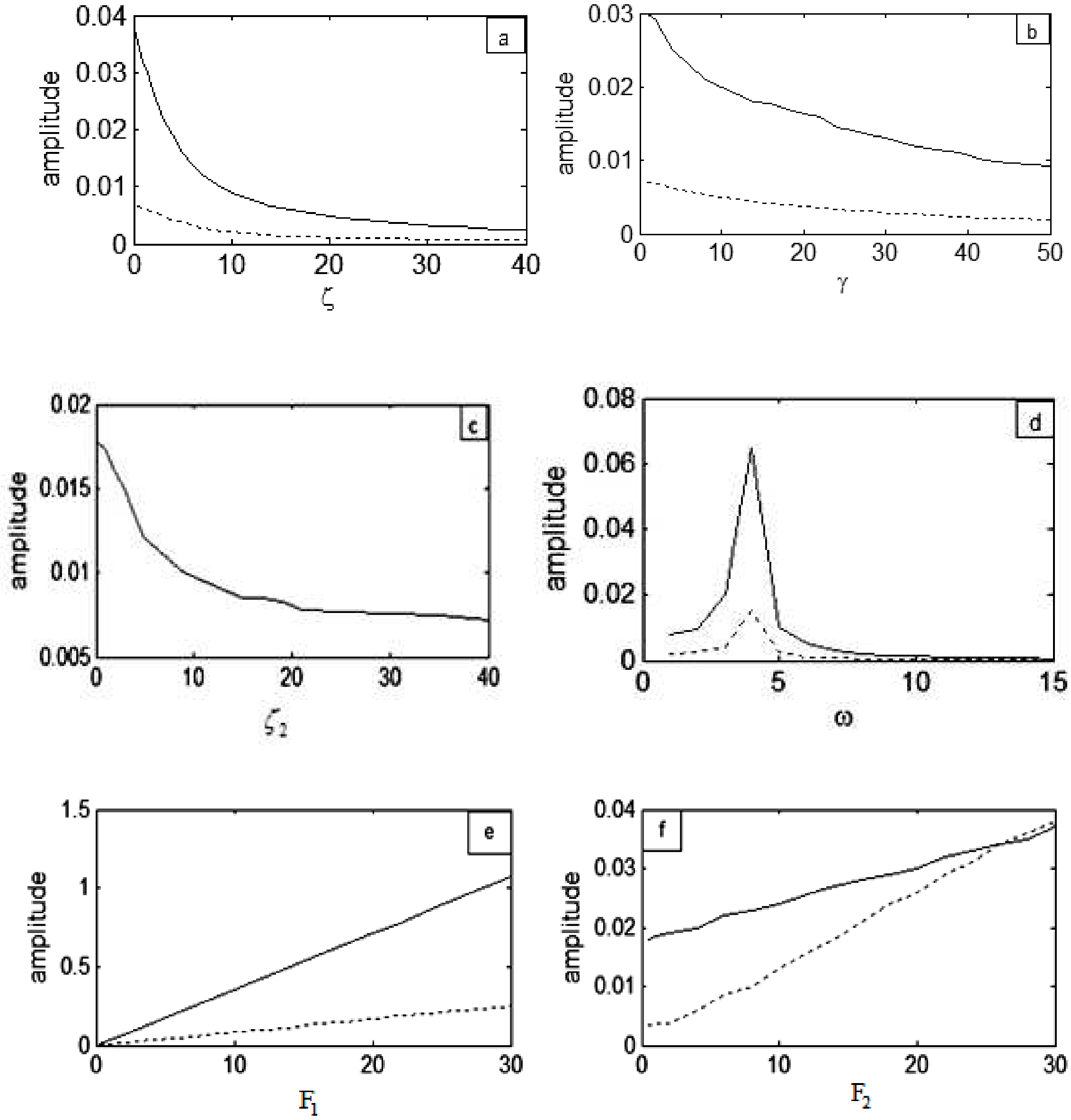


Figure 2 Effects of parameters (______U_1 (the absorber), ……U_2 (The main system)).

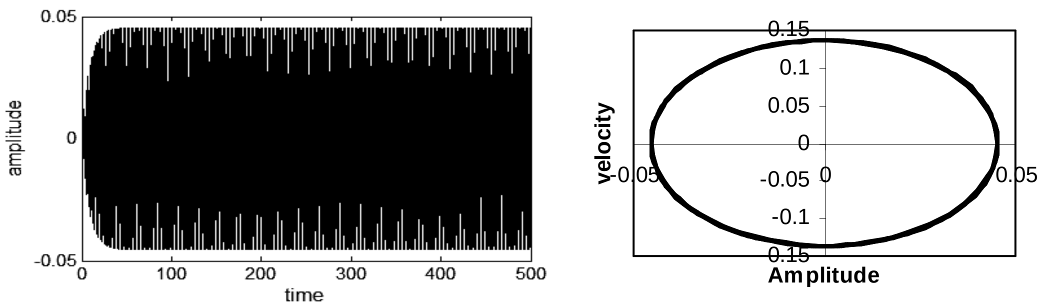


Figure 3. The steady state amplitude without absorber at simultaneous primary resonance $\Omega_1 \cong \omega, S = 2\omega$

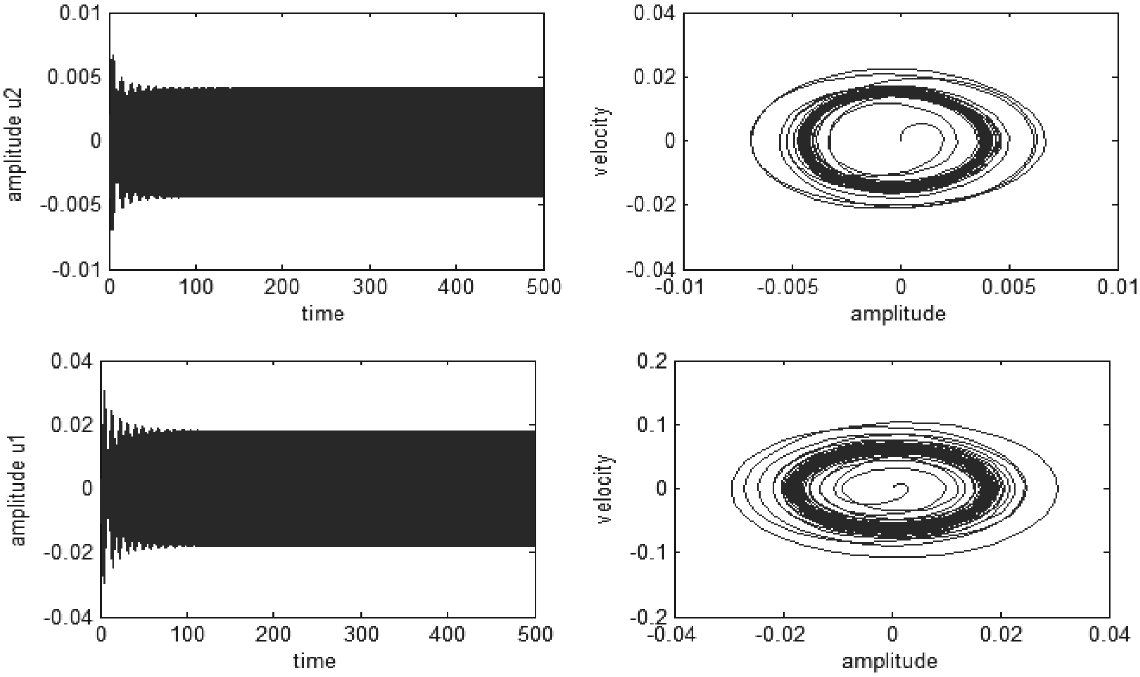


Figure 4. Non-resonant case (with absorber)

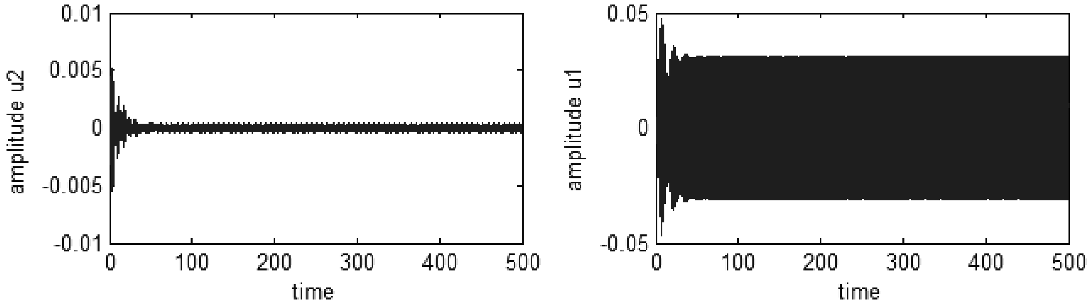
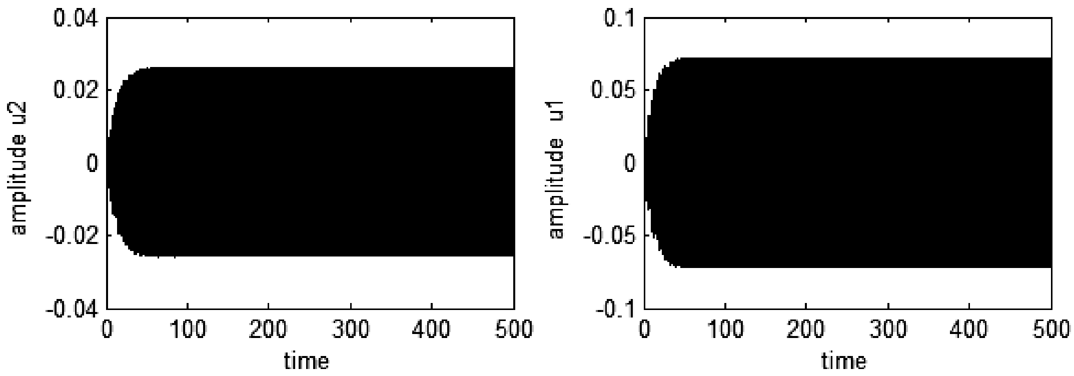
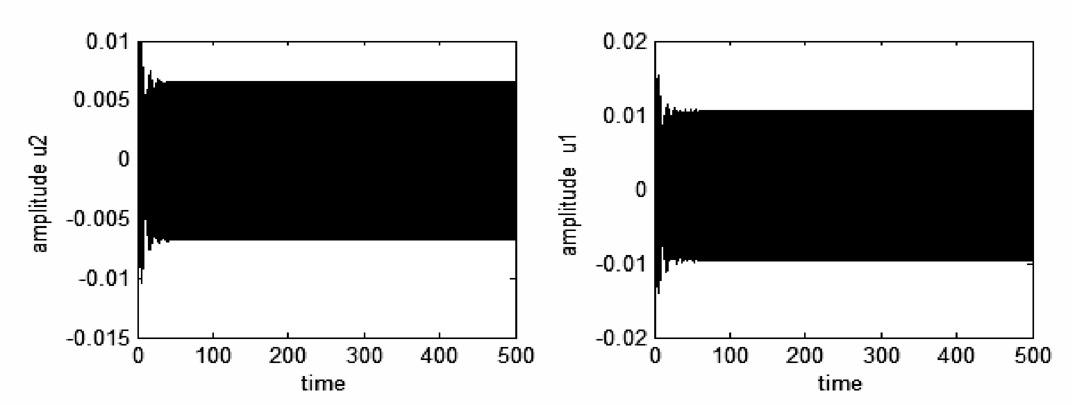


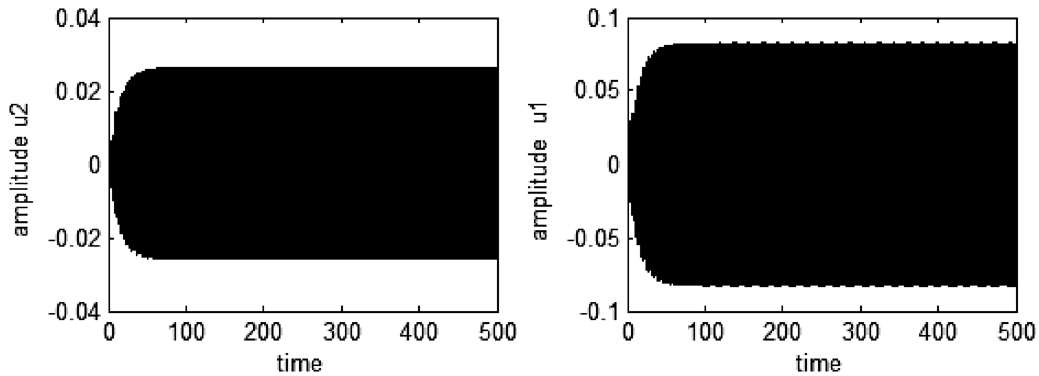
Figure 5. The response for the system with absorber at the simultaneous primary resonance $\Omega_1 \cong \omega \cong \omega_1, S = 2\omega$



(a) Primary resonance $\omega_1 \cong \omega$



(b) Sub-harmonic resonances $\omega_1 \cong 2\omega$



(c) Combined resonance $\omega_1 = \omega + S$

Figure 6. Some of selected resonance cases

3.4 Frequency response curves

The frequency response equation (18) is a nonlinear algebraic equation which can be solved numerically of a_1 against σ_1 as shown in Fig.7. From this Figure we see that the amplitude of the main system is monotonic decreasing function of the non linear coefficient γ and damping effect ζ_1 and β_2 as shown in Figs. 7a, 7b, 7c. But the amplitude is monotonic increasing function of natural frequency ω_1 and time stiffness coefficient β_1 as seen in Figs. 7d and 7e. The frequency response equation (19) is a nonlinear algebraic equation of the amplitude of the absorber a_2 against σ_2 which can be solved numerically as shown in Fig 8.

It can be seen that the amplitude a_2 is monotonic decreasing in coefficient ζ_1, ζ_2 and Ω_2 as shown in Figs. (8a-8c), and monotonic increasing in the excitation amplitude F_j as shown in Fig. 8d. If γ_2 increasing the frequency response curves are shifted to left in Fig. 8e but if ω_1 is increasing the curves are shifted to right as shown in Fig. 8f. The amplitude of the absorber is monotonic increasing function of natural frequency ω as shown in Fig. 8g.

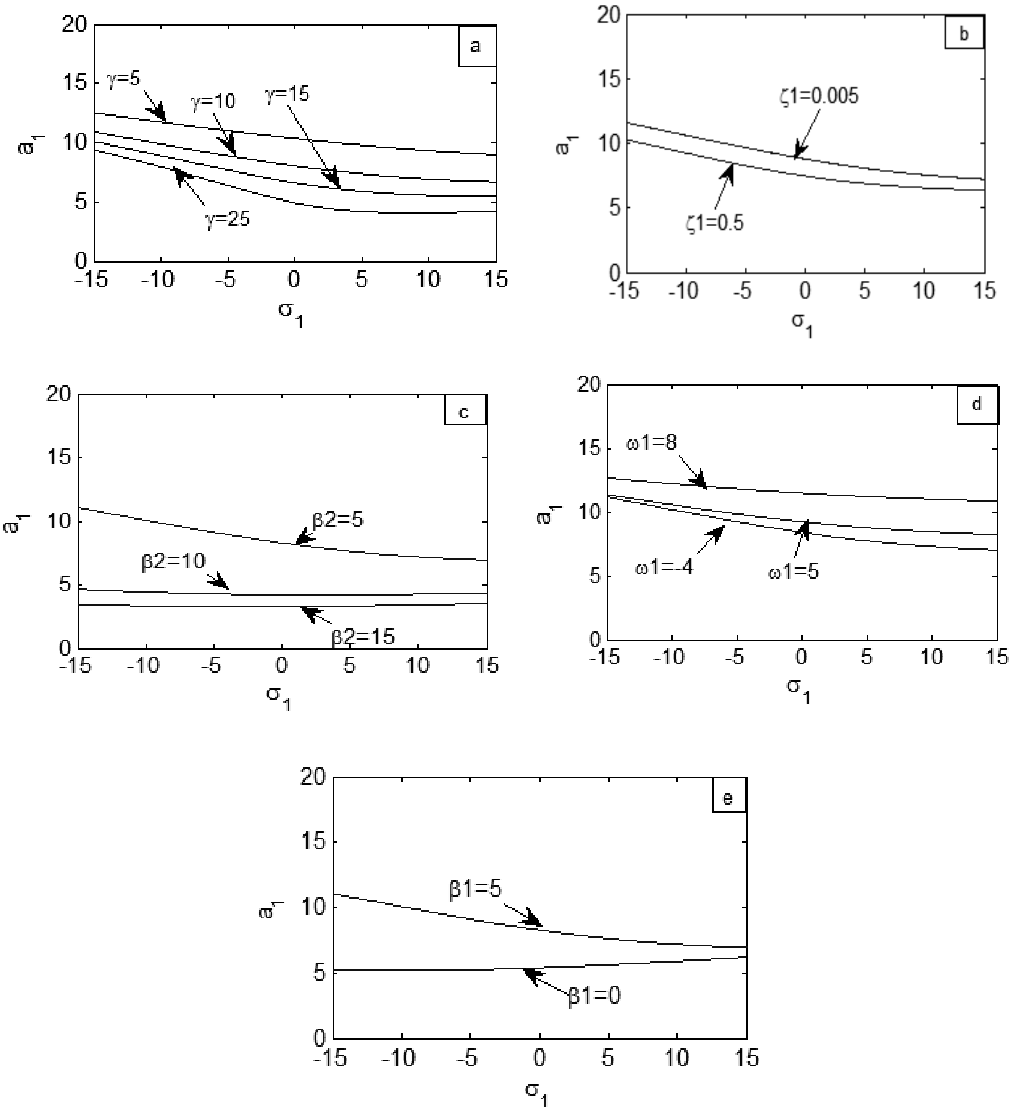


Figure7. Frequency response curves of a_1 against σ_1

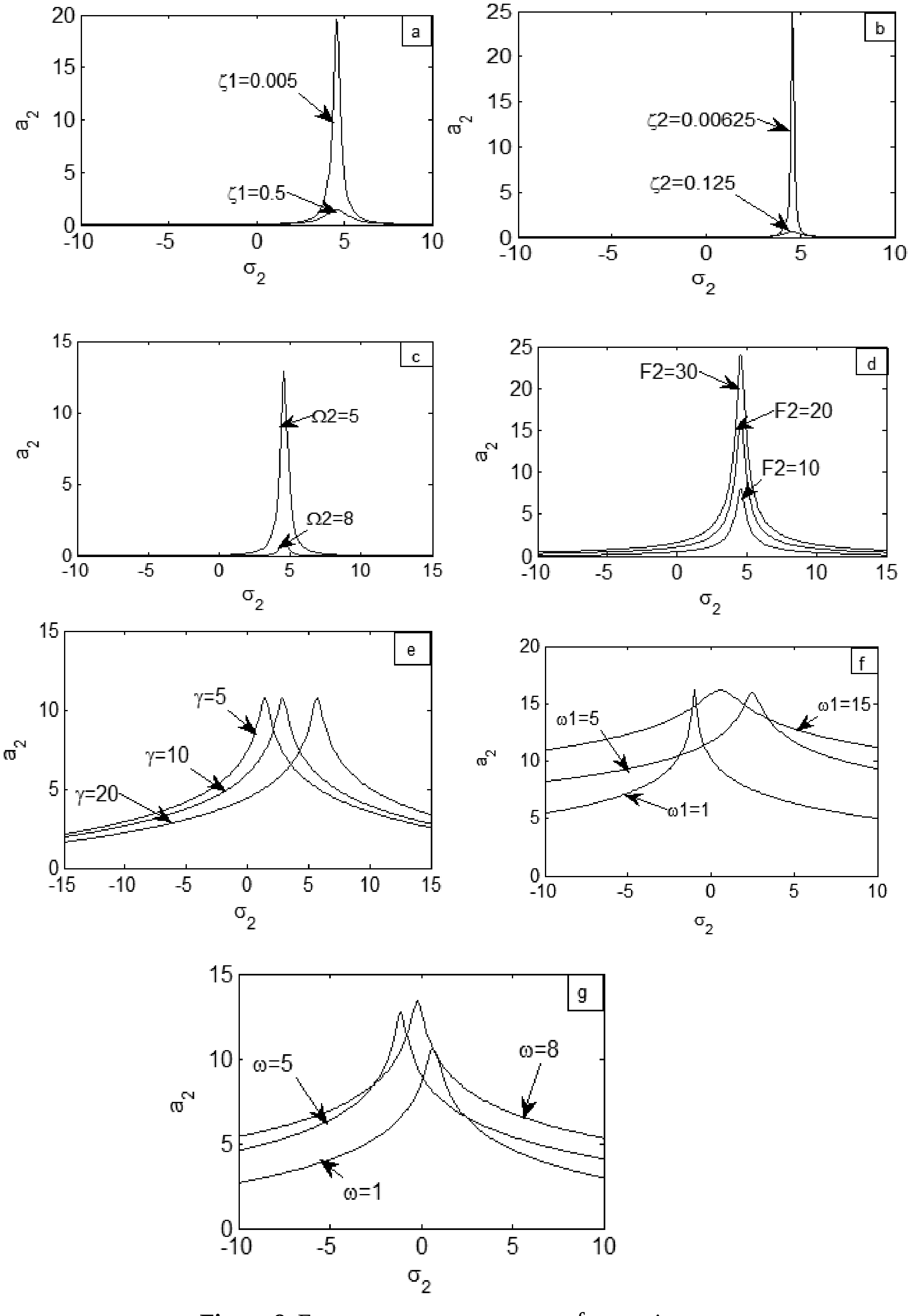


Figure 8. Frequency response curves of a_2 against σ_2

4. Conclusions

The vibration control of a system with the time varying stiffness are studied, and the analytical solution of the system is obtained using multiple scale method, also the stability of the system is studied from this studied the following are concluded

1- The maximum steady state amplitude is about 0.007 (1.4% of the maximum excitation amplitude F_1). The phase-plane shows approximately fine limit cycle denoting the system is free of chaos.
2-The steady state amplitudes of the system and the absorber are monotonic decreasing functions on the damping coefficient ζ and the nonlinear parameter γ.
3- The amplitudes are monotonic increasing functions of the excitation amplitudes F_j .
4- The worst resonance case is the simultaneous primary resonance case ($\Omega_1 \cong \omega$, $S \cong 2\omega$) which the steady state amplitude increases to about 700% of the basic case.
5-The amplitude of the main system is reduced to about 57% of the basic case which means that the control is active.
6- The amplitude of the system with absorber at the simultaneous primary resonance $\Omega_1 \cong \omega \cong \omega_1, S = 2\omega$ is reduced to about 14% compared with the basic case.

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