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ON MAGNETOHYDRODYNAMIC VERONIS'S THERMOHALINE CONVECTION

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Abstract

In this paper, the mathematically correct solution to more general physical situation such as magnetohydrodynamic Veronis's [17] thermohaline convection problem for the case of dynamically free, thermally insulating and electrically perfectly conducting boundaries is obtained. Some important results pertaining to the validity of principle of exchange of stabilities has been derived and discussed in detail.

Keywords: Convection; magnetohydrodynamic; principle of exchange of stability; viscosity.

1. Introduction

The principle of thermal convection is an important phenomenon that has applications to different areas such as geophysics, food processing, oil reservoir modelling and thermal insulator design etc. The classical theory of Bénard convection in horizontal layers of fluids heated from below has been treated both experimentally and theoretically by Pellew and Southwell [12]. The thermal convection of Newtonian fluid under various assumptions of hydrodynamics and hydromagnetics was discussed in detail by Thompson [15], Linhert and Little [9] and Chandrasekhar [4].

In the past few decades, considerable interest has been evinced in the study of magnetohydrodynamic thermohaline convection because it has various applications in oceanography, astrophysics, limnology and chemical engineering etc. A good account of thermohaline convection problems is as studied by Gupta et al. [6, 7, 8], Banerjee et al. [1, 2, 3] and Mohan [10]. For the magnetohydrodynamic Bénard convection problem, Banerjee et al. [3] noticed some shortcomings in the solution of problem as derived by Chandrasekhar [4]. These shortcomings have been removed and the correct solution has been constructed for the problem by Banerjee et al. [3] for the case of dynamically free, thermally insulating and electrically perfectly conducting boundaries.

In the present paper, the method of Banerjee et al. [3] is followed and mathematically correct solution is obtained to more general physical situation such as magnetohydrodynamic Veronis's [17] thermohaline convection problem for dynamically free, thermally insulating and electrically perfectly conducting boundaries.

2. The Physical System and the problem

Consider an electrically conducting viscous Boussinesq fluid confined between two boundaries $z = -\frac{1}{2}$ and $z = +\frac{1}{2}$ of infinite horizontal extension in the presence of uniform vertical magnetic field, acting parallel but opposite to the force field of gravity, and being acted upon by a uniform vertical adverse temperature gradient. Then under appropriate conditions, a phenomenon of more general convective motions, an outcome of hydrodynamic instability, is realised which is termed as magnetohydrodynamic Veronis's thermohaline convection (Veronis [17], [18]), Shirtcliffe [14], Turner [16], Normand et al. [11], Chen and Johnson [5], Rudraih and Shivkumara [13])

2. Governing equations and boundary conditions

The governing equations and boundary conditions in their non-dimensional forms for the magnetohydrodynamic Veronis's [17] thermohaline convection problem wherein dynamically free boundaries are thermally insulating and electrically perfectly conducting are given by (c.f. Gupta et al. [6], Banerjee et al. [3])

$$(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w = Ra^2 - \varphi - QD(D^2 - a^2)h_z, \tag{1}$$

$$(D^2 - a^2 - p)\theta = -w, \tag{2}$$

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\varphi = -\frac{R_s a^2}{\tau}w, \tag{3}$$

$$\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)h_z = -Dw, \tag{4}$$

$$w = 0 \quad \text{at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{5}$$

$$D^2 w = 0 \text{ at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{6}$$

$$D\theta = 0 \quad \text{at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{7}$$

$$\varphi = 0 \quad \text{at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{8}$$

$$h_z = 0 \quad \text{at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{9}$$

wherein the symbols used have the same meanings as given in Banerjee et al. [3] with the difference that the solute concentration φ and the relevant solute concentration equation (3) are also incorporated here ($R_s > 0$ and $\tau > 0$ for this problem).

Combining equations (1)-(4) and boundary conditions (5)-(9) in an appropriate manner we derive the following system of equations and the associated boundary conditions in terms of w alone (similarly for θ and h_z alone can be obtained), namely

$$Lw = 0. \tag{10}$$

$$w = 0 = D^2 w = L_1 w = L_2 w = L_3 w \quad \text{at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{11}$$

where

$$L = (D^2 - a^2)(D^2 - a^2 - p)\left(D^2 - a^2 - \frac{p}{\tau}\right)\left\{\left(D^2 - a^2 - \frac{p}{\sigma}\right)\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right) - QD^2\right\} + Ra^2\left(D^2 - a^2 - \frac{p}{\tau}\right) - \frac{R_3 a^2}{\tau}(D^2 - a^2 - p)\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right) \tag{12}$$

$$L_1 = D(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)\left(D^2 - a^2 - \frac{p}{\tau}\right) + \left(Ra^2 - \frac{R_3 a^2}{\tau}\right)D - QD\left[\frac{p^2\sigma_1}{\sigma}\left(\frac{\sigma_1}{\sigma} - \frac{1}{\tau}\right) + \left\{p\left(\frac{\sigma_1}{\sigma} - \frac{1}{\tau}\right) - a^2\right\}D^2 + D^4\right] \tag{13}$$

$$L_2 = D^3(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)\left(D^2 - a^2 - \frac{p}{\tau}\right) + Ra^2 D\left\{p\left(1 - \frac{1}{\tau}\right) + D^2\right\} - \frac{R_3 a^2}{\tau}D^3 + QD\left(a^2 + \frac{p\sigma_1}{\sigma}\right)\left\{a^2 + \frac{p}{\tau} - \frac{p\sigma_1}{\sigma}\left(a^2 + \frac{p\sigma_1}{\sigma}\right)\right\} + \left\{a^2 + \frac{p}{\tau} - \frac{p\sigma_1}{\sigma}\left(a^2 + \frac{p\sigma_1}{\sigma}\right)\right\}D^2 + \left\{a^2 + p\left(\frac{1}{\tau} - \frac{\sigma_1}{\sigma}\right)D\right\}\left(D^2 - a^2 - \frac{p}{\tau}\right) \tag{14}$$

$$L_3 = D^3(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)\left(D^2 - a^2 - \frac{p}{\tau}\right) + Ra^2 D\left(\left(p + \frac{p}{\tau}\right) + D^2\right) - \frac{R_3 a^2}{\tau}D^3 - QD\left\{D^4 + \frac{p\sigma_1}{\sigma}\left(D^2 + \left(a^2 + \frac{p\sigma_1}{\sigma}\right)\right)\right\}\left(D^2 - a^2 - \frac{p}{\tau}\right) \tag{15}$$

3. **Mathematical analysis and results**

The equations (2), (4)-(7) and (9) are the same as in Banerjee et al. [3], so the solutions for h_z, w and θ are the same which are given, respectively, by the following equations

$$h_z = \frac{1}{16}\sum_{n=0}^{\infty}\frac{C_n(-1)^{n+1}}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right]\left[\frac{3\{(2n+1)^2 - 5^2\}}{\{3^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\}}cos3\pi z + \frac{5\{(2n+1)^2 - 3^2\}}{\{5^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\}}cos5\pi z\right] + \sum_{n=0}^{\infty}C_n cos(2n+1)\pi z \tag{16}$$

$$w = \frac{1}{16\pi}\sum_{n=0}^{\infty}\frac{C_n(-1)^{n+1}}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right][\{(2n+1)^2 - 5^2\}sin3\pi z + \{(2n+1)^2 - 3^2\}cos5\pi z] + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{C_n}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right]sin(2n+1)\pi z \tag{17}$$

and

$$\theta = \frac{1}{16\pi}\sum_{n=0}^{\infty}\frac{C_n(-1)^{n+1}}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right]\left[\left\{\frac{(2n+1)^2 - 5^2}{3^2\pi^2 + a^2 + p}\right\}sin3\pi z + \left\{\frac{(2n+1)^2 - 3^2}{5^2\pi^2 + a^2 + p}\right\}cos5\pi z\right] + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{C_n}{(2n+1)}\left\{\frac{(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}}{(2n+1)^2\pi^2 + a^2 + p}\right\}sin(2n+1)\pi z \tag{18}$$

With w given by equation (18), equation (3) becomes

$$\left(D^2 - a^2 - \frac{p}{\tau}\right)\varphi = \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{C_n}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right]sin(2n+1)\pi z - \frac{R_3 a^2}{16\pi\tau}\sum_{n=0}^{\infty}\frac{C_n(-1)^{n+1}}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right][\{(2n+1)^2 - 5^2\}sin3\pi z + \{(2n+1)^2 - 3^2\}cos5\pi z] \tag{19}$$

Solving equation (19) by making use of boundary conditions (8), we get

$$\varphi = \frac{R_s a^2}{16\pi\tau}\sum_{n=0}^{\infty}\frac{C_n(-1)^{n+1}}{(2n+1)}\left[(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right]\left[\left\{\frac{(2n+1)^2-5^2}{3^2\pi^2+a^2+\frac{p}{\tau}}\right\}sin3\pi z + \left\{\frac{(2n+1)^2-3^2}{5^2\pi^2+a^2+\frac{p}{\tau}}\right\}cos5\pi z\right] + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{C_n}{(2n+1)}\left\{\frac{(2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}}{(2n+1)^2\pi^2+a^2+\frac{p}{\tau}}\right\}sin(2n+1)\pi z \tag{20}$$

Equation (20) gives the mathematical correct solution for the solute concentration variable φ by making use of the solution of Banerjee et al. [3].

With w given by equation (17), equation (10) becomes

$$\frac{1}{16\pi}\sum_{n=0}^{\infty}\frac{C_n(-1)^{n+1}}{(2n+1)}\left\{(2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right\}\Bigg[((2n+1)^2-5^2)\Bigg\{(3^2\pi^2+a^2)(3^2\pi^2+a^2+p)\left(3^2\pi^2+a^2+\frac{p}{\tau}\right)\left(\left(3^2\pi^2+a^2+\frac{p}{\sigma}\right)\left(3^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)+3^2\pi^2Q\right)+\left(3^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)\left(\frac{R_s a^2}{\tau}(3^2\pi^2+a^2+p)-Ra^2\left(3^2\pi^2+a^2+\frac{p}{\tau}\right)\right)\Bigg\}sin3\pi z + ((2n+1)^2-3^2)\Bigg\{(5^2\pi^2+a^2)(5^2\pi^2+a^2+p)\left(5^2\pi^2+a^2+\frac{p}{\tau}\right)\left(\left(5^2\pi^2+a^2+\frac{p}{\sigma}\right)\left(5^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)+5^2\pi^2Q\right)+\left(5^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)\left(\frac{R_s a^2}{\tau}(5^2\pi^2+a^2+p)-Ra^2\left(5^2\pi^2+a^2+\frac{p}{\tau}\right)\right)\Bigg\}sin5\pi z\Bigg] + \frac{1}{\pi}\sum_{n=0}^{\infty}\frac{C_n}{(2n+1)}\left\{(2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right\}\Bigg[((2n+1)^2\pi^2+a^2)((2n+1)^2\pi^2+a^2+p)\left((2n+1)^2\pi^2+a^2+\frac{p}{\tau}\right)\Bigg\{\left((2n+1)^2\pi^2+a^2+\frac{p}{\sigma}\right)\left((2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)+Q(2n+1)^2\pi^2\Bigg\}+\left((2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)\Bigg\{\frac{R_s a^2}{\tau}((2n+1)^2\pi^2+a^2+p)-Ra^2\left((2n+1)^2\pi^2+a^2+\frac{p}{\tau}\right)\Bigg\}\Bigg]sin(2n+1)\pi z = 0, \tag{21}$$

which can be written in the form

$$\sum_{n=0}^{\infty}C_n(\alpha_n S_1 sin3\pi z + \beta_n S_2 sin5\pi z) + \sum_{n=0}^{\infty}C_n\gamma_n S_n sin(2n+1)\pi z = 0, \tag{22}$$

where

$$\alpha_n = \frac{(-1)^{n+1}}{16(2n+1)\pi}\{(2n+1)^2-5^2\}\left\{(2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right\}, \tag{23}$$

$$\beta_n = \frac{(-1)^{n+1}}{16(2n+1)\pi}\{(2n+1)^2-3^2\}\left\{(2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right\}, \tag{24}$$

$$\gamma_n = \frac{(2n+1)^2\pi^2+a^2+\frac{p\sigma_1}{\sigma}}{(2n+1)\pi}, \tag{25}$$

and

$$S_n = \left\{((2n+1)^2\pi^2 + a^2)((2n+1)^2\pi^2 + a^2 + p)\left((2n+1)^2\pi^2 + a^2 + \frac{p}{\tau}\right)\right\}\left\{\left((2n+1)^2\pi^2 + a^2 + \frac{p}{\sigma}\right)\left((2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right) + Q(2n+1)^2\pi^2\right\} + \left((2n+1)^2\pi^2 + a^2 + \frac{p\sigma_1}{\sigma}\right)\left\{\frac{R_3 a^2}{\tau}((2n+1)^2\pi^2 + a^2 + p) - Ra^2\left((2n+1)^2\pi^2 + a^2 + \frac{p}{\tau}\right)\right\} \quad (26)$$

Multiplying equation (22) by $sin(2m+1)\pi z$ (since the first order derivative with respect to z of the left hand side of equation (22) vanishes at $z = \pm\frac{1}{2}$) and integrating the resulting equation over the range of z, we obtain

$$\sum_{n=0}^{\infty} C_n(\alpha_n S_1 \delta_{1m} + \beta_n S_2 \delta_{2m} + \gamma_n S_n \delta_{nm}) = 0, \quad \text{(m =0, 1, 2, 3…)}, \quad (27)$$

where δ_{nm} is the Kronecker's delta.

Equations (27) provide a set of linear homogenous equations for the constants C_n and the requirement that the determinant of this system of equations must vanish provides the characteristic equation for the determination of R and p_i when $p_r = 0$. We thus obtain

$$\|\alpha_n S_1 \delta_{1m} + \beta_n S_2 \delta_{2m} + \gamma_n S_n \delta_{nm}\| = 0. \quad (28)$$

The nth approximation to the characteristic values of R and p_i is obtained by setting the nth order determinant consisting of the first n rows and columns in the left hand side of equation (28) equal to zero, and this corresponds to the retention of the first n terms only in the Fourier expansion of the form

$$h_z - d_1 cos3z - d_2 cos5z = \sum_{n=0}^{\infty} C_n \cos(2n+1)\pi z. \quad \text{(c. f. Banerjee et al. [3])}$$

The corresponding result for Veronis's thermohaline convection is

$$\begin{vmatrix} \gamma_0 S_0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \alpha_0 S_1 & \alpha_1 S_1 + \gamma_1 S_1 & 0 & \alpha_3 S_1 & \alpha_4 S_1 & \alpha_5 S_1 & \dots & \alpha_{n-1} S_1 \\ \beta_0 S_2 & 0 & \beta_2 S_2 + \gamma_2 S_2 & \beta_3 S_2 & \beta_4 S_2 & \beta_5 S_2 & \dots & \beta_{n-1} S_2 \\ 0 & 0 & 0 & \gamma_3 S_3 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \gamma_4 S_4 & 0 & \dots & 0 \\ . & . & . & . & . & . & \dots & . \\ . & . & . & . & . & . & \dots & . \\ . & . & . & . & . & . & \dots & . \\ . & . & . & . & . & . & \dots & . \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots & \gamma_{n-1} S_{n-1} \end{vmatrix} = 0, \quad (29)$$

from which it follows uniquely that the lowest characteristic value of R and the associated value of p_i are given by the equation

$$S_0 = 0, \quad (30)$$

since α_n and β_n are non-zero numbers for every permissible value of n except for $n = 2$ and $n = 1$ respectively while γ_n does not vanish for any permissible value of n. Further, since equation (30) is valid whatever be the value of n, it follows that it is the unique solution that provides the lowest characteristic value of R and the associated value of p_i as given by the characteristic equation (28).

We complete the solution of the problem by demonstrating that h_z, w, θ and φ which are respectively given by equations by (16) – (18) and (20) and satisfy equations (2) – (4) along with boundary conditions (5) – (9) also satisfy equation (1).

To prove this we consider equation (10) which can be written in an alternative form as

$$(D^2 - a^2 - p)\left(D^2 - a^2 - \frac{p\sigma_1}{\sigma}\right)E = 0, \tag{31}$$

where $p = ip_i, p_i \neq 0$ and

$$E = \left(D^2 - a^2 - \frac{p}{\tau}\right)\left\{(D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w - Ra^2\theta + \varphi + QD(D^2 - a^2)h_z\right\},$$

while the boundary conditions on E are given by

$$DE = 0 = D^3E \text{ at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{32}$$

which follows from equation (20).

Multiplying equation (31) by E^* (the complex conjugate of E) throughout and integrating the resulting equation over the range of z by making use of boundary equations (32), we get upon equating imaginary part of this latter equation

$$p_i\left(\frac{\sigma+\sigma_1}{\sigma}\right)\int_0^1 (|DE|^2 + a^2|E|^2)\, dz = 0. \tag{33}$$

But, since $p_i \neq 0$, it follows from equation (33) that E = 0 for all z in $\left[-\frac{1}{2}, \frac{1}{2}\right]$, which can be written in the form as

$$\left(D^2 - a^2 - \frac{ip_i}{\tau}\right)F = 0 \text{ for all z in} \left[-\frac{1}{2}, \frac{1}{2}\right], \tag{34}$$

where $F = (D^2 - a^2)\left(D^2 - a^2 - \frac{p}{\sigma}\right)w - Ra^2\theta + \varphi + QD(D^2 - a^2)h_z$,

$$DF = 0 = D^3F \quad \text{at } z = -\frac{1}{2} \text{ and } z = +\frac{1}{2}, \tag{35}$$

Similarly, multiplying equation (34) by F^* (the complex conjugate of F) throughout and integrating over the range of z by making use of boundary conditions (35), we get upon equating the imaginary parts of this latter equation

$$\frac{p_i}{\tau}\int_0^1 |F|^2\, dz = 0. \tag{36}$$

But, since $p_i \neq 0$ it follows from equation (36) that F = 0 for all z in $\left[-\frac{1}{2}, \frac{1}{2}\right]$ which is in turn implies that equation (1) is also satisfied.

4. **Results and Discussions**

The above analysis leads to the following results:

Result 1. An exact solution of differential equations (1) – (4) and boundary conditions (5) – (9) is given by equations (16) – (18) and (20).
together with the characteristic equation (30) i. e. $S_0 = 0$, which reads

$$(\pi^2+a^2)(\pi^2+a^2+p)\left(\pi^2+a^2+\frac{p}{\tau}\right)\left[\left(\pi^2+a^2+\frac{p}{\sigma}\right)\left(\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right)+Q\pi^2\right]=$$
$$\left[Ra^2\left(\pi^2+a^2+\frac{p}{\tau}\right)-\frac{R_3a^2}{\tau}(\pi^2+a^2+p)\right]\left(\pi^2+a^2+\frac{p\sigma_1}{\sigma}\right) \tag{37}$$

Now, letting
$x=\frac{a^2}{\pi^2}, ip_i=\frac{p}{\pi^2}$ (p_i is real), $R_1=\frac{R}{\pi^4}, R_{s_1}=\frac{R_3}{\pi^4}$ and $Q_1=\frac{Q}{\pi^2}$ in equation (37), we get from the resulting equation after some simple arrangements

$$\left(\frac{1+x}{x}\right)\left[\left(\frac{1+x}{x}\right)\left\{1+\frac{Q_1}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}\right\}+\frac{ip_i}{\sigma}\left\{1-\frac{Q_1\sigma}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}\right\}\right]=(1+x)\left\{\frac{R_1}{(1+x)^2+p_i^2}-\right.$$
$$\left.\frac{R_{s_1}}{\tau(1+x)^2+\frac{p_i^2}{\tau}}\right\}-ip_i\left\{\frac{R_1}{(1+x)^2+p_i^2}-\frac{R_{s_1}}{\tau^2(1+x)^2+p_i^2}\right\} \tag{38}$$

Assuming $p_i \neq 0$, we get upon equating the real and imaginary parts of equation (39), respectively, as

$$\frac{R_1}{(1+x)^2+p_i^2}=\frac{R_{s_1}}{\tau(1+x)^2+\frac{p_i^2}{\tau}}+\left(\frac{1+x}{x}\right)\left\{1+\frac{Q_1}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}\right\}, \tag{39}$$

and

$$\frac{R_1}{(1+x)^2+p_i^2}=\frac{R_{s_1}}{\tau^2(1+x)^2+p_i^2}+\left(\frac{1+x}{\sigma x}\right)\left\{\frac{Q_1\sigma_1}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}-1\right\}. \tag{40}$$

Eliminating R_1 from equations (39) and (40) and then making some simple arrangements, we get

$$\frac{R_{s_1}(\tau-1)}{\tau^2(1+x)^2+p_i^2}=\left(\frac{1+x}{\sigma x}\right)\left\{\frac{Q_1(\sigma_1-\sigma)}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}-(1+\sigma)\right\}. \tag{41}$$

From equation (41) it follows that it cannot be satisfied if $\tau > 1$ and $\sigma_1 < \sigma$, for, p_i would then be purely imaginary which contradicts the hypothesis that p_i is real. We arrive at this contradiction on the assumption that $p_i \neq 0$. This leads to the following result:

Result 2: If $\tau > 1$ and $\sigma_1 < \sigma$ then $p_i = 0$ i. e. if $\tau > 1$ and $\sigma_1 < \sigma$ then the 'principle of exchange of stabilities' is valid or equivalently if $\tau > 1$ then the Thompson-Chandrasekhar sufficient condition for the validity of this 'principle' is true. Writing equation (41) in an alternative form as

$$1+\sigma=\frac{Q_1(\sigma_1-\sigma)}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}+\left(\frac{x}{1+x}\right)\left(\frac{R_{s_1}(1-\tau)\sigma}{\tau^2(1+x)^2+p_i^2}\right). \tag{42}$$

From equation (42) it follows that if $\tau \leq 1$ and $\sigma_1 > \sigma$, then

$$1<\frac{Q_1(\sigma_1-\sigma)}{(1+x)^2+\frac{p_i^2\sigma_1^2}{\sigma^2}}+\left(\frac{x}{1+x}\right)\left(\frac{R_{s_1}\sigma}{\tau^2(1+x)^2+p_i^2}\right)$$

$$< Q_1\sigma_1 + \frac{R_{s_1}\sigma x}{\tau^2(1+x)^3}$$
$$< Q_1\sigma_1 + \frac{R_{s_1}\sigma x}{3\tau^2 x}$$
$$< Q_1\sigma_1 + \frac{R_{s_1}\sigma}{2\tau^2}$$
$$= Q\frac{\sigma}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4}.$$

This leads to the following result:

Result 3. If $\tau \leq 1$ and $\sigma_1 > \sigma$ and $Q\frac{\sigma}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} \leq 1$ then $p_i = 0$ i. e. if $\tau \leq 1$ and $\sigma_1 > \sigma$ and $Q\frac{\sigma}{\pi^2} + \frac{R_s\sigma}{2\tau^2\pi^4} \leq 1$, then the 'principle of exchange of stabilities' is valid or equivalently if $\tau \leq 1$ and $\sigma_1 > \sigma$ then the Gupta et al. [8] sufficient condition for the validity of this 'principle' is true.
Also, from equation (42) it follows that if $\tau \leq 1$ and $\sigma_1 > \sigma$ then

$$1 + \sigma < Q_1(\sigma_1 - \sigma) + \frac{R_{s_1}(1-\tau)}{3\tau^2} = \frac{Q(\sigma_1 - \sigma)}{\pi^2} + \frac{R_s(1-\tau)}{3\tau^2\pi^4}$$

This leads to the following result:

Result 4. If $\tau \leq 1$ and $\sigma_1 > \sigma$ and $\frac{Q(\sigma_1-\sigma)}{\pi^2} + \frac{R_s(1-\tau)}{3\tau^2\pi^4} \leq 1 + \sigma$ then $p_i = 0$ i. e. if $\tau \leq 1$ and $\sigma_1 > \sigma$ and $\frac{Q(\sigma_1-\sigma)}{\pi^2} + \frac{R_s(1-\tau)}{3\tau^2\pi^4} \leq 1 + \sigma$ then the 'principle of exchange of stabilities' is valid.

Nomenclature

p Complex growth rate

d Depth of fluid layer

g Gravitational acceleration vector

D Linear differential operator

T Temperature

a Wave number

H Magnetic field

Greek Symbols

υ Kinematic viscosity

η Coefficient of magnetic diffusivity

κ Thermal diffusivity

κ_s Solute diffusivity

α Coefficient of volume expansion

α_s Coefficient volume of expansion due to solute gradient

β Adverse temperature gradient

β_s Non-adverse solute gradient

μ_e Magnetic permeability

$R = \frac{g\alpha\beta d^4}{\upsilon\kappa}$ Thermal Rayleigh number

$Q = \frac{\mu_e H^2 d^2}{4\pi\upsilon\rho_m\eta}$ Chandrasekhar number

$\sigma_1 = \frac{\upsilon}{\kappa}$ Thermal Prandtl number

$\sigma = \frac{\upsilon}{\eta}$ Magnetic Prandtl number

$\tau = \frac{\kappa_s}{\kappa}$ Lewis number

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